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ORIGINAL RESEARCH

A Step-by-Step Argument for Causal Finitism

Joseph C. Schmid[1]

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Abstract
I defend a new argument for causal finitism, the view that nothing can have an infinite causal history. I begin by defending a number of plausible metaphysical principles, after which I explore a host of novel variants of the Littlewood-Ross and Thomson's Lamp paradoxes that violate such principles. I argue that causal finitism is the best solution to the paradoxes.

1 Introduction

Many paradoxes involve infinity in some way. They can be killed in a number of ways, but a unified, elegant, and explanatorily powerful solution to them is preferred. *Causal Finitism (CF)* promises as much (Pruss, 2018). We can state CF as follows:

Causal Finitism (CF): Necessarily, every causal history is finite.[1]

According to CF, causal sequences leading up to any event, state, or substance must be finite. My aim in this paper is to argue in favour of CF. More precisely, I argue that CF is the best solution to a host of new paradoxes—or, at least, new *variants* of old paradoxes. The old paradoxes are the Littlewood-Ross and Thomsom's Lamp paradoxes, and the new variants involve the violation of plausible metaphysical principles.

I begin my investigation in Sect. 2 by articulating and defending the metaphysical principles that undergird the new paradoxes. Then, in Sect. 3, I articulate the new

[1] This rules out infinite regresses of causes. It's important to note that there are different ways of fleshing out this definition. Here are a few: (i) there is no possible world in which there exists an infinitely descending chain of causal-power dependencies; (ii) nothing can be affected (directly or indirectly) by infinitely many causes (Pruss, 2018, p. 2); and (iii) there cannot be infinitely many things that causally impact one target state (Pruss, 2018, p. 25). All of these ways accurately capture the general thrust of CF.

✉ Joseph C. Schmid
schmi215@purdue.edu

1 Purdue University, 100 N University St, West Lafayette, IN 47907, USA

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variants of the Littlewood-Ross and Thomson's Lamp paradoxes that violate the principles articulated in Sect. 2. In Sect. 4, I bring to light different versions of the paradoxes from Sect. 3 that vary aspects of the paradox (like the structure of space and time, the speed of causal influence, the magnitude of certain physical quantities, etc.). This paves the way for Sect. 5, wherein I argue that CF is the best solution to the variegated paradoxes explored in Sects. 3 and 4. Finally, I consider objections in Sect. 6.

I wish briefly to dwell on the significance of this investigation. First, I aim to enliven a debate that's stagnated, drawing on very recent proposals concerning the nature of infinity and potential solutions to infinitary paradoxes.[2] Second, I provide new variations on the Littlewood-Ross and Thomson's Lamp paradoxes that sidestep usual resolutions. Finally, I motivate a number of new principles and apply them to the Littlewood-Ross and Thomson's Lamp paradoxes.

2 Principles

Let's first consider the following intuitive principle:

> *Step Principle (SP):* If (i) each of the steps in two processes result in identical states of the processes' respective systems, (ii) the cardinalities of the steps in the two processes are the same, and (iii) the processes' respective systems are explanatorily closed, then the states of the two systems are identical at the end of the two processes.[3]

An *explanatorily closed system* is such that no causal or explanatory factors *outside* the process or system influence the state or condition of the system (along some specified axis or criterion). Explanatorily closed systems are ones whose end state depends only on the actions performed within the process that lead to the end state of the systems as a whole.

To better understand SP, let's consider an example. Suppose Smith and Jones begin a stepwise process. The process consists in traveling from Las Vegas to New York City (NYC). The processes are broken down into steps, each of which results in an *end state*. There is a procedure to follow for each step, and the procedures

[2] Discussion of the Littlewood-Ross paradox has (somewhat) stagnated, as the majority of substantive treatments of the paradox appear in the 1990s and early 2000s. Also, the recent proposals to which I'm referring are those contained in Pruss (2018), Koons (2014), and Huemer (2016) concerning CF and infinite intensive magnitudes, among others.

[3] Some of these terms merit definition. Roughly: (i) a *process* is a determinate sequence of actions on or within a system; (ii) a *system* is a bounded, specified area or domain; (iii) a *state* of a system is the condition or character of the system's contents at a specified point (e.g. point in time or space) and along a specified axis (e.g. mass, shape, numbered contents, etc.); (iv) a *step/sub-step* of a process is a single, definite action or closely related (in terms of space, time, or execution) sequence of such actions which—when conjoined with more steps—forms a whole process; (v) *identical* means *qualitatively* identical and is always understood as identical along a specified axis (e.g. global condition of the respective systems, or else some specified property or properties). If the rough characterizations prove ultimately unhelpful, we can rest content with an intuitive grasp and obvious examples.

are not necessarily identical. In the case of Smith and Jones, the end state of each step consists in the person's destination. There is also, of course, an end state of the *whole process*—namely, the destination at which each respective person ends.

Here's Jones' process:

> Step 1: Drive from Las Vegas to Dallas; Step 2: Fly from Dallas to Albany; Step 3: Drive from Albany to NYC.

Here's Smith's process:

> Step 1: Fly from Las Vegas to Dallas; Step 2: Drive from Dallas to Albany; Step 3: Fly from Albany to NYC.

Now, although the *procedures* for each step in their processes are not identical, it is nevertheless the case that the *end state* of each step is identical. Although in Step 1 Jones drives while Smith flies, the end state of their respective systems—the location—is the same.

I propose that one can just *see* the following: that Smith and Jones perform steps each of which has an identical end state—in conjunction with the fact that no *outside* factors (transporter devices, etc.) causally influenced their processes or systems—guarantees that the end states of the *whole processes* are identical. And that's precisely what we see here: both processes terminate in NYC.

Generalizing, I propose that one can just *see* that if each step of two processes results in end states that are wholly identical, then it simply cannot be the case that the end states of the two processes *as a whole* differ. In slogan form: if each step in the processes ends the same, the processes *themselves* end the same. Symmetry breeds symmetry.

Another reason (apart from its being intuitive and seemingly self-evident) favouring SP derives from *explicability*. For if a *different* end state is obtained, it's plausible that such a difference must obtain in virtue of some step along the way being different (even if that's the last step). After all, if the process' end state was different, but *every single step* in the process resulted in an identical end state, how could there be a difference in the end state? What could explain or account for it? Any proposed explanatory factor operative in one process but not the other would seem to render the end states of the sub-steps of each process no longer identical. Hence, if the end states of the sub-steps are, indeed, identical, there couldn't be an explanatory factor operative in one process but not the other. But in that case, a difference in end state would be inexplicable. This points, then, to another reason supporting SP: denying it seems to result in inexplicable, brute facts.[4]

One might worry that SP is tautologous. For every process has a final step. And if we are postulating that the end states of each step in the processes are identical, then

[4] Even if one thinks that symmetry can breed asymmetry—say, because one thinks this is entailed by indeterministic, stochastic quantum processes or libertarianly free actions—we can restrict the application of SP to deterministic processes. And this won't affect the arguments I'll give for CF, since the systems in which they operate are deterministic.

it is *guaranteed* (as a matter of definition!) that the end states of the processes as a whole are identical.

By way of response, note first that *not every process* has a final step. Suppose, for instance, that time is continuous, such that for any two times t and t', there's a distinct time between them. In that case, we could specify a process that occurs over a span from t_1 to t_2, where the latter is one minute after the former. Moreover, the process could be such that each of its (continuum many) sub-steps correspond to each time from (including) t_1 to (excluding) t_2. The interval is thus closed at t_1 but open at t_2. In this case, the process has an end state at t_2, but there is nevertheless no final step.[5]

So, while SP may be tautologous in cases of processes with a final step, it isn't tautologous for all processes *simpliciter*. Moreover, the considerations adduced in favour of SP equally apply to processes lacking a final step, since nothing hinged crucially on an inference from a final step to an identical end state of the system. Consider inexplicability and bruteness. Even if the processes lack a final step, whence come the differential end states of the processes as a whole? What could account for such a difference? Nothing in the processes could, since all end states of the sub-steps within the processes are identical. Any discrepancies across processes *within* a specified step are entirely *wiped away* (as it were) by the end of the step, since the step's end state (for both processes) is—*ex hypothesi*—identical. And nothing *outside* the process could, since we're concerned with explanatorily closed systems. In short, a different *process* end state despite complete, uniform identity in sub-step end states seems to amount to magic. The different end state would plausibly lack an explanation. Symmetry breeds further symmetry.

Overall, then, I propose that we have good reason to accept SP.[6] I now wish briefly to consider three further principles, each of which seems—like SP—quite plausible. The first principle runs:

> *Ineffective Principle (IP)*: Adding any number of interactions with a system does *not* affect the state/condition of the system if *none* of the aforementioned interactions changes the state/condition of the system.

IP states that adding interactions with a system that don't alter the system's state are *ineffective* in changing the system's state. Like SP, IP seems intuitively plausible, and it enjoys much the same justification. (Consider, e.g., that if IP is false—that is, if adding some number of interactions with a system altered the state of the system despite the fact that none of these interactions changes the state of the system—the

[5] Similar examples can be given in terms of spatially extended processes provided that space is continuous, as well as in terms of synchronous (at a single time) processes. More on this later.

[6] It's worth noting that other authors have hinted at something like SP—though, few have explored its motivations and implications. An example of an author who hints at (something like) SP is Forrest (1999), who espouses a principle called the *Impotence of Individuality*, "the principle that if qualitatively identical processes result in products which are not qualitatively identical then the results are not determined by which process is which," and he finds it plausible that such a principle is "a metaphysically necessary principle" (p. 445). This paper adds (what I take to be) much needed clarifications, motivations, and implications of SP.

system's altered state seems inexplicable. The only additions made to the system were the interactions; but each such interaction, *ex hypothesi*, made no difference to the system's state. Surely merely *compounding* things that make no difference to x *itself* makes no difference to x.)

Now let's consider the second principle. Unlike the other principles, this principle is specific to a particular system and particular interactions within that system. More precisely, this principle applies to a system in which the state of a lamp (*on* or *off*) is controlled by a switch. Toggling the switch causes the lamp to change its state (from *on* to *off* or from *off* to *on*). To be sure, the principle's specificity doesn't detract from the argument for CF that I shall develop. But its specificity contrasts with the generality of the other principles, and this is worth noting.[7] With that covered, we can now articulate the principle itself:

> *Toggle Principle (TP):* Identical sequences of individual switch togglings yield identical lamp states (provided the lamp is an explanatorily closed system).

Like SP and IP, TP seems eminently plausible. According to TP, if there's no difference between two (or more) sequences of switch togglings, then there is no difference between the lamp states that *result* from such sequences. I venture that TP is both plausible and motivated by explicability. What, we may ask, could explain a difference between resultant lamp states except for some difference in the sequences of switch togglings leading up to them? Plausibly, there could be such a difference in resultant lamp state *only if* there were some difference in the sequences of switch togglings leading up to them (assuming, of course, we have an explanatorily closed system).

The third and final principle runs:

> *Removal Ineffectiveness Principle (RIP):* Were any number of interactions with a system (performed before some time *t*)—none of which change the system's state—to be removed, the state of the system at *t* in the resulting world would be identical to the state of the system at *t* in the actual world.

Once again, RIP seems quite plausible. Instead of focusing on *adding* interactions with a system none of which alter, change, or make a difference to the system's state (as IP does), RIP focuses on a counterfactual removal, elimination, or subtraction of actual interactions with a system none of which alter, change, or make a difference to the system's state. And like IP, RIP seems clearly true. (And—again like IP—much of the same reasoning that supports SP likewise supports RIP.)

With these principles in hand, we can now shed new light on the Littlewood-Ross and Thomson's Lamp paradoxes.

[7] If one demurs at the specificity, we can modify the principle to a more general principle along the lines of: "Given an alternating process, identical sequences of individual alternations yield identical states (provided the process/system is explanatorily closed)." The argument for CF I develop can proceed (*mutatis mutandis*) with either formulation. (I stick with the specific formulation in the main text.) I am grateful to an anonymous referee for bringing the specificity of the principle—as well as this more general reformulation—to my attention.

3 Paradoxes

3.1 Littlewood-Ross

The first paradox is the *Littlewood-Ross Paradox*.[8] Part of the paradox lies in the fact that the number of balls in the urn grows without bound as the supertask proceeds but suddenly falls discontinuously to zero. Another part of the paradox arises when we consider similar ball-urn manipulations that lead to staggeringly different end results despite qualitatively identical states of the urn throughout their respective processes.

Consider an infinitely large urn with infinitely many numbered balls (each of which has a unique natural number). Here are a few stipulations for the construction. First, the urn is originally empty. Second, the urn and the balls persist in existence unless positively destroyed, and no destructive factors operate on the urn or the balls before or at noon. Third, nothing affects the urn *at* noon—no action inputs balls, removes balls, and so on. Fourth, the content of the urn (i.e. the number of balls, the labels on the balls, the trajectories or positions of the balls, etc.) persists *as it is* unless positively changed by some action specified by the process. Finally, the only manipulations that alter the state of the urn are manipulations of the balls specified by the steps.

Now consider the following process, *Process One*. For each step n, at 2^{1-n} minutes before noon, place balls numbered $10(n-1)+1$ through $10n$ into the urn and remove ball number n. Repeat this procedure for each natural number and perform infinitely many ($\aleph_0$ many) such steps. Do not replace balls. Consider next *Process Two*: For each step n, at 2^{1-n} minutes before noon, place balls numbered $10(n-1)+1$ through $10n-1$ into the urn but change ball n to ball $10n$. Repeat this procedure for each natural number and perform infinitely many ($\aleph_0$ many) such steps without removing balls.

Here we have two processes, and the cardinalities of their steps are identical. Moreover, the end states of the systems after each step are identical: balls numbered 2–10 after step 1; balls numbered 3–20 after step 2; and so on. But now a question arises: what are the respective end states of the systems as a whole (the urns and their contents) upon completion of the respective processes?

Let's begin with Process One. Importantly, there cannot be *any* balls left in the urn at the end. This is because at each step, the ball with that step's number on it is removed. And since there are an infinite number of steps, it follows that for any ball

[8] The paradox first appeared in Littlewood (1953, p. 26), though a description of the same paradox within Ross (1988, p. 46) captured philosophers' attention first. Both Littlewood and Ross argue that the urn is empty at noon, with Allis and Koetsier (1991; 1995) and Earman and Norton (1996) arguing likewise. Holgate (1994) adds mathematical clarifications on the debate, while van Bendegem (1994) argues for the impossibility of the paradox in virtue of the contradictory states obtained (both empty and infinitely full). Other authors have contributed to the debate concerning the Littlewood-Ross paradox (or versions of it) not only concerning the urn's end state but also the scenario's metaphysical possibility. In addition to those already mentioned, see (*inter alia*) van Bendegem (1995; 2003), Forrest (1999), Byl (2000), Friedman (2002), Oppy (2006), Huemer (2016), Manchak and Roberts (2016), and Cook (2020).

number, there is a step number at which it is removed. Indeed, if there's a ball left in the urn, then it will have some natural number n written on it. But if n is written on it, then that ball was removed on step n. But if that ball was removed, then (since it wasn't replaced) it's not in the urn. So, if there's a ball left in the urn, then there isn't a ball left in the urn—and that's absurd. So, there's no ball left in the urn. Thus, at the end of Process One, there are no balls inside the urn.

But now consider Process Two. Interestingly, there are *infinitely many* balls in the urn at the end. After all, we never removed a single ball—all we did was *add* balls. In fact, the contents of the urn at the end of Process Two are infinitely many balls on which are written all the natural numbers, each followed by an infinite string (i.e. omega sequence) of zeroes. This is because for each natural number n, we multiplied n by 10 to form $10n$; and since $10n$ is itself a natural number, we eventually multiplied $10n$ by 10 to form $100n$; and so on ad infinitum. So, for each natural number n, there is a ball in the urn on which is written n followed by an infinite string of zeroes.

Moreover, we can suppose that the respective systems are explanatorily closed: the only causal or explanatory factors that could influence the final outcome are the causal operations of the steps themselves.

This, then, is what I take to be *paradoxical* about the Littlewood-Ross paradox: we have a violation of SP. Despite having the same cardinality, being explanatorily closed, and having (qualitatively) identical end states for each of their respective steps, the end states of the systems are wildly different at the end of the respective processes. But we've already seen strong reasons to think SP is true. So, at least one of the assumptions that allowed for the possibility of the infinite urn processes must be wrong (so the argument goes).

Which assumption? I return to this question in Sect. 5. Before we get there, we need to consider Thomson's Lamp as well as a host of other new paradoxical variants of the Littlewood-Ross paradox and Thomson's Lamp.

3.2 Thomson's Lamp

In the Thomson's Lamp paradox, we suppose that a lamp is off at 11:00 and that its switch is toggled infinitely many times before noon at 11:30, 11:45, 11:52:30, and so on. The question arises concerning the state of the lamp (*on* or *off*) at noon, with neither option seeming satisfactory. Importantly, though, there appears to be nothing incoherent or absurd to suggest that the lamp is in one of the specified states (*on*, say); as such, it may be a *puzzle*, but it seems not to be a strict paradox.

But with the principles explored in Sect. 2, we can construct versions of Thomson's Lamp that *do* engender paradox. Consider again IP, which states that adding interactions with a system each of which doesn't alter the system's state is *ineffective* in changing the system's state. Next consider the following interactions with the lamp system (encapsulated in the earlier language of 'steps'), where the lamp is originally off at 11:00. Call these interactions (steps) the *First Task*:

Step 1: Toggle the switch at 11:30 and 11:45.
Step 2: Toggle the switch at 11:52:30 and 11:56:15.

…
Step n: Toggle the switch at 2^{-2n+1} and 2^{-2n} hours before noon (for every n).

Because two successive togglings do not affect the state or condition of the lamp, each of these interactions or steps likewise do not affect the state or condition of the lamp. And from this it follows (by IP) that adding them all makes no difference to the state of the system.[9] But if adding them all makes no difference, then since the lamp was off at 11:00, it follows that the lamp is off at noon.

Consider now, though, the following interactions with the lamp system, where the lamp happened to be off at 11:00 and toggled on at 11:30. Call these interactions (steps) the *Second Task*:

Step 1: Toggle the switch at 11:45 and 11:52:30.
Step 2: Toggle the switch at 11:56:15 and 11:58:07.5
…
Step n: Toggle the switch at 2^{-2n} and 2^{-2n-1} h before noon (for every n).

As before, none of these interactions effects a difference to the lamp's on/off state, and hence adding all of them won't alter or affect the lamp's state (per IP). But adding all of *these* interactions means that the lamp is *on* immediately after 11:30 (since we still have the 11:30 toggling, as in the original setup) and hence *remains* in such a condition (since adding the subsequent interactions make no difference to the system's condition). It follows, then, that the lamp is on at noon.

We're only one step away from paradox. We need only add to the mix TP (which, recall, states that identical sequences of individual switch togglings yield identical lamp states, provided the lamp is an explanatorily closed system). Supposing that the tasks we've considered in this section occur in explanatorily closed systems, we have a full-blooded paradox. Given IP, we found that the First Task yielded an *off* state while the Second Task yielded an *on* state. Crucially, though, the *sequences* of individual switch togglings from 11:00 to 12:00 in both tasks were identical. Per TP, it follows that the lamp states yielded by such tasks are identical. But (as we've seen) they are decidedly *not*. The conjunction of Thomson's Lamp, IP, and TP yield a contradiction. Since IP and TP are (or at least strongly seem) true (as I argued in Sect. 2), it follows that the Thomson's Lamp scenario couldn't obtain.

In fact, we actually don't require TP to engender paradox here. Recall RIP: *were* any number of interactions with a system performed before t (none of which change the system's state) to be removed, the state of the system at t in the resulting world would be identical to the state of the system at t in the actual world.

Because RIP deals with counterfactual removal, we don't require the addition of TP to generate a contradiction. Simply consider a world in which the original Thomson's Lamp supertask is performed (with switch togglings at 11:30, 11:45, 11:52:30, etc.). In such a world, the state of the system at noon is either on or off. Perhaps it's on. Perhaps it's off. We need not specify that here. All we need is that the lamp is

[9] By 'adding them', I simply mean adding (i.e. actually performing) such interactions to an original state of the system (such as being off at 11:00).

either on or off. (Nor do we need to say that the specification of the togglings from [11:00, 12:00) is sufficient to determine one end state of the lamp at noon.) Now we can consider two counterfactual worlds (relative to the Thomson's Lamp world)—one wherein the First Task is removed and another wherein the Second Task is removed. As we saw earlier, the First Task and Second Task are such that they are interactions with the system none of which change the system's state. It follows (by RIP) that the states of the systems at noon in the resulting worlds are identical to the state of the system in the 'actual' (Thomson's Lamp) world. But the lamp is clearly *off* upon removal of the First Task but *on* upon removal of the Second Task. Hence, the lamp is both on and off in the 'actual' world. Contradiction.

I've thus uncovered two new paths to generating paradox in the case of Thomson's Lamp. In the following sub-section, I address questions concerning the paradoxes' end states.

3.3 Underdetermined end State?

One response to infinitary paradoxes like the ones I've considered is found in Benacerraf (1962). Benacerraf points out that the description of such paradoxes (in our case, Littlewood-Ross and Thomson's Lamp) only specifies the operations performed on the lamp/urn at times *before* noon—nothing is specified or entailed about the state of the lamp/urn *at* noon. This is especially serious given the absence of a convergent limit of the series in question. As Pruss puts the objection as applied to Thomson's Lamp, "there is just no absurdity whether or not the lamp is on or off at the end of the experiment. Both outcomes are compatible with the story as given. Neither gives rise to a contradiction. The story doesn't determine which of the two outcomes will happen, but underdetermination is no paradox" (2018, p. 43).

Let's apply this approach to the Littlewood-Ross and Thomson's Lamp paradoxes, beginning with the former.[10] First, surely the scenarios involving ball manipulation simply *underdetermine* the state of the urn at noon (i.e. upon completion of the supertask). For we are only given information about the state of the urn (and the manipulations of balls) *before* noon. Second, suppose we add further specifications to the situation. Suppose, for instance, that we specify enough details about the locations and trajectories of the balls in the urn. In that case, it's not clear that we get any damaging violations of SP (because it is not clear that we have (i) the required identity/indiscernibility at all steps between the two processes and (ii) convergence).

Now let's apply the approach to Thomson's Lamp. Recall Benacerraf's view: the specification of the case is simply incomplete or underdetermined. Part of what needs to be specified is what the infinite series of switches converges to (as the displacements of the switch go to zero). Suppose that it is part of the initial set up that an infinite series of switches converges to an *on-state*. In that case, it doesn't matter whether the initial state is on or off. Nor does it matter how we group pairs of successive togglings. Pairwise grouping or bracketing will not change the divergent

[10] I'm thankful to an anonymous referee for bringing these approaches to my attention.

nature of the series, and hence it will not change the fact that the end state is underdetermined by the specification of the scenario.

What to make of these responses to the infinitary paradoxes?

The responses are valuable, as they invite further clarifications and specifications of the paradoxes. I will begin with the Littlewood-Ross paradox. The response's first point was the underdetermination of the urn's contents at noon. This response helps sharpen the specification of the paradox. Recall that, in setting up the paradox, I made a variety of stipulations (e.g. the urn and the balls persist in existence unless positively destroyed, etc.). The various specifications here are *continuity* or *persistence conditions* and are needed in order to make inferences about the system *at* noon. Without these, the state of the urn at noon is, indeed, underdetermined, since the domain of times for which the contents of the urn are specified is given by the closed-open interval [11:59, 12:00). We therefore only know—solely on the basis of the domain of times—the contents of the urn for each time *before* noon. Mathematicians Allis and Koetsier (1995) have shown how such persistence conditions lead to an empty urn in Process One.[11] They also lead to an infinitely full urn in Process Two, since (i) no balls are ever removed from the urn and (ii) the balls persist in their locations (i.e. the urn) unless removed. The addition of persistence conditions, then, suffices to allay the first point concerning underdetermination.

The second point was: If we specify enough details about the locations and trajectories of the balls in the urn, it's not clear that we get any damaging violations of SP (because it is not clear that we have (i) the required identity/indiscernibility at all steps between the two processes and (ii) convergence). In response, I make two notes.

First, as before, it seems that *convergence* is addressed by appeal to persistence conditions. In Process One, each ball is removed from the urn before twelve and then never replaced or moved again; given the persistence conditions, it follows that each ball is outside the urn at noon. In Process Two, each ball is within the urn at some point before noon and then never removed from the urn; given the persistence conditions, it follows that each ball is inside the urn at noon.

Second, it seems that qualitative identity (along some specified axis) at each step can be preserved. (I discuss an objection similar to this in Sect. 6, but a few comments here suffice.) For we can ensure system-wide qualitative identity in numbered contents, locations, and trajectories by *adding to or modifying* the paradox's specifications. It is easy to see that the *numbered contents* are identical at each step. Let us consider, then, the locations and trajectories.

One might think we can specify that the mechanism that prints numbers on the balls places each ball in precise urn locations L_1, L_2, … L_n corresponding to balls labeled 1, 2, … *n* (for every *n*), and one might conclude—on that basis—that identity in numbered contents would entail identity in positions. But this would be

[11] Most others working on the paradox agree. Among others, these include Littlewood and Ross themselves as well as Huemer (2016), Oppy (2006), Cook (2020), Holgate (1994), and Earman and Norton (1996).

mistaken. For in the case at hand, Process Two is simply inconsistent from the get-go. None of the L_n could be occupied by a ball, since each L_n is such that (i) ball *n* is placed in L_n, (ii) ball *n* is then moved from L_n to L_{10n}, and (iii) no ball is ever placed back into L_n. Together with the persistence conditions, (i)–(iii) entail that each L_n is unoccupied upon completion of Process Two. But *ex hypothesi*, all the balls are in the urn upon completion of Process Two, and hence it cannot be the case that *none* of the L_n—the only urn locations in which the balls could be located, we are supposing—is occupied by a ball. This is a contradiction: upon completion of Process Two, *none* of the balls are in any of L_n, and yet *all* of the balls are in the urn—and, since we are supposing that the urn is composed only of the L_n, it follows that each ball occupies some L_n. So, none of the balls are in any of the L_n, and each ball is in some L_n—a contradiction.

We need some way, then, to avoid the contradictory set-up for Process Two that nevertheless preserves identical locations and trajectories of the balls in both processes. Here's one way this could go. Suppose that the balls can occupy the same location simultaneously—they can all be co-located. (This may seem extravagant, but remember that we are concerned with *logical possibility*. Also: the constructions are not essentially tied to balls. The 'balls' play the functional role of numerically distinct items that can (i) exemplify some unique, label-like property and (ii) occupy some region. Thus, even if ball co-location is logically impossible, surely other kinds of co-location are logically possible provided that we use other logically possible entities (e.g., quantum fields, or electromagnetic waves, or photons[12] or photon-like entities, or whatever) and logically possible physical, label-like properties.) In that case, we avoid the contradiction from the previous paragraph, since the contradiction is predicated on the balls continually re-locating as the supertask proceeds. (More precisely, the contradiction is predicated on locational divergence.) We also ensure that the locations of the balls at each step of both processes are *identical*.

What about the trajectories of the balls during the steps? Two responses. First, this is, strictly speaking, irrelevant to SP. For SP only concerns the *end state* of the system upon completion of each step. Recall the example of Jones and Smith, whose trajectories were very different—what mattered was the identity between the *end states* of their steps. Second, we can actually suppose that the balls are special insofar as their numbers are essential to them.[13] In that case, we can actually secure identical trajectories of numbered balls between the two processes, assuming that the mechanism can cause ball *n* to substantially change to ball 10*n* without causing it to change location (i.e. without making it non-co-located with the other balls in the urn).

Thus, plausibly, we can secure global, system-wide qualitative identity between each of the steps of the two processes. For the above paragraphs illustrate situations

[12] Why photons? Because "[i]t is possible for two or more photons to share the same physical state, a condition that would not be possible for [say] electrons. To have a large cardinality of photons in a space–time such as ours would require that some photons be in the same place, and indeed in the same state" (Pruss 2006, p. 100). Note, though, that in this context we need not restrict ourselves to 'space-times such as ours'. Nevertheless, this point should help assuage worries about co-location.

[13] See Sect. 6 for more details. Recall: these need only be logically possible.

wherein the numbers, locations, *and* trajectories of *each* ball in *each* step for *both* processes are identical.

But suppose one is unconvinced by my points about the system-wide qualitative identity of each of the steps of the two processes. (Suppose, for instance, that one thinks co-location is logically impossible, full-stop.) *Even still*, I think my SP-based argument for CF argument can get up-and-running. For SP doesn't necessarily require *complete*, *global* qualitative identity of the contents of the systems at the end of each step. Rather, it simply requires qualitative identity *along a specified axis*. And this condition is satisfied so long as we specify that the qualitatively identical respect is the *numbered contents* of the urn. Thus, even if the trajectories or positions must be different at steps between the two processes (contrary to what I've argued), the argument for CF can still go through.[14]

Now I shall respond to the worry concerning Thomson's Lamp. This worry invites a clarification of my new variants of Thomson's Lamp. Importantly, I *agree* that the end state of the Thomson's Lamp scenario, as originally described by Thomson and others, is *underdetermined*. But this is where the introduction of IP/TP becomes integral. For such principles actually *do* facilitate inferences to a determinate state of the lamp at noon. Thus, they make Thomson's Lamp *truly paradoxical*. Allow me to explain.

To set up the paradoxes, we can *grant* that—solely on the basis of the infinitely many togglings between [11:00, 12:00)—the end state is underdetermined. It is indeed true that *one way* to infer a determinate lamp state at noon is to add—as part of the initial set up—that an infinite series of switches converges to (say) an *on-state*. But this is not *needed* to infer a determinate end state, as there are other ways available. In particular, IP/TP and RIP *themselves*—in conjunction with persistence conditions—allow us to make inferences about the lamp's state at noon.

The persistence conditions are relevantly similar to the Littlewood-Ross case (e.g. the lamp persists in existence unless positively destroyed; it retains its state unless positively acted upon to change its state; the only factors that can change the lamp's state are the switch togglings; and so on). What matters is that such conditions—in conjunction with IP/TP or RIP—facilitate inferences about the lamp's state at noon.

Suppose we take the initial Thomson's Lamp specification which underdetermines the state at noon. Crucially, we *need not* add some further specification about the lamp's state at noon. Suppose, instead, that we simply let the lamp's state at 11:00 be *off*. By IP, adding any number of interactions with this system won't change the lamp's state provided that none of these interactions change the lamp's

[14] Recall that in motivating SP, the intuitive plausibility and explicability considerations didn't hinge on total or global qualitative identity, in every single respect, of the respective systems. I was cautious to articulate the principle *along some specified axis*. (One might object: if there is some *other* axis, O*, along which the respective systems differ, might that end up providing an explanation for why the end states of the respective systems differ along the *original* axis, O, upon completion of the relevant processes? I don't think so. This could only be the case if O is *dependent on* O*. But at least in the cases I have specified, the numbers on the balls are *independent* of the positions and trajectories. Changing the positions and trajectories does nothing to alter the numbers; only the mechanism can change the numbers.)

state. We can then add infinitely many interactions—[on, off], [on, off], and so on—between 11:00 and noon. So long as none of these interactions change the lamp's state (which is true if each of them is [on, off]), it simply follows (from the persistence conditions plus IP) that the lamp is *off* at noon. We therefore don't need to initially specify—*pace* the worry I'm responding to—the state of the lamp *at* noon. It just falls out of the IP plus the persistence conditions.

And the same reasoning applies to a case wherein we have a lamp that was off at 11:00 and toggled on at 11:30. In this case, the lamp's state (at 11:30) is originally on. We can then add infinitely many interactions—[off, on], [off, on], and so on—between 11:30 and noon. So long as none of these interactions change the lamp's state (which is true if each of them is [off, on]), it simply follows (from the persistence conditions plus IP) that the lamp is *on* at noon. And now we have a violation of TP on our hands, since these two scenarios I've just specified have identical sequences of individual switch togglings but end—determinately—in different lamp states. And this is true *without* adding to our initial setup anything about the urn's state at noon.[15]

For these reasons, I conclude that Benacerraf's 'underdetermined end state' worry doesn't present a problem for the paradoxes I've articulated. In the next section (Sect. 4), I bring to light a host of new variants of the paradoxes from this section (Sect. 3). As we shall see in Sect. 5, these new variants avert usual solutions to the paradoxes.

4 Variants

Each of the variants in Sects. 4.1 through 4.4 are developed for the Littlewood-Ross paradox. I then briefly connect such variants to Thomson's Lamp in Sect. 4.5.

4.1 Finite Magnitudes

The original formulation of the Littlewood-Ross paradox required certain physical magnitudes to increase without bound—for instance, the balls are moved a finite distance over a smaller and smaller period of time, meaning the speed at which they move increases boundlessly. Moreover, whatever mechanism transports the balls moves an infinite number of finite distances (an infinite distance in total) in a finite period of time, meaning that the average speed of such a mechanism is *infinite*. It would also seem that the mechanism(s) would need an infinite store of energy to complete the task. An infinitely large urn, in addition, would require an infinite quantity of matter.

[15] What about RIP? I have addressed its relation to the 'underdetermined end state' worry at the end of Sect. 3.2. In short, RIP engenders paradox regardless of whether the end state in the original Thomson's Lamp story is underdetermined—all RIP needs to get off the ground is that the lamp is either on or off at noon in the 'actual' world.

It would be important, then, if we could construct versions of the paradox whose magnitudes are thoroughly finite in nature. Now, there are two relevant kinds of magnitudes to consider: extensive magnitudes (which can be defined by adding up the values of some magnitude) and intensive magnitudes (which are non-extensive magnitudes). I'm primarily concerned with ridding the paradoxes of infinite *intensive* magnitudes, since Huemer (2016) attempts to rule out all infinitary paradoxes by showing that they each contain some infinite intensive magnitude or other.[16] It would be important, then, if infinitary paradoxes remain despite the removal of infinite intensive magnitudes. This would establish the need for a different approach to ruling out such paradoxes. And, indeed, there are straightforward and logically coherent constructions of the paradox satisfying this.

To rid the infinite quantity of matter, simply consider a world wherein the first ball has finite quantity of matter q, the second ball ½ of q, the third ball ¼ of q, and so on.[17] Then simply make the finite urn sufficient to accommodate the finite quantity of matter 2q, as 2q is the sum of the quantities of all the balls. While quantity of matter is extensive, certain quantities that multiply or divide by it are intensive.

As for the speeds of (i) the mechanism(s) that adds and removes balls, (ii) the mechanism(s) that draws/prints zeroes on balls, and (iii) the total distance (over finite time) traveled by the balls, Oppy (2006, pp. 73–77) has argued that there are perfectly logically coherent mechanisms of these sorts that *avoid* unbounded or infinite magnitudes. While a characterization of all the details is beyond the present inquiry, I'll nevertheless provide a glimpse into how it could work.

First, the distance that each machine (or collection of machines) moves decreases according to a geometric proportion.[18] The weight and size of the balls likewise decrease in geometric proportion (as described before). The surface area of the mechanism that comes into contact with the balls likewise decreases in geometric proportion as time progresses. The distance traveled by the mechanism in drawing zeroes likewise decreases in geometric proportion, and we can also suppose that prior to the procedure each ball is aligned closer and closer to the urn in geometric proportion. This means that both the 'adding/removing mechanism' as well as the balls only move a finite distance in total (since the sums of such decreasing geometric series are finite).

Once again, this is nowhere near exhaustive, and the details get highly technical very quickly. Suffice it to note for now that there seem to be perfectly coherent ways of spelling out the mechanisms that avoid infinite (intensive) magnitudes.

[16] A second reason (with which Huemer agrees) to restrict our focus to *intensive* magnitudes is that it *seems* possible for there to be infinite extensive magnitudes. For instance, absent strict finitist qualms, plausibly space could be infinite in extent, and plausibly there could be a universe with an infinite amount of mass or infinitely many electrons. Plausibly, moreover, for philosophers of a realist bent, it's at least possible for there to be infinitely many abstracta (numbers, say).

[17] Note that as a zero is printed on ball n, the ball shrinks to the size of ball $10n$. This stipulation ensures the absolute qualitative identity between the two urns' contents. Though, absolute qualitative identity is technically not required; all we need is qualitative identity along some dimension or axis (with respect to numbered contents, say). That suffices for a violation of SP.

[18] I'm using 'machine' for simplicity, but keep in mind that I'm referring to any mechanism that performs the steps.

4.2 Infinite Time

While it may seem strange, there seems to be nothing incoherent about enjoying two eternal lives, where an eternal life is one wherein every day of one's life is followed by another day of life. (One proposal for how this could go involves *hypertime*—one's second eternal life would be hyper-after one's first.[19]) As Pruss (2018) has pointed out, there exists a mathematically coherent specification of a time sequence that includes two eternal lives: "We just suppose that the temporal dimension is molded by two copies of an ordinary timeline, with every point of the second timeline coming after the first" (p. 61). If we mark members of the second timeline with asterisks, it could look like:

$$0, 1, 2, 3, \ldots ; 0^*, 1^*, 2^*, 3^*, \ldots$$

Now we can consider an urn (including its contents) that enjoys two eternal lives. Just suppose that the urn's life includes the times 0, 1, 2, 3, … (and all intermediate times), and then all the times of the asterisked timeline. While this doubled timeline is certainly strange, it's "difficult to see why one could rule out the possibility of it, apart from some finitist or causal finitist considerations about the events in it" (Pruss, 2018, p. 61).

With this groundwork laid, we can consider a variant of the paradox according to which each step n is performed n days from today, where today is day 0 and where the timeline matches the first, non-asterisked one above. Since there are infinitely many future days (corresponding to each natural number n), each step of the originally specified process is (or will be) performed. The paradox then arises concerning the state of the urn on day 0* in the second, asterisked timeline. As in the original reasoning, the first process will lead to an empty urn (since each natural number is eventually removed in the first timeline and not replaced) while the second process will lead to an infinitely full urn.

Note that this version requires neither continuous time nor continuous space nor anything that possesses an infinite intensive magnitude (since (i) the speeds aren't condensed into a supertask, (ii) no mechanism requires an infinite storage of energy, etc.).

4.3 Infinite Space, Infinite Past, Finite Magnitudes

Here's one version that doesn't require continuous space. Consider a scenario wherein each mechanism (corresponding to each natural number n) is located n meters from the urn.[20] Then simply suppose that each mechanism acts instantaneously at its designated time (where mechanism n is set to act at 2^{1-n} minutes before noon to perform step n). This, of course, would require a world wherein space is

[19] For more on hypertime, see Hudson (2014) and Lebens and Goldschmidt (2017).

[20] Again, the 'mechanisms' could be point particles with the causal power to produce the (to-be-specified) effect, or whatever.

infinite in extent and instantaneous action at a distance are possible.[21] Again, though, there seems to be nothing impossible in principle in either of these stipulations. A violation of SP ensues.

There's yet another version of the paradox that makes use of infinite space but that requires neither continuous space nor instantaneous action at a distance. Suppose that causal influences can only travel at or below the speed of light. Let the spatial region of the urn be L, and suppose that each mechanism *n* is located *n* light years away from L. Now suppose that each mechanism *n* sent a signal (traveling at the speed of light) exactly *n* years in the past relative to their designated step-performing times (as before, 2^{1-n} minutes before noon for each *n*), and that such a signal is powerful enough to cause step *n* to be performed in the urn.[22] Once again, a violation of SP results.

4.4 Angels

The Littlewood-Ross paradox doesn't seem essentially tied to matter or the spatiotemporal world of material things. Simply imagine (say) an infinite number of angels, each of which thinks of a unique natural number.[23] Then suppose two angelic processes (A-Process One and A-Process Two) are performed that are isomorphic to Process One and Process Two.

In A-Process One, step *n* consists in adding ten angels (the ten angels thinking about the numbers $10(n—1)+1$ through $10n$) to some designated angel area sufficient to accommodate infinitely many angels, and then destroy the angel thinking of number *n* (or simply make it think about some non-number, or remove it from the angel area).[24] Repeat this procedure for infinitely many steps. In A-Process Two, step *n* consists in adding nine angels (the nine thinking about the numbers $10(n—1)+1$ through $10n—1$) to some area (as before), and then cause the angel thinking of number *n* to multiply its number by 10. Repeat this for infinitely many steps.

[21] We seem to have *some* reason to think (on the basis of quantum entanglement phenomena) that some kind of influence or action at a distance is not only possible but *actual*.

[22] We need not worry about the details of the signal—it suffices to note simply that the causal influence travels across space and time to cause a specific change in L, namely the addition (and/or subtraction) of labeled balls. Again: the balls are not essential to the story—particles could take their place, where a label on the particles could be represented by some special physical property or quantity.

[23] There will also be *angelic* variants on the persistence conditions. E.g., nothing destroys them throughout the task or at any point shortly thereafter; if an angel thinks of a number, the angel *persists* in thinking of that number unless caused (internally or externally) *not* to think that number; etc. Again, the argument only requires the *logical possibility* of angels (or something like angels, such as distinct thoughts in the divine intellect). As I use it, an angel is just a non-divine, non-physical, non-embodied, non-human mind.

[24] I don't have any worked out account of what an *area* for angels would be. This is only a functional term that serves the role of the urn (i.e. the system). Maybe angels are (or would be) utterly non-spatial, in which case we could simply define the system as the totality of the number-thinking angels, or as the non-spatial realm in which they reside, or whatever.

Once again, this will result in wildly different system end results. The system in A-Process One will end in no number-thinking angels, whereas the system in A-Process Two will end in infinitely many number-thinking angels.[25] Like the original urn case, we have a violation of SP on our hands.

Now, one might think that this angelic paradox is ruled out by Huemer's infinite intensive magnitude approach. For surely the cognitive power of some of these intellects would have be infinite.

But this is incorrect. For starters, Huemer's theory is restricted to infinite intensive *natural* magnitudes—ones that factor into scientific explanations (2016, pp. 135–137). Without the restriction—that is, if all intensive magnitudes whatsoever must be finite—strict finitism simply follows (something Huemer and others want to avoid). "For if finitism is false," writes Pruss, "then the log-count of objects in existence is infinite, where the log-count of Fs is the logarithm of the number of Fs, but is not defined as a sum" (2018, p. 153). Huemer is only able to accommodate this result (since he wishes to avoid strict finitism) by restriction to natural magnitudes, since the log-count of objects certainly doesn't enter into natural, scientific explanations. But, of course, the magnitudes in the angelic paradox aren't natural ones; incorporeal minds are non-natural in the sense of not entering into scientific explanation. It follows, then, that Huemer's approach fails to rule out this version of the paradox.

Moreover, it's unclear whether the implicated magnitudes are even *intensive* or *infinite*. For the *number* of angels is extensive; each one thinks of a finite number, or at least has a single thought whose single conceptual content is a natural number followed by an omega sequence of zeroes; none of them have brains with electrochemical signals whose average speed grows and grows; and so on. At the very least, significant work lies ahead for those who wish to rule out the angelic version by appeal to the impossibility of infinite intensive magnitudes (even ignoring the criticisms leveled in the previous paragraph).

4.5 Thomson's Lamp Variations

As with the Littlewood-Ross paradox, we can construct versions of Thomson's Lamp that don't require the continuity of time, the continuity of space, infinite (intensive) magnitudes, and so on. It would, however, take me too far afield to spell these out in detail here.

But what is the purpose of all these paradoxical variations? Do they shed any light on *solutions* to the paradoxes? I consider this question in the next section.

[25] Or, rather, infinitely many angels each of which thinks of a natural number followed by an omega sequence of zeroes.

5 CF as Best Solution

I aver that the simplest, most elegant, and most unifying explanation or account of the impossibility of the Littlewood-Ross paradoxes and the new Thomson's Lamp paradoxes is CF. In other words, CF is the *best solution* to such paradoxes (collectively). There are lots of ways to appreciate this, but I'll survey just a few. First, each version of the paradox fundamentally relies on infinitely many causes bearing on one thing: the state of the urn (or lamp) at noon. CF thereby kills each such paradox. Moreover, if it's genuinely possible for infinitely many causes to bear on one state, what would *prevent* them bearing on such a state in the way that the paradoxes (Littlewood-Ross, Thomson's Lamp, and variants thereof) describe? It seems that there would be no reason why infinite causal chains—if they are genuinely possible—couldn't be arranged using the mundane causal powers of things so as to generate such paradoxical situations.[26]

Another way to appreciate the inference to CF is by means of (versions of) modal patchwork principles (Lewis, 1983, pp. 76–77). A decent portion of our modal knowledge seems to derive from (implicit or explicit) use of such patchwork principles, as we lack direct access to non-actual possibilities. Plausibly, we only have *direct* knowledge of actual facts. How, then, do we come to know possible but non-actual ones? One way is plausibly the license to combine, recombine, duplicate, and cut-and-paste various spatiotemporal regions (localized 'patches') of the actual world into a new arrangement in another possible world (space, time, and geometry permitting). And while such rearrangements won't be *guaranteed* to be possible, it's plausible that there's *defeasible reason* to think that rearrangements of actual spatiotemporal regions are possible.

For sake of space, it suffices for present purposes to note that if infinite causal chains are indeed possible, then (given a patchwork principle) we should be able (i) to use as a *framework* the causal, temporal, and spatial structure of a world in which there exists an infinite causal chain and (ii) to duplicate and/or rearrange actual mechanisms (with intrinsic causal powers to add and remove balls form urns, print/draw zeroes, etc.) in order to *form* another possible world in which Littlewood-Ross and Thomson's Lamp paradoxes are realized. The possibility of infinite causal regresses—in conjunction with plausible patchwork principles—engenders paradox. As such, we should reject the possibility of infinite causal regresses.[27]

Another way to see the inference to CF as the best solution is simply to compare it with competing hypotheses that likewise aim to rule out paradoxes like the ones I've considered. To rule out such paradoxes, we need to find some false assumption or assumptions upon which the paradoxical constructions rest. It seems, though,

[26] See Erasmus and Luna (2020) and Pruss (2018, ch. 3) for more precise and extended discussions of the inference from the possibility of ungrounded chains (i.e. a chain, sequence, or series with (i) a strict total ordering (e.g. an 'earlier than' or 'caused by' relation) among its members and (ii) no 'first' member) to the possibility of paradoxes (viz. Benardete-type paradoxes involving Grim Reapers, Deafening Peals, etc.) analogous to the ones explored in my paper.

[27] See Koons (2014, esp. pp. 257–260) for an elaboration and defence of this inference.

that an infinite causal history is the only common denominator among the variety of paradoxes.

It won't do merely to adopt the necessarily discrete nature of time or space, since paradoxes still arose even without assumptions like continuous time or continuous space. For instance, the infinite time version from Sect. 4.2 or the angelic version required neither, and numerous versions made no use of continuous space. More importantly, such approaches simply fail when applied to infinitary paradoxes not considered in this paper. For there are *synchronic* versions of many such paradoxes. Consider, for instance, the Grim Reaper paradox wherein the victim, Fred—who only dies (we can suppose) if someone kills him, and the only things around that can kill him are the Grim Reapers—cannot survive past midnight despite no Grim Reaper killing him. In an original version, Grim Reaper n (for each natural number n) is set to kill Fred exactly 2^{1-n} minutes after midnight if and only if no earlier Grim Reaper (i.e. no Grim Reaper i, $i > n$) kills Fred.

But the same paradoxical result can obtain in a *synchronic* version. Let the infinitely many Grim Reapers be arranged to the right of Fred, such that Grim Reaper n is located 2^{1-n} meters away from Fred.[28] At midnight, each Grim Reaper activates and, moreover, instantaneously kills Fred if and only if no Grim Reaper located to its left is killing Fred at midnight. Once again, Fred doesn't survive past midnight, but no Grim Reaper kills him. Fred's worries are both diachronic *and* synchronic.

Not only can CF kill all the paradoxes surveyed in this paper, then, but it can also kill such synchronic paradoxes of the infinite—unlike a view that merely aims to rule out infinitary paradoxes by means of discrete time and/or space. For the synchronic versions require neither continuous time nor continuous space.

Nor will it do merely to say that space and time are necessarily finite in extent, since paradoxes arose even *within* such constraints. And as we saw earlier, uniformly ruling out the possibility of infinite intensive natural magnitudes likewise won't do as a solution, since many paradoxes still arose without such magnitudes (e.g. the finite magnitudes version, the angel version, etc.).

Of course, one could adopt an *array* of such disparate and seemingly unrelated approaches, but this approach would be less elegant, less explanatory, more complex, and less unifying than CF. The more metaphysical necessities postulated, the less probable the solution is, especially if such metaphysical necessities are not explained in virtue of more fundamental logical or metaphysical facts. All else being equal, we should minimize brute, unexplained things. In fact, it seems that an array of these disparate approaches has to be posited "precisely in order to rule out infinite causal histories," and as such it "is preferable simply to accept CF by itself, unless we have independent arguments" for the conjunction of approaches within the array (Pruss, 2018, p. 160).

[28] While some such synchronic versions employ continuous space, others don't require it and instead employ strategies like infinite space plus instantaneous action at a distance (as in Sect. 4.3) or even infinite space plus an infinite past (with the speed of light as the speed limit to causal interaction, again as in Sect. 4.3).

One could also adopt some other form of finitism, but nearly all such approaches won't serve as *competitors* to CF but actually *entail* it. For instance, strict finitism clearly entails CF; additionally, the impossibility of there being infinitely many prior conditions for the obtaining of some event, state, or substance plausibly entails CF; and so on.

Overall, then, I submit that the best solution to the paradoxes of this paper is CF. Other solutions either *entail* CF (e.g. strict finitism, no-infinite-prior-conditions, etc.), or fail to *rule out* a number of the paradoxes (e.g. banning spatiotemporal continuity, banning infinite intensive magnitudes, etc.), or introduce needless and inelegant complexity (e.g. a combination of the aforementioned approaches). We also saw two reasons at the beginning of this section favouring CF (viz. (i) the seeming absence of any *prevention* of the paradoxes from arising given the possibility of infinite causal chains, and (ii) the modal patchwork principles). I venture, then, that the paradoxes I've considered in this article provide support for the truth of CF.

In the next section, I consider objections to my case for CF.

6 Objections

Objection One Instead of adopting CF, we should simply reject SP in the case of the Littlewood-Ross paradoxes and IP or TP (or both)—as well as RIP—in the case of the new Thomson's Lamp paradoxes. While SP may hold for finite processes, the falsity of SP is precisely what we should *expect* if infinite processes were possible. And the same holds for IP/TP as well as RIP. Indeed, there is no mystery as to why SP doesn't hold for infinite processes, as we have a seemingly perfectly adequate explanation for why Process One ends with zero balls while Process Two ends with infinitely many.

Reply First, it may be true that a violation of SP, IP or TP, and RIP is what we should expect if there could be such infinite causal processes. But this doesn't *resolve* or even *undermine* the paradoxical nature of the situations; rather, it merely *illustrates* the seeming absurdities derivable from infinite causal chains. And while we do have an account as to why Process One ends in an empty urn while Process Two doesn't, this merely explains the *means* by which we arrived at the absurd result (viz. a violation of SP)—it doesn't *remove* the absurdity.

Second, we've actually seen *reasons* for thinking that SP is true *simpliciter*—not merely true for finite causal processes. Moreover, the primary reasons in its favour don't hinge on application to finite cases. And as we saw in the section on SP, the intuitive obviousness and high plausibility of SP gives us reason to think SP is true absent defeaters. We would therefore need some *positive reason* for thinking that SP is false or that CF is false. Absent such a positive reason, SP's truth is undefeated; and we've seen how the truth of SP gives reason to accept CF.

Third, the truth of CF seems to be a simpler, more elegant, and more unifying solution to such paradoxes. In order for this objection to avoid CF, it would essentially require rejecting all three of the following plausible principles: (i) SP, (ii) IP or TP (or both), and (iii) RIP. In itself, this seems to make the truth of CF a more parsimonious and attractive account. Unlike an approach to killing the paradoxes that merely denies each of (i)–(ii), an approach like CF that kills all the paradoxes uniformly with one fell swoop seems preferable.

Finally, even if this objection succeeds, the result is still highly significant: if one or more of (i) SP, (ii) IP and TP, and (iii) RIP is true, then CF is probably true. (Recall that my argument from Sect. 5 is non-deductive.) Even if we negate the antecedent (and thereby block the inference to CF), establishing the conditional is itself a very significant result.

Objection Two The end states of the steps of the two processes are decidedly *not* (qualitatively) identical. Consider, for instance, step two of both processes. In Process One, the urn contains balls 3–20, while in Process Two the urn contains balls *labeled* 3–20 but that are really the original balls 1–9 and 11–19. And the same holds for each other step. Hence, there's simply no isomorphism between the two processes. SP therefore simply fails to apply, and so the inference to CF—at least by means of the Littlewood-Ross paradox—fails.

Reply First, note that this objection—if successful—only targets the argument from the Littlewood-Ross paradox; it targets neither of the arguments from IP/TP and RIP.

Second, the objection doesn't actually engage SP. Recall that SP doesn't necessarily require *complete*, *global* qualitative identity of the contents of the systems at the end of each step. Rather, it simply requires qualitative identity *along a specified axis*. And this condition is satisfied so long as we specify that the qualitatively identical respect is the *numbered contents* of the urn. Note, moreover, that the arguments in favour of SP apply equally to a specified axis—not just complete, global qualitative identity.

Third, the following scenario seems logically possible. Suppose these balls are special. Part of these special balls' identity conditions is the *number* written on their respective surface, such that a change in a ball's number constitutes a substantial change. In essence, relabeling ball 1 to ball 10 on a ball's surface causes ball 1 to substantially change into ball 10. In this way, we actually preserve global (i.e. system-wide) qualitative identity.

Moreover, keep in mind that the paradoxes aren't crucially tied to balls, and so the (admittedly somewhat extravagant) scenario thus described isn't absolutely necessary. Instead, the 'balls' could be some kind of particle, with numbers represented as some kind of unique physical property or other. And it certainly doesn't strain credulity to think—as a matter of logical possibility—that there could be such particles whose identity conditions are fixed by the unique physical property (or properties) they instantiate.

But perhaps this objection is getting at a more fundamental worry, namely that there *is* a difference-maker between the two processes that accounts for the discrepancy in system end states upon completion of the processes. As we saw in Sect. 2, however, this response won't do. This is because every alleged difference-maker within steps is *wiped away* (as it were) by the end states of such steps.[29] Just return

[29] And the difference maker couldn't come upon *completion* of the process, since that's 'too late' in the order of explanation.

to the flying-driving example—any differences in the manner or way in which Jones or Smith reached their destination were irrelevant, since despite such differences, they were ensured to arrive at the exact same location at the end of each step.

Objection Three The argument for CF from Thomson's lamp is mistaken. Suppose that the lamp is on at 11:00, off at 11:30, on at 11:45, off at 11:52:30, and so on. Then we can describe what happens to the lamp as a succession of toggles spanning the intervals [11:30, 11:45], [11:52:30, 11:56:15], and so on. According to the argument, we can infer that the lamp is *on* at 12:00. But we can equally describe what happens to the lamp as a succession of toggles spanning the intervals [11:45, 11:52:30], [11:56:15, 11:58:07.5], and so on (subsequent to the state of the lamp being *off* at 11:30). Per the argument, we can infer that the lamp is *off* at 12:00. So, per the argument, we can correctly infer that the lamp is both *on* and *off* at 12:00—a contradiction.

The underlying problem with the argument is that the sequence 1, −1, 1, −1, 1, −1, … is *divergent*: $1+(-1+1)+(-1+1)+\ldots=1$, but $(1-1)+(1-1)+(1-1)+\ldots=0$. This doesn't tell us anything about infinite causal histories; it just tells us that using IP in conjunction with the persistence conditions in divergent infinite cases *yields a contradiction*. But no true friends of infinity would suppose otherwise. For the friend of infinity, this argument should simply teach us that it is a mistake to apply the conjunction of IP and persistence conditions to divergent infinite cases, *not* that divergent infinite cases (and/or infinite causal histories) are impossible.[30]

Reply This objection is valuable because it brings to light both the modesty and significance of my argument.

Any valid argument with premises p_1, p_2, … p_n and conclusion *c* can only show that the conjunction (p_1 & p_2 & … & p_n & ~c) is logically inconsistent. Crucially, this doesn't tell you what to believe. Maybe you reject one or more of the premises. Or maybe you believe all the premises, but upon discovering that they entail *c*, you simply decide to reject one or more of the premises. The argument itself is silent about what you should do. One's modus ponens, so the saying goes, is another's modus tollens.

With this in mind, my IP-based Thomson's Lamp argument for CF can be understood as saying that the following conjunction is logically inconsistent:

(a) IP;
(b) If there could be infinite causal histories, then there could be some variant of the Thomson's Lamp construction (where the persistence conditions are part of the specification of the construction); and
(c) There could be infinite causal histories.[31]

[30] Thanks to an anonymous referee for this perceptive point.

[31] More precisely, I have only argued that CF is the *best solution* to the paradoxes of this paper. I have here put the argument in deductive form for the sake of simplicity.

For if (b) and (c) are true, then some variant (cf. Section 4.5) of the Thomson's Lamp construction could obtain. And by (a), it follows that any such construction results in a lamp that is both *on* and *off* at noon—a contradiction.

Crucially, though, nothing in this argument tells us what to accept. My argument wishes to privilege (a) and (b) as premises to derive the negation of (c). But this will be thoroughly unconvincing to someone who (say) accepts (c) and (b) but denies (a)—or at least denies (a) when applied to divergent infinite series.[32]

Here is where the objection highlights my argument's modesty. The fact that a contradiction ensues from (a)–(c) tells us *neither* that we should reject (a) as applied to divergent infinite series *nor* that we should reject (c). But what it *does* tell us is that to the extent that one finds (a) and (b) plausible, and to the extent that one has no comparable reasons for thinking (c) is true, *to that extent* one is justified in denying (c) and thereby adopting CF.

This, moreover, is deeply significant—not only does it open up new dialectical avenues in the CF and Thomson's Lamp literature, but it also allows each person to consult their own evidence base and update their beliefs accordingly. Suppose Sophia is not possessed of strong reason to think that there could be infinite causal histories. Suppose, further, that Sophia finds (b) plausible for reasons adumbrated in Sect. 5.[33] Finally, suppose Sophia finds (a) deeply intuitively plausible. She also thinks it is supported by explicability considerations: if (a) were false—that is, if adding some number of interactions with a system altered the state of the system despite the fact that none of these interactions changes the state of the system—the system's altered state would seem inexplicable. In this case, my argument can *extend Sophia's vision of the world* to include a denial of (c).

Not everyone will be like Sophia. Some might, for instance, be possessed of independent reasons for thinking there could be infinite causal chains. Others may not find (a) intuitively plausible, or they may not find the explicability point convincing, or they may antecedently think (a) is false. Each person is invited to consult their own epistemic resources. To the extent that one finds (a) and (b) more plausible than (c), *to that extent* they are equipped with reason to reject (c). And this is true even if the friend of infinity should not be moved to deny (c) on the basis of the argument.

Moreover, the argument retains its value for such friends of infinity who are not moved to deny (c). First, the argument *focuses* debates about CF and Thomson's Lamp by incorporating new principles and exploring their consequences. Second, the novel variants of the paradoxes illuminate CF's comparative strength over non-CF solutions *conditional* on the metaphysical principles at play (e.g. IP). The novel variants also expose potential flaws in competing solutions to CF insofar as paradoxical constructions arise *even within the constraints* of such solutions. Third, the argument might *lower the friend-of-infinity's credence* in (c) to the extent that they find (a) and (b) plausible.

[32] I shall hereafter speak simply of 'rejecting (a)', but note that this *includes* merely rejecting (a) as applied to divergent infinities.

[33] For instance, (i) the seeming absence of any *prevention* of the construction's arising given the possibility of infinite causal chains, (ii) patchwork principles, etc.

I think an analogy will help draw out my point. Jc Beall has recently defended a ‘contradictory Christology’—that is, a glut-theoretic view on which “Christ is a contradictory being” (2021, p. 3). He continues: “At the crux of christian theology is a contradiction: namely, Christ Jesus is a being of whom some claims are both true and false. … The simple thesis of this book is that some claims are both true and false of Christ—full stop, no new meanings, no playing with words” (*ibid*). For instance, Christ—because he is possessed of two natures, one human and the other divine—is both peccable and impeccable, mutable and immutable.

Suppose Sophia is convinced (by Beall) that the early church councils are best understood as teaching that Christ is a contradictory being. (We can call what the councils teach about Christ ‘conciliar Christology’.) Sophia can then construct an inconsistent triad as follows:

(d) The Law of Non-Contradiction (LNC) is true;
(e) If conciliar Christology is true, then LNC is not true; and
(f) Conciliar Christology is true.

As with (a)–(c), nothing here tells us what to accept or reject. Suppose that Sophia finds LNC deeply intuitively plausible. She may therefore wish to privilege (d) and (e) as premises to derive the negation of (f). But this will be thoroughly unconvincing to someone (like Beall) accepts (f) and (e) but denies (d)—or at least denies (d) when applied to Christ.

I don’t think this is a mark against Sophia’s argument; it just highlights her argument’s modesty. The fact that (d)–(f) are inconsistent tells us *neither* that we should reject (d) as applied to Christ *nor* that we should reject (f). But what it *does* tell us is that to the extent that one finds (d) and (e) plausible, and to the extent that one has no comparable reasons for thinking (f) is true, *to that extent* one is justified in denying (f). This is precisely the kind of modest conclusion I wish to draw in my argument for CF. And as we’ve seen, modesty should not be equated with insignificance. Indeed, the *opposite* is true—the modesty highlights and complements the significance.

Objection Four Intimately related to Objection Three is the following. Since the conjunction of IP, persistence conditions, and divergent infinite processes yields a contradiction, it clearly follows that the conjunction of TP, IP, persistence conditions, and divergent infinite processes likewise yields a contradiction. But there is nothing here that impugns *convergent* infinite processes; if we let the switching converge to a suitable final state, we can restore consistency by rejecting one or more of TP, IP, and the persistence conditions. But, the objection continues, it is clear that friends of infinity will *simply reject* one or more of TP, IP, and the persistence conditions.

Reply In response, I express agreement: friends of infinity will simply reject TP or IP (assuming we build the persistence conditions into the set-up of the paradoxical constructions), just as friends of Beall’s contradictory Christology will simply reject LNC. And such friends may very well be *justified* in such a rejection (assuming, of course, that they aren’t friends of such views for no reason!). But this does not impugn the argument for CF; it highlights its modesty and significance. The argument unveils a new, hitherto unappreciated connection between various novel metaphysical principles and various paradoxes (including novel variants thereof). It thereby shows that *to the extent* that one finds the principles plausible, *to that extent* one has reason to embrace CF. It's similar to Sophia's argument against conciliar Christology: *to the extent* that one finds LNC plausible, *to that extent* one has reason to reject conciliar Christology. (Assuming, along with Beall (2021), that the early church councils are best interpreted as teaching a contradictory Christology.) And as before, this argument can affect the credence of a friend of conciliar Christology (analogy: a friend of infinity). For if this friend does, indeed, find LNC (analogy: TP, IP, RIP, SP, etc.) independently plausible in its own right, then this friend has *some* weight of a reason against conciliar Christology (analogy: against the possibility of infinite causal histories).

7 Conclusion

I've uncovered a number of significant results in this article. After articulating CF, I explored the motivations and implications of a number of new principles (SP, IP, TP, and RIP). I also combined these principles with the Littlewood-Ross and Thomson's Lamp paradoxes (as well as new variations thereof). Which assumption(s) facilitated the construction of such paradoxical situations? After examining different alternatives, I argued that infinite causal chains are the most likely culprit. Finally, I addressed four objections to my arguments.[34]

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