

WHAT IS AN ELEMENTARY PARTICLE?¹

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1. A PARTICLE IS NOT AN INDIVIDUAL

Atomism in its latest form is called quantum mechanics. It has extended its range to comprise, besides ordinary matter, all kinds of radiation, including light—in brief, all forms of energy, ordinary matter being one of them. In the present form of the theory the “atoms” are electrons, protons, photons, mesons, etc. The generic name is elementary particle, or merely particle. The term “atom” has very wisely been retained for chemical atoms, though it has become a misnomer.

This essay deals with the elementary particle, more particularly with a certain feature that this concept has acquired—or rather lost—in quantum mechanics. I mean this: That the elementary particle is not an individual; it cannot be identified, it lacks “sameness.” The fact is known to every physicist, but is rarely given any prominence in surveys readable by nonspecialists. In technical language it is covered by saying that the particles “obey” new-fangled statistics, either Einstein-Bose or Fermi-Dirac statistics. The implication, far from obvious, is that the unsuspected epithet “this” is not quite properly applicable to, say, an electron, except with caution, in a restricted sense, and sometimes not at all. My objective here is to explain this point and to give it the thought it deserves. In order to create a foil for the discussion, let me summarize in sections 2–5 what we are usually told about particles and waves in the new physics.

2. CURRENT VIEWS: THE AMALGAMATION OF PARTICLES AND WAVES

Our image of the material world had been made up of two kinds of “fittings”—waves and particles. The former were instanced mainly, if not exclusively, by Maxwell’s waves of electromagnetic energy, comprising such as are used in radio, light, X-rays, and gamma-rays. Material bodies were said to consist of particles. One

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was also familiar with jets of particles, called corpuscular rays, such as cathode rays, beta-rays, alpha-rays, anode rays, etc. Particles would emit and absorb waves. For instance, cathode rays (electrons), when slowed down by colliding with atoms, emit X-rays. The distinction between particles and waves was, however, considered as clear-cut as that between a violin and its sound. An examinee who alleged cathode rays to be waves, or X-rays to be jets of particles, would have got very bad marks.

In the new setting of ideas the distinction has vanished, because it was discovered that all particles have also wave properties, and vice versa. Neither of the two concepts must be discarded; they must be amalgamated. Which aspect obtrudes itself depends not on the physical object, but on the experimental device set up to examine it. A jet of cathode rays, for example, produces in a Wilson cloud chamber discrete tracks of water droplets—curved tracks if there is a magnetic field to deflect the electrons, otherwise straight alignments of droplets. We cannot but interpret them as traces of the paths of single electrons. Yet the same jet, after crossing a narrow tube placed at right angles to it and containing crystal powder, will produce on a photographic plate at some distance behind the tube a pattern of concentric circles. This pattern can be understood in all its details when looked upon as the interference pattern of waves, and in no other way. Indeed, it bears a close resemblance to similarly produced X-ray patterns.

The suspicion arises: are the conical jets that impinge on the photographic plate and form the pattern of circles really cathode rays; are they not perhaps secondary X-rays? The suspicion has to be dismissed, for the whole system of circles can be displaced by a magnet, while X-rays cannot; moreover, by putting a lead screen with a small hole in it in the place of the photographic plate, a jetlet can be isolated from one of the conical jets and made to display any of the typical particle characters of cathode rays; it will produce discrete tracks in a cloud chamber; bring about discrete discharges in a Geiger-Müller counter; and charge up a Faraday cage in which it is intercepted.

A vast amount of experimental evidence clinches the conviction that wave characteristics and particle characteristics are never encountered singly, but always in a union; they form different aspects of the same phenomenon, and indeed of all physical phenomena. The union is not a loose or superficial one. It would be quite unsatisfactory to consider cathode rays to consist both of particles and of waves. In the early days of the new theory it was suggested that the particles might be singular spots within the waves, actually singularities in the meaning of the mathematician. The white crests on a moderately rough sea

would be a fairly adequate simile. The idea was very soon abandoned. It seems that both concepts, that of waves and that of particles, have to be modified considerably, so as to attain a true amalgamation.

3. CURRENT VIEWS: THE NATURE OF WAVES

The waves, so we are told, must not be regarded as quite real waves. It is true that they produce interference patterns—which is the crucial test that in the case of light had removed all doubts as to the reality of the waves. However, we are now told that all waves, including light, ought rather to be looked upon as “probability waves.” They are only a mathematical device for computing the probability of finding a particle in certain conditions, for instance (in the above example), the probability of an electron hitting the photographic plate within a small specified area. There it is registered by acting on a grain of silver bromide. The interference pattern is to be regarded as a statistical registration of the impinging electrons. The waves are in this context sometimes referred to as guiding waves—guiding or directing the particles on their paths. The guidance is not to be regarded as a rigid one; it merely constitutes a probability. The clear-cut pattern is a statistical result, its definiteness being due to the enormous number of particles.

Here I cannot refrain from mentioning an objection which is too obvious not to occur to the reader. Something that influences the physical behavior of something else must not in any respect be called less real than the something it influences—whatever meaning we may give to the dangerous epithet “real.” It is certainly useful to recall at times that all quantitative models or images conceived by the physicist are, epistemologically, only mathematical devices for computing observable events, but I cannot see that this applies more to, say, light waves than to, say, oxygen molecules.

4. CURRENT VIEWS: THE NATURE OF PARTICLES (UNCERTAINTY RELATION)

As regards the modification required in the concept of a particle, the stress is on Heisenberg’s uncertainty relation. The so-called classical mechanics hinged on Galileo’s and Newton’s discovery that the thing which in a moving body is determined at any instant by the other bodies in its environment is only and precisely its acceleration, or, in mathematical terms, the second derivatives with respect to time of the coordinates. The first derivatives, commonly called the velocity, are therefore to be included in the description of the momentary state of the body, together with the coordinates themselves which label its momentary place in space or “whereness” (or ubiety, to use an antiquated but convenient word). Thus, to describe the momen-

tary state of a particle, two independent data were required: its coordinates and their first time derivatives, or ubiety and velocity. According to the new theory less is required, and less is obtainable. Either of the two data can be given with arbitrary accuracy, provided that no store is set on the other, but both cannot be known together with absolute precision. One may not even conceive of both as having absolutely sharp values at the same instant. They mutually blur each other, as it were. Broadly speaking, the product of the latitudes of their respective inaccuracies cannot be reduced below a fixed constant. For an electron, this constant happens to be about 1 if the units centimeter and second are used. Thus, if the velocity of an electron is considered sharp with a latitude of only 1 centimeter per second, its location has to be considered as blurred within the latitude of 1 centimeter. The strangeness does not lie in the mere existence of inaccuracies, for the particle might be a thing of vague and changeable extension, within which slightly different velocities prevailed at different spots. Then, however, a sharp location or ubiety would probably entail a sharply defined velocity and vice versa. Actually it is just the other way round.

5. CURRENT VIEWS: THE MEANING OF THE UNCERTAINTY RELATION

Two links connect this strange and certainly very fundamental statement to other parts of the theory. It can be arrived at by declaring that a particle is equivalent to its guiding wave, and has no characteristics save those indicated by the guiding wave according to a certain code. The code is simple enough. The ubiety is indicated by the extension of the wave, the latitude in the velocity by the range of wave numbers. "Wave number" is short for reciprocal of the wavelength. Each wave number corresponds to a certain velocity proportional to it. That is the code. It is a mathematical truism that the smaller a wave group, the wider is the (minimum) spread of its wave numbers.

Alternatively, we may scrutinize the experimental procedure for determining either the ubiety or the velocity. Any such measuring device implies a transfer of energy between the particle and some measuring instrument—eventually the observer himself, who has to take a reading. This means an actual physical interference with the particle. The disturbance cannot be arbitrarily reduced, because energy is not exchanged continuously but in portions. We are given to understand that, when measuring one of the two items, ubiety or velocity, we interfere with the other the more violently the higher the precision we aim at. We blur its value within a latitude inversely proportional to the latitude of error allowed in the first.

In both explanations the wording seems to suggest that the uncertainty or lack of precision refers to the attainable knowledge about a particle rather than to its nature. Indeed, by saying that we disturb or change a measurable physical quantity we logically imply that it has certain values before and after our interference, whether we know them or not. And in the first explanation, involving the wave, if we call it a guiding wave how should it guide the particle on its path, if the particle has not got a path? If we say the wave indicates the probability of finding the particle at A, or at B, or at C—this seems to imply that the particle is at one, and one only, of these places; and similarly for the velocity. (Actually the wave does indicate both probabilities simultaneously, one by its extension, the other by its wave numbers.) However, the current view does not accept either ubiquity or velocity as permanent objective realities. It stresses the word “finding.” Finding the particle at point A does not imply that it has been there before. We are more or less given to understand that our measuring device has brought it there or “concentrated” it at that point, while at the same time we have disturbed its velocity. And this does not imply that the velocity “had” a value. We have only disturbed or changed the probability of finding this or that value of the velocity if we measure it. The implications as to “being” or “having” are misconceptions, to be blamed on language. Positivist philosophy is invoked to tell us that we must not distinguish between the knowledge we can obtain of a physical object and its actual state. The two are one.

6. CRITICISM OF THE UNCERTAINTY RELATION

I will not discuss here that tenet of positivist philosophy. I fully agree that the uncertainty relation has nothing to do with incomplete knowledge. It does not reduce the amount of information attainable about a particle as compared with views held previously. The conclusion is that these views were wrong and we must give them up. We must not believe that the completer description they demanded about what is really going on in the physical world is conceivable, but in practice unobtainable. This would mean clinging to the old view. Still, it does not necessarily follow that we must give up speaking and thinking in terms of what is really going on in the physical world. It has become a convenient habit to picture it as a reality. In everyday life we all follow this habit, even those philosophers who opposed it theoretically, such as Bishop Berkeley. Such theoretical controversy is on a different plane. Physics has nothing to do with it. Physics takes its start from everyday experience, which it continues by more subtle means. It remains akin to it, does not transcend it generically, it cannot enter into another realm. Discoveries in physics cannot in

themselves—so I believe—have the authority of forcing us to put an end to the habit of picturing the physical world as a reality.

I believe the situation is this. We have taken over from previous theory the idea of a particle and all the technical language concerning it. This idea is inadequate. It constantly drives our mind to ask information which has obviously no significance. Its imaginative structure exhibits features which are alien to the real particle. An adequate picture must not trouble us with this disquieting urge; it must be incapable of picturing more than there is; it must refuse any further addition. Most people seem to think that no such picture can be found. One may, of course, point to the circumstantial evidence (which I am sorry to say is not changed by this essay) that in fact none has been found. I can, however, think of some reasons for this, apart from the genuine intricacy of the case. The palliative, taken from positivist philosophy and purporting to be a reasonable way out, was administered fairly early and authoritatively. It seemed to relieve us from the search for what I should call real understanding; it even rendered the endeavor suspect, as betraying an unphilosophical mind—the mind of a child who regretted the loss of its favorite toy (the picture or model) and would not realize that it was gone forever. As a second point, I submit that the difficulty may be intimately connected with the principal subject of this paper, to which I shall now turn without further delay. The uncertainty relation refers to the particle. The particle, as we shall see, is not an identifiable individual. It may indeed well be that no individual entity can be conceived which would answer the requirements of the adequate picture stated above.

It is not at all easy to realize this lack of individuality and to find words for it. A symptom is that the probability interpretation, unless it is expressed in the most highly technical language of mathematics, seems to be vague as to whether the wave gives information about one particle or about an ensemble of particles. It is not always quite clear whether it indicates the probability of finding “the” particle or of finding “a” particle, or indicates the likely or average number of particles in, say, a given small volume. Moreover, the most popular view on probability tends to obliterate these differences. It is true that exact mathematical tools are available to distinguish between them. A point of general interest is involved, which I will explain. A method of dealing with the problem of many particles was indicated in 1926 by the present writer. The method uses waves in many-dimensional space, in a manifold of $3N$ dimensions, N being the number of particles. Deeper insight led to its improvement. The step leading to this improvement is of momentous significance. The many-dimensional treatment has been superseded by so-called second quantization, which is mathematically equivalent to uniting into one three-dimen-

sional formulation the cases $N=0, 1, 2, 3 \dots$ (to infinity) of the many-dimensional treatment. This highly ingenious device includes the so-called new statistics, with which we shall have to deal below in much simpler terms. It is the only precise formulation of the views now held, and the one that is always used. What is so very significant in our present context is that one cannot avoid leaving indeterminate the number of the particles dealt with. It is thus obvious that they are not individuals.

7. THE NOTION OF A PIECE OF MATTER

I wish to set forth a view on matter and the material universe, to which Ernst Mach [1],² Bertrand Russell [2], and others were led by a careful analysis of concepts. It differs from the popular view. We are, however, not concerned with the psychological origin of the concept of matter but with its epistemological analysis. The attitude is so simple that it can hardly claim complete novelty; some pre-Socratics, including the materialist Democritus [3], were nearer to it than were the great men who resuscitated science and molded it during the seventeenth to nineteenth centuries.

According to this view, a piece of matter is the name we give to a continuous string of events that succeed each other in time, immediately successive ones being as a rule closely similar. The single event is an inextricable complex of sensates, of associated memory images, and of expectations associated with the former two. The sensates prevail in the case of an unknown object, say a distant white patch on the road, which might be a stone, snow, salt, a cat or a dog, a white shirt or blouse, a handkerchief. Even so, within the ensuing string of events we usually know from general experience how to discount the changes caused by motions of our own body, in particular of our direction of sight. As soon as the nature of the object is recognized, images and expectations begin to prevail. The latter concern sensations as hard, soft, heavy, flexible, rough, smooth, cold, salty, etc., associated with the image of touching and handling; they also concern spontaneous movements or noises such as barking, mewing, shouting, etc. It should be noted that I am not speaking of our thoughts or considerations about the object, but of what forms part and parcel of our perception of it—of what it is to us. However, the limit is not sharp. As our familiarity with a piece of matter grows, and in particular as we approach its scientific aspect, the range of expectations in regard to it widens, eventually to include all the information science has ascertained, e. g., melting point, solubility, electric conductivity, density, chemical and crystalline structure, and so on. At the same time, the momentary sensational core recedes in relevance the more

² Numbers in brackets refer to authorities cited at the end of this article.

the object becomes familiar to us, whether by scientific knowledge or by everyday use.

8. INDIVIDUALITY OR "SAMENESS"

After a certain wealth of association has come to outshine the core of sensates, the latter is no longer needed to keep the complex together. It persists even when the contact of our senses with the object temporarily ceases. And more than that: the complex is latently conserved even when the whole string is interrupted by our turning away from the object to others and forgetting all about it. Indeed, this is not exceptional, but a rule which—since we sometimes sleep—has no exception. But we have adopted the useful device of filling these gaps. We supplement the missing parts of the strings relating to pieces of matter in our nearer and farther surroundings, to cover the periods when we neither watch them nor think of them. When a familiar object re-enters our ken, it is usually recognized as a continuation of previous appearances, as being the same thing. The relative permanence of individual pieces of matter is the most momentous feature of both everyday life and scientific experience. If a familiar article, say an earthenware jug, disappears from your room, you are quite sure somebody must have taken it away. If after a time it reappears, you may doubt whether it really is the same one—breakable objects in such circumstances are often not. You may not be able to decide the issue, but you will have no doubt that the doubtful sameness has an indisputable meaning—that there is an unambiguous answer to your query. So firm is our belief in the continuity of the unobserved parts of the strings!

No doubt the notion of individuality of pieces of matter dates from time immemorial. I suppose animals must have it in some way, and a dog, when seeking for his ball that has been hidden, displays it very plainly. Science has taken it over as a matter of course. It has refined it so as safely to embrace all cases of apparent disappearance of matter. The idea that a log which burns away first turns into fire, then into ashes and smoke, is not alien to the primitive mind. Science has substantiated it; though the appearance in bulk may change, the ultimate constituents of the matter do not. This was (in spite of his occasional skepticism mentioned above) the teaching of Democritus. Neither he nor Dalton doubted that an atom which was originally present in the block of wood is afterward either in the ashes or in the smoke.

9. THE BEARING ON ATOMISM

In the new turn of atomism that began with the papers of Heisenberg and of de Broglie in 1925 such an attitude has to be abandoned. This is the most startling revelation emerging from the ensuing de-

velopment, and the feature which in the long run is bound to have the most important consequences. If we wish to retain atomism we are forced by observed facts to deny the ultimate constituents of matter the character of identifiable individuals. Until recently, atomists of all ages, for all I know, had transferred that characteristic from visible and palpable pieces of matter to the atoms, which they could not see or touch or observe singly. Now we do observe single particles; we see their tracks in the cloud chamber and in photographic emulsions; we register the practically simultaneous discharges caused by a single swift particle in two or three Geiger counters placed at several yards' distance from each other. Yet we must deny the particle the dignity of being an absolutely identifiable individual. Formerly, if a physicist were asked what stuff the atoms themselves were made of, he might smile and shirk the answer. If the inquirer insisted on the question whether he might imagine them as small unchangeable bits of ordinary matter, he would get the smiling reply that there was no point in doing so but that it would do no harm. The formerly meaningless question has now gained significance. The answer is definitely in the negative. An atom lacks the most primitive property we associate with a piece of matter in ordinary life. Some philosophers of the past, if the case could be put to them, would say that the modern atom consists of no stuff at all but is pure shape.

10. THE MEANING OF THE NEW STATISTICS

We must at last proceed to give the reasons for this change of attitude in a more comprehensible form than at the end of section 6. It rests on the so-called new statistics. There are two of them. One is the Bose-Einstein statistics, whose novelty and relevance were first stressed by Einstein. The other is the Fermi-Dirac statistics, of which the most pregnant expression is Pauli's exclusion principle. I shall try to explain the new statistics, and its relation to the old classical or Boltzmann statistics, to those who have never heard about such things and perhaps may be puzzled by what "statistics" means in this context. I shall use an instance from everyday life. It may seem childishly simple, particularly because we have to choose small numbers—actually 2 and 3—in order to make the arithmetic surveyable. Apart from this, the illustration is completely adequate and covers the actual situation.

Three schoolboys, Tom, Dick, and Harry, deserve a reward. The teacher has two rewards to distribute among them. Before doing so, he wishes to realize for himself how many different distributions are at all possible. This is the only question we investigate (we are not interested in his eventual decision). It is a statistical question: to count the number of different distributions. The point is that the

answer depends on the nature of the rewards. Three different kinds of reward will illustrate the three kinds of statistics.

(a) The two rewards are two memorial coins with portraits of Newton and Shakespeare respectively. The teacher may give Newton either to Tom or to Dick or to Harry, and Shakespeare either to Tom or to Dick or to Harry. Thus there are 3 times 3, that is 9, different distributions (classical statistics).

(b) The two rewards are two shilling-pieces (which, for our purpose, we must regard as indivisible quantities). They can be given to two different boys, the third going without. In addition to these three possibilities there are three more: either Tom or Dick or Harry receives 2 shillings. Thus there are six different distributions (Bose-Einstein statistics).

(c) The two rewards are two vacancies in the football team that is to play for the school. In this case two boys can join the team, and one of the three is left out. Thus there are three different distributions (Fermi-Dirac statistics).

Let me mention right away: the *rewards* represent the particles, two of the same kind in every case; the boys represent states the particle can assume. Thus, "Newton is given to Dick" means: the particle Newton takes on the state Dick.

Notice that the counting is natural, logical, and indisputable in every case. It is uniquely determined by the nature of the object—memorial coins, shillings, memberships. They are of different categories. Memorial coins are individuals distinguished from one another. Shillings, for all intents and purposes, are not, but they are still capable of being owned in the plural. It makes a difference whether you have 1 shilling, or 2 or 3. There is no point in two boys exchanging their shillings. It does change the situation, however, if one boy gives up his shilling to another. With memberships, neither has a meaning. You can either belong to a team or not. You cannot belong to it twice over.

Experimental evidence proves that statistical counts referring to elementary particles must never follow the pattern (a), but must follow either (b) or (c). Some hold that for all genuinely elementary particles (c) is competent. Such particles, electrons for instance, correspond to membership in a club; I mean to the abstract notion of membership, not to the members. Any person eligible to membership in that club represents a well-defined state an electron can take on. If the person is a member, that means there is an electron in that particular state. According to Pauli's exclusion principle, there can never be more than one electron in a particular state. Our simile renders this by declaring double membership meaningless—as in most clubs it would be. In the course of time the list of members changes,

and membership is now attached to other persons: the electrons have gone over into other states. Whether you can, in a loose way, speak of a certain membership going over from Dick to Tom, thence from Tom to Harry, etc., depends on the circumstances. They may suggest this view, or they may not, but never in an absolute fashion. In this our simile is perfect, for it is the same with an electron. Moreover, it is quite appropriate to consider the number of members as fluctuating. Indeed, electrons too are created and annihilated.

The example may seem odd and inverted. One might think, "Why cannot the people be the electrons and various clubs their states? That would be so much more natural." The physicist regrets, but he cannot oblige. And this is just the salient point: the actual statistical behavior of electrons cannot be illustrated by any simile that represents them by identifiable things. That is why it follows from their actual statistical behavior that they are not identifiable things.

The (*b*), illustrating Einstein-Bose statistics, is competent for light quanta (photons), inter alia. It hardly needs discussion. It does not strike us as so strange for the very reason that it includes light, i. e., electromagnetic energy; and energy, in prequantum times, had always been thought of in very much the way our simile represents it, viz, as having quantity, but no individuality.

11. RESTRICTED NOTION OF IDENTITY

The most delicate question is that of the states of, say, an electron. They are, of course, to be defined not classically, but in the light of the uncertainty relation. The rigorous treatment referred to at the end of section 6 is not really based on the notion of "state of one electron" but on that of "state of the assembly of electrons." The whole list of members of the club, as it were, has to be envisaged together—or rather several membership lists, corresponding to the several kinds of particles that go to compose the physical system under consideration. I mention this, not to go into details about it, but because, taken rigorously, the club simile has two flaws. First, the possible states of an electron (which we had assimilated to the persons eligible for membership) are not absolutely defined; they depend on the arrangement of the—actual or imagined—experiment. Given this arrangement, the states are well-defined individuals, which the electrons are not. They also form—and this is the second flaw of the simile—a well-ordered manifold. That is, there is a meaning in speaking of neighboring states as against such as are more remote from each other. Moreover, I believe it is true to say that this order can be conceived in such a fashion that, as a rule, whenever one occupied state ceases to be occupied, a neighboring state becomes occupied.

This explains that, in favorable circumstances, long strings of successively occupied states may be produced, similar to those contemplated in sections 7 and 8. Such a string gives the impression of an identifiable individual, just as in the case of any object in our daily surrounding. It is in this way that we must look upon the tracks in the cloud chamber or in a photographic emulsion, and on the (practically) simultaneous discharges of Geiger counters set in a line, which discharges we say are caused by the same particle passing one counter after another. In such cases it would be extremely inconvenient to discard this terminology. There is, indeed, no reason to ban it, provided we are aware that, on sober experimental grounds, the sameness of a particle is not an absolute concept. It has only a restricted significance and breaks down completely in some cases.

In what circumstances this restricted sameness will manifest itself is fairly obvious: namely, when only few states are occupied in the region of the state-manifold with which we are concerned, or, in other words, when the occupied states are not too crowded in that region, or when occupation is a rare event—the terms “few,” “crowded,” and “rare” all referring to the state-manifold. Otherwise, the strings intermingle inextricably and reveal the true situation. In the last section we shall formulate the quantitative condition for the prevailing of restricted individuality. Now we ask what happens when it is obliterated.

12. CROWDEDNESS AND WAVE ASPECT

One gains the impression that according as the individuality of the particles is wiped out by crowding, the particle aspect becomes altogether less and less expedient and has to be replaced by the wave aspect. For instance, in the electronic shell of an atom or molecule the crowding is extreme, almost all the states within a certain region being occupied by electrons. The same holds for the so-called free electrons inside a metal. Indeed, in both cases the particle aspect becomes entirely incompetent. On the other hand, in an ordinary gas the molecules are extremely rare in the wide region of states over which they spread. No more than one state in 10,000 or so is occupied. And indeed, the theory of gases, based on the particle aspect, was able to attain great perfection long before the wave nature of ordinary matter was discovered. (In the last remark I have been speaking of the molecules as if they were ultimate particles; this is legitimate as far as their translatory motion is concerned.)

It is tempting to assign to the two rivals, the particle aspect and the wave aspect, full competences in the limiting cases of extreme “rarefaction” and extreme “crowding” respectively. This would separate them, as it were, with only some sort of transition required for the

intermediate region. This idea is not entirely wrong, but it is also far from correct. One may remember the interference patterns referred to in section 2 in evidence of the wave nature of the electron. They can be obtained with an arbitrarily faint bundle of cathode rays, provided the exposure is prolonged. Thus a typical wave phenomenon is produced here, irrespective of crowding. Another instance is this. A competent theoretical investigation of the collision of two particles, whether of the same or of different kind, has to take account of their wave nature. The results are duly applied to the collisions of cosmic-ray particles with atomic nuclei in the atmosphere, both being extremely rarefied in every sense of the word. But perhaps this is trivial; it only means that even an isolated particle, which gives us the illusion of transitory individuality, must yet not be likened to a classical particle. It remains subject to the uncertainty relation, of which the only tolerable image is the guiding wave group.

13. THE CONDITION FOR THE PARTICLE ASPECT

The following is the quantitative condition for strings to develop which counterfeit individuals and suggest the particle aspect: the product of the momentum p and the average distance l between neighboring particles must be fairly large compared with Planck's constant h ; thus

$$pl \gg h.$$

(The momentum p —and not the velocity—is the thing we should really have referred to when, in sections 4 and 5, we dealt with the uncertainty relation; p is simply the product of the mass and the velocity, unless the latter is comparable with that of light.)

A large l means a low density in ordinary space. What matters, however, is the density in the manifold of states—or phase space, to use the technical term. That is why the momentum p comes in. It is gratifying to remember that those very obvious strings—visible tracks in the cloud chamber or in the photographic emulsion, and simultaneous discharges of aligned counters—are all produced by particles with comparatively very large momentum.

The above relation is familiar from the theory of gases, where it expresses the condition which must be fulfilled in very good approximation in order that the old classical particle theory of gases should apply in very good approximation. This theory has to be modified according to quantum theory when the temperature is very low and at the same time the density very high, so that the product pl is no longer very large compared with h . This modification is called the theory of degenerate gases, of which the most famous application is that by A. Sommerfeld to the electrons inside a metal; we have mentioned them before as an instance of extreme crowding.

There is the following connection between our relation and the uncertainty relation. The latter allows one at any moment to distinguish a particle from its neighbors by locating it with an error considerably smaller than the average distance l . But this entails an uncertainty in p . On account of it, as the particle moves on, the uncertainty in the location grows. If one demands that it still remain well below l after the particle has covered the distance l , one arrives precisely at the above relation.

But again I must warn of a misconception which the preceding sentences might suggest, viz, that crowding only prevents us from registering the identity of a particle, and that we mistake one particle for the other. The point is that they are not individuals which could be confused or mistaken one for another. Such statements are meaningless.

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