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Neutrosophic Set and Neutrosophic Topological Spaces

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Abstract: *Neutrosophy has been introduced by Smarandache [7, 8] as a new branch of philosophy. The purpose of this paper is to construct a new set theory called the neutrosophic set. After given the fundamental definitions of neutrosophic set operations, we obtain several properties, and discussed the relationship between neutrosophic sets and others. Finally, we extend the concepts of fuzzy topological space [4], and intuitionistic fuzzy topological space [5, 6] to the case of neutrosophic sets. Possible application to superstrings and* ζ^{∞} *space–time are touched upon.*

Keywords: *Fuzzy topology; fuzzy set; neutrosophic set; neutrosophic topology*

I. Introduction

The fuzzy set was introduced by Zadeh [9] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. After the introduction of the neutrosophic set concept [7, 8]. In recent years neutrosophic algebraic structures have been investigated. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory.

II. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7, 8], and Atanassov in [1, 2, 3]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]0^-,1^+\right[$ is nonstandard unit interval.

2.1 Definition. [3,4]

Let T, I,F be real standard or nonstandard subsets of $\left]0^-,1^+\right[$, with
Sup_T=t_sup, inf_T=t_inf
Sup_I=i_sup, inf_I=i_inf
Sup_F=f_sup, inf_F=f_inf
n-sup=t_sup+i_sup+f_sup
n-inf=t_inf+i_inf+f_inf,
T, I, F are called neutrosophic components

III. Neutrosophic Sets and Its Operations

We shall now consider some possible definitions for basic concepts of the neutrosophic set and its operations.

3.1 Definition

Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A=\{\langle x,\mu_A(x),\sigma_A(x),\gamma_A(x)\rangle : x\in X\}$ Where $\mu_A(x),\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x\in X$ to the set A .

3.1 Remark

A neutrosophic $A=\{< x,\mu_A(x),\sigma_A(x),\gamma_A(x) >: x\in X\}$ can be identified to an ordered triple $<\mu_A,\sigma_A,\gamma_A>$ in $\left]^-0,1^+\right[$ on. X .

3.2 Remark

For the sake of simplicity, we shall use the symbol $A = < x, \mu_A, \sigma_A, \gamma_A >$ for the

NS $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$

3.1 Example

Every IFS A a non-empty set X is obviously on NS having the form

$A = \{< x, \mu_A(x), 1-(\mu_A(x)+\gamma_A(x)), \gamma_A(x) >: x \in X\}$

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the NSS 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

1_N may be defined as:

(1_1) $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

3.2 Definition

Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ a NS on X, then the complement of the set A $(C(A)$, for short $)$ maybe defined as three kinds of complements

(C_1) $C(A) = \{\langle x, 1-\mu_A(x), 1-\gamma_A(x)\rangle : x \in X\}$,

(C_2) $C(A) = \{\langle x, \gamma_A, \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

(C_3) $C(A) = \{\langle x, \gamma_A, 1-\sigma_A(x), \mu_A(x)\rangle : x \in X\}$

One can define several relations and operations between NSS follows:

3.3 Definition

Let x be a non-empty set, and NSS A and B in the form $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$, $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle$, then we may consider two possible definitions for subsets $(A \subseteq B)$

$(A \subseteq B)$ may be defined as

(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \gamma_A(x) \ge \gamma$ and $\sigma_A(x) \le \sigma_B(x)$ $\forall x \in X$

(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \gamma_A(x) \ge \gamma_B(x)$ and $\sigma_A(x) \ge \sigma_B(x)$

3.1 Proposition

For any neutrosophic set A the following are holds

(1) $0_N \subseteq A$, $0_N \subseteq 0_N$

(2) $A \subseteq 1_N$, $1_N \subseteq 1_N$

3.4. Definition

Let X be a non-empty set, and $A = < x, \mu_A(x), \gamma_A(x), \sigma_A(x) >$, $B = < x, \mu_B(x), \sigma_B(x), \gamma_B(x) >$ are NSS. Then

(1) $A \cap B$ maybe defined as:

(I_1) $A \cap B = < x, \mu_A(x).\mu_B(x), \sigma_A(x).\sigma_B(x),$

$\gamma_A(x).\gamma_B(x) >$

(I_2) $A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x),$

$\gamma_A(x) \vee \gamma_B(x) >$

(I_3) $A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x),$ $\gamma_A(x) \vee \gamma_B(x) >$

(2) $A \cup B$ may be defined as:

(U_1) $A \cup B = < x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x),$
$\gamma_A(x) \wedge \gamma_B(x) >$

(U_2) $A \cup B = < x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x),$
$\gamma_A(x) \wedge \gamma_B(x) >$

(3) $[\ ]A = < x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) >$

(4) $< > A = < x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) >$

We can easily generalize the operations of intersection and union in definition 3.4 to arbitrary family of NSS as follow:

3.5 Definition

Let $\{Aj : j \in J\}$ be a arbitrary family of NSS in X, then

(1) $\cap Aj$ maybe defined as:

(i) $\cap Aj = \left\langle x, \underset{j\in J}{\wedge} \mu_{Aj}(x), \underset{j\in J}{\wedge} \sigma_{Aj}(x), \vee \gamma_{Aj}(x) \right\rangle$

(ii) $\cap Aj = \left\langle x, \wedge \mu_{Aj}(x), \vee \sigma_{Aj}(x), \vee \gamma_{Aj}(x) \right\rangle$

(2) $\cup Aj$ maybe defined as:

(i) $\cup Aj = \langle x, \vee, \wedge, \wedge \rangle$

(ii) $\cup Aj = \langle x, \vee, \wedge, \wedge \rangle$

3.6. Definition

Let A and B are neutrosophic sets then
$A|B$ may be defined as

$A|B = \left\langle x, \mu_A \wedge \gamma_B, \sigma_A(x) \sigma_B(x), \gamma_A \vee \mu_B(x) \right\rangle$

3.2. Proposition

For all A, B two neutrosophic sets then the following are true

(1) $C(A \cap B) = C(A) \cup C(B)$

(2) $C(A \cup B) = C(A) \cap C(B)$

IV. Neutrosophic Topological Spaces

Here we extend the concepts of fuzzy topological space [4], and intuitionistic fuzzy topological space [5, 7] to the case of neutrosophic sets.

4.1 Definition

A neutrosophic topology (NT for short) an a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms

(NT_1) $0_N, 1_N \in \tau$,

(NT_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(NT_3) $\cup G_i \in \tau \quad \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is known as neuterosophic open set (NOS for short) in X. The elements of τ are called open neutrosophic sets, A neutrosophic set F is closed if and only if it C (F) is neutrosophic open.

4.1 Example

Any fuzzy topological space (X, τ_0) in the sense of Chang is obviously a NTS in the form $\tau = \{A : \mu_A \in \tau_0\}$ wherever we identify a fuzzy set in X whose members ship function is μ_A with its counterpart.

4.1. Remark Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology.

4.3 Example

Let $X = \{x\}$ and

$A = \{\langle x, 0.5, 0.5, 0.4 \rangle : x \in X\}$

$B = \{\langle x, 0.4, 0.6, 0.8 \rangle : x \in X\}$

$D = \{\langle x, 0.5, 0.6, 0.4 \rangle : x \in X\}$

$C = \{\langle x, 0.4, 0.5, 0.8\rangle : x \in X\}$

Then the family $\tau = \{O_n, 1_n, A, B, C, D\}$ of NSs in X is neutrosophic topology on X

4.4 Example

Let (X, τ_0) be a fuzzy topological space in changes sense such that τ_0 is not indiscrete suppose now that $\tau_0 = \{0_N, 1_N\} \cup \{V_j : j \in J\}$ then we can construct two $NTSS$ on X as follows

a) $\tau_0 = \{0_N, 1_N\} \cup \{< x, V_j, \sigma(x), 0 >: j \in J\}$.

b) $\tau_0 = \{0_N, 1_N\} \cup \{< x, V_j, 0, \sigma(x), 1 - V_j >: j \in J\}$.

4.1 Proposition

Let (X, τ) be a NTS on X, then we can also construct several $NTSS$ on X in the following way:

a) $\tau_{o,1} = \{[\,]G : G \in \tau\}$,

b) $\tau_{o,2} = \{<> G : G \in \tau\}$,

Proof

a) (NT_1) and (NT_2) are easy.

(NT_3) Let $\{[\,]G_j : j \in J, G_j \in \tau\} \subseteq \tau_{0,1}$.Since

$\cup G_j = \left\{\left\langle x, \vee\mu_{G_j}, \vee\sigma_{G_j}, \wedge\gamma_{G_j}\right\rangle\right\} or \left\{\left\langle x, \vee\mu_{G_j}, \wedge\sigma_{G_j}, \wedge\gamma_{G_j}\right\rangle\right\} or \left\{\left\langle x, \vee\mu_{G_j}, \wedge\sigma_{G_j}, \vee\gamma_{G_j}\right\rangle\right\} \in \tau$, we have

$\cup([\,]G_j) = \{x, \vee\mu_{G_j}, \vee\sigma_{G_j}, \wedge(1 - \mu_{G_j})\} or \{x, \vee\mu_{G_j}, \vee\sigma_{G_j}, (1 - \vee\mu_{G_j})\} \in \tau_{0,1}$

b) This similar to (a)

4.2 Definition

Let $(X, \tau_1), (X, \tau_2)$ be two neutrosophic topological spaces on X. Then τ_1 is said be contained in τ_2 (in symbols $\tau_1 \subseteq \tau_2$) if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

4.2 Proposition

Let $\{\tau_j : j \in J\}$ be a family of $NTSS$ on X. Then $\cap \tau_j$ is A neutrosophic topology on X .Furthermore, $\cap \tau_j$ is the coarsest NT on X containing all. τ_j, s

Proof. Obvious

4.3 Definition

The complement of A (C (A) for short) of NOS. A is called a neutrosophic closed set (NCS for short) in X .

Now, we define neutrosophic closure and interior operations in neutrosophic topological spaces:

4.4 Definition

Let (X, τ) be NTS and $A =< x, \mu_A(x), \gamma_A(x), \sigma_A(x) >$ be a NS in X .

Then the neutrosophic closer and neutrosophic interior of Aare defined by

$NCl(A) = \cap\{K : K \text{ is an NCS in X and } A \subseteq K\}$

$NInt(A) = \cup\{G : G \text{ is an NOS in X and } G \subseteq A\}$.It can be also shown that

It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X

a) A is in X if and only if $NCl(A)$.

b) A is NCS in X if and only if $NInt(A) = A$.

4.2 Proposition

For any neutrosophic set A in (x, τ) we have

(a) $NCl(C(A) = C(NInt(A)$,

(b) $NInt(C(A)) = C(NCl(A))$.

Proof.

a) Let $A = \{< x, \mu_A, \sigma_A, \upsilon_A >: x \in X\}$ and suppose that the family of neutrosophic subsets contained in A are indexed by the family if NSS contained in A are indexed by the family $A=\{< x,\mu_{G_i},\sigma_{G_i},\upsilon_{G_i} >: i\in J\}$. Then we see that $NInt(A)=\{< x,\vee\mu_{G_i} ,\vee\sigma_{G_i},\wedge\upsilon_{G_i} >\}$ and hence $C(NInt(A))=\{< x,\wedge\mu_{G_i},\vee\sigma_{G_i},\vee\upsilon_{G_i} >\}$. Since $C(A)$ and $\mu_{G_i}\le\mu_A$ and $\nu_{G_i}\ge\nu_A$ for each $i\in J$, we obtaining $C(A)$. i.e $NCl(C(A))=\{< x,\wedge\upsilon_{G_i},\vee\sigma_{G_i},\vee\mu_{G_i} >\}$. Hence $NCl(C(A)=C(NInt(A),$ follows immediately

b) This is analogous to (a).

4.3 Proposition

Let (x,τ) be a NTS and A,B be two neutrosophic sets in X. Then the following properties hold:

(a) $NInt(A)\subseteq A,$

(b) $A\subseteq NCl(A),$

(c) $A\subseteq B\Rightarrow NInt(A)\subseteq NInt(B),$

(d) $A\subseteq B\Rightarrow NCl(A)\subseteq NCl(B),$

(e) $NInt(NInt(A))=NInt(A)\wedge NInt(B),$

(f) $NCl(A\cup B)=NCl(A)\vee NCl(B),$

(g) $NInt(1_N)=1_N,$

(h) $NCl(O_N)=O_N,$

Proof (a), (b) and (e) are obvious (c) follows from (a) and Definitions.

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