ORIGINAL ARTICLE

The Principle of Indifference and Imprecise Probability

Susanna Rinard

University of Missouri – Kansas City

Sometimes different partitions of the same space each seem to divide that space into propositions that call for equal epistemic treatment. Famously, equal treatment in the form of equal point-valued credence leads to incoherence. Some have argued that equal treatment in the form of equal interval-valued credence solves the puzzle. This paper shows that, once we rule out intervals with extreme endpoints, this proposal also leads to incoherence.

The *Principle of Indifference* (POI) states that if one has no more reason to believe R than S, and no more reason to believe S than R, then one's (point-valued) credence in S should equal one's (point-valued) credence in R. POI is highly plausible, but it seems to lead to incoherence in cases like the following.[1]

Suppose you are told of a factory that produces identical cubes. The only other information you are given is that the length of a side of a cube is *at most* 2 feet. So, the length in feet is either between 0 and 1 or between 1 and 2. You have no more reason to believe one of these than the other, so, according to POI, you should assign them equal credence: $\Pr(0 < L \leq 1) = \Pr(1 < L \leq 2) = 1/2$.

But now think about the same example a bit differently. Your information—that $0 < L \leq 2$—is equivalent to the proposition that the *area* of a side is at most 4 square feet, that is, $0 < A \leq 4$. We can divide this into four possibilities: $(0 < A \leq 1)$, $(1 < A \leq 2)$, $(2 < A \leq 3)$, and $(3 < A \leq 4)$. You have no more reason to believe any one of these than any one of the others, so POI says you should assign them all equal probability, that is, $1/4$.

Now we have a problem, because $0 < L \leq 1$ is equivalent to $0 < A \leq 1$. Our first application of POI had us assign probability $1/2$ to this proposition; our second application had us assign probability $1/4$ to the very same proposition. This leaves us with a puzzle. POI is highly plausible but it seems to lead to incoherence.

Some authors, including Joyce (2005, 2010) and Weatherson (2007), have argued that the puzzle can be solved by moving to a model on which one's doxastic state is represented by a *set* of probability functions, rather than a *single* function. On this model, one's

Correspondence to: E-mail: srinard@gmail.com

credence in a proposition P is the *interval* $[c, d]$ just in case, for every r, r is in $[c, d]$ if and only if there is some function in one's set with $Pr(P) = r$.[2]

Joyce holds that POI is partly correct. It's true that, when we have no more reason to believe R than S (and vice versa), we ought to give R and S equal treatment. The mistake in POI, however, is to assume that equal treatment must come in the form of equal *point-valued* credence. Rather, equal treatment can come in the form of equal *interval-valued* credence.

So, Joyce and Weatherson agree that we should treat $(0 < L \leq 1)$ and $(1 < L \leq 2)$ equally *and* that we should treat each of the following equally: $(0 < A \leq 1)$, $(1 < A \leq 2)$, $(2 < A \leq 3)$, and $(3 < A \leq 4)$. We can do this by assigning to each proposition in both partitions the same interval-valued credence. This solution is possible *only* if we employ the set of functions model, rather than the single function model. So, as Joyce and Weatherson point out, if the solution succeeds, it constitutes a powerful reason in favour of the set of functions model.

However, the solution does not succeed. First, the particular interval recommended by Joyce (2005) and Weatherson (2007) is $[0, 1]$. But it is never rational to have an interval-valued credence with an extreme endpoint (i.e., an endpoint equal to 0 or 1). I have defended this claim in Rinard (2013), and Joyce (2010) also presents strong reasons in favour of it (though he is neutral on whether they are decisive). For example, sometimes extreme endpoints render inductive learning impossible. There are other, equally serious problems. However, I will not repeat the case against extreme endpoints here.

This may seem like a minor detail, however. Surely we can reinstate the solution by assigning some narrower interval to each cell in both partitions. Indeed, this proposal appears in Joyce (2010).[3]

Unfortunately, however, this is probabilistically incoherent, as I will now show. This is the main claim of the paper, so it bears emphasizing:

For any interval $[c, d]$, (c, d), $(c, d]$, or $[c, d)$ with $c > 0$ and $d < 1$, it is probabilistically incoherent to assign that same interval to each of $(0 < L \leq 1)$ and $(1 < L \leq 2)$ as well as $(0 < A \leq 1)$, $(1 < A \leq 2)$, $(2 < A \leq 3)$, and $(3 < A \leq 4)$.

The proof has two stages. First I show that if the same interval can be assigned to some proposition and its negation (such as $(0 < L \leq 1)$ and $(1 < L \leq 2)$), then that interval must be symmetric around .5. Next, I show that if some interval with nonextreme endpoints is symmetric around .5, it cannot be assigned to all four propositions in the second partition.

The proof of the first claim proceeds by reductio. Suppose, for reductio, that some interval is assigned to both A and $\sim A$ but is not symmetric around .5 — it is shifted high, so that $1 - d < c$. First, suppose the interval is *closed* — $[c, d]$. Consider a function in the set with $Pr(A) = d$. (There must be one, since we are assuming that A is assigned the interval $[c, d]$.) Since $Pr(\sim A) = 1 - Pr(A)$, this function must have $Pr(\sim A) = 1 - d$. But now we have a contradiction, because $1 - d$ is not in the interval $[c, d]$ (since, we are assuming, the interval is asymmetric with $1 - d < c$). So $[c, d]$ cannot be assigned to both A and $\sim A$. Parallel reasoning shows that an asymmetric interval shifted low (with $1 - d > c$) cannot

be assigned to both A and $\sim A$. I conclude that if the same closed interval $[c, d]$ is assigned to both A and $\sim A$, then it must be symmetric around .5.

Now suppose the interval is *open*—(c, d). As before, assume for reductio that it is not symmetric around .5, but rather is shifted high, with $1 - d < c$. This time, consider a function in the set with $\Pr(A) = d - \epsilon$, where ϵ is some tiny real number greater than 0. Suppose ϵ is sufficiently tiny that $d - \epsilon > c$; this ensures that $d - \epsilon$ is in (c, d). The function with $\Pr(A) = d - \epsilon$ has $\Pr(\sim A) = 1 - (d - \epsilon)$. But this yields a contradiction, if ϵ is sufficiently tiny that $1 - (d - \epsilon) < c$ because then $1 - (d - \epsilon)$ is not in (c, d). Since there is some sufficiently tiny ϵ, (c, d) cannot be assigned to both A and $\sim A$. Once again, parallel reasoning shows that an asymmetric open interval shifted low (with $1 - d > c$) cannot be assigned to both A and $\sim A$.

I conclude that if an interval of the form $[c, d]$ or (c, d) is assigned to both A and $\sim A$, it must be symmetric around .5.

What about intervals of the form $(c, d]$ or $[c, d)$? Reasoning similar to that just given shows that no interval of this kind that is shifted either high or low (i.e., with $1 - d < c$ or $1 - d > c$) can be assigned to both A and $\sim A$. However, in these cases, it is also impossible to assign the same interval to both A and $\sim A$ when $1 - d = c$. Consider an interval $(c, d]$ with $1 - d = c$. Suppose, for reductio, that you have assigned this interval to both A and $\sim A$. Some function in your set has $\Pr(A) = d$, so $\Pr(\sim A) = 1 - d$, so $\Pr(\sim A) = c$. But now we have a contradiction because c is not a member of $(c, d]$. Similar reasoning shows that $[c, d)$ also cannot be assigned to both A and $\sim A$ when $1 - d = c$. The upshot is that there is *no way* to assign an interval of the form $[c, d)$ or $(c, d]$ to both A and $\sim A$.

Now I will prove that no interval that is symmetric around .5 with nonextreme endpoints can be assigned to each of four partition cells. Let A, B, C, and D be these cells and assume for reductio that we have assigned the same interval to each of them.

Once again, we start with a *closed* interval $[c, d]$. Consider a function in the set with $\Pr(A) = d$. $\Pr(\sim A) = 1 - d$. $\sim A$ is equivalent to (B or C or D), so $\Pr(B \text{ or } C \text{ or } D) = 1 - d$. So $\Pr(B) + \Pr(C) + \Pr(D) = 1 - d$. Since the interval is symmetric around .5, $1 - d = c$, so $\Pr(B) + \Pr(C) + \Pr(D) = c$. But now we have a contradiction because we assumed that $[c, d]$ was assigned to each of these propositions, which means that, for each, c is the lowest possible value it can be assigned by a function in the set. But then the sum of the three values cannot *equal* c. (Recall our stipulation that $c > 0$.) So we cannot assign $[c, d]$ to all four cells.

Now consider an *open* interval (c, d), again symmetric around .5 with nonextreme endpoints. Consider a function in the set with $\Pr(A) = d - \epsilon$. Once again, $\epsilon > 0$, and we stipulate that $d - \epsilon > c$ to ensure that the set contains some such function. That function has $\Pr(B) + \Pr(C) + \Pr(D) = 1 - (d - \epsilon)$. Since the interval is symmetric around .5, $1 - (d - \epsilon) = c + \epsilon$. So $\Pr(B) + \Pr(C) + \Pr(D) = c + \epsilon$. But now we have a contradiction, if ϵ is less than c. This is because $\Pr(B)$, $\Pr(C)$, and $\Pr(D)$ must each individually be greater than c. But if $\epsilon < c$, there are no three real numbers, each greater than c, whose sum equals $c + \epsilon$. Since there is some ϵ less than c, we cannot assign (c, d) to all four partition cells.

I conclude that no interval — either open or closed — that is symmetric around .5, with nonextreme endpoints, can be assigned to all four partition cells.

This completes the proof of the main claim of the paper. Once we rule out extreme endpoints, there is no interval that we can coherently assign to both of the two partition cells concerning length *and* each of the four partition cells concerning area.[4]

The upshot is that the puzzles surrounding the *POI* cannot be solved simply by moving from the single function model to the set of functions model. This doesn't mean that the set of functions model should be rejected. It simply means that cases like the cube example are exceedingly difficult for both models, and, *contra* Joyce and Weatherson, cannot be used to motivate one over the other.

Acknowledgements

Notes

1 See Gillies (2000) and van Fraassen (1989). The cube example which follows is adapted from the latter.

2 As argued in Joyce (2010), this interval alone may not constitute a *complete* characterization of one's doxastic attitude towards P. One's entire set of functions may be needed for this.

3 Joyce (2010) proposes the following principle (restated in the terminology used here): *Symmetry*: *Let* $\{E_1, \ldots, E_n\}$ *be a partition such that you have no more reason to believe one member of the partition than any other. If you are rational, then for every function Pr in your set and permutation* σ *of* $\{1, \ldots, n\}$, *there will also be some function* Pr_σ *in your set with* $Pr_\sigma(E_{\sigma(n)}) = Pr(E_n)$. Let A and B be arbitrary members of $\{E_1, \ldots, E_n\}$. Symmetry entails that for every real number r, if some function in your set assigns r to A, then there is some function in your set that assigns r to B. Since A and B are arbitrary, the reverse also follows. So, for every real number r, there is some function in your set that assigns r to A if and only if there is some function in your set that assigns r to B. But this is just to say that you assign the same interval to A and B. Since A and B were arbitrary, we conclude that according to Symmetry, you have the same interval-valued credence for every member of $\{E_1, \ldots, E_n\}$. Joyce suggests that we retain Symmetry even if we adopt the view that rationality requires us never to have an interval-valued credence with an extreme endpoint. This, as I show in the main text, is probabilistically incoherent (assuming we retain the view that you have no more reason to believe $(0 < L \leq 1)$ over $(1 < L \leq 2)$, and vice versa; and no more reason to believe any of the following over any of the others: $(0 < A \leq 1)$, $(1 < A \leq 2)$, $(2 < A \leq 3)$, and $(3 < A \leq 4)$).

4 It is standard to assume that the set representing one's credence will always be an interval. But in this case, there is one non-interval option with some plausibility. Perhaps you should have the set $\{1/4, 1/2\}$ as your credence for each of $(0 < L \leq 1)$ and $(1 < L \leq 2)$ as well as $(0 < A \leq 1)$, $(1 < A \leq 2)$, $(2 < A \leq 3)$, and $(3 < A \leq 4)$. But this is also incoherent. Your set would contain a function with $Pr(0 < L \leq 1) = 1/2$, so $Pr(0 < A \leq 1) = 1/2$, so $Pr(1 < A \leq 2) + Pr(2 < A \leq 3) + Pr(3 < A \leq 4) = 1/2$. But then it can't be that each of these is assigned either $1/4$ or $1/2$. (If one is assigned $1/2$, the others must be 0. If each is $1/4$, their sum would be $3/4$, not $1/2$.)

References

van Fraassen, B. *Laws and Symmetry*. Oxford: Clarendon Press, 1989.

Gillies, D. *Philosophical Theories of Probability*. London: Routledge, 2000.

Joyce, J. "How Probabilities Reflect Evidence." *Philosophical Perspectives* **19**.1 (2005): 153–78.

———. "A Defence of Imprecise Credences in Inference and Decision Making." *Philosophical Perspectives* **24**.1 (2010): 281–323.

Rinard, S. "Against Radical Credal Imprecision." *Thought* **2**.1 (2013): 157–65.

Weatherson, B. "The Bayesian and the Dogmatist." *Proceedings of the Aristotelian Society* **107** (2007): 169–85.