

SOME ENDEAVORS AT SYNTHESIZING A SOLUTION TO THE SORITES

By Shane Ralston

“Puzzles,” “word games,” “logical anomalies,” whatever we call them, they perplex us and challenge our familiar patterns of reasoning. One of these puzzles, among many others, originated from the mind of an ancient Megarian logician, Eubulides of Miletus, and endures to the modern day.¹ Its name, “sorites,” can be traced to the Greek word *soros*, meaning ‘heap.’ The answer to whether one grain of sand “is a heap” or “is not a heap” seems quite simple: it is not a heap. However, as we add grains to the one, at what future point does the non-heap become a heap? Our decision is fraught with uncertainty. Are the objects or the language we are using to describe them vague?

In academic philosophy, the ancient Greek puzzle has gained the status of a paradox, as philosophers apply stoic and modern logic to these propositions considered to have vague predicates. The current debate has developed quite serious and wide-ranging implications, such as whether sorites issues provide adequate grounds for abandoning our standard ontology (or our understanding of what really exists),² and (germinating into another discipline) whether vagueness in the

language of legal rules can generate disagreement as to whether there are right answers to questions of law.³

Several unique solutions to the paradox have been proposed, yet all suffer from specific inadequacies that might, upon further reflection, disappear in the event that we endeavor to produce a synthesis. When we say that we are attempting to derive a synthesis, we seek to combine elements of two solutions for the sake of creating a new (and hopefully, though not necessarily, a better) one. The paradox of the sorites, or “the heap,” appeals to us because it challenges our assumption that we may categorically describe clear cases and their negations, yet fail to distinguish the borderline ones. I will demonstrate that some combinations of solutions are easy fits while others are extremely weak and awkward as attempts at resolving the paradox. Nonetheless, the process of synthesis produces three noteworthy combinations.

The sorites paradox will be discussed in terms of categories, their negations and the extent to which our reasoning about them succeeds or fails to accord with the laws of logic. For example, to call something a “heap,” “bald” or “short” is to differentiate it from apparent cases of a thing that is a “non-heap,” “not bald” or “not short.” In keeping with formal logic and the law of non-contradiction, we might gainsay that a thing could evince both its positive instance and its negation, “P and not P” or “P and ~P.” According to the more debatable law of excluded middle, a thing must either display its positive property or the property’s negation (e.g. bald or not bald). *Prima facie*, denying the law of excluded middle has an appeal that

rejection of the law of non-contradiction lacks. This claim originates with the intuition that while a thing cannot be both itself and its opposite [intuitively and by the law of non-contradiction], when we attempt to designate borderline cases, it might not clearly match the criteria for either category. Where do we specify that a thing has effectively crossed the categorical boundary between its positive and negative instance, e.g. from bald to not bald?

In more formal terms, we might state the Sorites as follows:

[1] The first, or categorical, premise declares that “1000 grains of sand is a heap.”

[2] The second premise holds that “if x is a heap then $x-1$ is also a heap.”

[3] Then, (through multiple operations of *modus-ponens*) we arrive at the paradoxical conclusion that “1 grain of sand is a heap.” [Sainsbury 1995: 29]

Since we cannot determine the point at which subtracting one grain of sand converts the heap to a non-heap, it seems inevitable that we must either accept the vagueness of the term “*heap*” or slide back to the position that anything resembling sand [one grain or more] constitutes a heap. The paradox seems to revolve around the issue of whether there is an absolute boundary between categories and their negations, such that x elements makes a thing a member and $x - 1$ makes it a non-member. In short, we would want to know whether there exists a grain of sand, a piece of hair or a one-millimeter [or other] difference between positive cases of something being a heap, bald or short and it being a non-heap, not bald or not short.

One attempt at a solution has already been hinted at in the process of defining the paradox. The notion that the linguistic term “*heap*” is vague begs the question of whether or not further elaboration of our existing language, or the inclusion of a meta-language, could remove the vagueness. We might call this strategy the “semantic approach.” While mathematics is precise in its form and operations, language invariably suffers in this respect because of its open texture; ⁴ the impreciseness of its meaning and of the rules that govern it invariably force the speaker’s hermeneutic license. Even though we may add infinitely many descriptive adjectives before a word, each with increasing levels of precision, it still proves impossible to escape a certain level of distortion inherent in practical language’s construction. Premised on this notion of the imperfection of practical language is Tarski’s view that we should embrace a meta-language encompassing greater logical rigor. [Tarski 1943: 603] However, it is uncertain whether the meta-language may prove useful for any purpose other than to avoid problems of self-reference.

While some attempts at amalgamating solutions to the paradox produce natural unions, others result in less rewarding and often strange combinations. For example, joining the supervaluationist account with the epistemic solution would only create obfuscation since aspects of both are clearly antithetical to each other: the former holds that there is no definite boundary between a category and its negation, unless it is arbitrarily stipulated, while the latter states that such a boundary exists regardless of any stipulation. The same would hold true of an attempted pairing of the degrees of truth theory with the epistemic view. To suggest that the law of excluded middle should be rejected in favor of assigning degrees of

truth admits that the boundary cannot be sharp, though it can be known to what extent it is vague; the epistemic position, however, defends a conception of the boundary that it is neither vague nor known. On the epistemic account, there are definite boundaries, e.g. one hair, one grain of sand or one-millimeter differences, between categories and their negations, yet it is impossible for us to know where these boundaries lie. The “supervaluationist,” “degrees of truth” and “rejecting the categorical” accounts, on the other hand, synthesize easily with the “semantic approach” because each of them reveals to some extent that the sorites owes its paradoxical quality to the limitations of natural language.

In the supervaluationist model, boundaries between positive categories and their negations blur into an area of uncertainty. Supervaluationists do not intend that vagueness should correspond to an object as a property does, but instead believe that vagueness resides in language. Vague terms correspond to things in such a way that they either do or could partake in the penumbra, an area where we could say something is or is not the case without designating a truth-value.

[Sainsbury 1995: 33] More specifically, in the case of the heap, for any specific number of grains of sand x which is in the penumbra, “ x grains of sand make a heap” and “ x grains do not make a heap” are both propositions which are neither true nor false. But “there is some number of grains that make a heap where that number minus one grain does not make a heap” is a proposition that comes out as true. According to the supervaluationist’s reasoning, vagueness is avoided only when human beings arbitrarily stipulate meaning that serves to sharpen the category from its negation in the penumbral area. For example, a person may be

clearly short in the positive extension, clearly not short in the negative extension, or either short or not short in the penumbra depending on where one freely decides to make a sharp distinction.

To its merit, the supervenient position seems to accord best with our intuition that absolute boundaries—like one millimeter, one hair or one grain differences between categories and their negations—do not exist except by stipulation. However, when put in concert with the semantic approach, the strong criticism that the approach levels at language, and specifically at any attempt to precisely delineate meaning, seems to backfire on the supervenient. According to the semantic approach, natural language is inherently defective to such a degree that even descriptively elaborate or scientifically exact terminology may fail to formulate an effective distinction or absolute boundary between a category and its negation. Does the supervenient's attempt at a sharpening, or precisification, in the penumbra likewise prove impossible? ⁵

We can only say that a statement is true, e.g. "x is a heap," if it is true for all sharpenings, false if it is false for all sharpenings. Statements in the penumbral area, on the other hand, are neither true nor false until we delineate a clearer, or less ambiguous, meaning. Still, since we can make sharpenings wherever we like, the truth or falsity of a statement falling within the penumbra will only be so relative to a sharpening, which any further sharpening may controvert. If we decide that a panel of the sunset having predominant tones of purple is yellow by way of a sharpening then we are completely justified in stating that any panel which is

yellow than it, though still predominantly purple, is yellow. In addition, higher order vagueness has a way of infecting the tripartite nature of the model of the supervenient. We may ask whether the division between the penumbra and the positive and negative extensions is itself vague, so that a higher order uncertainty spreads outwards and entreats us to stipulate whether the sky is ‘certainly yellow’ or ‘certainly not yellow’ in order to avoid further vagueness. [Sainsbury 1995: 39]

Laurence Goldstein addresses this particularly unfavorable result of applying the sorites to events of differentiating color gradation. He claims that the problem is based on a mistake assumption about the revisability of our judgements:

...there will be intersubjective and diachronic discrepancies in judging that a particular shade is properly termed ‘red’ or ‘yellow’. Adjacent shades [shades which, when placed together, cannot be told apart] will be judged red by some, yellow by others, and one red, one yellow by some who see them spatially or temporally separated. What leads one to the incorrect view that the succession of steps in the reasoning commits one to a conclusion that is at odds with the evidence of one’s eyes is only the incorrect thought that observational judgements are irrevocable. [Goldstein 1988: 451]

According to Goldstein, we may revise our judgements without reference to the standardized constraints of logical reasoning; hence, the process of revising judgments reflects the way in which we, as individuals [and not always logical ones], perceive and cognitively evaluate data that does not come to us in ‘neatly packaged objective categories.’ [Goldstein 1988: 454]

We can also effectively synthesize the ‘rejecting the categorical’ and the ‘semantic’ approaches. The negative sorites states that given a single grain of sand

you cannot make a heap; if you cannot make a heap out of x grains of sand, then you cannot make a heap out of $x + 1$ grains; therefore, you cannot construct a heap no matter how many grains of sand you have. When we take the bold position that a negative sorites argument proves that there are no heap-like, bald or short things, then we deprive the standard sorites of its first premise, namely the categorical. Rejecting the categorical thereby takes the conclusion of the negative sorites quite literally; it extends the vagueness that we ascribe to categories to the point of ridiculousness, to say that there are no real objects to which they correspond.⁶ Mark Heller takes a slightly more mitigated approach by claiming that objects are not vague, though our ontological claims about them are. [Heller 1988: 110-2] What these two perspectives share in common is that they both point to the relation between real things and our claims about them. Language affects our perception of reality, but according to Heller and contrary to the “rejecting the categorical” approach, it cannot eliminate real objects or insert the quality of vagueness into them.⁷

To review, the ‘semantic approach’ states that practical language is inadequate as a means to describe vague concepts. A synthesis of “rejecting the categorical” with the “semantic” approach could attack the vagueness of natural language generally and facilitate an effort to criticize the vagueness of categories (expressed in language) specifically. Yet, since we know that there exist real objects (the contrary conclusion, of course, being absurd) and that categories, though inherently flawed by their construction in language, do help differentiate those objects, the combined solution of the “semantic approach” and “rejecting the

categorical” strategies, while coherent, would be unconvincing since it defies our standard ontology.

The last synthesis of possible solutions to the sorites paradox ties the “degrees of truth” to the “semantic” strategy. To state that it is seventy-percent [.7] true that a person is inside another room, thirty-percent [.3] true that the room is vacant does not mean that the idea of a person or a room is vague. Uncertainty about the location of a person differs from vagueness at the borderline of categories and their negations in the sense that the former may be removed by placing oneself in a better epistemic position (e.g. inside the room) while the latter cannot be remedied no matter how much further knowledge we attain. [Schiffer 1998: 204] Assigning degrees of truth is therefore a non-epistemic approach to solving the paradox. [Sainsbury 1995: 44]

Let us examine another standard scenario in which degrees of truth would commonly be assigned and ask whether it involves the paradox: A carpenter might say that it is ninety-nine percent likely that a house is infested with termites. On the basis of this assignment of a high degree of truth to this determination, he would recommend that the house be razed and rebuilt. However, the owner infers that there is a one-percent likelihood that the house is not infested and insists that if the carpenter is not entirely certain that the house is infested he will not authorize the demolition. How is this different from an assignment of degrees of truth to a category and its negation? The carpenter or the owner could hypothetically assume a better epistemic position. However, what one carpenter defines as “is infested”

and another defines as “is not infested” might be vague at the borderline. Indeed, Sainsbury suggests that a “degree of truth” approach is grounded on a semantic distinction between vague and precise predicates. [Sainsbury 1995: 43] To combine the “degrees of truth” with the “semantic” approach would in essence reaffirm the semantic nature of the “degrees of truth” approach. In short, this is because the rationale for assigning degrees of truth originates with an observance of the irreparably defective nature of language.

As long as vagueness in language persists, though, assigning a truth-value of one hundred percent (1 or absolute certainty) will be impossible, even in clear cases. Assigning degrees of truth thereby becomes more like assigning degrees of belief than determining probabilities that one prediction (e.g. 10 grains of sand is a heap) or the other (e.g. 10 grains of sand is not a heap) will prove true. We must likewise ask what is meant when we speak of degrees of truth. Two candidates will be discussed here: (1) degrees of truth as degrees of logical truth in propositions, and (2) degrees of truth as degrees of belief.

Understood as a determination about the truth of propositions, the “degrees of truth” approach leads to a denial of the difference of logical operators. The truth-value of a proposition formed by two conjuncts is always the value of the lesser conjunct, while for disjuncts it is the value of the greater. Yet a statement such as “that person is bald and not bald” would then have the same degree of truth as “that person is bald or not bald” when we assign equivalent degrees of truth to the conjuncts and disjuncts of each statement, or assign equivalent though contextually

low and high degrees of truth to one of the conjuncts in the former and disjuncts in the latter statements.

Additional problems in the “degrees of truth” approach take hold, though, when we consider degrees of truth as degrees of belief. To the credit of this approach, borderline cases like the previously discussed sunset do not succumb (as they do under the supervaluationist account) to over-generalising categorization. We might state that we have a .75 degree of belief in the truth that the sky is yellow and a .25 degree of belief that it is not yellow (or that it is purple). However, when we observe the discordance between the logic of propositions and the logic of rational beliefs, the weak underbelly of equating degrees of truth with degrees of belief becomes apparent. From a belief in “M” and belief in “N” it should follow that we believe “M and N.” A clear example where such a belief in “M and N” would be rationally inconsistent surfaces in the case of a lottery. We may have a rational belief that “Every purchaser of a lottery ticket is justified in believing the ticket which he or she buys is not the winning ticket” on the ground that the probability of winning is one in one million (belief M). In addition, we may believe that there is a winning lottery ticket (belief N), although we might want to say that the more tickets that are purchased the weaker our first rational belief becomes. [Sorenson 1988: 240] In other words, it is possible to have an inconsistent set of rational beliefs, resulting in an inconsistent conjunctive belief (in the prior example, the belief in “M and N”). Of course, it is also possible that not all our beliefs will accommodate such a rational epistemology; that is, we may have irrational beliefs that by their nature entail inconsistent reasoning. One such irrational belief

manifests in holding that modus ponens is an invalid method of deduction. This tactic stymies the emergence of the paradoxical conclusion that “One grain of sand is a heap” by preventing the operation of the mechanism that lands us there. However, it fails to preserve such a vital intuition about the truth of conditional statements that in the end it only alienates potential adherents. At the most, it might seem intuitively satisfying to state that modus ponens is redundant, as some skeptics have, but this move does not serve to deny the operation's validity, nor does it defeat the paradox. Either way, understanding degrees of truth as degrees of belief—whether rational or irrational—fails to secure veridical solutions to the sorites paradox.

In concluding the discussion, we might generalize that the process of synthesizing solutions of the sorites seems most effective when accomplished in combinations between the overall semantic approach and the more semantically-based attempts at solving the paradox. This exercise could then yield to the criticism that any argument for a synthesis of the solutions is specious when accomplished between an umbrella solution and a derivative solution. However, what the exercise serves to accomplish is to point out the semantic nature of the three solutions (as well as the overall paradox) and to delineate their individual failings. While their failings are not all due to their semantic underpinnings, the highlight of discussing each plausible solution and the paradox in general is perhaps to point out the vagueness in meanings that we attach to words. We might learn something from Alice's conversation with Humpty Dumpty in *Through the Looking Glass*:

'When I use a word,' Humpty Dumpty said in rather a scornful tone, 'it means just what I choose it to mean -- neither more nor less.'
'The question is,' said Alice, 'whether you can make words mean so many different things.'
'The question is,' said Humpty Dumpty, 'which is to be master—that's all.'⁸

The semantic approach might yield such a 'master', a meta-language free from vague terms and borderline cases. However, we could only hope, perhaps in vain, for a meta-language free from the vagueness of our own natural language.⁹

Instead, we might return to the conclusion that the sorites paradox is just another “word game” or “puzzle” meant to perplex us, but with little or no serious implications for our ways of life. In mathematics and science, for example, sorites issues do not affect measurements of real objects, such as determining that there are two molecules of oxygen and one of carbon in carbon dioxide. Yet, in areas such as applied ethics, law, and politics, intersubjective agreement depends on the clear categorization of things vis-à-vis organizing terms and concepts. This fact invites us to further investigate just how the sorites underlies problems in these and other practical areas. All in all, the sorites paradox could therefore have a more profound influence than we think, more than just another “word game” or “puzzle,” on our individual and collective lives.

NOTES

1. Eubulides of Miletus also produced the puzzles of 'The Liar', 'The Hooded Man', and 'The Bald Man.' [Burnyeat 1982] In the last ten years, examples of the modern debate persist in Laurence Goldstein's paper on the revisability of judgements involving vague predicates, 'The Sorites as a lesson in Semantics', [Goldstein 1988], Diana Raffman's article on a proposed solution, 'Vagueness without Paradox' [Raffman 1994] and Mark Heller's paper 'Vagueness and the Standard Ontology' [Heller 1988: 109-131].

2. See Mark Heller, 'Vagueness and the Standard Ontology.' [Heller 1988: 109-131]

3. H.L.A. Hart refers to the 'open texture' of language in describing the interpretive demands placed on citizens and judges to determine what the law, or the legislature, requires. He concurs that 'there is a limit, inherent in the nature of language, to the guidance which general language can provide.' [Hart 1961: 123-125] See also Ronald Dworkin, 'No Right Answer?' [Dworkin 1977: 83-4].

4. *Supra* note 2.

5. According to Schiffer: 'A precisification is a model-theoretic interpretation of the language wherein the set assigned as an extension to a vague term includes everything to which the term definitely applies, nothing to which it definitely doesn't apply, and may include none, some, or all of the term's borderline applications. [Schiffer 1998: 195]

6. This is the view of Unger, presented by Sainsbury as showing that what 'we should draw from the existence of the sorites paradoxes is that vague concepts are deeply flawed: they commit us to absurdities.' [Sainsbury 1995: 31]

7. This brings up the issue of whether vagueness is a primary or secondary quality, meaning whether it exists in the object or in the percipient.

8. Lewis Carrol, *Through the Looking Glass*, text posted on the web at <http://www.student.kun.nl/l.derooy/>.

9. As Sainsbury points out, describing one vague language with another vague language would only engender higher order vagueness and lead back to the sorites' familiar slippery slope. [Sainsbury 1995: 46]

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