Book of Abstracts
Livro de Resumos

19th Brazilian Logic conference
EBL 2019

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19TH BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XIX ENCONTRO BRASILEIRO DE LÓGICA

LIVRO DE RESUMOS

XIX EBL
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19TH BRAZILIAN LOGIC CONFERENCE:

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XIX ENCONTRO BRASILEIRO DE LÓGICA:

LIVRO DE RESUMOS

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To
GIOVANNI QUEIROZ,
in memoriam

Contents / Sumário

Preface

The Brazilian Logic Conferences (EBL) is one of the most traditional logic conferences in South America. Organized by the Brazilian Logic Society (SBL), its main goal is to promote the dissemination of research in logic in a broad sense. It has been occurring since 1979, congregating logicians of different fields — mostly philosophy, mathematics and computer science — and with different backgrounds — from undergraduate students to senior researchers. The meeting is an important moment for the Brazilian and South American logical community to join together and discuss recent developments of the field. The areas of logic covered in the conference spread over foundations and philosophy of science, analytic philosophy, philosophy and history of logic, mathematics, computer science, informatics, linguistics and artificial intelligence. Previous editions of the EBL have been a great success, attracting researchers from all over Latin America and elsewhere.

The 19th edition of EBL takes place from May 6-10, 2019, in the beautiful city of João Pessoa, at the northeast coast of Brazil. It is conjointly organized by Federal University of Paraíba (UFPB), whose main campus is located in João Pessoa, Federal University of Campina Grande (UFCG), whose main campus is located in the nearby city of Campina Grande (the second-largest city in Paraíba state) and SBL. It is sponsored by UFPB, UFCG, the Brazilian Council for Scientific and Technological Development (CNPq) and the State Ministry of Education, Science and Technology of Paraíba. It takes place at Hotel Luxxor Nord Tambaú, privileged located right in front Tambaú beach, one of João Pessoa's most famous beaches.

This 19th Brazilian Logic Conference is peculiarly important for a number of reasons. First, it is the first time that an EBL occurs in the state of Paraíba, and the third time that it occurs in northeast of Brazil (the other two occasions were in 1979 in Recife and 1996 in Salvador). Second, the state of Paraíba, in particular the Philosophy Department of UFPB, has been historically one of the most important centers for the development of logic in northeast of Brazil. During its more than four decades of existence, the Philosophy Department of UFPB has been crucial for the formation and capacitation of dozens of professional logicians. During this period, it has had a fruitful relationship with the Centre for Logic, Epistemology and History of Science (CLE) of the State University of Campinas; many of its logic professors, including the late Giovanni Queiroz, to whom this volume is dedicated, have studied at CLE.

Third, this EBL is a continuity of a series of recent developments related to the field of logic which have been taken place in the state of Paraíba. In 2010 and 2011, for example, the Philosophy Department of UFPB has organized two editions of a meeting focused at non-classical and paraconsistent logics entitled Newton da Costa Seminar. Four years later, in 2015, UFPB, UFCG and SBL organized in João Pessoa the 1st World Congress on Logic and Religion, which was a crucial step in the direction of stablishing a new and wanting series of events (the 2nd World Congress on Logic and Religion took place in Warsaw, Poland, in 2017; its 3rd edition will happen in Varanasi, India, in 2020).

The call for papers of this 19th Brazilian Logic Conference has asked for submissions on general topics of logic, including philosophical and mathematical logic and applications, history and philosophy of logic, non-classical logic and applications, philosophy of formal sciences, foundations of computer science, physics and mathematics, and logic teaching. The abstracts of the talks contained in this volume reflect this diversity of topics. Besides the communications, the volume contains the abstracts of eight keynote talks and five round tables. The keynote talks range from philosophical issues such as the relation between logic and metaphysics to more technical and computationally applied ones such as coalgebra to teaching logic to undergraduate students. As a novelty, this EBL has a keynote talk on Indian logic, an important but underappreciated topic.

Parallel to the EBL there is a Logic School, aimed mainly at undergraduate and graduate students, which offers tutorials on different logical subject matters by renowned experts. This year there are six tutorials, whose abstracts are also included in this volume.

The conference was only possible due to the sponsorship of UFCG, UFPB, CNPq and the State Ministry of Education, Science and Technology of Paraíba. We thank immensely to these institutions and their representatives for their sensibility regarding the importance of the EBL. We would also like to thank all the people involved in putting together this conference. In special, we would like to thank Classic Turismo, the official travel agency of the conference; Professor Lucídio Cabral, from the Computer Science Department of UFPB; Professor Tiago Massoni, from the Computer Science Department of UFCG; Professors Bruno Petrato Bruck and Teobaldo Leite Bulhões Júnior, from the Computer Science Deapartment of UFPB, who have provided an invaluable help managing the website; and Guilherme Buriti Vasconcelos, from the Philosophy Department of UFCG, who helped in formatting many of the abstracts which appear in this volume.

Ana Thereza Dürmaier

Cezar Mortari

Ricardo Silvestre

Prefácio

O Encontro Brasileiro de Lógica (EBL) é uma das conferências de lógica mais tradicionais da América do Sul. Organizado pela Sociedade Brasileira de Lógica (SBL), seu principal objetivo é promover a disseminação de pesquisas em lógica em sentido amplo. Ele vem ocorrendo desde 1979, reunindo lógicos de diferentes áreas — principalmente filosofia, matemática e ciência da computação — e com diferentes formações — desde estudantes de graduação a pesquisadores seniores. O encontro é um momento importante para a comunidade lógica brasileira e sul-americana se unir e discutir os desenvolvimentos recentes na área de lógica. As áreas de lógica cobertas pela conferência se espalharam sobre fundamentos da filosofia da ciência, filosofia analítica, filosofia e história da lógica, matemática, ciência da computação, informática, linguística e inteligência artificial. As edições anteriores da EBL foram um grande sucesso, atraindo pesquisadores de toda a América Latina e de outros lugares.

A décima-nona edição do EBL acontece de 6 a 10 de maio de 2019, na bela cidade de João Pessoa, no litoral nordeste do Brasil. É organizada conjuntamente pela Universidade Federal do Paraíba (UFPB), cujo campus principal está localizado em João Pessoa, Universidade Federal de Campina Grande (UFCG), cujo campus principal está localizado na vizinha cidade de Campina Grande (a segunda maior cidade do estado da Paraíba) e SBL. É patrocinada pela UFPB, UFCG, pelo Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) e pela Secretaria de Estado da Educação e da Ciência e Tecnologia da Paraíba. Acontece no Hotel Luxxor Nord Tambaú, localizado em frente à praia de Tambaú, uma das praias mais famosas de João Pessoa.

Este XIX Encontro Brasileiro de Lógica é particularmente importante por vários motivos. Em primeiro lugar, é a primeira vez que uma edição do EBL ocorre no estado da Paraíba, e a terceira vez que ocorre no Nordeste do Brasil (as outras duas ocasiões foram em 1979 em Recife e 1996 em Salvador). Em segundo lugar, o estado da Paraíba, em particular o Departamento de Filosofia da UFPB, tem sido historicamente um dos centros mais importantes para o desenvolvimento da lógica no Nordeste. Em suas mais de quatro décadas de existência, o Departamento de Filosofia da UFPB tem sido fundamental para a formação e capacitação de dezenas de profissionais da lógica. Nesse período, teve uma relação frutífera com o Centro de Lógica, Epistemologia e História da Ciência (CLE) da Universidade Estadual de Campinas; muitos de seus professores de lógica, incluindo o falecido Giovanni Queiroz, a quem este volume é dedicado, estudaram no CLE.

Em terceiro lugar, este EBL é a continuidade de uma série de desenvolvimentos recentes relacionados ao campo da lógica que tem ocorrido na Paraíba. Em 2010 e 2011, por exemplo, o Departamento de Filosofia da UFPB organizou duas edições de um encontro focado em lógicas não clássicas e paraconsistentes intitulado Seminário Newton da Costa. Quatro anos depois, em 2015, a UFPB, UFCG e SBL organizaram em João Pessoa o 1º Congresso Mundial de Lógica e Religião, que foi um passo crucial na direção de uma nova e carente série de eventos (o 2º Congresso Mundial de Lógica e Religião ocorreu em Varsóvia, na Polônia, em 2017, e sua 3ª edição acontecerá em Varanasi, na Índia, em 2020).

A chamada de artigos deste XIX Encontro Brasileiro de Lógica solicitou submissões em tópicos gerais da lógica, incluindo lógica filosófica e matemática e aplicações, história e filosofia da lógica, lógicas não clássicas e aplicações, filosofia das ciências formais, fundamentos de ciência da computação, física e matemática e ensino da lógica. Os resumos das comunicações contidos neste volume refletem essa diversidade de tópicos. Além dos resumos submetidos e selecionados, o volume contém os resumos de oito palestras principais e cinco mesas redondas. Os tópicos das palestras principais vão desde questões filosóficas como a relação entre lógica e metafísica, passando por tópicos mais técnicos como coalgebra, até o ensino da lógica para alunos de graduação. Como novidade, este EBL contém uma palestra sobre lógica indiana, um tópico importante, mas pouco apreciado.

Paralelamente ao EBL, acontece a Escola de Lógica, voltada principalmente para alunos de graduação e pós-graduação, que oferece tutoriais sobre diferentes assuntos lógicos por especialistas renomados. Este ano há seis tutoriais, cujos resumos também estão incluídos neste volume.

Esta conferência só foi possível graças ao apoio financeiro da UFCG, UFPB, CNPq e da Secretaria de Estado da Educação e da Ciência e Tecnologia da Paraíba. Agradecemos imensamente a essas instituições e seus representantes por sua sensibilidade em relação à importância do EBL. Gostaríamos também de agradecer a todas as pessoas envolvidas na organização desta conferência. Em especial, gostaríamos de agradecer à Classic Turismo, agência de viagens oficial da conferência; Professor Lucídio Cabral, do Departamento de Ciência da Computação da UFPB; Professor Tiago Massoni, do Departamento de Ciência da Computação da UFCG; Professores Bruno Petrato Bruck e Teobaldo Leite Bulhões Júnior, do Departamento de Ciência da Computação da UFPB, que prestaram inestimável auxílio gerenciando o site; e Guilherme Buriti Vasconcelos, do Departamento de Filosofia da UFCG, que auxiliou na formatação de muitos dos resumos que aparecem neste volume.

Ana Thereza Dürmaier

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I

KEYNOTE TALKS

What Coalgebra Can Do for You?

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Often referred to as 'the mathematics of dynamical, state-based systems', coalgebra claims to provide a compositional and uniform framework to specify, analyse and reason about state and behaviour in computing. This lecture addresses this claim by discussing why Coalgebra matters for the design of models and logics for computational phenomena. To a great extent, in this domain one is interested in properties that are preserved along the system's evolution, the so-called 'business rules', as well as in 'future warranties', stating that e.g. some desirable outcome will be eventually produced. Both classes are examples of modal assertions, i.e. properties that are to be interpreted across a transition system capturing the system's dynamics. The relevance of modal reasoning in computing is witnessed by the fact that most university syllabi in the area include some incursion into modal logic, in particular in its temporal variants. The novelty is that, as it happens with the notions of transition, behaviour, or observational equivalence, modalities in Coalgebra acquire a shape . That is, they become parametric on whatever type of behaviour, and corresponding coinduction scheme, seems appropriate for addressing the problem at hand.

In this context, the lecture revisits Coalgebra from a computational perspective, focussing on models, their composition and behavioural properties. An effort will be made to help building up the right intuitions, often at the expense of a completely self-contained exposition.

What Makes a Logic Dynamic?

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In recent years, many dynamic logics have been proposed in fields like Computer Science, Philosophy, Physics and Formal Biology. In this talk, we discuss three broad categories where dynamic logics have been developed: dynamic logics for program/process specification, dynamic Logics for reasoning about actions in AI, multi-agent epistemic logic and dynamic epistemic logics. First, we present some standard extension of Propositional Dynamic Logic. Second, we introduce a Dynamic Logic in which the programs are terms in some process algebras: CCS (Calculus for Communicating Systems) and pi-Calculus specifications. We discuss how to match the notion of bisimulation between two processes in CCS with the notion of logically equivalent processes in PDL. Finally, we briefly discuss other possibilities to extend PDL, for instance adding data structure, with Petri nets and with fuzzy programs..

Paradoxes and Structural Rules from a Dialogical Perspective[*]

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In recent years, substructural approaches to paradoxes have become quite popular. But whatever restrictions on structural rules we may want to enforce, it is highly desirable that such restrictions be accompanied by independent philosophical motivation, not directly related to paradoxes. Indeed, while these recent developments have shed new light on a number of issues pertaining to paradoxes, it seems that we now have even more open questions than before, in particular two very pressing ones: what (independent) motivations do we have (if any) for restrictions on structural rules, and what to make of the plurality of new logics emerging from these restrictions, i.e. how to 'choose' among the different options. In this paper, we address these two questions from the perspective of a dialogical conception of logic that we've been advocating in recent years. We will argue that dialogical interpretations of structural rules, that is, as rules determining specific properties of the dialogues in question, provide a conveniently neutral framework to adjudicate between the different substructural proposals that have been made in the literature on paradoxes.

[*]Joint work with Rohan French, UC Davis, rfrench@ucdavis.edu.

Logic as a Modelling Tool

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There are many reasons to study logic, but the one which we shall discuss in this talk is the use of logic to model thought processes in mathematics and to obtain new theorems as a consequence. One of the most beautiful instances of this is the first order logic which can be used to prove and unify many theorems in mathematics. For example, the compactness of first order logic has numerous applications, from Ramsey's theorem on. Yet, there are many mathematical processes which first order logic is not sufficient to express, for example even the notion of convergence of real sequences needs an infinite conjunction to be expressed in logical terms. Therefore one needs to study the so called strong logics, with more expressive power. Gaining in expressibility often means losing the pleasing properties that we are used to have in the first order case, such as compactness. Yet, surprisingly, there are certain very strong compactness results happening at unexpectedly high cardinals: singular cardinals, the smallest of which is $\aleph_\omega$. We try to explain this phenomenon in terms of strong logics.

Defeasible Reasoning in Navya-Nyāya

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The present talk is about affinities between John L. Pollock's theory of defeasible reasoning and the doctrine of the so-called "associate condition" (upādhi) in Navya-Nyāya, a school of classical Indian philosophy. We will show that the defeasible character of the five-membered inferences (anumāna), which Navya-Naiyāyikas regard as a knowledge source, can be explained in terms of enumerative induction and nomic generalization. Moreover, our analysis of an upādhi's function as a vitiator (dūṣaṇa) and of Navya-Nyāya definitions of the concept of upādhi will give a clue to relations between upādhis and "defeaters" in the sense of Pollock's theory. Although it seems tempting to conceive of an upādhi as an undercutting defeater (cf. Stephen Phillips' interpretation of the doctrine of upādhi), we will challenge this view here. upādhis are objects of the domain. The equivalents of defeaters in the Navya-Nyāya doctrine of upādhi are rather certain propositions which can be gleaned from specifications of an upādhi's vitiating function in Navya-Nyāya sources. Not all of these propositions are undercutters. Some are only rebutters and some are rebutters and undercutters with respect to different prima facie reasons which are involved in a five-membered inference. It is important to note that the defeater-related vitiating function of an upādhi, which aims at overruling an inference, applies only to the so-called "ascertained upādhis" (niścitopādhi). The so-called "dubious upādhis" (saṃdigdhopādhi) are relevant to situations which Pollock describes as "collective defeat". We will see that the distinction between a skeptical and a credulous reasoner can help to understand in what sense this type of upādhi is also regarded as a vitiator in Navya-Nyāya.

Modalities as Prices: a Game Model of Intuitionistic Linear Logic with Subexponentials

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Various kinds of game semantics have been introduced to characterize computational features of substructural logics, in particular fragments and variants of linear logic (LL) [5]. This line of research can be traced back to Lorenzen's dialogues for intuitionistic logic [6] and to the works of Blass [2,3], Abramsky and Jagadeesan [1], among several others. We look at substructural calculi from a game semantic point of view, guided by certain intuitions about resource conscious and, more specifically, cost conscious reasoning. To this aim, we start with a game for affine intuitionistic linear logic, where player I defends a claim corresponding to a (single-conclusion) sequent, while player II tries to refute that claim. Branching rules for additive connectives are modeled by choices of II, while branching for multiplicative connectives leads to splitting the game into parallel subgames, all of which have to be won by player I to succeed. The game comes into full swing by using subexponentials ([4] [7]) for representing two types of options — volatile and permanent — for purchasing resources. This leads to a new type of subexponential calculus where costs are attached to sequents. Different proofs are interpreted as more or less expensive strategies to obtain a certain resource from a bunch of resources (priced options). We also generalize the concept of costs and option's prices in proofs by using a semiring structure. This allows for the interpretation a wider range of subexponential systems, giving meaning to resources in proofs in a more flexible way. We conclude by studying some proof-theoretical properties of the proposed systems, justifying our intended meaning for costs and resources.

This is a joint work with Carlos Olarte, Timo Lang and Christian Fermüller.

References

[1] Samson Abramsky and Radha Jagadeesan. Games and full completeness for multiplicative linear logic. *J. Symb. Log.*, 59(2):543–574, 1994.

[2] Andreas Blass. A game semantics for linear logic. *Ann. Pure Appl. Logic*, 56(1-3):183–220, 1992.

[3] ——. Some semantical aspects of linear logic. *Logic Journal of the IGPL*, 5(4):487–503, 1997.

[4] Vincent Danos, Jean-Baptiste Joinet, and Harold Schellinx. The structure of exponentials: Uncovering the dynamics of linear logic proofs. In Georg Gottlob, Alexander Leitsch, and Daniele Mundici, editors, *Kurt Gödel Colloquium*, volume 713 of *Lecture Notes in Computer Science*, pages 159–171. Springer, 1993.

[5] Jean-Yves Girard. Linear logic. *Theor. Comput. Sci.*, 50:1–102, 1987.

[6] Paul Lorenzen. Logik und agon. *Atti Del XII Congresso Internazionale di Filosofia*, 4:187–194, 1960.

[7] Vivek Nigam and Dale Miller. Algorithmic specifications in linear logic with subexponentials. In António Porto and Francisco Javier López-Fraguas, editors, *PPDP*, pages 129–140. ACM, 2009.

On Teaching Logic for Undergraduate Philosophy Students

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I will explore four topics of interest in teaching logic: (1) the use of formal logic to deal with dynamic aspects of argumentation; (2) the value of recreational logic; (3) diagrammatic methods for evaluating the validity of arguments; (4) the problems arising from the use of different types of examples.

Logic and Existence: How Logic and Metaphysics are Entangled

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The thesis that logic and metaphysics are entangled is cashed out as: logic and metaphysics have to be developed simultaneously – to develop logic one needs to develop metaphysics and to develop metaphysics one needs to develop logic. In particular, to develop logic, one needs logical and abstract objects of some sort: sentence and symbol types (not tokens), truth-values, possible worlds, properties or the extensions of properties, natural numbers (as conceived by Frege), etc. I try to show that one must simultaneously develop a theory of these metaphysical objects while one is formulating the foundations of logic.

II

TUTORIALS

Classical Propositional Logic – A Universal Logic Approach

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Classical Propositional Logic (hereafter CPL) is the main logic system of modern logic. The objective of this tutorial is to show on the one hand how we can have a general vision of logic from the perspective of CPL (Local to Global), on the other hand how we can have an understanding of CPL from the perspective of universal logic, understood as a general theory of logical systems (Global to Local), both directions being explored philosophically, mathematically, semiologically and computationally. Logic is at the crossroads of these fields and CPL at the crossroads of logic systems, theories and concepts.

We will examine the notion of *proposition*, which is at the heart of CPL, discussing its relation with other notions such as *sentence*, *fact*, *thought*, *information*, *assertion*. In CPL a proposition is by definition something than can be true or false, therefore a question is not a proposition from the point of view of CPL. But there are non-classical systems dealing with questions, this is the field of "erotetic logic". A question can be defined as a unary operator. We are then facing the other central notion of CPL, the notion of *connective*. Why in CPL do we have only these 16 connectives, how does this work and what does this mean? In which sense CPL can be extended despite Post maximality? Is it possible to define a concept of absolute maximality encompassing both expressive and logical power?

We will investigate the fundamental metatheorems of CPL such as completeness, replacement, compactness decidability, on the one hand studying the relations between them and on the other hand examining what is fundamentally and essentially classical in them, seeing how they can be generalized or not to non-classical systems. And this double investigation will be carried out at the same time concerning the tools used to deal with CPL: proof-theory, semantics, intermediate techniques, like tableaux.

We will finally study the relation between CPL and First-Order Logic (FOL), examining in which sense CPL is a subsystem of FOL, in which sense it makes sense to call CPL, "Zero-Order Logic", and how we can apply FOL to CPL. This naturally will lead us to notions such as combination, decomposition, translation of logics, which are fundamental notions of universal logic.

References

[1] Beziau, J.-Y., Universal Logic, in *Logica'94 — Proceedings of the 8th International Symposium*, T. Childers & O. Majer (eds), Prague, 1994, pp.73-93.

[2] Beziau, J.-Y., The mathematical structure of logical syntax. In *Advances in contemporary logic and computer science*, W. A. Carnielli and I. M. L. D'Ottaviano (eds), American Mathematical Society, Providence, 1999, pp.1-17.

[3] Beziau, J.-Y., La véritable portée du théorème de Lindenbaum-Asser, *Logique et Analyse*, 167-168 (1999), pp.341-359.

[4] Beziau, J.-Y. R. P. de Freitas and J. P. Viana, What is classical propositional logic? (A study in universal logic), *Logical Studies*, 7 (2001).

[5] Beziau, J.-Y., Sequents and bivaluations, *Logique et Analyse*, 44 (2001), pp.373-394.

[6] Beziau, J.-Y., A paradox in the combination of logics, in *Workshop on Combination of Logics: Theory and Applications*, W. A. Carnielli, F. M. Dionisio and P. Mateus (ed), IST, Lisbon, 2004, pp.75-78.

[7] Beziau, J.-Y., Is the Principle of Contradiction a Consequence of $xx = x$?, *Logica Universalis*, 12 (2018), pp.58-81.

[8] Beziau, J.-Y., An unexpected feature of classical propositional logic in the Tractatus, in G. Mras, P. Weingartner and B. Ritter (eds), *Philosophy of Logic and Mathematics: Proceedings of the 41st International Ludwig Wittgenstein Symposium*, De Gruyter, Berlin, Munich, Boston, 2019.

[9] Church, A. *Introduction to Mathematical Logic*, Princeton Univ. Press, Princeton, 1956.

[10] Enderton, H. B., *A Mathematical Introduction to Logic*, Academic Press, New York, 1972.

[11] Humberstone, L., Béziau's Translation Paradox, *Theoria* 71 (2005), 138-181.

[12] Post, E., Introduction to a General Theory of Elementary Propositions, *American Journal of Mathematics*, 43 (1921), pp.163–185.

Lógica, Probabilidade: Encontros e Desencontros

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Discutiremos o relacionamento nem sempre amoroso entre lógica e probabilidade enfatizando o universo das teorias probabilísticas sobre lógicas não-clássicas e algumas relações espinhosas entre probabilidade e filosofia.

Non-deterministic Semantics: Theory and Applications

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Non-deterministic semantics are useful tools for describing logics which are not algebraizable by means of the standard tools, and that cannot be characterized by a single finite logical matrix. A good source of examples of this kind of logics can be found in the class of paraconsistent systems known as *Logics of Formal Inconsistency* (**LFI**s), introduced in [6] (see also [5] and [4]). The aim of this tutorial is analyzing several approaches to non-deterministic semantics for non-classical logics. The relationship between them will be discussed as well.

The first non-deterministic semantical framework to be discussed is called *possible-translations semantics* (PTSs), introduced by Carnielli in 1990 in an attempt to offer a more acceptable philosophical interpretation for some non-classical logics (see [3] and [10]). The basic idea of this method is interpreting a given logic $\mathscr{L}$ (with an unknown semantic characterization) as the combination of an appropriate set of translations of the formulas of $\mathscr{L}$ into a class of "simpler" logics with known semantic characterization (typically, a family of finite-valued logics).

Another interesting paradigm to be discussed is a generalization of the notion of logical matrix proposed by Avron and Lev in 2001 called *Non-deterministic matrices*, or simply *Nmatrices* (see for instance [1] and [2]). Nmatrices are logical matrices in which each entry of the truth-tables returns a non-empty set of possible values. In more precise terms, Nmatrices are logical matrices in which the underlying algebra is replaced by a multialgebra, that is, an algebra in which the operations interpreting the connectives are multiple-valued: a non-empty set of outputs is obtained from a single input. In this context, the valuations choose, for any complex formula, a possible value generated by the multioperators from the values already given to its subformulas.

In [4, Chapter 6], Carnielli and Coniglio propose a Nmatrix semantics for **LFI**s in such a way that the underlying multialgebras, called *swap structures*, are formed by triples over a given Boolean algebra. For each **LFI**, say $\mathscr{L}$, the Nmatrix associated to the two-valued Boolean algebra coincides with the finite-valued Nmatrix proposed by Avron and his collaborators for $\mathscr{L}$, hence swap structures semantics generalizes, in this sense, these characterization results. The advantage of considering a wider class of multialgebraic models for **LFI**s was explored in [8], showing that it is possible to obtain an algebraic theory based on swap structures, by adapting concepts of universal algebra to multialgebras in a suitable way. In particular, representation theorems similar to the Birkhoff's decomposition theorem in traditional algebraic logics where obtained for several **LFI**s. The wide possibilites of swap structures semantics are not limited to **LFI**s, as it was shown in [9], where several non-normal modal logics were semantically characterized by this kind of structures.

One of the first non-deterministic semantical structures for non-classical logics proposed in the literature were introduced by M. Fidel in 1977, in order to show the decidability of da Costa's paraconsistent logics C_n. These structures are nowadays known as

Fidel structures, or simply **F**-structures. Suppose that $\mathscr{L}$ is a logic in which a fragment of its language is characterized by a class $\mathbb{A}$ of algebras. Suppose that, in addition, $\mathscr{L}$ has unary connectives which do not admit any algebraic characterization. For instance, some basic **LFI**s like **mbC** have a fragment characterized by the class $\mathbb{A}$ of Boolean algebras, but they also have the paraconsistent negation and the consistency operator, which are not truth-functional. The idea of **F**-structures is taking models formed by an algebra $\mathscr{A}$ in $\mathbb{A}$ expanded with relations which interpret the non-truth-functional connectives. It is worth noting that **F**-structures can be seen as Tarskian first-order structures satisfying certain coherence axioms, and as such, they can be studied from the perspective of model theory (see [7]).

As shown in [1], not every **LFI** (including the well-known da Costa's logic C_1) can be characterized by a single finite Nmatrix (this is a Dugundji-like theorem for Nmatrix semantics). However, each of these logics can be fully characterized by a single finite-valued **F**-structure. This shows that **F**-structures semantics have more expressive power than Nmatrices, as discussed in [4, Chapter 6]. In the last part of this tutorial, a comparison between the expressive power of the three paradigms of non-deterministic semantics presented above will be discussed.

References

[1] Arnon Avron. Non-deterministic semantics for logics with a consistency operator. *Journal of Approximate Reasoning*, 45:271–287, 2007.

[2] A. Avron and A. Zamansky. Non-deterministic semantics for logical systems. In: *Handbook of Philosophical Logic*, vol. 16, pp. 227–304. Eds.: D. Gabbay; F. Guenthner. Springer, 2011.

[3] W. A. Carnielli. Possible-translations semantics for paraconsistent logics. In D. Batens, C. Mortensen, G. Priest, and J.-P. Van Bendegem, editors, *Frontiers in Paraconsistent Logic: Proceedings of the I World Congress on Paraconsistency*, pages 149–163. Research Studies, Hertfordshire, 2000.

[4] W. Carnielli and M. E. Coniglio. *Paraconsistent Logic: Consistency, Contradiction and Negation*. Volume 40 of *Logic, Epistemology, and the Unity of Science*, Springer, 2016.

[5] W. A. Carnielli, M. E. Coniglio and J. Marcos. Logics of Formal Inconsistency. In: *Handbook of Philosophical Logic*, vol. 14, pp. 1–93. Eds.: D. Gabbay; F. Guenthner. Springer, 2007.

[6] W. A. Carnielli and J. Marcos. A taxonomy of C-systems. In: W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The Logical Way to the Inconsistent*. Volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pages 1–94. Marcel Dekker, New York, 2002.

[7] M. E. Coniglio and A. Figallo-Orellano. A model-theoretic analysis of Fidel-structures for mbC. In: C. Baskent and T. Ferguson, editors, *Graham Priest on Dialetheism and Paraconsistency*. To appear in *Outstanding Contributions to Logic*, Springer, 2019.

[8] M. E. Coniglio, A. Figallo-Orellano and A. C. Golzio. Non-deterministic algebraization of logics by swap structures. *Logic Journal of the IGPL*, to appear. First published online: Nov. 29, 2018. DOI: 10.1093/jigpal/jzy072. Preprint available at *arXiv*:1708.08499 [math.LO]

[9] M. E. Coniglio and A. C. Golzio. Swap structures semantics for Ivlev-like modal logics. *Soft Computing*, 23(7):2243–2254, 2019.

[10] J. Marcos. Possible-translations semantics for some weak classically-based paraconsistent logics. *Journal of Applied Non-Classical Logics*, 18(1):7–28, 2008.

Some Highlights of Indian Logic

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The present tutorial comprises 3 parts:

1.) The first part deals with the concept of emptiness (śūnyatā) in Mādhyamaka and Graham Priest's ideas how to model it mathematically. It should be noted that Priest's approach does not involve any commitment to his more controversial dialethic interpretation of Mādhyamaka Buddhism. In order to represent the idea of universal emptiness mathematically Priest replaces the ontology of independent objects by an ontology of loci (i.e., non-empty sets of relation instances) in a field of relations where each relation is a locus in another field of relations and where each of these relations is a locus in yet another field of relations etc. Priest's mathematical construction is an appropriate means to model an object's purely relational existence. Instead of focussing on the aspect of relational existence, we can also conceive of an object's emptiness in another plausible way, namely by envisaging it as a composite entity whose components can be analyzed into further components and these again into further components etc. We never reach a bottom line of primary existents. Priest claims that the mathematical counterpart of such an entity is a non-well-founded set. There is good reason to endorse this idea if we interpret an object as a set of relation instances which constitute its parts. The part-whole-relationship can then be understood in terms of set-membership ($\in$). Two aspects of non-well-foundedness are at stake here, namely infinitely descending $\in$-chains and looping $\in$-chains. While Priest focusses on the former aspect, we will deal especially with the latter. Apart from pointing out some minor mistakes in Priest's exposition, we suggest also an alternative idea how to model the aspect of non-well-foundedness which is associated with an object's ungroundedness or its infinite divisibility. Non-well-founded relations like the part-whole-relation, as it is conceived of in Mādhyamaka, can also be defined in a well-founded set-theoretic framework. Moreover, we critically examine Priest's idea to understand the Mādhyamika's claim of the emptiness of emptiness in the sense that the universal class (i.e., the class of all empty objects as a set-theoretic correlate of the the concept of emptiness) is an element of itself. While this is indeed possible in a non-well-founded system of set theory like "Quine's New Foundation", we might alternatively interpret emptiness as a property which is characterized as empty in the sense of having a purely relational existence, for (here I am quoting Westerhoff) ". . . as long as objects exist, and are conceived of by deluded minds more or less like ours, then these objects will be empty."

2.) The second part will be about a very interesting account of the reference of number words in classical Indian philosophy which was given by Maheśa Chandra (1836–1906) in his *Brief Notes on the Modern Nyāya System of Philosophy and its Technical Terms* (BN), a primer on Navya-Nyāya terminology and doctrines. Despite its English title BN is a Sanskrit work. The section on "number" (saṃkhyā) provides an exposition of a theory of number which can account for both, the adjectival and the substantival use of number words in Sanskrit. According to D. H. H. Ingalls some ideas about the reference of number words in BN are close to the Frege-Russell theory of natural number. Ingalls's comparison refers to a concept of number in Navya-Nyāya which is related to the things numbered via the so-called "circumtaining relation" (paryāpti). Although there is no theory of sets in Navya-Nyāya, Navya-Naiyāyikas do have a realist theory of properties (*dharma*) and their theory of number is a theory of properties as constituents of empirical reality, anchored to their system of ontological categories. As shown by George Bealer, properties can serve the same purpose as sets in the Frege-Russell theory of natural number. In the present paper we attempt a formal reconstruction of Maheśa Chandra's exposition of the Navya-Nyāya theory of number, which accounts for its affinity to George Bealer's neo-Fregean analysis. As part of our appraisal of the momentousness and robustness of the "circumtaining" concept of number we show that it can be cast into a precise recursive definition of natural number and we prove property versions of Peano's axioms from this definition.

3.) The third part will be about the Navya-Nyāya contribution to a logic of questions. According to Maheśa Chandra, the author of the Navya-Nyāya manual *Brief Notes on the Modern Nyāya System of Philosophy and its Technical Terms*, "certitude" (niścaya) and "doubt" (saṃśaya) are the two varieties of "cognition" (jñāna). He illustrates the verbal expression of certitudes by means of declaratives and the verbal expression of doubts by means of interrogatives (functioning as polar or alternative questions). He notes also that different credence levels might be associated with the alternatives involved in a speaker's doubt. A biassed question in the form of a tag interrogative might be an appropriate way to express such a doubt. In Western logic the idea to treat declaratives and interrogatives on a par, which is anticipated by the Navya-Naiyāyikas' use of the unifying concept of "cognition", goes back to Frege's distinction between the semantic content (the "thought") of a sentence and its force and it was recently elaborated by Ciardelli, Farkas, Groenendijk, Roelofsen et al., the founders of a new branch in logic called "inquisitive logic". We will discuss Maheśa Chandra's succinct exposition of the Navya-Naiyāyikas' innovative approach from the perspective of this type of logic.

Uma Miscelânea de Aplicações de Ultrafiltros em Matemática

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Filtros e ideais podem ser entendidos como formalizações, em Teoria de Conjuntos (ou, mesmo mais abstratamente, em Álgebras Booleanas e estruturas assemelhadas) das noções de maioria e minoria, respectivamente; os elementos de filtros são conjuntos "grandes", enquanto que os elementos de ideais são conjuntos "pequenos". A existência de ultrafiltros — e, mais precisamente, de ultrafiltros livres — é reconhecida como sendo um princípio maximal, e sob esse ponto de vista é frequentemente encarada como sendo uma versão estritamente mais fraca do Axioma da Escolha. Ultrafiltros possuem aplicações profundas em quase todas as áreas da Matemática. Neste minicurso, vamos focar em questões relacionadas à Topologia e à Análise, com um ponto de vista bastante topológico/combinatório. A noção de convergência de filtros e ultrafiltros nos permite definir, construir e verificar as propriedades da compactificação de Stone-Cech do espaço discreto dos naturais; a noção de convergência de ultrafiltros também caracteriza a compacidade em espaços topológicos, tarefa que a convergência de sequências não é capaz de fazer — e ultrafiltros são, ainda, capazes de testemunhar tal incapacidade. Também é comum que os ultrafiltros livres desempenhem, por si sós, papel decisivo em certas questões que são mais normalmente associadas a aplicações do Axioma da Escolha: exemplos disso incluem a prova da existência de subconjuntos da reta com propriedades especiais (não-mensuráveis, não-determinados, etc.), ou mesmo a prova do Teorema de Ramsey para conjuntos infinitos em geral. Finalizaremos com uma aplicação de ultrafiltros em Ciências Sociais, demonstrando que, sob um determinado e preciso ponto de vista, é impossível que eleições de candidatos em uma lista sejam completamente justas e reflitam, fiel e necessariamente, as opiniões da maioria dos votantes.

Bibliografia

[1] Arrow, K. J., A Difficulty in the Concept of Social Welfare. *Journal of Political Economy 58*, (4) 1950, 328-346.

[2] Bell, J. L. e Slomson, A. B.,*Models and ultraproducts: An introduction.* North-Holland Publishing Co., Amsterdam-London, ix + 322 pp., 1969

[3] Engelking, R., *General Topology.* (rev. compl. ed.), Berlin, Heldermann (Sigma Series in Pure Mathematics, 6), viii + 529 pp., 1989

[4] Galvin, D., Ultrafilters, with applications to analysis, social choice and combinatorics. Notas de um Curso ministrado no Combinatorics and Logic Reading Seminar na Universidade de Notre Dame. Disponível em: `https://www3.nd.edu/ dgalvin1/pdf/ultrafilters.pdf`, 2009.

[5] García-Ferreira, S.; Sanchis, M. Ultrafilter-limit points in metric dynamical systems. *Commentationes Mathematicae Universitatis Carolinae* 48 (2007), no. 3, 465–485.

[6] Jech, T. *Set theory – The third millennium edition,* revised and expanded. Springer Monographs in Mathematics, Springer-Verlag, Berlin, xiv+769, 2003.

[7] Komjáth, P., e Totik, V. Ultrafilters. *American Mathematical Monthly* 115 (1) 2008, 33-44

[8] Kunen, K. *Set theory – An introduction to independence proofs.* Studies in Logic and the Foundations of Mathematics, 102, North-Holland Publishing Co., Amsterdam, xvi+313, 1983

[9] Odifreddi, P. Ultrafilters, dictators, and gods. Finite versus infinite, *Discrete Mathematics and Theoretical Computer Science* (London), Springer, London, 239-245, 2000

[10] Tao, Terence. Ultrafilters, nonstandard analysis, and epsilon management. *What's New [Blog da Internet]*, 25 de Junho de 2007. Disponível em: `https://terrytao.wordpress.com/2007/06/25/ultrafilters-nonstandard-analysis-and-epsilon-management/`.

Logics in Artificial Intelligence

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When John McCarthy coined the term Artificial Intelligence, in 1956, he brought together scientists working with different aspects of intelligence. Two years later, he published "Programs with Common Sense", the first paper to propose the use of logic to represent knowledge and reason with it. After a period of glory, logic gave place to machine learning as the most visible part of AI. Recently, people started looking back at symbolic AI, as machine learning fails to explain why how decisions are made. In this tutorial, we will explore some of the logics used for knowledge representation and their applications.

III

ROUND TABLES

III.1 Algumas abordagens para disciplinas de Lógica em cursos de graduação

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Um problema que enfrentamos com frequência é como preparar uma apresentação para não-especialistas. Nesta mesa redonda vamos discutir um caso extremo deste problema: como apresentar Lógica para turmas de alunos de graduação — "especialistas em nada" — de formas que maximizem a chance deles aprenderem boa parte do conteúdo necessário? E que "conteúdo necessário" é esse, já que temos bastante liberdade de escolher o programa dos nossos cursos?

Nesta mesa redonda vamos apresentar três abordagens bem diferentes para este problema. O João Marcos vai falar sobre as três disciplinas de Fundamentos Matemáticos no curso de TI da UFRN e sobre como elas se conectam com as outras disciplinas do curso ("O lugar da Lógica em um percurso de formação em Fundamentos da Computação"); o Eduardo Ochs ("eu") vai falar sobre uma disciplina de Matemática Discreta de primeiro período para um curso de Ciência da Computação ("Ensinando Matemática Discreta para calouros com português muito ruim"), e o Jean-Yves Beziau vai falar sobre um curso de Lógica para filósofos ("Como ensinar a lógica aos filósofos?").

Ensinando Matemática Discreta para calouros com português muito ruim

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Considere a seguinte situação: um curso de Matemática Discreta para calouros, com conteúdo muito grande, num campus em que entram muitos alunos muito fracos com muito pouca base matemática e português muito ruim; além disso nos pedem pra que ensinemos o máximo possível nesse curso e não deixemos passar alunos que não tiverem aprendido o suficiente, porque este curso de Matemática Discreta é pré-requisito para cursos que exigem uma certa maturidade matemática, e ele deve servir como "filtro"... como estruturar um curso assim pra que ele funcione bem?

O problema mais básico aqui é: que linguagem utilizar? Os alunos sempre começam tentando discutir suas idéias em português, tanto entre si quanto com o professor, mas o português deles é impreciso demais, e não há tempo hábil para debugá-lo no curso! Então precisamos ir introduzindo desde o início notações matemáticas precisas que sejam fáceis o suficiente de usar, e aí usar sempre exemplos em notação matemática... a linguagem matemática adequada vira a base que torna as discussões em português possíveis.

Nesta palestra vou mostrar como esta "linguagem matemática adequada para calouros" pode ser dividida em três camadas: uma camada mais básica, em que tudo pode ser calculado até o resultado final num número finito de passos sem precisar de criatividade praticamente nenhuma; uma camada média, em que alguns objetos infinitos, como $\mathbb{N}$ ou $\mathbb{Z}$, passam a ser permitidos; e uma camada acima destas que inclui uma linguagem para provas formais. Além disto vou mostrar modos de visualizar vários dos objetos matemáticos do curso, e discutir alguns pontos em que esta abordagem do curso ainda tem "buracos".

Algumas das idéias daqui foram apresentadas e discutidas, com exemplos e figuras, no Logic Day 2019 no Rio de Janeiro. Os slides estão disponíveis em:

http://angg.twu.net/LATEX/2019logicday.pdf

Como ensinar a lógica aos filósofos?

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Nesta palestra vou tratar da questão do ensino da lógica. Primeiro argumentarei que não é bom ensinar a lógica da mesma forma em todos os departamentos. Na sequencia discutarei de como apresentar um curso de introdução a lógica num departamento de filosofia. Examinarei as perguntas seguintes que são relativas a que deve ser o conteúdo do curso e a metodologia usada.

1. É bom falar da história da lógica? Em que proporção?
2. Deve-se explicar o que é o raciocínio? Como fazer isso?
3. Que sistemas apresentar? Silogística? Lógica proposicional? Lógica de primeira ordem? Lógica modal? Teoria dos conjuntos?
4. É necessario provar teoremas? Quais e até que nível de sofisticação?
5. É importante falar do teorema de Gödel? De que forma?
6. Faz sentido fazer exercício de tradução da língua natural para a língua formal?
7. Deve-se incluir pensamento crítico e estudo de falácias?
8. Deve-se usar livros e artigos de apoio? quais?
9. Como devem ser formuladas as provas?

O lugar da Lógica em um percurso de formação em Fundamentos da Computação

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Nesta contribuição eu vestirei a carapuça do cientista da computação e apresentarei o bem-sucedido percurso de formação atual dos alunos da área de Tecnologia da Informação (TI) da UFRN, na grande área de Teoria da Computação.

Nossos alunos de TI (um curso de bacharelado de 7 semestres) cursam hoje duas disciplinas obrigatórias de "Fundamentos Matemáticos da Computação"; aqueles que escolhem continuar sua formação em segundo ciclo em Ciência da Computação (um curso com um total de 10 semestres) fazem uma terceira disciplina de "Fundamentos"; cada uma destas disciplinas tem uma carga horária de 90h. As disciplinas de "Fundamentos" partilham uma mesma estrutura geral: para além dos **tópicos centrais** que constam de suas respectivas ementas, há uma série de **conteúdos transversais** que *devem* ser explorados. Os tópicos centrais de "Fundamentos 1" são elementos de Teoria dos Números e de Contagem; em "Fundamentos 2" cobrimos Teoria dos Conjuntos e elementos de Álgebra. Em ambas as disciplinas é essencial que a tecnologia matemática necessária às demonstrações dos principais resultados e exercícios seja apresentada de forma transversal, à medida em que esta se faz necessária, e que seja dominada minimamente pelos alunos.

A disciplina de "Fundamentos 3" atende um público já selecionado, e cobre temas ligados a Tipos Recursivos de Dados, Álgebras Heterogêneas, e Lógica de Primeira Ordem. O foco aqui é a especificação de diversos tipos de conjuntos indutivamente definidos e as verificações de suas propriedades, com ênfase na fina separação entre sintaxe e semântica, cuja compreensão é essencial ao cientista da computação. Após as disciplinas de "Fundamentos" nossos alunos ainda cursarão disciplinas de Lógica Computacional, Linguagens Formais e Autômatos, e Especificação e Verificação de Programas, cada uma das quais com uma carga horária de 60 horas-aula, dentre outras.

Na minha contribuição eu mostrarei como a Lógica surge de maneira natural, nas disciplinas de Fundamentos, a partir de um percurso de abstração (de números e conjuntos para álgebras) e de imposição crescente de estrutura (até chegar à Álgebra Universal sobre múltiplos Tipos de Dados). Discutirei também na minha contribuição algumas das dificuldades que temos enfrentado em garantir que este modelo de formação seja bem implementado.

III.2 Mulheres na Lógica (e no Brasil)

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A parca presença de mulheres no exercício das artes, da política e das ciências — humanas e exatas — é um problema que tem recebido crescente atenção nas últimas décadas. Nesse contexto, o enfrentamento do fato de que as mulheres são cronicamente sub-representadas na comunidade lógica mundial parece uma empreitada além de importante, urgente. Muitas vezes, as raras praticantes de lógica se sentem "visíveis demais". Outras, invisíveis e isoladas. Em qualquer caso, há o risco de que essa sub-representação endêmica se perpetue.

Em um recente trabalho sobre estratégias de motivação e ensino de lógica para mulheres, destaca-se que "[S]egundo o relatório da UNESCO (2018) sobre educação de meninas e mulheres em STEM (Science, Technology, Engineering and Mathematics), as mulheres representam apenas 35% de todos os estudantes matriculados nos campos de estudo relacionados. A diferença quantitativa entre mulheres e homens em cursos de STEM é tão relevante mundialmente que é contemplada na Agenda 2030 para o Desenvolvimento Sustentável da ONU". [1] No Brasil, ainda segundo os últimos dados da Pesquisa Nacional por Amostra de Domicílios reportados em [1], das mulheres ocupadas com 16 anos ou mais de idade, 18,8% têm ensino superior completo, enquanto que, entre os homens na mesma categoria, esse percentual é de 11%.

A escolaridade das mulheres é maior também na esfera profissional. Elas são maioria nos cursos de qualificação de mão de obra, de acordo com estudo do Plano Nacional de Qualificação, do Ministério do Trabalho e Previdência Social (MTPS). Os números são claros: de 2003 a 2012, do 1,8 milhão de alunos e alunas dos cursos de qualificação, 713 mil eram mulheres, ou seja, mais de 60% do total.

Apesar destes e outros dados, como os reportados em [2] — que indicam aspectos positivos da inclusão de mulheres nos processos de formação educacional e profissional — muitas barreiras permanecem. A "inclusão com segregação" permanece: apesar de um aumento no número de ingressantes na graduação, poucas mulheres chegam ao doutorado, menos mulheres chegam ao exercício da docência, e menos ainda conquistam espaço como pesquisadoras, em algumas áreas mais do que em outras. Como modificar este quadro em busca de mais diversidade de gênero nas carreiras científicas — e em especial na Lógica? Além dos movimentos de escala mundial, são necessárias iniciativas locais que impulsionem e possibilitem a manutenção da participação de mulheres em nossa comunidade de interesse. Esta mesa redonda visa apresentar e discutir diferentes estratégias para motivar e desenvolver a atuação das mulheres na lógica brasileira.

Referências

[1] Sass, C. et. al. Um relato sobre estratégias de motivação e ensino de lógica de programação para e por mulheres. *Anais dos Workshops do VII Congresso Brasileiro de Informática na Educação* (WCBIE, 2018), 659-668: 2018.

[2] Brito, C.; Pavani, D.; Lima Jr., Paulo. Meninas na ciência: atraindo jovens mulheres para carreiras de ciência e tecnologia. *Gênero – Revista do Núcleo Transdisciplinar de Estudos de Gênero do Programa de Estudos Pós-Graduados em Política Social*, 16: 33-50, 2018

Mulheres na Lógica: oficina de trabalho, blog e grupo

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Como muitas pesquisadoras de nossa geração, pensávamos que nossas avós já haviam vencido a luta pela igualdade das mulheres nas ciências exatas. Claro, não há lei que impeça as mulheres de frequentarem a Universidade, obterem diplomas universitários avançados ou trabalharem na maioria das carreiras. Talvez não fosse assim no princípio do século passado (Emmy Noether não podia assistir aulas!), mas isso não é mais o caso e há muito tempo. Pelo menos nos lugares que conhecemos. Também pensávamos (erroneamente) que a razão para o número reduzido de mulheres nas ciências exatas era uma questão de tempo: que as coisas estavam mudando para melhor e logo haveria uma melhor distribuição entre os gêneros em todas as ciências. Estávamos enganadas. As coisas não estão melhorando, nem ficando consistentemente mais igualitárias. Elas estão piorando. Pelo menos, com certeza, na Computação nos EUA, onde temos dados. Em particular na Lógica, as coisas estão muito ruins. Conferências e painéis sem uma única mulher apresentadora especial convidada (Invited Speaker), sem mulheres (ou com um número extremamente baixo delas) nos comitês de programa, são muito comuns. E os números parecem estar piorando, em vez de melhorando.

Por isso estamos tentando fazer algo sobre essa situação de falta de mulheres nas ciências exatas. A lógica é um pouco mais complicada, pois há lógicas em Matemática, em Filosofia e na Computação, e essas áreas possuem problemáticas diversas. Mas ainda assim o assunto que estudamos e os problemas que resolvemos são semelhantes. Então decidimos seguir a liderança de outras mulheres na Computação e criamos uma oficina de trabalhos (um workshop) sobre e para mulheres, “Mulheres na Lógica”. Esta segue o padrão de outros encontros tais como “Mulheres no Aprendizado de Máquina” [1] e “Mulheres na Engenharia” [2] que já estão ocorrendo há alguns anos, com muito sucesso e devidamente apreciados pela audiência alvo.

Estamos agora organizando o terceiro encontro “Women in Logic” [3] que acontecerá em Vancouver, BC, Canadá, em 23 de junho de 2019, associado ao LiCS (Logic in Computer Science). Esse encontro é o sucessor de “Women in Logic 2018” em Oxford, parte da FLoC (Federated Logic Conference) e do encontro “Women in Logic 2017” em Reykjavik, Islândia. Nós temos um Grupo do Facebook “Women in Logic” com cerca de 400 membros [7]. E também temos um blog “Women in Logic” [6] onde nós expomos opiniões diversas sobre os problemas que afligem e soluções descobertas para o problema de falta de diversidade da lógica. No blog queremos também manter listas de estudos sérios e indicações para artigos disponíveis sobre as muitas facetas do problema da falta de mulheres na lógica. Lá estamos tentando registrar os números de mulheres em conferências e painéis, bem como manter registros de nossas heroínas.

Referências

[1] Workshops sobre mulheres em aprendizado de máquina,
`http://wimlworkshop.org/`

[2] Women in Engineering,
`http://www.ieee-ras.org/membership/women-in-engineering`

[3] Women in Logic 2019,
`https://sites.google.com/site/womeninlogic2019/home`

[4] Women in Logic 2018,
`https://sites.google.com/site/womeninlogic2018/welcome`

[5] Women in Logic 2017,
`https://sites.google.com/site/firstwomeninlogicworkshop/`

[6] Women in Logic – blog,
`http://womeninlogic.blogspot.com/`

[7] Women in Logic – grupo do Facebook,
`https://www.facebook.com/groups/WomenInLogic/`

Mulheres, Lógica e Matemática

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É com muito interesse que venho acompanhado, nos últimos dois anos, a proposta de diversas ações no sentido de discutir e propor mecanismos que visem a diminuição do hiato que existe entre a participação feminina e masculina em áreas de Ciências Exatas, em especial em Matemática e Lógica. Seja em grupos organizados (como *Association for Women in Mathematics* [1], *Women in Computing* [2], ou Women in Logic [3]) ou em ações individuais (como Sharon Okpoe [4]), diversas propostas têm sido formuladas para estimular, de alguma maneira, o crescimento do interesse de meninas por Matemática e áreas afins.

Para entender as questões de gênero em Matemática no Brasil, talvez a melhor referência seja o excelente texto "O 'dilema de Tostines' das mulheres na Matemática" [5]. No nosso país, várias iniciativas vêm tendo sucesso na discussão e implementação de ações nesse sentido. Citando três:

1. o ciclo de debates "Matemática: substantivo feminino" [6] realizado em 2017-2018 em diversas universidades teve, com o objetivo principal, discutir as questões de gênero na comunidade Matemática brasileira;
2. o projeto "Mulheres na Matemática da UFF" [7], coordenado pela Profa. Cecília de Souza, divulga, por meio da sua página web e redes sociais, notícias, biografias e entrevistas sobre/com mulheres na Matemática, buscando a transmissão e a difusão da participação feminina na Matemática;
3. a conferência internacional "World Meeting for Women in Mathematics — $(WM)^2$" [8], organizada como um evento satélite do ICM 2018 (International Conference in Mathematics) trouxe matemáticas de todo o mundo para discutir questões de gênero em Matemática, desafios, iniciativas e perspectivas para o futuro, com foco na América Latina. Neste encontro, pretendemos discutir as ações já consolidadas, projetos em desenvolvimento e novas atividades que colaboram com o objetivo de incentivar e despertar o interesse de meninas por Matemática, com especial ênfase em lógica.

Referências

[1] https://sites.google.com/site/awmmath/home

[2] https://women.acm.org

[3] http://womeninlogic.blogspot.com

[4] https://edition.cnn.com/2018/09/13/world/cnnheroes-abisoye-ajayi-akinfolarin-pearls-africa-foundation/index.html

[5] Christina Brech. O 'dilema de tostines' das mulheres na matemática. *Revista Matemática Universitária*, 54:1–5, 2018.

[6] https://matematicasf.wordpress.com

[7] http://mulheresnamatematica.sites.uff.br

[8] https://www.worldwomeninmaths.org

Meninas na lógica e na filosofia, e a arte graça

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Muitas das pessoas que optam por um curso superior de filosofia imaginam que jamais sofrerão novamente as aflições experimentadas ao longo do ensino básico em suas aulas de matemática ou de ciências naturais. No mais das vezes, entretanto, já no primeiro período de sua formação universitária, deparam-se com aulas de lógica. O impacto causado pela familiarização com a disciplina de lógica, tal como lecionada nos cursos de graduação em filosofia, pode ter diversas consequências: da pura e simples ojeriza à lógica (e, como corolário, aos estilos de filosofia que dela mais explicitamente se valem) — até o mais completo despreparo de futuros professores de ensino médio (EM) para a transposição didática dos conhecimentos elementares de lógica. Fecha-se, assim, um círculo não virtuoso: as licenciaturas em filosofia não preparam seus estudantes para uma docência que comporte boas relações com a lógica, o que acaba induzindo-a, senão à completa exclusão das práticas didático-filosóficas no EM, ao menos a um tratamento inadequado (porque desarticulado do currículo da filosofia e do currículo escolar como um todo). Consequentemente, segue-se alimentando as *expectativas antimatemáticas* daqueles estudantes com alguma tendência a escolher pela filosofia como curso superior. A pergunta que naturalmente se impõe diante deste quadro é: como dissolver um tal círculo? Considerando-se, por outro lado, o tema desta mesa redonda — a baixa densidade de mulheres no domínio da lógica — outra importante pergunta se pronuncia: como incluir mais meninas no mundo da lógica?

Minha comunicação vai sugerir uma resposta, em três tempos: num momento preliminar, oferecerei algumas clarificações conceituais que orientam a proposta de uma *didática mínima da lógica para o ensino médio* (formulada preliminarmente em [1]). A seguir, apresentarei exemplos de práticas didáticas – realizadas em dois contextos de ensino médio brasileiro (descritas em [2] e [3]) e dois contextos de ensino no Reino Unido (relatadas em [4]) — que ilustram pontos importantes da proposta, e ademais contribuem para o que se poderia chamar de "empoderamento lógico" de estudantes do sexo feminino. Por fim, indicarei como tais exemplos sugerem uma agenda de pesquisa e de desenvolvimento de estratégias para uma didática da lógica que faça jus a seu título de *arte da graça* [5] e, complementarmente, contribua para uma formação escolar integral, interdisciplinar, e inclusiva do ponto de vista do gênero.

Referências

[1] Secco, G. D. Filosofia no Ensino Médio: Distinções preliminares para uma *didática mínima* da lógica. *Controvérsia.* 9, 2: 89-102, 2013.

[2] Nunes, R.; Laux, M. The role of Logic in the genesis of a feminist study group: Philosophy and Logic as tools to fight against oppression. (Aceito para publicação nos *Anais do XX Encuentro Internacional de Didáctida de la Lógica - 7o SIILA.*)

[3] Dietrich, B. "Falar em Filósofas é Falar em Revolução" — Representação feminina no ensino de Filosofia. *Trabalho de conclusão de curso — Licenciado em Filosofia*, UFRGS. Orientador: Prof. Dr. Leonardo Sartori Porto, 2019.

[4] Jansset-Laurent, F. Making Room for Women in our Tools for Teaching Logic: A Proposal for Promoting Gender-Inclusiveness. *Proceedings of the 4th International Conference on Tools for Teaching Logic* pp. 65–74, 2015.

[5] Rocha, R. P. da. A didática como arte da graça. In: M. Carvalho e G. Cornelli (Orgs.) *Ensinar filosofia: volume 2*. Central de Texto, 2013.

III.3 Logic and Analytic Philosophy: Past, Present and Future

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This panel will assess historically, philosophically and logically the seminal relationship between Logic and Analytic Philosophy by examining relevant episodes and prospects for mutual enrichment and growth in both areas. It will bring together distinguished researchers who are members of the Brazilian Society for Analytic Philosophy (SBFA), the National Council for Scientific and Technological Development (CNPq) and the Brazilian Society for Logic (SBL). This panel will critically examine some procedural and conceptual paths to promote the interchange between both areas and its respective representative institutions for future projects.

Navigating between dichotomies: are we doomed to a binary thought?

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In the three lectures on Carnap's *Logical Syntax of Language* (1934) [2], instead of emphasising the differences between the more abstract sentences of formal languages and those of informal, Quine [1], paradoxically, emphasised the importance of informal discourse. He then identifies therein, contrary to what is the case in classic empiricism and in Carnap's work, the same clarity which could supposedly be found only in formal languages. What emerges from this — throughout Quine's work — is, on the one hand, the claim of the inseparability of formal and empirical aspects in sentences and, on the other hand, the claim of the impossibility of determining the meaning of sentences in isolation — two claims that are still seen as very controversial by formal sciences. At the same time, in the preface to the first edition of *Elementary Logic*(1941) [3], Quine states that to ignore the new logical techniques would be to ignore what is revolutionary in the history of philosophy. However, once again, in addition to stressing the importance of formalisation and logical mechanisms of transformation, Quine calls our attention to the challenging relationship between formal and informal languages — logical forms are symbolic abstractions of structures that are inseparable from the original grammatical content-full representations.

We have already entered the 21st century. In three years, we will be celebrating a century of Ludwig Wittgenstein's *Tractatus Logico-Philosophicus* [4]. How much research and philosophical writing has fallen under this category: analytic philosophy — which initially was defined by its use of modern logic as a method. Some say that the founder of analytic philosophy, Gottlob Frege, was not an analytic philosopher at all, for he did not use the term "analysis" to refer to his chosen logical-philosophical method. Moreover, who would argue that the best definition of the initial analytic method is in the last aphorisms of the *Tractatus*? It is also there that the fate of the analysis of language is drawn: to be blamed for the end of philosophy. Fear of analysis would permeate the entire twentieth century. One must concede that analytic philosophy often succumbed to the aridity of analysis and that its critics were right to condemn its more significant concern with pruning than with fruits. However, at the same time, in a distinctly Kantian spirit, it was responsible for indicating the exaggerations of philosophical discourses that believed to be able to fly in a vacuum. Admired and feared, it has proved to be the source of many successful therapies of natural languages: everyday, scientific, or even philosophical language. Its initial aversion to traditional metaphysics was not perennial, however. Analysis unrelated to scientism — to the empirical basis of natural sciences — could be used in *a priori* conceptual arguments and networks. Besides that, the limits of philosophical analysis applied to different content-full discourses have been expanding, and now encompass, for example, theories of mind and perception, political and ethical theories, aesthetics, beyond the initially occupied more formal scopes of logic, philosophy of language and epistemology. We could say that one of the most striking features of analytical philosophy of the twenty-first century is its capacity not only to propose methods to the sciences or to analyze its discourses but to propose concepts, arguments and theories, either by an *a priori* bias or by a naturalistic one, where the dichotomy formal/informal is not explicitly sustained. Should we demand this dichotomy to be explicitly sustained to call a philosophical activity analytical philosophy?

We are still dependent on the formal/informal dichotomy that moved so many debates one century ago. There might be no escape from it since we see the same dilemma in the sciences, where mathematical systems cohabit with empirical theories, but one can never say they merge one into the other. Maybe the only apparent way out is a pragmatic one: using languages — that can be decomposed artificially into structures and contents — as wholes to understand experienced and investigated phenomena that we call the world.

References

[1] Quine, W. V. O. Lectures on Carnap. In: W. V.O. Quine; R. Carnap. *Dear Carnap, Dear Van: The Quine-Carnap correspondence and related work*. Edited, with an introduction by Richard Creath. Berkeley: the University of California Press, pages 45–103, 1990.

[2] Carnap, R. (1934) *Die Logische Syntax der Sprache*. Wien/New York: Springer Verlag, 1968.

[3] Quine, W. V. O. (1941) *Elementary logic*. 2. ed. Cambridge, Harvard University Press, 1966.

[4] Wittgenstein, L. (1921) *Tractatus Logico-Philosophicus*. Edição Bilíngue. Tradução Luiz Henrique Lopes dos Santos. São Paulo: EDUSP, 1993.

Logic and analytic philosophy: siblings, twins, or Siamese twins?

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Analytic philosophy (or 'analytical' philosophy, as some people like to say) is somehow one hundred years old, but I don't know what it is. And I am not alone: several contemporary books and papers try to explain what is analytic philosophy. "What is Analytic Philosophy?" by Hans-Johann Glock (Cambridge University Press, 2008) is one of them, and is not that successful, according to the review of Steven D. Hales in [2]. An online text with the same title, "What is Analytic Philosophy?", by Michael Beaney (The Oxford Handbook of The History of Analytic Philosophy, 2013) tries to get a handle on the same question.

The tradition of analytic philosophy is usually regarded as circumscribed to the Anglo-Saxon and the German world, having Kant as a hero and Frege honored as the father of analytic philosophy.

But is there really a divide between continental and analytic philosophy,or is it just a matter of style? Even if I don't know what is analytic philosophy, or even if there is such a thing, as a logician I feel that the analytic style certainly has more connections to logic (if any branch of philosophy does) than its so-called continental counterpart, sometimes interested in topics as German idealism, existentialism, hermeneutics, and so on.

If analytic philosophy focuses on the problem of knowing the external world and in thought independent of historical ties, logic must play a role therein. Frege, the father of analytic philosophy, is also the founder of quantificational logic, introduced in his Begriffsschrift of 1879. In his insistence that thought, and not sentences, are the element that convey the essence of the subject -matter of his investigation, Frege regarded natural language more as an obstacle than a guide. In 1906 he wrote to Husserls that "the main task of the logician consists in the liberation from language". If we recall Russell's paradox and his influence on Frege (even if the paradox was already implicit in Cantor, and originated from the Liar' s paradox of antiquity) we will readily conclude that a kind of philosophical inquiry, more "analytic" than anything, led to the foundations of modern mathematics. Together wiht such foundations came the ambition of Hilbert to show that the mathematical castle could be proved consistent by secure, finitary means. The Hilbertian ambitions were frustrated by Gödel, whose famous incompleteness theorems of 1931, as Ulf Persson says in reviewing "A Hundred Years of Philosophy" by J. Passmore, "really belongs to philosophy, although philosophers refer to him as a mathematician, maybe because they cannot understand his proof".

One of the most relevant achievements of analytic philosophy was the establishment of the Vienna Circle in the 1920s, which was also at the vanguard of developments in logic. And the most famous American visitor to the Vienna Circle was W. V. O. Quine, who spent a year in Europe in 1932–3. Quine stayed for some time in São Paulo, where he published in Portuguese O sentido da nova lógica (1944), recently translated ([3]), a work in the crossroads of logic and analytic philosophy that influenced all the subsequent development of Brazilian logic. In Brazil, siblings at least.

References

[1] G. Frege, *Philosophical and and Mathematical Correspondence*, ed. B. MacGuinness. Oxford, 1980

[2] S. D. Hales, Review of Hans-Johann Glock, What is Analytic Philosophy?, Notre Dame Phil Reviews, 2008.10.09.
`https://ndpr.nd.edu/news/what-is-analytic-philosophy/`

[3] W. V. Quine (2018) 'The Significance of the New Logic', in *The Significance of the New Logic* (ed. and tr. W. Carnielli, F. Janssen-Lauret, and W. Pickering), Cambridge University Press.

Part of Logic is Part of Philosophy

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In 1914 (*Our Knowledge of the External World*), Bertrand Russell, an analytical philosopher if there ever was one, wrote of logic as indistinguishable from philosophy: logic is the essence of philosophy, ‘’all philosophy is logic”. Some thirty years later, however, in the preface to *Human Knowledge: Its Scope and Limits* (1948), he apparently recants this position: “Logic, it must be admitted, is technical in the same way as mathematics is, but logic, I maintain, is not part of philosophy.” This echoes two conceptions of logic we find in Greek philosophy: for Aristotle, logic is an instrument for philosophy; for the Stoics, a proper part of philosophy.

I would like to take the opportunity of this joint session of the Brazilian Logic Society and the Brazilian Society for Analytic Philosophy to talk a bit about the relations between logic and philosophy, maintaining that, at least, part of logic is part of philosophy. The question about the relevance of logic to philosophy is for me a pragmatic one: what should a philosophy student know about logic?

Logic and Philosophy in Frege's System

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As a matter of fact, contemporary logic is deeply connected with mathematics in various ways. The main contributions to logic are made by mathematicians that publish primarily in mathematical journals, and the main institutions at which logic is taught are mathematical institutions. From this perspective, it might seem to be an anachronism when logic is still considered to be also a philosophical discipline. Continental philosophers frequently question the philosophical relevance of logic. Many hold that logic does not have any special place in philosophy. We can divide the question of the relationship between logic and philosophy into the question of the relevance of logic for philosophy and the question of the relevance of philosophy for logic. My talk will be concerned with the latter question. I shall focus on Frege's conception. It seems to me that Frege took an ambivalent stance on the relevance of philosophy for logic. On the one hand, he considered it to be a serious error of a logical theory when it gets involved with metaphysical questions. Logic should not depend on any metaphysical assumptions (GGI, p. XXI). For, in his view, the laws of logic are analytic, that is, they are self-evident in the sense that, in order to justify them, it is already sufficient to make their meaning explicit. Since metaphysical assumptions are not analytic, the truth of a logical law cannot depend on such assumptions. Moreover, Frege considers the logical laws to be basic laws. This means that they are so fundamental that they cannot somehow be grounded on any non-logical principles. On the other hand, Frege clearly tackles on philosophical questions in the context of his foundation of logic. He seems to see the introduction of the truth-values (and the value courses) into his system as a metaphysically significant step that must be justified in an extra-logical way. The truth of his system depends on the existence of these objects. The aim of my talk is to clarify for what reason Frege's logic loses its metaphysical innocence. I shall argue that it is the conception of logic as a universal medium (and not as a calculus) that is responsible for the intrusion of philosophical questions into Frege's logic. According to this approach, the laws of logic do not speak about sentences, but about all objects. They are, properly speaking, metaphysical laws describing the most general features of all things.

References

[1] Frege, Gottlob 1893. *Grundgesetze der Arithmetik. Begriffsschriftlich abgeleitet*, Vol. 1. Jena: Hermann Pohle. Reprint: Darmstadt: Wissenschaftliche Buchgesellschaft, 1962.

Proof theory and its contribution to semantics

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The history of analytic philosophy merges with the very history of logic, at least of the new logic that arose in the late nineteenth / early twentieth century. To say the least, the central characters were the same. This is history. Important philosophical thoughts and ideas were clearly related to the new logic, to the new concept of logical form that then emerged. The relation between logic and Philosophy in general is not so clear nowadays (the very expression "Philosophy of Logic" is a strange sign of these strange days): logic is a discipline that can also be taught at the departments of Computer Science, Mathematics, and Linguistics. The present relation between Logic and Philosophy in general is a deep question. My aim in this (very) presentation is much simpler: I will just try to suggest how some results in Proof Theory can contribute to constructive semantics, and in particular to the justification of classical (non-constructive) principles of reasoning.

III.4 Logic and Religion

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There is a perennial philosophical debate on the relation between reason and religion. Many theologians have struggled to show that there is place, and perhaps need, to rational thought in religion. Indeed, it can be reasonably defended that only by making use of critical rational thought could we have hope of developing theology into a mature and fruitful discipline.

Logic is the discipline that studies sound arguments. In one sense, it is the canon of rationality. As such, it is essential for the critical reflection on religion, which includes, but is not restricted to, the following questions: Can there be a rational justification for the belief in God? How can we objectively appraise arguments for the existence and non-existence of God? Can we show that the several properties traditionally attributed to God, taken one by one or collectively, are consistent? To what extent is religion compatible with science and our current scientific worldview?

Alike to what happened with physics in the sixteenth and seventeenth centuries, logic was mathematized at the beginning of the twentieth century. This gave it an impressive power to deal with its basic issues as well as a remarkable interdisciplinary cross-fertilization. It also turned it into a powerful theory of representation, able to foster our understanding of a wide range of concepts and principles, such as the ones present in the living religious traditions.

Academic work done in the past halt-century, of which Gödel's famous proof for the existence of God is one of the best examples, evidences that the tools developed inside the field of logic might allow us to pose in precise terms, and alas provide fruitful answers to relevant questions related to reason and religion. Despite of this and that much of the research done in philosophy of religion in the past six decades makes use of modern logic, there is no stablished field connecting these two important facets of human thought.

Things started to change in 2015, when it took place in João Pessoa, Brazil, the 1st World Congress on Logic and Religion. The congress was followed by the publication of two special issues in two Springer journals [1] [2] and a soon-to-be-published Springer anthology [3]. In 2017 the 2nd World Congress on Logic and Religion was held in Warsaw, Poland. Selected papers delivered at the congress are being published in several special issues of the Journal of Applied Logics (College Publications) [4] [5]. The 3rd World Congress on Logic and Religion will take place in 2020 in Varanasi, India.

The purpose of this round table is to take part in these developments and contribute to the debate on logic and religion. It is composed by four contributors: Ricardo Silvestre (UFCG), who will deliver a talk entitled*Logic and Religion: An Overview*, Desidério Murcho (UFOP), who will talk about *Epistemic Responsibility and Faith*, Jean-Yves Beziau (UFRJ), who will deliver a talk entitled *Is it Possible to Prove the Existence of God* *in First-Order Logic?* and Francisco de Assis Mariano (UFPB), who will deliver a talk entitled *Assessing Possible Objections to the Structure of Probabilistic Arguments for Theism.*

References

[1] Beziau, J-Y; Silvestre, R. Special Issue on Logic and Religion. *Logica Universalis*, volume 11, issue 1, Springer, 2017.

[2] Silvestre, R.; Beziau, J-Y. Special Issue on Logic and Philosophy of Religion, *Sophia*, volume 56, issue 2, Springer, 2017.

[3] Silvestre, R.; B. Goecke; Beziau, J-Y; Bilimoria, P. (editors). *Beyond Faith and Rationality: Essays on Logic, Religion and Philosophy*. Springer, forthcoming.

[4] Silvestre, R.; Beziau, J-Y. Special Issue on Formal Approaches to the Ontological Argument, *Journal of Applied Logics*, volume 5, issue 6, College Publications, 2018.

[5] Silvestre, R.; Krajewski, S. Special Issue on Logic and the Concept of God, *Journal of Applied Logics*, College Publications, forthcoming.

Logic and Religion: An Overview

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From a historical point of view, logic has been a constant companion of philosophical reflections about religion. Arguments for and against the existence of God have been proposed and subjected to logical analysis in different periods of the history of philosophy. In discussions on the concept of God too logic has played a considerable role. With the rise of modern logic, in the beginning of twentieth century, and the analytic philosophy of religion, in the fifties, the connection between logic and religion has become much more established. The purpose of this talk is to present a general overview of these connections from the perspective of philosophical inquiry; nonetheless, something will be said about the role played by logic in world religious traditions.

Epistemic Responsibility and Faith

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Clifford [1] claimed very simply that believing without sufficient evidence is always wrong. Nineteen years later, James [2] would counter Clifford's thesis vigorously, asserting that one is entitled to believe without evidence if matters are intellectually undecided and the option at hand is genuine, in a sense he himself defines. Clifford's simple claim and James' forceful rejoinder are still good points of entry to a debate about faith and epistemic responsibility — a term neither uses, however. This talk starts by offering some remarks on the concept of evidence; to be plausible, Clifford's claim needs to rest on a generous understanding of 'evidence' that encompasses anything that appropriately counts for a claim. The difference between logical and mathematical monotonic proofs, on the one hand, and nonmonotonic reasoning, on the other, is crucial here, and this will also be spelled out. An upshot of this discussion is that (1) except in the case of logical and mathematical reasoning, and even in this case only under certain restrictions, there is always a plurality of appropriate reasons for and against any claim, and (2) error-control is always required to appropriately assess any epistemic source as well as any reasoning. Due to time restrictions the concept of rationalization will not be developed, but some remarks will be required to fully understand the challenges epistemic responsibility poses.

In the second part James's rejoinder will be used to show that taken at face value his claim is simply wrong and his arguments are far from being cogent. The main upshot here is that taken at face value his main thought turns out to have the opposite effect James intends: instead of allowing more knowledge, it blocks it. However, other written sources [3] show that James had other claim in mind, that will be labeled 'epistemic pluralism'. After briefly characterizing this concept, the aim is to argue that epistemic pluralism assumes epistemic responsibility, along the lines Clifford so forcefully insisted, instead requiring its rejection, as James might seem to believe.

In the third part, religious belief will finally be the main focus. The starting point is that religious belief is to be sharply contrasted with faith. On this basis, it will be argued that in order to be epistemically responsible faith requires stronger evidence than simple belief, religious or not, not weaker. This might be a surprise for those who hold various forms of the claim that no Cliffordian evidence, or no strong Cliffordian evidence, is needed to have an epistemically responsible faith. Plantinga's [4] properly basic belief is then briefly discussed, as an important example of an anti-Cliffordian approach. It will be argued that if no epistemic source is error-free then no purported belief is basic in any non-relative and strong sense. The general and surprising conclusion is that any account of the epistemology of faith along the lines of Plantinga's, or that follows James at face value, is a powerful enemy of faith. Clifford is the friend of true faith here, contrary to what might seem at first.

References

[1] Clifford, W. K. (1877) “The Ethics of Belief.” *Contemporary Review* 29 (December 1876–May 1877): 289–309..

[2] James, W. (1896a) “The Will to Believe” in *Writtings 1878–1899*, ed. G. E. Meyers. New York: The Library of America, 1992: 445–704.

[3] James, W. (1896b) Preface to The Will to Believe and Other Essays in Popular Philosophy, in *Writings 1878–1899*, ed. G. E. Meyers. New York: The Library of America, 1992: 447–452.

[4] Plantinga, A. (1981) “Is Belief in God Properly Basic?” *Noûs* 15.1: 41–51.

Is it Possible to Prove the Existence of God in First-Order Logic?

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Many different proofs of the existence of God have been presented, from St Anselm [1] to Gödel [5], through Descartes [3], Leibniz [7] or Kant [6]. The meaning of such a thought experiment depends in which context it is carried out. We focus here on the most famous system of logic of modern logic, namely First-Order Logic (FOL) ([4] is a good canonical presentation of it).

For examining if it is possible to proof the existence of God in FOL, we need to analyze what is a proof in FOL (we here consider "proof" from the proof-theoretical point of view as well as from the model-theoretical point of view), what is existence in FOL and how to express God in FOL.

Our research is interesting both from the perspective of theology, giving a better understanding of what proving God can mean, and from the perspective of philosophy of logic, giving a better understanding of how FOL works, its qualities, its limits, and what a proof of existence (of God, or other entities) in FOL is.

In order to develop our investigation we study some specific concrete properties which have been attributed to God, such as being the first cause. To do that we have to see how we can model causality in FOL, if this possibly makes sense (cf. [2]).

Though the traditional proofs of the existence of God are of course not presented in FOL, they are relevant for our discussion. For example Kant argues that St Anselm's proof is wrong because existence is not a predicate, which is indeed the case in FOL where existence is a quantifier.

References

[1] St. Anselm, *Proslogion*, 1077. (Trans. M.J. Charlesworth), University of Notre Dame Press, 1965.

[2] Beziau, J.-Y., Modelling causality. In *Conceptual Clarifications Tributes to Patrick Suppes (1922-2014)*, College Publication, London, 2015, pp.187-205.

[3] Descartes, R., *Meditations on First Philosophy in which the existence of God and the immortality of the soul are demonstrated*, 1641. (Trans. by J.Cottingham), Cambridge University Press, 1996.

[4] Enderton, H. B., *A Mathematical Introduction to Logic*, Academic Press, 1972.

[5] Gödel, K. *Collected Works – Vol. III* (Unpublished essays and lectures). Oxford University Press, 1995.

[6]] Kant. I., *The Only Possible Argument in Support of a Demonstration of the Existence of God*, 1763. In David Walford, editor and translator, *Theoretical Philosophy, 1755-1770. The Cambridge Edition of the Works of Immanuel Kant*, Cambridge University Press, pp. 107–201.

[7] Lenzen, W., Leibniz's Ontological Proof of the Existence of God and the Problem of "Impossible Objects", *Logica Universalis*, March 2017, Volume 11, Issue 1, pp 85–104.

Assessing Possible Objections to the Structure of Probabilistic Arguments for Theism

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The task of this paper is to present and examine possible objections to the structure of probabilistic arguments for Theism. First, I will introduce an objection to the explanatory power of theism. As Gregory Dawes [1, p. 78-99] explain, since the explanatory power of theism is a matter of how likely God would create a world such as ours, to know the explanatory power of theism, one needs to make very ambitious judgments about how God would act. But how can one really make such judgments? Second, I will develop an objection against the a priori probability of theism. Since it is a matter of how simple theism is, many philosophers, like John Mackie [2, p. 149], have attempted to either discredit the principle of simplicity or to demonstrate that theism does not meet this criterion. Third, I will present an objection against the reliability of the conclusion of the probabilistic arguments for theism. As Robert Prevost [3, p. 175] argues, it is an objection related to the disagreement many theists and atheists have on the result of their arguments, namely, the probability of the existence of God. If the structure is the same, the outcome should be the same too. Finally, I will argue that none of these objections are strong enough to knockdown the structure of probabilistic arguments for theism.

References

[1] Dawes, G. W. *Theism and explanation.* Routledge, 2009.

[2] Mackie, J. *The miracle of theism: Arguments for and against the existence of God.* Oxford University Press, 1982.

[3] Prevost, R. *Probability and theistic explanation.* Oxford University Press, 1990. (Oxford Theology and Religion Monographs)

III.5 Speech Acts in Mathematics

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Contemporary theory of illocutionary acts was originally developed inspired by Austin (1962) and further elaborated by Searle (1969, 1975, 1979) as an account of the illocutionary aspects of utterances produced in the concrete uses of language. In particular, this theory searched for a foundational account of the possibility of promises, orders, statements, suggestions, etc., and of the differences between these different acts. It was originally thought as a theory belonging only to the pragmatics and devoted solely to linguistic aspects of human actions. Later it found widespread application in the philosophy of mind, philosophy of law and, more recently, in the foundation of social sciences. However, in the philosophy of mathematics very little attention has been paid to pragmatic phenomena; indeed, pragmatic aspects of mathematical language are almost universally ignored. This is in part understandable. Mathematics is usually seen as the realm of objective truths and truth-functional propositions, and ideally its results are expressed in a purely formalized language. Typically pragmatic phenomena such as implicatures, presuppositions and illocutionary acts are ubiquitous in ordinary language but are far less evident in mathematics. However, this picture overlooks many important (and, in some cases, essential, as we shall argue) aspects of mathematical theories and mathematical practice. It is our working hypothesis that the activity of discovering and proving theorems is impregnated with some illocutionary acts perpetrated by mathematicians (either as a group or individually or through the projection of an "ideal"' subject with "ideal" judgments). For instance, they must contain some initial stipulations (definitions, postulates, choice of vocabulary, rules of inference, etc.), and include in its metalanguage typically performative terms ('therefore', 'we conclude', etc.). These illocutionary acts create a network of what Searle calls "institutional facts" (i.e., non-natural facts) that do not belong originally to the mathematical realm, but interact with that realm and are used as a kind of platform for the study of that realm. Our working hypothesis should not be understood as a defense of an anti-realist ontology of mathematical entities or propositions. Indeed, as we shall argue, this hypothesis is largely independent of any such ontology. Even if one adopts a strict realist view of mathematical entities, the discovery of these entities and of their structure depends largely on some illucutionary acts. It should also not be confounded with the trivial claim that communication among mathematicians is done in part through natural language and, as such, it is impregnated with illocutionary acts (questions, assertions, promises, praises, invitations, etc.). What we mean is that even at the level of perfectly formalized language there are some essential illocutionary acts as well as some illocutionary force indicators.

Frege's conception of the Speech Acts in Logic and Arithmetic

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In Frege's system, speech acts play an important role, at least for three reasons. First, in his logical language, the speech acts of making an assertion and of making a definition are represented syntactically by special signs, the assertion sign and the definition sign, respectively. Second, in the metalogical part of his system, the success conditions and the logical role of these speech acts are described to some extent. Thus, Frege established rules for the correct making of definitions, and he argued that, in a logical inference, all premises must be asserted. Third, in his philosophy of logic, Frege characterized, at some places, the essence of logic in terms of the assertoric force, and not in terms of the notion of truth. The aim of my talk will be to reconstruct Frege's conception of the speech acts in logic in detail and to compare it with the more modern conceptions.

References

[1] Frege, G. *Basic Laws of Arithmetic*, ed. by M. Furth, Berkeley and Los Angeles: University of California Press. 1964.

[2] Frege, G. *Posthumous Writings*. Ed. by H. Hermes, F. Kambartel and F. Kaulbach, and translated by P. Long and R. White, Oxford: Basil Blackwell. 1979.

[3] Greimann, D. 2012. A Caraterização da Lógica pela Força Assertórica em Frege. Resposta ao Marco Ruffino, *Manuscrito*, 35, 2012, pp. 61–83.

[4] Greimann, D. The Judgement-Stroke as a Truth-Operator: A New Interpretation of the Logical Form of Sentences in Frege's Scientific Language, *Erkenntnis*, 52, 2000, pp. 213–238.

Directive Speech Acts in Mathematics

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Directive speech acts (orders, requests, permissions) are ubiquitous in ordinary language. Less attention has been paid to whether there are directive speech acts in formal sciences (like logic and mathematics). In this talk I shall deal with this kind of illocutionary force indicators in formal languages. As it seems, some postulates take this form ("Let there be such and such entities? construct such and such figures… etc."), and even some inference rules (given such and such, infer this and that). I shall ask whether there is a characteristically mathematical directive.

Axioms and Postulates as Speech Acts

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In this talk we analyze axioms and postulates as speech acts, finding a substantial difference between the two. This distinction has roots in the history of mathematics and is exemplified by the difference of Euclid's and Hilbert's axiomatic methods. We will propose a classification of axioms and postulates as assertive-declartive and, respectively, directive speech acts. We show how the debate on the nature of the axioms in Hilbert's Grundlagen der Geometrie is illuminating both for understanding the role of speech acts in mathematics and the underlying ontological problem in mathematics. We will end with a broader discussion on the effect that speech act theory has on the ontology of mathematics.

Speech Acts and Fregean Realism

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The presence of speech acts is a common feature of the mathematical practice. However, only now philosophers of mathematics are paying attention to its importance and possible consequences, specially in ontology. Speech acts, like assertions and declarations, can be helpful to understand the creative features that mathematicians usually take for granted in the practice. Nonetheless, the presence of both were already recognized by Frege in the late nineteenth century. Frege introduced in its logical system a sign for assertions and a sign for definitions, therefore taking speech acts as part of logical practice. However, he often criticized the creative usage of definitions and defended that content are rather to be grasped than created by the language. Thus, Frege seems to have connected speech acts with a realist ontology. In this talk, I aim to clarify this position, showing not only how Frege anticipated the importance of speech acts, but how he believed to reconcile it with logicism.

IV

COMMUNICATIONS

Teaching Fragments of Natural Deduction

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When teaching logic using Natural Deduction, it is possible to consider fragments and, as is the case with classical logic, ask if they have a finite semantics. For example, one may take the conjunction fragment, meaning the logic resulting from just having the introduction and elimination rules for conjunction, and ask whether it has a finite semantics. Another particular case is when one considers intuitionistic logic, proved by Gödel in 1932 not to have a finite semantics, proof that also applies to positive logic, that is, the conjunction-disjunction-conditional fragment. In this direction, we prove that all fragments with the conditional do not have a finite semantics. In general, we consider all 16 possible fragments, and find finite semantics in the case of all fragments included in the conjunction-disjunction fragment and also in the case of all the fragments included in the conjunction-negation fragment. The remaining fragments, that is, the fragments with disjunction and negation, similarly to the fragments with conditional, do not have a finite semantics.

References

[1] Gentzen, G. Untersuchungen über das logische Schliessen. I. *Mathematische Zeitschrift* 39:176–210, 1935.

[2] Gödel, K. Zum intuitionistischen Aussagenkalkül. *Anzeiger der Akademie der Wissenschaften in Wien* 69:65–66, 1932; translated in Gödel (1986), pp. 222-225.

[3] Gödel, K. *Collected Works*. Vol. I. Oxford University Press, 1986.

Utilizando iALC para Formalizar a Legislação Brasileira

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Realizar uma análise mais precisa e automatizada de sistemas legais não é uma tarefa simples, uma vez que leis são criadas e alteradas continuamente no tempo, de acordo com a sociedade na qual estão inseridas. Há, claro, métodos de formação e padronização do processo de criação de leis, mas eles ocorrem de modo não centralizado, podendo, inclusive, variar em função do tempo. Com isso, os textos que representam diferentes sistemas legais, além de poderem apresentar formatos diferentes entre si, apresentam muitas distinções entre várias de suas partes, e atuar umas sobre as outras, o que dificulta ainda mais tentativas de formalização via análise textual, puramente. Isso nos leva a buscar soluções que não se limitam apenas a uma análise textual lei por lei, uma vez que é necessário saber como elas se relacionam e como podem ser aplicadas a casos reais i. e. como se realiza o processo de jurisdição, especialmente em casos onde ocorrem as ditas *antinomias reais*. Um dos métodos para se realizar isso é via formalização por lógica, que além de contribuir para a formalização dos textos legais, nos permite transformar o raciocínio jurídico em raciocínio dedutivo [12], deixando-o mais analítico.

Há propostas que envolvem formalização de leis em lógica, a grande maioria com foco central em lógica deôntica, como em [2] e [11], e poucas com outros tipos, como [1], onde é utilizada uma lógica de descrição fuzzy. A lógica deôntica toma sentenças normativas (como, por exemplo, leis), e atribui a elas valores-verdade. Há diversas variantes, como a Lógica Deôntica Padrão (SDL) (uma variante da lógica modal KD), a Lógica Deôntica Dinâmica (DDL), e a Lógica de Input/Output (I/O).

A lógica iALC, introduzida em [5] e expandida em [6] é uma lógica de descrição de caráter intuicionista, criada para lidar com textos jurídicos como alternativa à mais comumente utilizada lógica deôntica, por conseguir contornar problemas que se encontram ao utilizar essa última, em especial os chamados paradoxos *contrary-to-duty*, apresentados em [6]. Outro aspecto é sobre não ser possível atribuir valores-verdade diretamente a sentenças imperativas, e a possibilidade de lógica deôntica não lidar diretamente com normas, mas sim com seus *conteúdos*, chamados de *norm propositions* (proposições em normas), discorrido por Jørgensen em [9] e Hilpinen em [8]. Algumas dessas limitações são explicadas em maior detalhe em [7].

Nós argumentamos a favor do uso de iALC no lugar de demais lógicas para formalização de leis, em especial às lógicas de base deôntica, de modo comparativo. Também mostramos exemplos de formalização de leis brasileiras em iALC, apresentando as heurísticas encontradas para tal, focando em artigos presentes na Lei 8906, que diz respeito aos direitos e deveres de advogados. Por fim, mostramos um exemplo de aplicação desta formalização para resolução de questões de múltipla escolha da primeira fase do exame da OAB, que tem por objetivo avaliar a aptidão dos candidatos para a prática da advocacia no Brasil.

References

[1] Ioan A. Letia and Adrian Groza. Modelling imprecise arguments in description logic. *Advances in Electrical and Computer Engineering*, 9, 10 2009.

[2] Martin Mose Bentzen. Action type deontic logic. *Journal of Logic, Language and Information*, 23(4):397-414, 12 2014.

[3] Cleo Condoravdi, Dick Crouch, Valeria de Paiva, Reinhard Stolle, and Daniel G. Bobrow. Entailment, intensionality and text understanding. In *Proceedings of the HLT-NAACL 2003 Workshop on Text Meaning - Volume 9*, HLT-NAACL-TEXTMEANING '03, pages 38-45, Stroudsburg, PA, USA, 2003. Association for Computational Linguistics.

[4] Pedro Delfino, Bruno Cuconato, Edward Hermann Haeusler, and Alexandre Rademaker. Passing the brazilian oab exam: Data. *Legal Knowledge and Information Systems*, page 89, 2017.

[5] Edward Hermann Haeusler, Valéria de Paiva, and Alexandre Rademaker. Intuitionistic logic and legal ontologies. In *Proc. JURIX 2010*, pages 155-158. IOS Press, 2010.

[6] Edward Hermann Haeusler and Alexandre Rademaker. On how kelsenian jurisprudence and intuitionistic logic help to avoid contrary-to-duty paradoxes in legal ontologies. Proc. *Journal of Applied Non-Classical Logics*, 2016.

[7] Jörg Hansen, Gabriella Pigozzi, and Leendert van der Torre. Ten philosophical problems in deontic logic. In Guido Boella, Leon van der Torre, and Harko Verhagen, editors, *Normative Multi-agent Systems*, number 07122 in Dagstuhl Seminar Proceedings, Dagstuhl, Germany, 2007. Internationales Begegnungs- und Forschungszentrum f́ur Informatik (IBFI), Schloss Dagstuhl, Germany.

[8] Risto Hilperin. Norms, normative utterances, and normative propositions. *Análisis Filosófico*, 26(2):229-241, 2006.

[9] Jørgen Jørgensen. Imperatives and logic. *Erkenntnis*, pages 7: 288-96, 1937-38.

[10] S. A. Kripke. Semantical analysis of intuitionistic logic i. In J. N. Crossley and M. A. E. Dummett, editors, *Formal Systems and Recursive Functions*, pages 92-130, Amsterdam: North-Holland, 1965.

[11] L. L. Royakkers. *Extending Deontic Logic for the Formalisation of Legal Rules*. Springer Publishing Company, Incorporated, 1st edition, 2011.

[12] Vern R. Walker. Discovering the logic of legal reasoning. *Hofstra Law Review*, 35(4), 2007.

Teaching Formal Logic through Formal Proof

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Despite the importance of logic for Computer Science (CS) professionals be accepted almost universally, assuring that students acquire a good know-how in *logical deduction* usually fails in regular courses on computational logic. Teaching first-order logic to CS students tends to follow old precepts without emphasizing the importance of mastering *deduction* itself. However, motivating the necessity of a deep knowledge about the available deductive frameworks is sometimes hard, if no practical context is provided. For doing this, we have contextualized our courses of computational logic with activities concerning the formal verification of basic properties involving simple but relevant algorithms. Following this motivational premise, the foundational aspects of both natural deduction and deduction *à la* Gentzen are taught and, in parallel, the operational premises of deduction are put into practice in proof assistants.

Sorting algorithms are adequate for this purpose because students must only master primary concepts such as comparisons between non-interpreted total ordered sets (as numbers), lists and/or arrays and, at the implementational level, iteration and recursion. The proposed tasks to the students include the proof of non-trivial properties of algorithms, which require the use of induction principles, a fundamental topic for computer science professionals. Algorithmic properties are verified using the *Prototype Verification System* (PVS), that is a higher-order proof assistant based on deduction à la Gentzen with a functional specification language that supports dependent types. Despite the importance of classical results of mathematical logic should not be neglected, such as Gödel's completeness, incompleteness and undecidability theorems, as well as expressiveness bounds of first-order logic, the most important target when teaching *computational* logic is to provide enough background so that CS students can master a good understanding of mathematical deduction and computational abilities to apply it in real computational problems [2].

We know that asking students with no previous knowledge on formal methods to build a complete formalization is not, in general, a realistic and feasible task for an one semester course. In order to circumvent this problem, we have provided formalizations with holes, i.e. without some definitions and/or proofs that are supposed to be completed by the students. In fact, the declarative approach of PVS allows an easy understanding of the code since it is close to the paper-and-pencil way of writing it. In this way, students can grasp the syntax needed for that specific task, and feel motivated to fulfill these holes. The choice of the proof assistant at this initial level is not important because, after this first contact, it is not so difficult for the students to move between different systems. After introducing some concepts and relations through some easy examples, we have introduced some PVS functional specifications of sorting algorithms that we developed and are available in the sorting theories as part of the NASA LaRC PVS libraries, including the proofs of correctness of *Maxsort*, *Mergesort*, *Insertion sort*, *Quicksort*, *Bubblesort* and *Heapsort* algorithms over keys in a non-interpreted type with an abstract dichotomous preorder [1].

No statistical evidence of the effectiveness of this approach is given, but under the current crisis made evident by the well-known large CS dropout rates, following such instrumental teaching approach is important to start the preparation of all professionals that will work with the construction and design of computer systems. In fact, the need of correct software with certifica- tion is no longer restricted to critical systems, as the ones used in avionics, banks and hospitals, usually developed by super programmers. Therefore, a long term, continuous and strong prepa- ration of all CS professionals should start during undergraduate courses, as advocated in this work.

References

[1] A. A. Almeida, A. C. Rocha-Oliveira, T. M. F. Ramos, N. Fernandes, and M. Ayala-Rincón. sorting: a PVS Theory for Sorting Algorithms. Available at `http://shemesh.larc.nasa.gov/fm/ftp/larc/PVS-library/library.html` - NASA Langley Research Center PVS libraries, Last visited: September, 2016.

[2] M. Ayala-Rincón and F. L. C. de Moura. *Applied Logic for Computer Scientists – Computational Deduction and Formal Proofs*. UTCS. Springer, 2017.

Provas e sinonímia de derivações

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Em sua versão mais discutida na literatura considerada, a chamada questão da identidade de provas se deixa formular nos seguintes termos: dadas duas derivações e.g. em dedução natural, a que condições estas devem atender para que se diga que elas representam a mesma prova? Em [7], graças ao desenvolvimento ulterior dos importantes resultados de normalização obtidos em [6], Prawitz encontra oportunidade de formular uma das principais teses a respeito do tema — a saber, a "tese da normalização", enunciada ali em caráter conjectural e segundo a qual duas derivações representam a mesma prova se e somente se se reduzem a uma mesma forma normal (cf. [7] e [11]). Essa tese parece sugerir a possibilidade de uma avaliação semântica de derivações em dedução natural por meio das reduções envolvidas no processo de sua normalização, de tal forma que derivações normais passam a exercer o papel de representantes canônicos dos objetos (i.e. provas) denotados pelas diferentes derivações que a elas se reduzem — uma solução algo fregeana (ver [4]) em espírito para o problema de como explicar a possibilidade de se declarar verdadeiramente a identidade entre provas representadas por derivações diferentes.

Nesta comunicação, a "tese da normalização" será brevemente apresentada, discutida e comparada a propostas alternativas que evidenciam alguns de seus aspectos conceituais críticos. Importantes críticas presentes na literatura direcionadas à cláusula "se" da tese, habitualmente considerada livre de problemas por seus proponentes e entusiastas, serão apresentadas, e.g. a de Feferman, apresentada em [3] e respondida por Prawitz em [8], bem como algumas observações feitas por Došen em [2]. Em especial, será dada atenção a uma questão levantada em [2] concernente à relação entre a "tese da normalização" e a noção de generalidade de uma derivação. Em conjunto com alguns resultados apresentados em [5] (adequadamente transferidos para o contexto formal da dedução natural), questões dessa natureza ensejam argumentos interessantes e ainda mais fortes do que o apresentado em [2] contra a plausibilidade da tese. Além disso, será feita uma comparação entre uma noção de sinonímia de derivações baseada no conceito de isomorfismo intensional definido por Carnap em [1] — e portanto pouco dependente do formalismo específico para o qual é definida (ver [1]) — e a noção de equivalência entre derivações obtida a partir da "tese da normalização". Aplicado a derivações, o conceito de sinonímia mantém seu significado habitual, grosso modo descritível como uma forma estrita de equivalência semântica. Será facilmente percebido que a noção de sinonímia a ser apresentada, pouco extravagante e estrita às raias da esterilidade, representa um empecilho à pretensão de verdade da tese sob escrutínio — sobretudo quando levados em consideração resultados de maximalidade como o obtido por Wideback em [12].

Bibliografia

[1] R. Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*, The University of Chicago Press, Chicago, (1947).

[2] K. Došen, Identity of proofs based on normalization and generality, *The Bulletin of Symbolic Logic*, vol. 9 (2003), pp. 477-50.

[3] S. Feferman, Review of Prawitz 1971. *J. Symbolic Logic* 40 (1975), pp. 232-234.

[4] G. Frege, Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik*, NF 100, (1892) pp.25-50.

[5] J. R. Hindley. *Basic Simple Type Theory*. Cambridge University Press, (1997)

[6] D. Prawitz. *Natural Deduction. A Proof-Theoretical Study*. Stockholm (1965), Almqvist & Wiksell

[7]] D. Prawitz, Ideas and results in proof theory, in: J.E. Fenstad ed., *Proceedings of the Second Scandinavian Logic Symposium*, North-Holland, Amsterdam (1971), pp. 235-307.

[8] D. Prawitz, Philosophical aspects of proof theory, in: G. Fløistad ed., *Contemporary Philosophy: A New Survey*, Vol. 1, Nijhoff, The Hague (1981), pp. 235-277.

[9] G. Sundholm, Identity: Absolute. Criterial. Propositional. in: Timothy Childers (ed.), *The Logica Yearbook 1998*, The Institute of Philosophy, Academy of Sciences of the Czech Republic, Prague, (1999) pp. 20-26.

[10] M. Tatsuta. Uniqueness of normal proofs of minimal formulas. *The journal of symbolic logic*, vol. 58 (1993), pp. 789-799.

[11] A.S. Troelstra, Non-extensional equality, *Fund. Math.* 82 (1975), pp. 307-322.

[12] F. Widebäck, Identity of Proofs, doctoral dissertation, University of Stockholm, Almqvist & Wiksell, Stockholm (2001).

Some contributions to Boolean-valued set theory regarding arrows induced by morphisms between complete Boolean algebras

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The expression "Boolean-valued set theory" in the title has two (related) meanings, both parametrized by a complete Boolean algebra $\mathbb{B}$:

(i) The canonical Boolean-valued models in set theory, $V^{\mathbb{B}}$, as introduced in the 1960s by D. Scott, P. Vopěnka and R. M. Solovay in an attempt to help understand the, then recently introduced, notion of *forcing* in ZF set theory developed by P. Cohen ([4], [5], [1]);

(ii) The (local) "set-like" behavior of categories called *topoi*, specially in the case of the (Boolean) topoi of the form $Sh(\mathbb{B})$ ([3], [2]).

The concept of a Boolean-valued model is nowadays a general model-theoretic notion, whose definition is independent from forcing in set theory: it is a generalization of the ordinary Tarskian notion of structure where the truth values of formulas are not limited to "true" and "false", but instead take values in some fixed complete Boolean algebra $\mathbb{B}$. More precisely, a $\mathbb{B}$-valued model M in a first-order language L consists of an underlying set M and an assignment $[\varphi]_{\mathbb{B}}$ of an element of $\mathbb{B}$ to each formula φ with parameters in M, satisfying convenient conditions.

The canonical Boolean-valued model in set theory associated to $\mathbb{B}$ is the pair $\big(V^{\mathbb{B}}, [\;]_{\mathbb{B}}\big)$, where both components are recursively defined. Explicitly, $V^{\mathbb{B}}$ is the proper class $V^{\mathbb{B}} := \bigcup_{\beta \in On} V^{\mathbb{B}}_{\beta}$, where $V^{\mathbb{B}}_{\beta}$ is the set of all functions f such that $dom(f) \subseteq V^{\mathbb{B}}_{\alpha}$, for some $\alpha < \beta$, and $range(f) \subseteq \mathbb{B}$. $(V^{\mathbb{B}}, [\;]_{\mathbb{B}})$ is a model of ZFC in the sense that for each axiom σ of ZFC, $[\sigma]_{\mathbb{B}} = 1_{\mathbb{B}}$.

On the other hand, it is well known that $V^{\mathbb{B}}$ gives rise to a Boolean topos, $Set^{(\mathbb{B})}$, that is equivalent to the (Grothendieck) topos $Sh(\mathbb{B})$ of all sheaves over the complete Boolean algebra $\mathbb{B}$ ([1], [2]). The objects of $Set^{(\mathbb{B})}$ are equivalence classes of members of $V^{\mathbb{B}}$ and the arrows are (equivalence classes of) members f of $V^{\mathbb{B}}$ such that "$V^{\mathbb{B}}$ believes, with probability $1_{\mathbb{B}}$, that f is a function". A general topos encodes an internal (higher-order) intuitionistic logic, given by the "forcing-like" Kripke-Joyal semantics, and some form of (local) set-theory ([2], [3]); a Boolean Grothendieck topos is guided by a much more well behaved (Boolean) internal logic and set theory.

All the considerations above concern a fixed complete Boolean algebra $\mathbb{B}$. However, to the best of our knowledge, there are very few results on how Boolean-valued models are affected by the morphisms on the complete Boolean algebras that determine them: the only cases found are concerning automorphisms of complete Boolean algebras and complete embeddings (*i.e.*, injective Boolean algebra homomorphisms that preserves arbitrary suprema and arbitrary infima). In the present (ongoing) work, we consider and explore how more general kinds of morphisms between complete Boolean algebras $\mathbb{B}$ and $\mathbb{B}'$ induce arrows between $V^{\mathbb{B}}$ and $V^{\mathbb{B}'}$, and between their corresponding Boolean toposes $Set^{(\mathbb{B})}$ and $Set^{(\mathbb{B}')}$. In particular, we verify that these induced arrows are useful to understand and connect the corresponding Tarskian semantics, Boolean-valued semantics and Kripke-Joyal semantics.

References

[1] J. L. Bell, *Set Theory: Boolean-Valued Models and Independence Proofs*, Oxford Logic Guides v. 47, Clarendon Press, 2005.

[2] J. L. Bell, *Toposes and local set theories: An introduction*, Oxford Logic Guides v. 14, Clarendon Press, 1988.

[3] F. Borceux, *Handbook of Categorical Algebra 3: Categories of Sheaves*, Encyclopedia of Mathematics and its Applications v. 50, Cambridge University Press, 1994.

[4] T. Jech, *Set Theory: Third Millennium Edition*, revised and extended, Springer Monographs in Mathematics, Springer-Verlag, 2003.

[5] K. Kunen, *Set Theory*, Studies in Logic v. 34, Lightning Source, Milton Keyne, UK.

A resolution-based E-connected calculus

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We introduce a resolution-based calculus to reason about $\mathcal{E}$-connections, which is a method proposed in [1] for combining Abstract Description Systems (ADS), a generalisation of description and modal logics. The method of connections or $\mathcal{E}$connected logics has gained attention in the KR community, mainly for its intuitive semantics, generality and robustness regarding decidability preservation.

We provide a modular resolution-based calculus to deal with the global satisfiability problem for $\mathcal{E}$connections, extending the work in [2] which could only deal with the local satisfiability problem of the $\mathcal{E}$connection of a specific multimodal logic. In this work, we do not specify the component logics, but only require that the normal modal logics to be combined have a decidable global satisfiability problem. We propose a normal form for $\mathcal{E}$connected formulae, which results in the separation of syntactical elements related to different components. Given a formula in this normal form, the calculus is applied only to the connection between the logics and relies on the existence of complete, terminating calculi for its components. We show that the resolution calculus for $\mathcal{E}$connected logics is sound, complete and terminating.

Our approach allow us to focus on reasoning only about the restrictions imposed by the $\mathcal{E}$connections, leaving domain-specific reasoning to the component logic. More specifically, the proof search can be carried out by *querying* the components? calculi for the satisfiability of the (sub)formulae in the domain of their respective language and transfering information between the components.

Besides the correctness, completeness and termination results for the proposed calculus and its underlying normal form, we also provide a proof-of-concept implementation, based on the K_SP prover [3]. We discuss the results of the experimental evaluation and suggest some modifications that can be made to improve performance, paving the way for the development of a future modular and efficient implementation of fully automated provers for $\mathcal{E}$ connected logics.

References

[1] Kutz, Oliver; Lutz, Carsten; Wolter, Frank; Zakharyaschev, Michael. (2004). *$\mathcal{E}$-connections of abstract description systems*. Artif. Intelligence. 156. 1-73. 10.1016/j.artint.2004.02.002.

[2] Nalon, Cláudia; Kutz, Oliver. (2014). *Towards resolution-based reasoning for connected logics*. Electr. Notes in Theor. Computer Science. 305. 85-102. 10.1016/j.entcs.2014.06.007.

[3] Nalon, C.; Hustadt, U.; Dixon, C. *K_SP: A resolution-based prover for multimodal K*, Coimbra, Portugal, 2016. Automated Reasoning: 8th International Joint Conference, IJCAR 2016. pp 406-415.

Combined Proof Methods for Multimodal Logic

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In [2], a clausal resolution-based calculus for the multimodal logic K_n was proposed. Clauses are labelled by the modal level at which they occur, that is, the number of modal operators in which the scope a clause is. The calculus was implemented in the form of the automatic theorem-prover KSP [1]. The experimental evaluation of KSP shows that the prover performs very well if the propositional variables are uniformly distributed over the modal levels. However, when there is a high number of variables in just one particular level, the performance deteriorates. One reason is that the specific normal form used always generates satisfiable sets of propositional clauses. As resolution relies on saturation, this can be very time consuming.

In order to try to ameliorate the performance of KSP, we are currently investigating the use of the combination of our resolution procedure and *Boolean Satisfiability Solvers*, or SAT solvers, for short. SAT solvers can often solve hard structured problems with over a million variables and several million constraints in reasonable time. Some of the fastest implementations of such solvers rely on the well-known Conflict-Driven Clause Learning (CDCL) algorithm [3]. Very briefly, in the attempt of finding an assignment which satisfies the input, a CDCL-based SAT prover analyses the clauses and the partial assignments which have generated a conflict (i.e. a contradiction) by applications of unit resolution. From such analysis, a new clause may be learnt and is added to the clause set, prunning the search space and often abbreviating the time spent in the search for further satisfiable assignments.

Our implementation, which is work in progress, modifies KSP to invoke a SAT solver based on clause learning. We use the set of satisfiable clauses generated by KSP at each modal level as the input of the solver. As we already know that these sets are satisfiable, we are not particularly interested in the model generated by the SAT solver, but only in the learnt clauses, which are then fed back to KSP in order to guide the application of the modal resolution rules. We believe that by carefully choosing the set of clauses we feed the SAT solver, and appropriately using the learnt clauses generated, we may be able to reduce the time the prover spends during saturation and, thus, the overall performance of KSP.

References

[1] Nalon, C.; Hustadt, U.; Dixon, C. *KSP: A resolution-based prover for multimodal K*, Coimbra, Portugal, 2016. Automated Reasoning: 8th International Joint Conference, IJCAR 2016. pp 406–415.

[2] Nalon, C.; Hustadt, U.; Dixon, C. *A modal-layered resolution calculus for K*, International Conference on Automated Reasoning with Analytic Tableaux and Related Methods, 185–200, 2015.

[3] Biere, A.; Heule, M.; Maaren, H; Walsh, T. *Conflict-driven clause learning SAT solvers*, Handbook of Satisfiability, Frontiers in Artificial Intelligence and Applications 131–153, 2009.

Podem Máquinas Entrar no Jogo da Linguagem?

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Pensar é uma dádiva humana. É o que separa nós das demais inteligências. Na verdade, que provas existem que, além de nós mesmos (neste ponto, até uma visão individualista), algo mais possua uma vida mental? Essa visão wittgensteiniana nos leva ao cerne deste trabalho.

O Processamento de Linguagem Natural é uma das subáreas mais encantadoras da Inteligência Artificial, encontrando-se em uma intersecção com a linguística: trata-se da parte que lida com a linguagem. De uma maneira extremamente simplificada e em modelos de aprendizado supervisionados, opera analisando textos com inputs humanos ao algoritmo, e em modelos não-supervisionados, atua localizando padrões. Apesar de magnífico, ainda soa muito mecânico, algo que não pertence ao universo humano.

No entanto, assim como diversos aspectos do comportamento humano não encontram explicação plausível, em deep learning, o aprendizado profundo da Inteligência Artificial (I.A), a premissa também pode servir. Também, em 2017 foi publicado pela Revista Nature um artigo sobre uma rede neural capaz de treinar a si própria, sem dados humanos, guia ou conjunto de regras, isto é, sem tabula rasa, para predizer os movimentos no jogo Go.

Quando transferido para a área linguística, seria o processo epistemológico autônomo da máquina, bem como a interação além de uma linguagem privada uma das chaves para equiparar computadores ao pensamento humano?

A ideia principal de Wittgenstein era que o significado de palavras, sentenças e quaisquer expressões de linguagem é determinado por seu uso cultural estabelecido ou estabelecido. O significado e a compreensão das expressões são, portanto, fundamentados no comportamento social, não na estrutura sintática / semântica / linguística ou na gramática complexa das frases ou frases. A gramática só nos dá meios e ferramentas para construir novas expressões de linguagem e desconstruir expressões, mas não determina o significado ou a compreensão de expressões. Uma máquina capaz de captar além de caracteres, entraria no Jogo?

Este trabalho pretende compreender o deep learning como um método que pode entrar no Jogo de Linguagem, inclusive desenvolvendo contextos, brincadeiras e ambiguidades, e, por fim, tenta vislumbrar se haveria algo que diferenciaria o modo de Jogo entre homens e máquinas.

Bibliografia

[1] Anscombe, Elizabeth. *Eine Einführung in Wittgensteins "Tractatus"*. Themen in der Philosophie Witgensteins. Aus dem Englischen von Jürgen Koller. Turia + Kant, Wien/Berlin 2016

[2] Burkert, Herbert. Systemvertrauen: Ein Versuch über einige Zusammenhänge zwischen Karte und Datenschutz. 1991. 12 p. Artigo. *Card Euro-Jornal*, Heft 1, 1991.

[3] Diamond, Cora (1989). *Wittgenstein's Lectures on the Foundations of Mathematics*. University Of Chicago Press.

[4] Kaczynski, Theodore John. *Industrial Society and Its Future*. EUA: Pub House Books, 2018.

[5] Lange, Ernst. *Ludwig Wittgenstein. Philosophische Untersuchungen, eine kommentierte Einführung*. Schöningh, Paderborn 1998,

[6] Loper, Edward. *Natural Language Processing with Python*. EUA, O'Reilly 2009.

[7] Luhmann, Niklas. *Vertrauen: ein Mechanismus der Reduktion sozialer Komplexität*. 3. ed.: Stuttgart, 1973.

[8] Nagel, Thomas. "What is it like to be a bat?" *The Philosophical Review*, Vol. LXXXIII, No 4, p. 435-50, 1974.

[9] Rosenthal, David (ed.). *Materialism and the mind-body problem*. Indianapolis: Hackett, 2000.

[10] Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus*. Disponível no texto original em: `http://tractatus-online.appspot.com/Tractatus/jonathan/D.html`

[11] Žižek, Slavoj. *Menos Que Nada: Hegel e a Sombra do Materialismo Dialético*. SP: Bomtempo Editorial, 2013.

Compiling C code to PDL

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In critical systems failure or malfunction may result in a big outcome, like deaths or several financial damage. Model checking provide an automated method for proof the correctness regarding the requirements. It is a convenient technique to be used on systems that require reliability.

Propositional Dynamic Logic (PDL) is a formal system tailored to reason about programs. It has a simple Kripke model and good model checking performance. This work presents the implementation of a compiler from a subset of the C language to PDL, integrated with a model checker. It leads to an environment to reason about C code. The implementation is open source and is available at `https://github.com/phgeraldeli/CtoPDLCompiler`.

References

[1] M. J. Fischer, R. E. Ladner, Propositional dynamic logic of regular programs. *J. Comput. System Sci.* 18(2): 194–211, 1979

On the very idea of choosing a logic

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Antiexceptionalism is the view that theory choice in logic is not special, it just follows a rational procedure of theory choice as in other branches of empirical science. In this paper, we present a challenge to the view based on what was dubbed "Kripke's adoption problem". The challenge stems from the very fundamentality of logic: choosing a logic requires that some evidence for a specific choice of logical theory be rationally evaluated. Evaluation, on its turn, requires that logical inferences be made, which on its turn requires that a logic is available to begin with. Then, one cannot choose a logic unless one has already chosen a logic. In these circumstances, it seems two related problems will appear: i) the data available is theory laden (logic-laden, that is) and judgment is relative to a logic or ii) we will assume a logic without having to run the very method of rational choice to begin with. Both horns bring a lot of trouble for antiexceptionalism. Briefly put, in the first case, a non-question begging debate seems precluded by the very nature of the method employed for theory choice and also, of course, by the nature of the subject of debate,, which is all pervasive. In the second case, adopting a logic without running the method seems to speak against the very idea of having a method for logical choice. We shall propose that the trouble arise from the presupposition that natural or intuitive reasoning have a logic, which must be settled by such methods of theory choice. We shall highlight to what extent this presupposition appears in the works of philosophers such as Priest, Hjortland and Williamson.

Horn filter pairs and Craig interpolation property

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The notion of filter pair, introduced in [2], is based on the categorial behavior of the lattice of theories associated to a logic and is used as a tool for creating and analyzing logics. Given a signature Σ, a filter pair consists of a contravariant functor G from Σ-algebras to the category of algebraic lattices together with a natural transformation $i : G \to P(-)$ from G to the functor taking an algebra to the power set of its underlying set, which preserves arbitrary infima and directed suprema. We have showed that every Tarskian logic arises from a filter pair and that translations of logics arise from morphisms of filter pairs. Congruence filter pairs are filter pairs for which the functor $G = Co_K$ is the relative congruence functor on a class K of Σ-algebras, i.e., on objects Co_K sends a Σ-algebra M to the lattice of congruences θ such that $M/\theta \in K$. Congruence filter pairs are useful to treat algebraizable, equivalential, protoalgebraic and truth-equational logics.

Horn filter pairs are a generalization of congruence filter pairs, which allow to encode not just algebraic semantics of a logic but also semantics in a class of first order structures axiomatized by universal Horn sentences. In this work, besides introducing Horn filter pairs, we give a criterion for when amalgamation in the class of structures implies the Craig interpolation property of the associated logic. This criterion subsumes the case of algebraizable logics and the left variable inclusion companions of [1].

References

[1] S. Bonzio, T. Moraschini and M. Pra Baldi, Logics of left variable inclusion and Plonka sums of matrices, arXiv preprint, https://arxiv.org/abs/1804.08897, 2018

[2] P. Arndt, H.L. Mariano and D.C. Pinto. Finitary Filter Pair and Propositional Logics To appear in: South American Journal of Logic (Proceedings of EBL 2017), 2019.

Um Estudo Comparativo Sobre Técnicas de Compressão de Provas

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Provas lógicas podem ser utilizadas no desenvolvimento de softwares validando o funcionamento de porções de código, na fabricação de hardwares, um projeto de circuito pode ser validado através da prova de uma conjectura que o descreve, além de outros exemplos. A Prova Automática de Teoremas (*Automated Theorem Proving* - ATP) é a área da Ciência da Computação que usa programas de computador para a geração automatizada, ou semi-automatizada, de provas. No entanto, um provador de teoremas pode produzir provas demasiadamente grandes. O tamanho de uma prova pode prejudicar sua utilização prática, visto que pode ser inviável extrair algum significado, além de que manipular grandes volumes de dados pode ocasionar problemas de implementação para os provadores. O tamanho das provas também possui algumas importantes implicações teóricas na área da complexidade computacional. O problema de determinar se uma fórmula é uma tautologia da Lógica Proposicional Intuicionista e do seu fragmento puramente implicacional (Lógica Proposicional Minimal Implicacional - $M\supset$) é PSPACE-Completo. Apesar de ser um fragmento, $M\supset$ é capaz de simular a Lógica Proposicional Intuicionista através de uma tradução polinomial. Qualquer lógica proposicional com um sistema de dedução natural que satisfaça o princípio da subfórmula possui o problema de determinar tautologias em PSPACE. Saber se qualquer tautologia em $M\supset$ admite provas de tamanho polinomialmente limitado está relacionado com saber se NP = PSPACE. Provas em dedução natural podem ser representadas em diferentes formatos. No estilo de Gentzen-Prawitz, as provas possuem o formato de uma árvore, onde as fórmulas são os nós, e as regras e os números de descartes são as arestas. No estilo de Jakowski-Fitch, as provas são sequências de passos numerados, seguido pela identificação da regra e sua referida justificativa. O tamanho das provas podem ser aferidos a partir de diferentes pontos de vista. A quantidade de linhas, e até a quantidade de símbolos podem ser utilizados para mensurar o tamanho de uma prova. Técnicas de compressão de provas reportadas na literatura utilizam duas abordagens principais para comprimir provas: gerar provas já compactadas, ou comprimir uma prova já gerada. Nosso trabalho realiza um estudo comparativo sobre as técnicas de compressão de provas reportadas na literatura e as técnicas tradicionais de compressão de dados, ressaltando as características das técnicas e os respectivos impactos causados no tamanho resultante das provas. Mostramos os resultados da aplicação dos métodos de compressão sobre provas de tautologias da $M\supset$, ressaltando que os métodos de compressão de provas são as melhores opções em relação aos métodos de compressão de dados tradicionais, a principal vantagem dos primeiros métodos é a possibilidade de manipulação do dado compactado sem a necessidade de descompressão.

Referências

[1] Statman, R. Intuitionistic propositional logic is polynomial-space complete. *Theoretical Computer Science*. Vol. 9. p. 67-72. 1979.

[2] Haeusler, E, H. Propositional logics complexity and the sub-formula property. In: Proceedings International Workshop on Developments in Computational Models (DCM 2014), Vienna, Austria, 2014.

[3] Gordeev, L.; Haeusler, E. H. Proof Compression and NP Versus PSPACE. *Studia Logica*. Special Issue: General Proof Theory. 2017. https://doi.org/10.1007/s11225-017-9773-5.

Von Neumann Regular $\mathscr{C}^\infty$-Rings and Applications to Boolean Algebras

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In this paper we introduce and explore the notion of a "von Neumann regular $\mathscr{C}^\infty$-ring", as well as some of its applications to the theories of Boolean spaces and Boolean algebras.

A $\mathscr{C}^\infty$-ring can be considered, from an universal algebraic viewpoint, as a pair $\mathfrak{A} = (A, \Phi)$, where the carrier A is a (non-empty) set and Φ is a function that assigns to every smooth $n-$ary function, $f : \mathbb{R}^n \to \mathbb{R}$, an $n-$ary function symbol, $\Phi(f) : A^n \to A$, preserving all the equational relationships between real smooth functions. Given two $\mathscr{C}^\infty$-rings $\mathfrak{A} = (A, \Phi)$ and $\mathfrak{B} = (B, \Psi)$, a homomorphisms from $\mathfrak{A}$ to $\mathfrak{B}$ is a function $\varphi : A \to B$ such that for every smooth function $f : \mathbb{R}^n \to \mathbb{R}$, for every $n \in \mathbb{N}$, we have $\Psi(f) \circ \varphi^{(n)} = \varphi \circ \Phi(f)$. These data - together with the ordinary composition of functions and identity morphisms - compose a category that we denote by $\mathscr{C}^\infty$**Rng**.

There is a natural "forgetful functor" from $\mathscr{C}^\infty$**Rng** to **CRing**, obtained by "forgetting" the interpretations of all smooth functions, except for the sum, the product, the opposite and the interpretations of $0, 1 \in \mathscr{C}^\infty(\{*\}, \mathbb{R})$. Such a forgetful functor, that we denote by $\widetilde{U} : \mathscr{C}^\infty\mathbf{Rng} \to \mathbf{CRing}$, provides us with a convenient definition of a *von Neumann regular $\mathscr{C}^\infty$-ring* as a $\mathscr{C}^\infty$-ring $\mathfrak{A} = (A, \Phi)$ whose underlying ring is a von Neumann regular ring in the ordinary sense, that is,

$$\widetilde{U}(\mathfrak{A}) \models (\forall a \in A)(\exists e \in \mathrm{Idemp}(A))(\exists x, y \in A)(a \cdot x = e \,\&\, e \cdot y = a),$$

where $\mathrm{Idemp}(A) = \{b \in A : b^2 = b\}$ is the Boolean algebra consisting of all idempotent members of A.

The category of all von Neumann regular $\mathscr{C}^\infty$-rings, together with their homomorphisms, compose the category we denote by $\mathscr{C}^\infty$**vNRng**. We prove, using different methods, that $\mathscr{C}^\infty$**vNRng** is a reflective subcategory of $\mathscr{C}^\infty$**Rng** and we show, among other things, that the "Moerdijk-Reyes" Zariski spectrum functor (first defined in[13]) restricted to this subcategory, $\mathrm{Spec}^\infty \restriction : \mathscr{C}^\infty\mathbf{vNRng} \to \mathbf{Top}$ has, as its essential image, the category of all Boolean spaces. Also, in this case, the structure sheaf (first defined in [14]) is such that its stalks are $\mathscr{C}^\infty$-fields. In fact, we prove that this property characterizes *all* von Neumann regular $\mathscr{C}^\infty$-rings. Moreover, the subcategory of $\mathscr{C}^\infty$**Rng** consisting of all von Neumann regular $\mathscr{C}^\infty$-rings is characterized as the closure under small limits of the category of $\mathscr{C}^\infty$-fields, *i.e.*, it is the smallest subcategory of $\mathscr{C}^\infty$**Rng** which contain all $\mathscr{C}^\infty$-fields and is closed under small limits.

Finally we show that von Neumann regular $\mathscr{C}^\infty$-rings classify Boolean spaces in the following strong sense: for a fixed $\mathscr{C}^\infty$-field, $\mathbb{K}$, for each pair of Boolean algebras B, B'

and each Boolean algebra homomorphism, $h : B \to B'$, there is a pair of von Neumann-regular $\mathscr{C}^\infty$-rings, V, V', that are also $\mathbb{K}$-algebras, and a $\mathscr{C}^\infty$-homomorphism $f : V \to V'$ such that: $f \restriction_{\mathrm{Idemp}(V)}: \mathrm{Idemp}(V) \to \mathrm{Idemp}(V')$ is (naturally) isomorphic to $h : B \to B'$.

References

[1] P. Arndt, H.L. Mariano, The von Neumann-regular Hull of (preordered) rings and quadratic forms, *South American Journal of Logic*, Vol. X, n. X, pp. 1-43, 2016.

[2] J.C. Berni. Alguns Aspectos Algébricos e Lógicos dos Anéis $\mathscr{C}^\infty$ (*Some Algebraic and Logical Apects of $\mathscr{C}^\infty$–Rings*, in English). PhD thesis, Instituto de Matemática e Estatística (IME), Universidade de São Paulo (USP), São Paulo, 2018.

[3] F. Borceux, *Handbook of Categorical Algebra*, 2, Categories and Structures, Cambridge University Press, 1994.

[4] F. Borceux, *Handbook of Categorical Algebra*, 3, Categories of Sheaves, Cambridge University Press, 1994.

[5] M. Bunge, F. Gago, A.M. San Luiz, *Synthetic Differential Topology*, Cambridge University Press, 2017.

[6] D. van Dalen, *Logic and Structure*, Springer Verlag, 2012.

[7] P. Johnstone, Rings, Fields and Spectra, *Journal of Algebra* 49, 1977.

[8] D. Joyce, Algebraic Geometry over $\mathscr{C}^\infty$–Rings, volume 7 of *Memoirs of the American Mathematical Society*, arXiv: 1001.0023, 2016.

[9] A. Kock, *Synthetic Differential Geometry*, Cambridge University Press, 2006.

[10] S. MacLane, I. Moerdijk, *Sheaves in Geometry and Logic, A first introduction to Topos Theory*, Springer Verlag, 1992.

[11] M. Makkai, G.E. Reyes, *First Order Categorical Logic - Model-Theoretical Methods in the Theory of Topoi and Related Categories*, Springer Verlag, 2008.

[12] R. S. Pierce, *Modules over Commutative Regular Rings*, Memoirs of the AMS 70, American Mathematical Society, Providence, USA, 1967.

[13] I. Moerdijk, G. Reyes, *Rings of Smooth Functions and Their Localizations I*, Journal of Algebra, 99, 324-336, 1986.

[14] I. Moerdijk, G. Reyes, N.v. Quê, *Rings of Smooth Functions and Their Localizations* II, Mathematical Logic and Theoretical Computer Science, Lecture Notes in Pure and Applied Mathematics 106, 277-300, 1987.

[15] I. Moerdijk, G. Reyes, *Models for Smooth Infinitesimal Analysis*, Springer Verlag, 1991.

[16] H. Schubert, *Categories*, Springer Verlag, 1972.

Strong necessity and consistency

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We present a formal treatment of the modality of *strong necessity*, denoted by $\boxtimes$, in order to investigate in what sense this modality can have a *formal/mathematical* interpretation. The alethic interpretation of $\boxtimes$ is that '$\boxtimes\phi$' means that ϕ is both necessary and possible. But we propose a formal interpretation close to the provability interpretation of modalities. Since Gödel's work ([4]) on provability interpretation of the Intuitionistic Propositional Calculus, several proposals of provability interpretations of modal logic have been made ([5,2,1]). For example, Solovay ([5]) proposes to interpret the operator $\Box$ of the modal logic **KGL** as provability in Peano Arithmetic (PA). That is, '$\Box\phi$' means that "ϕ^t' is provable in PA', where *t* is a translation of the sentences of **KGL** to sentences of PA. Our proposal is to give a formal interpretation for the operator $\boxtimes$ as follows: '$\boxtimes\phi$' means that "ϕ' has a model and no countermodels'. The operator $\boxtimes$ is not normal due to the failure of the rule of necessitation with respect to the class of all models. The logic we will consider has the operator $\boxtimes$ as primitive. and the logics with $\boxtimes$ will be called $\boxtimes$-logics. First, we introduce the basic $\boxtimes$-logic, called $\mathbf{B}^{\boxtimes}$ as well as its soundness and completeness. Then, we present some expressivity results concerning the language of these logics, comparing them to the language of normal modal logics ($\Box$-logics). We apply a general method provided by Gilbert & Venturi [3] in order to prove completeness and soundness theorems for $\boxtimes$-logics which extend $\mathbf{B}^{\boxtimes}$ through a specific translation $()^{\boxtimes}$ of $\Box$-logics. For completeness, the normal modal logic **L** is required to be canonical. In what concerns soundness, the class of frames $\mathbb{C}_{\mathbf{L}}$ which characterizes **L** is required to be *robust under seriality*, that is, the frame $\mathbb{C}_{\mathbf{L}}$ plus the condition of seriality, $\mathbb{C}_{\mathbf{LD}}$, also characterizes **L**. Thus, given a canonical $\Box$-logic **L**, the translation $()^{\boxtimes}$ applied to **L** produces a sound and complete $\boxtimes$-logic $\mathbf{L}^{\boxtimes}$. Then, from these results we intend investigate how $\boxtimes$-logics can shed light on the concept of consistency.

References

[1] S. Artemov and T. Straßen. The basic logic of proofs. In *International Workshop on Computer Science Logic*, Springer (28): 14–28, 1992.

[2] G. Boolos. The logic of provability. *Cambridge university press*, 1995.

[3] D. Gilbert, G. Venturi. Reflexive insentive modal logics. *The Review of Symbolic Logic*, 9(1): 167–180, 2016.

[4] K. Gödel. Eine interpretation des intuitionistischen aussagenkalküls. *Ergebnisse eines mathematisches Kolloquiums* (4): 39–40, 1933.

[5] R. Solovay. Provability interpretations of modal logic. *Israel journal of mathematics*, 25(3-4): 287–304, 1976.

Tarski's two notions of consequence

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In the 1930s Tarski introduced two notions of consequence. We present and compare them. We also examine their relations with similar notions presented by others.

The first notion is connected with the consequence operator theory and was presented in (Tarski 1928, 1930a,b). The second one is based on the concept of model and was presented in (Tarski 1936a,b,c, 1937).

We argue that it is misleading to understand and qualify the first as a syntactic or proof-theoretical notion. A more appropriate qualification is "abstract consequence". The word "abstract" has indeed been later on used by Suszko for his theory of abstract logics, a continuation of the consequence operator theory (see Brown and Suszko, 1973).

Regarding the second notion, we point out that besides Bolzano, already notified by Scholz (1937), other people had similar ideas, in particular Abu'l-Barakāt (see Hodges 2018) and Wittgenstein (1921, 5.11). And we compare this notion, in particular using (Corcoran-Sagüillo 2011), with the one later on developed in model theory by Tarski himself (1954-1955).

We discuss the relations between the two notions, emphasizing that the model-theoretical one is a particular case of the consequence operator theory one, and discussing fundamental features of them that can be used to prove a general completeness theorem, following the line of Gentzen's work (1932) about Hertz's Satzysteme (1929), framework connected with the consequence operator.

References

[1] J. Brown et Roman Suszko, 1973, "Abstract logics", *Dissertationes Mathematicae*, 102, 9-41.

[2] J. Corcoran and J. M. Sagüillo, 2011, "The absence of multiple universes of discourse in the 1936 Tarski consequence-definition paper", *History and Philosophy of Logic*, 32, pp.359-374.

[3] G. Gentzen, 1932, "Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen", *Matematische Annalen*, 107, 329-350.

[4] P. Hertz, 1929, "Über Axiomensysteme für beliebige Satzsysteme", *Matematische Annalen*, 101, 457-514.

[5] W. Hodges, 2018, "Two early Arabic applications of model-theoretic consequence", *Logica Universalis*, 12, 37-54.

[6] H. Scholz, 1937, "Die Wissenschaftslehre Bolzanos. Eine Jahrhundert–Betrachtung", *Abhandlungen der Fries'schen Schule*, 6, 399-472.

[7] A. Tarski, 1928, "Remarques sur les notions fondamentales de la méthodologie des mathématiques", *Annales de la Société Polonaise de Mathématique*, 7, 270-272.

[8] A. Tarski, 1930a, "Über einige fundamenten Begriffe der Metamathematik", *C. R. Soc. Sc. et Lett. de Varsovie XXIII*, Classe III, 23, 22-29

[9] A. Tarski, 1930b, “Fundamentale Begriffe der Methodologie der deduktiven Wissenschaften. I”, *Monatshefte für Mathematik und Physik*, 37, 361-404.

[10] A. Tarski, 1936a, “Über den Begriff der logischen Folgerung”, *Actes du Congrès International de Philosophie Scientifique*, vol.7, Hermann, Paris, pp.1–11.

[11] A. Tarski, 1936b, “O pojẹciu wynikania logicznego”, *Przeglad filozoficzny* 39, 58-68.

[12] A. Tarski, 1936c, O Logice Matematycznej i Metodzie Dedukcyjnej, Atlas, Lvov- Warsaw. 1936. Eng. ed., *Introduction to Logic and to the Methodology of Deductive Sciences*, OUP, Oxford, 1941.

[13] A. Tarski, 1937, “Sur la méthode déductive”, *Travaux du IXe Congrès International de Philosophie*, vol.6, Hermann, Paris, pp.95-103.

[14] A. Tarski, 2003, “On the concept of following logically”, Translated from the Polish and German by M. Stroińska and D. Hitchcock, *History and Philosophy of Logic* 23, 155-196.

[15] A. Tarski, 1954-55, “Contributions to the theory of models. I, II, III”, *Indigationes Mathematicae*, 16, 572-588; 17, 56-64.

[16] L. Wittgenstein, 1921, “Logisch-Philosophische Abhandlung”, *Annalen der Naturphilosophie*, 14, 185-262.

The Capture of Collection, Replacement, Specification and Choice

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Let W be the weak set theory Zermelo set theory Z without the axiom schema of specification. We isolate an operation the capture of $\alpha(x,y)$, for $\alpha(x,y)$ any first order condition in the language of set theory on the two indicated free variables, by presupposing an axiom schema of capture:

Axiom 1. $\forall v \exists w \forall x(x \in w \leftrightarrow \exists y(y \in v \wedge \alpha(y,x) \wedge \forall z(\alpha(y,z) \to x = z)))$

LetZF be Zermelo-Fraenkel set theory. We can show:

Theorem 2. $ZF = W +$ **Axiom 1**

If one has an adequate binary well-ordering constant $\trianglelefteq$ one may adjust **Axiom 1** so as to also obtain a Theorem of Choice. In some set theories, like e.g. the author's £ as partially set out in [1] and [2], capture with well-ordering in contexts where extensionality fails is stronger than ordinary capture in that the former provides collection and the latter not. Capture is advantageous in the elegance that it avoids the cumbersome restriction to *functional* condition, and because it allows for a more natural acquisition of choice principles. Capture is also helpful in the context of £ because it allows for more flexibility in expressing useful closure principles. We will present some of the proofs and discuss more on motivation and potential applications.

References

[1] Bjørdal, F. A. Librationist Closures of the Paradoxes. *Logic and Logical Philosophy*, 21(4), 323-361, 2012.

[2] Bjørdal, F. A. *Elements of Librationism*, arXiv:1407.3877v8.

What is a logical theory? On theories containing assertions and denials

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The standard notion of formal theory, in Logic, is in general biased exclusively towards assertion: it commonly refers only to collections of assertions that any agent who accepts the generating axioms of the theory should also be committed to accept. In reviewing the main abstract approaches to the study of logical consequence, we point out why this notion of theory is unsatisfactory at multiple levels, and introduce a novel notion of theory that attacks the shortcomings of the received notion by allowing one to take both assertions and denials on a par. This novel notion of theory is based on a bilateralist approach to consequence operators, which we hereby introduce, and whose main properties we investigate in the present paper.

References

[1] Blasio, C.; Caleiro, C.; Marcos, J. What is a logical theory? On theories containing assertions and denials. *Synthese*, 2019, in print.

The classical foundations of mathematics and the representation of directionality

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Russell took the coherentization of calculus by Cauchy, Weierstrass and Cantor as pointing in the direction of the so called at-at theory of motion (and change), now the received view about the nature of change and time: to say that an object moves during a time period Δt is to say no more than it occupies different positions at different instants of times during Δt. Thus, far from providing us with a coherent notion of a state of change, the rigorized notions of limit and derivative, Russell thought, give us reasons to deny that there are any such things: "Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest." ([4], 353)

Russell's idea, however, was not intended to concede to Zeno that change is thereby unreal: "[p]eople used to think that when a thing changes, it must be in a state of change, and when a thing moves, it is in a state of motion. This is now known to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another." (ibid. p. 66.). So the view has it that things do indeed change. Only, they do not change by being changing.

I argue that this contention is untenable. Not only does the classical foundation deprives velocity of its putative causal role (see [1]) but, furthermore, if one takes the received view to its coherent consequences, one has to abandon the idea that time is real at all, and hence that things can change in any literal sense.

Change and the passage of time are essentially directed (*from* earlier times to *later* times). If one cannot represent directionality, one has thereby lost any hope of representing time and change, since to represent time as flowing and things as changing one has to represent time as flowing and things as changing in one direction only.

My argument starts by showing that the classical foundation of calculus makes mathematical structures unsuited to represent directed physical magnitudes, such as velocities, forces and accelerations. Drawing on a recent literature on the metaphysics of asymmetrical relations ([3], [5]), the notion of 'senses' of asymmetrical relations, on which alone the possibility to represent directed magnitudes depends, is argued to be essentially different from the relevant notion of directionality, and unsuited to stand for the latter. Finally, the idea that mathematical vectors provide magnitudes with a direction is considered and dismissed.

My argument provides both a vindication and an explanation for the fact, often emphasized by Bergson ([2]), and vigorously denied by Russell and his followers, that mathematics "spatializes" time.

References

[1] Arntzenius, F., 2000, Are There Really Instantaneous Velocities? *The Monist* 83: 187–208.

[2] Bergson, H., 1998 [1911], *Creative Evolution*, tr., Arthur Mitchell, New York: Dover.

[3] Fine, K. 2000, Neutral Relations, *The Philosophical Review*, 109, pp. 1–33.

[4] Russell, B., 1938, *Principles of mathematics*, W. W. Norton & Company, inc, New York.

[5] Williamson, T. 1985, Converse Relations, *The Philosophical Review*, 94, pp. 249–262.

Recursion on the Inferentialist Approach to Language

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Traditional presentations of formal logic typically begin with what is taken to be the simplest elements of language: singular terms (constructed with constants, variables and functions) and predicates. Next, those elements are combined to form atomic sentences, which are further combined with the help of logical connectives to form arbitrarily complex sentences. Finally, one introduces a relation among sentences which is considered the fundamental concept to be studied in logic: the notion of logical inference or logical consequence (see Figure 1).

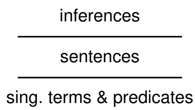

Figure 1. Traditional model of language

But this is by no means the only possible way to understand language. Kant and Frege, for example, question the assumption that singular terms and predicates can be made sense of outside the context of a sentence, and propose to take judgments or propositions (i.e., sentences) as the fundamental unit of language (see Figure 2, left). And Robert Brandom takes one step further, questioning the assumption that sentences can be made sense of outside the context of a language game, whose basic unit is the notion of material inference (see Figure 2, right).

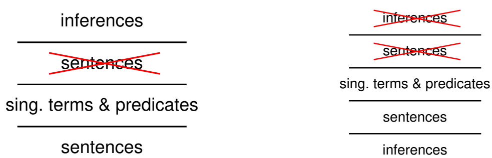

Figure 2. Alternative models of language

Now, after turning the traditional model upside down, Brandom has to face a natural question and a more serious problem for inferentialism. The question is: what is the role of the singular terms and predicates at the top of the model? And the problem is the issue of the productivity of the language: how to explain our ability to produce and understand novel sentences which have never been used in the language game before? Brandom's ingenious idea is to use the singular terms and predicates to solve the productivity problem through a so-called "two-stage compositional strategy": from an initial basis of sentences and inferences, one may project semantic relations involving pairs of singular terms and predicates, which can then be used to derive inferential relations between (novel) sentences formed by arbitrary combinations of singular terms and predicates (see Figure 3, left). The interesting observation is that, with this move, Brandom basically recovers the original layers of the traditional model, which is now seen to be as an elaboration of a more fundamental inferential basis of language.

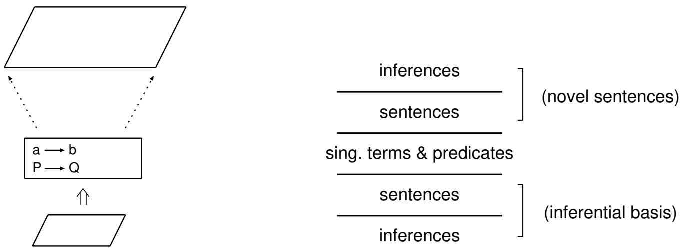

Figure 3. Two-stage compositional strategy

In this work, we propose a variation of Brandom's scheme in the form of a so-called "three-stage structural-compositional strategy". The basic idea of this strategy is to retro-project the relations among singular terms and predicates back on the inferential basis to induce structure on it. With this structure in place, we then make two crucial observations. The first one amounts to a navigational procedure which finds paths on the structure which can naturally be interpreted as 'higher level inferences' (in the sense that they correspond to an entire chain of reasoning). The second observation consists in interpreting equivalence classes of singular terms (vertical moves in the structure) — which Brandom takes to be objects "that singular terms are purport to refer to" — as 'higher level concepts'. The easy conclusion, next, is that the notions of higher level concepts and inferences instantiate a language game at another level of abstraction (see Figure 4, left). The interesting observation is that our approach also recovers the original layers of the traditional model, but this time they correspond to a different conceptual level (see Figure 4, right).

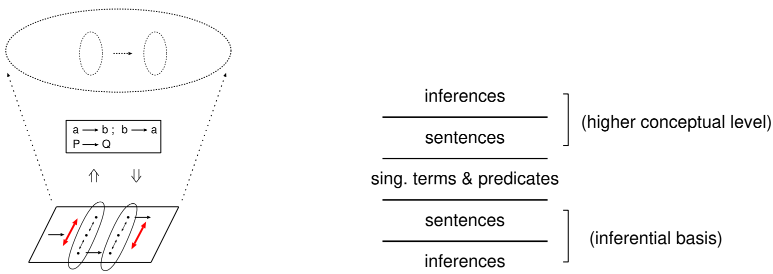

Figure 4. Three-stage structural-compositional strategy

Beyond the categorial forms of the Axiom of Choice

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After working in [1] and [2], on new categorial forms of the axiom of choice and some *small* versions of Zorn's lemma, which are based on partially ordered sets viewed as categories, we will go on to explore some more categorial forms of statements of Zorn's lemma and Hausdorff maximal principle.

We will introduce some versions of Zorn's Lemma and compare these with the categorial forms presented in [KS] and in [2]. In [3], the authors have introduced a notion of categorial Zorn's Lemma that is: "*in a category C if every filtered diagram has an inductive limit, then C has an almost terminal object*". We realized that an inductive limit is able to translate the notion of *upper bound*, but there are other possibilities to do this. So, we introduce another categorial Zorn's Lemma that states: "*if every filtered diagram has a cocone in C, then it has an almost maximal object*". In the case, C is a poset viewed as a category, both categorial notions coincide and are equivalent to Zorn's Lemma. Considering other categories, these two versions of Zorn's Lemma can be incomparable.

We also introduce versions of the categorial Hausdorff Maximal Principle that is: "*the category of filtered subcategories of C has a almost terminal (almost maximal) object*". In category Set, these versions give the classical version of Hausdorff maximal principle, i.e., every partial order contains a maximal chain. We will investigate the relation between the versions of Zorn's Lemma and the Hausdorff maximal principles for categories.

References

[1] A. B. M. Brunner, H. L. Mariano, S. G. da Silva, Categorial forms of the axiom of choice, *Logic Journal of the IGPL* 25 (4), 387–407, 2017.

[2] A. B. M. Brunner, H. L. Mariano, D. C. Pinto, S. G. da Silva, More on categorial forms of the axiom of choice, to appear in *South American Journal of Logic*, 2019.

[3] M. Kashiwara, P. Schapira, *Categories and Sheaves*, Springer-Verlag Berlin Heidelberg 2006.

The Total Probability Theorem as a key to justify a pluralist view on probability theory

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The Total Probability Theorem (TPT) is a fundamental tool to define Bayes' rule in classical probability theory, as it permits to deal with problems involving conditional probability.

In logical terms we can say that probability theory is related to classical propositional logic, but it is also possible to relate it with logics other than classical logic. In [1] a paraconsistent probability theory was proposed based on the logic **Ci** (a paraconsistent system belonging to the family of Logics of Formal Inconsistency cf. [3]). It was also shown that various paraconsistent logics can be used to define different versions of paraconsistent probability theory (cf. [2]), so a natural question is: what is the fundamental difference among such approaches?

In this talk I will consider such a question, showing that it is possible to make a distinction among different kinds of probability theory by analyzing extensions of TPT. I will show that in distinct paraconsistent probability theories it is possible to define different versions of TPT, where each one can be applied in different circumstances. In the limit case (i.e, in classical probability theory) all versions of TPT collapse into a unique version: the standard one.

In philosophical terms we can argue in favor of a pluralist view of probability theory by showing that non-classical probability theory is more sensible than classical theory in the sense that some more sophisticate applications can be envisaged within certain paraconsistent probability theories.

References

[1] Bueno-Soler, J.; Carnielli, W. Paraconsistent probabilities: consistency, contradictions and Bayes' theorem. In: J. Stern (Editor), *Special Issue Statistical Significance and the Logic of Hypothesis Testing*. On line Entropy. Volume 18(9), 2016.

[2] Bueno-Soler, J.; Carnielli, W. Paraconsistent probabilities and their significance and their uses. In: P. Gouveia, C. Caleiro and F. Dionísio (Editors), *Logic and Computation: Essays in Honour of Amilcar Sernadas*. Pages 197–230, 2017.

[3] Carnielli, W.A.; Coniglio, M.E.; Marcos J. Logics of Formal Inconsistency. In: D. Gabbay and F. Guenthner (Editors), *Handbook of Philosophical Logic*. Volume 14, pages 1–93, 2007. Amsterdam, 2007.

Fixed-Parameter Tractability of Some Classes of Formulas in FO with Functions

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We consider the parameterized complexity [4] of the satisfiability problem for the classes $[\text{all}, (\omega), (\omega)]$, the fragment of first-order Logic (FO) in the prenex normal form with arbitrary prefix, a countable number of monadic relation and unary function symbols, and $[\exists^*, \text{all}, \text{all}]_=$, the class of formulas in FO in the prenex normal form with only existential quantifiers, and an arbitrary number of relation, function symbols, and the equality symbol. Here we extend the fixed-parameter analyses addressed in [5] for these two classes, showing that that satisfiability problem is fixed-parameter tractable with a suitable choice of parameters.

The decidability of the satisfiability problem for many prefix-vocabulary fragments of FO has a long tradition in the branch of the Mathematical Logic. In [3], the classification of the prefix-vocabulary fragments of FO is made into reduction classes (undecidable cases), and decidable classes. The computational complexity for the decidable cases was investigated in [1,2,6], and, for almost all maximal decidable classes, the computational complexity for the satisfiability problem is in NEXPTIME.

We obtain the fixed-parameter tractability, the existence of an algorithm that solves the problem in at most $f(k) \cdot |x|^{O(1)}$ steps for some computable function f, where k is the parameter and $|x|$ is the input size, by means of a fpt-reduction and the closure of FPT, the class problems that are fixed-parameter tractable under this kind of reduction. In [5], the fixed-parameter tractability results were obtained through finite model property. Then, to handle prefix-vocabulary classes with function symbols, someone has to replace the function symbols to reduce to a relational class with the finite model property or achieve the finite model property based on some structure with grounded terms. Then, we can attest the fixed-parameter tractability of $[\text{all}, (\omega), (\omega)]$ and $[\exists^*, \text{all}, \text{all}]_=$ with respect to some parameters.

For the class $[\text{all}, (\omega), (\omega)]$, we consider the quantifier rank qr, the number of monadic relation symbols nr, the number of unary function symbols nf, and the maximum size

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of a term t. We just need to adapt a Lemma from [6] to discard the unary function symbols.

Lemma 1. [6] Let φ be a formula in the prenex normal form with size n, quantifier rank qr, nr monadic relations, nf unary function symbols, and with terms of the form $f^i x_j$ such that $i < t$ for some constant $t < n$. Then, there is an equivalent formula φ' without functions.

The previous lemma shows that every monadic formula φ of length n can be converted into a formula $\psi \in [\text{all}, (n), (0)]$ satisfiable over the same domains as ψ with at most $O(nr + nr \cdot (nf)^t)$ monadic relations, and with quantifier rank bounded by $qr + nf + 1$. Using the previous lemma, we can achieve the fixed-parameter tractability.

Theorem 2. The satisfiability problem $p\text{-}(\mathrm{qr} + \mathrm{nr} + \mathrm{nf} + \mathrm{t})\text{-SAT}([\text{all}, (\omega), (\omega)])$ is in FPT.

The existential fragment with equality is one of the decidable cases that are maximal with respect to the finite model property, and its satisfiability is NP-complete [3, pg. 304]. The finite model property, then, is obtained by the set of terms T occurring in a given existential formula. In this case, we add the number of terms $|T|$ as a parameter. For all terms in the form $s = f s_1 \dots s_r$, we also consider $s_1 \dots s_r$ and their sub-terms in the set T. We also consider the maximum arity a with respect to relation and function symbols.

Lemma 3. [3] Let φ a first-order sentence in $[\exists^*, \text{all}, \text{all}]_=$ with quantifier rank qr, and let be T the set of terms occurring in φ. Then φ has a model with size $qr + |T|$.

Theorem 4. $p\text{-}(\mathrm{qr} + \mathrm{nr} + \mathrm{nf} + \mathrm{a} + |T|)\text{-SAT}([\exists^*, \text{all}, \text{all}]_=)$ is in FPT.

References

[1] Fürer, M. Alternation and the Ackermann case of the decision problem. *L'Enseignement Math* 27:137–162, 1981.

[2] Lewis, H. R. Complexity results for classes of quantificational formulas. *Journal of Computer and System Sciences* 21:317–353, 1980.

[3] Börger, E.; Grädel, E.; Gurevich, Y. *The classical decision problem.* Springer Science & Business Media, 2001.

[4] Flum, J.; Grohe, M. *Parameterized complexity theory.* Springer Science & Business Media, 2006.

[5] Bustamante, L. H.; Martins, A. T.; Ferreira, F. M. Parameterized Complexity of Some Prefix-Vocabulary Fragments of First-Order Logic. n: Moss L., de Queiroz R., Martinez M. (eds), *Logic, Language, Information, and Computation.* Springer, Berlin, Heidelberg, WoLLIC 2018. Lecture Notes in Computer Science, Volume 10944, pages 163–178, 2018.

[6] Grädel, E. Complexity of formula classes in first order logic with functions. *Fundamentals of Computation Theory.* Springer, Berlin, Heidelberg, pages 224–233, 1989.

Using Dependent Type Theory and $LE\forall N$ to prove that every huge normal proof is redundant

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Some results on the size of logical proofs seem to point out towards structured proofs in logic. The academic community seems to notice recently that the use of graphs instead of trees or lists, to represent logical proofs provides shorter proofs. The *size* ($|\Pi|$) of a (formal) proof Π is considered to be the number of occurrences of letters in the word obtained by linearizing the graph/tree/list. A proof/derivation Π of α from Γ is short whenever $|\Pi| \leqslant (|\alpha| + |\Gamma|)^{\kappa}$, for some $1 \leqslant \kappa$, with the $|\alpha|$ and $|\Gamma|$ are defined analogously to $|\Pi|$. As opposed to a short proofs/derivations, a derivations Π is *huge*, iff, there is a real number $\lambda > 1$ and a natural number $\kappa, 1 \leqslant \kappa$, such that $\lambda^{|\alpha|^{\kappa}} \leqslant |\Pi|$. In [1] one the authors present a method for converting any Natural Deduction derivation of α from Γ, in $M_{\supset}$, to a Dag-like proof of α from Γ. The important feature of this conversion is that the size of this Dag-like derivation is polynomially bounded concerning $|\alpha| + |\Gamma|$ and a linearly bounded certificate $c(\alpha, \Gamma)$. Nowadays the conversion procedure evolved to a stage that it does not need the certificate $c(\alpha, \Gamma)$ anymore. Besides that, it has been proved that the verification that the Dag-like derivation is valid can be performed in polynomial time concerning $|\alpha| + |\Gamma|$. The improvements mentioned previously entails that $NP = PSPACE$, although this is not the focus of the work present here. This article reports our experience on the use of interactive theorem proving (ITP) in formalizing parts of our proof of $NP = PSPACE$. The (new) conversion procedure relies on a set of 32 conversion rules, so that, in the proof of the soundness of them is enough to prove that each of these rules preserves the minimal logical consequence already obtained in the original treelike (Natural Deduction) derivation. Due to many details and the higher number of cases and subcases that appear in the proof, we use $LE\forall N$ ([2]) to prove the soundness of the conversion. However, due to its big size, we choose to report the experience in the use of ITP in the result that shows that any proof Π in $M_{\supset}$ that is huge has a subderivation Π^{e} tha occurs exponentially many times in Π. We use $LE\forall N$ to formally show this result that is originally presented, in an informal way, in [3]. This result was chosen because it is central in the understanding that the gap in the size of the original and the converted proof.

References

[1] L. Gordeev and E. H. Haeusler. Proof compression and np versus pspace. *Studia Logica*, Dec 2017.

[2] Leonardo Mendonça de Moura, Soonho Kong, Jeremy Avigad, Floris van Doorn, and Jakob von Raumer. The lean theorem prover (system description). In CADE, volume 9195 of *Lecture Notes in Computer Science*, pages 378-388. Springer, 2015.

[3] E. H. Haeusler and L. Gordeev. Huge proofs, redundant proofs and some reasons in favor of np=pspace. In *Proceedings of the Third Tübingen Conference on Proof-Theoretic Semantics*. Tübingen Universität, 2019.

Verdades analíticas ou intuições sintéticas a priori? Considerações sobre Frege, os fundamentos da aritmética e a filosofia de Kant

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A falência do projeto logicista inaugura uma nova fase no pensamento de Frege. Nesta nova fase, diversas teses fundamentais para a redução das verdades aritméticas às leis lógicas são abandonadas. Em particular, Frege abandona a defesa do caráter analítico das verdades aritméticas, e apresenta em seu lugar uma tese segundo a qual a aritmética possuiria como sua origem mais própria a geometria. Em termos filosóficos, isto significa que o espaço ocupado pelas intuições sintéticas a priori na teoria de Frege teria de ser completamente reavaliado. Em nossa exposição, pretendemos construir a hipótese de acordo com a qual ambas as fases do pensamento de Frege - tanto a fase logicista, quanto a fase posterior — podem ser reunidas através de uma observação mais atenta das relações estabelecidas por Frege com a doutrina de Kant.

Reasoning about evidence: a logico-probabilistic viewpoint

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Evidence, probability, logic, and information are related in several ways, and reasoning with and about evidence has becoming an increasingly hot topic in machine learning and AI, with reflexes on philosophy. Yet no agreement among researchers on the conceptual relationship involving such concepts has been found.

In spite of some possibilities of application, the Dempster-Shafer theory of evidence (cf. [6]) suffers sharp criticism from several sides. One of its weakest points is the relationship between belief functions, its basic concept, and probability, as Wasserman contends in [7]. For this reason, Dempster-Shafer theory of evidence is of little interest for us.

Halpern and Pucella in [5] offer a logic for reasoning about evidence, but it is so complicated, involving quantification over real numbers, that it seems hardly applicable in real contexts.

In this talk I discuss a probabilistic semantics for LET_F, an extension of the well-known logic of First-Degree Entailment (*FDE*) plus the operators for consistency $\circ\alpha$ and inconsistency $\bullet\alpha$. I show that LET_F is suitable for an intuitive interpretation in terms of preservation of non-conclusive and conclusive evidence, the later being understood as truth. Moreover, continuing the work done in [4], evidence can be quantified by giving a probabilistic semantics for LET_F in terms of measures of evidence.

By accepting that α and $\neg\alpha$ does not mean that both are true, or are 'real contradictions', our approach emphasizes that the available information, constituted by positive and negative evidence about some collection of events, is what can be contradictory. Thus $P(\alpha) = \epsilon$, where P is a probability measure, means that the amount of evidence for α is ϵ, a notion explicitly *weaker* than truth.

In this way we can express incomplete situations in which there are little or no evidence for and against α, as well as contradictory situations in which there may be conflicting evidence for α. When there is low or no evidence for α, $P(\alpha) + P(\neg\alpha) < 1$, and when there is conflicting evidence about α, $P(\alpha) + P(\neg\alpha) > 1$. The former is a paracomplete, and the later a paraconsistent configuration. When evidence available for α behaves classically, this is logically expressed by $P(\circ\alpha) = 1$, and then $P(\alpha) + P(\neg\alpha) = 1$. This gives prominence to the consistency and inconsistency operators of LFIs (cf. [3]), now under the role of controlling evidence.

The logic properties of such construals and their possible extensions will be clarified with some realistic applications. In particular, we will test the 'paradefinite' logics of Arieli and Avron [1] (designed for handling contradictory or partial information), a class of paraconsistent and paracomplete four-valued logics that expand the framework of first degree entailment.

References

[1] Arieli, O.; Avron, A. Four-Valued Paradefinite Logics. *Studia Logica* 105:1087–1122, 2017.

[2] Belnap, N. How a computer should think, in G. Ryle (Editor), *Contemporary Aspects of Philosophy*, Oriel Press, Stocksfield, pages 30–56, 1977

[3] Carnielli, W.A.; Coniglio, M.E.; Marcos J. Logics of Formal Inconsistency. In: D. Gabbay and F. Guenthner (Editors), *Handbook of Philosophical Logic*. Volume 14, pages 1–93, 2007. Amsterdam, 2007.

[4] Carnielli, W.A.; Rodrigues, A. An epistemic approach to paraconsistency: a logic of evidence and truth. *Synthese*. Available from `https://link.springer.com/article`, accessed in 23/01/2019.

[5] Halpern, J. Y; Pucella, R. A logic for rasoning about evidence. *Journal of Artificial Intelligence* 26(1):1–34, 2006.

[6] Shafer, G. *A Mathematical Theory of Evidence*. Princeton University Press, Princeton, 1976.

[7] L. Wasserman. Comments on Shafer's "Perspectives on the theory and practice of belief func- tions". *International Journal of Approximate Reasoning* 6(3):367–3, 1992.

Some developments on the logic $\mathbf{G}_3'$

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In 2001, W. Carnielli and Marcos [2] considered a 3-valued logic in order to prove that the schema $\varphi \vee (\varphi \rightarrow \psi)$ is not a theorem of da Costa's logic C_ω. In 2006 this logic was studied (and baptized as $\mathbf{G}_3'$) by Galindo et al. [4] as a tool to define semantics of Logic Programs. It is known that the truth-tables of $\mathbf{G}_3'$ have the same expresive power than the ones of Lukasiewicz 3-valued logic Ł$_3$ –hence, to the ones of Gödel 3-valued logic **G3**. From this, the three logics coincide up-to language, taking into acccount that **1** is the only designated truth-value in these logics.

Two different Hilbert-style systems for $\mathbf{G}_3'$ were introduced in [5] and [6], respectively. However, both approaches assume the validity of the deduction metatheorem, which is not the case in $\mathbf{G}_3'$. In this paper we fix this problem by presenting a Hilbert-style system which is sound and complete for $\mathbf{G}_3'$. In addition, a novel semantics of twist-structures is given for $\mathbf{G}_3'$, showing that it is sound and complete.

From the algebraic point of view, Canals-Frau and Figallo have studied in [1] the 3-valued modal semilattices with infimum, Gödel implication and the Moisil-Monteiro-Baaz Delta operator (the supremum is definable from this). We prove that the subvariety obtained by adding a bottom element 0 is term-equivalent to the variety generated by the 3-valued algebra of $\mathbf{G}_3'$. From this, we present the identities which axiomatize the variety generated by $\mathbf{G}_3'$ (the $\mathbf{G}_3'$algebras) as an equational class. Moreover, we prove that this variety is semisimple, and the 3-element and the 2-element chains are the unique simple algebras of the variety.

Finally an extension of $\mathbf{G}_3'$ to first-order languages is presented, with an algebraic semantics based on complete $\mathbf{G}_3'$-algebras. The proof of soundness and completeness is obtained by adapting the techniques introduced in [3].

References

[1] M. Canals-Frau, and A. V. Figallo. Modal 3-valued implicative semilattices. *Preprints Del Instituto de Ciencias Básicas* 1, 1992.

[2] W. A. Carnielli and J. Marcos. A taxonomy of C-systems. In: W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The Logical Way to the Inconsistent*, volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pp. 1–94. Marcel Dekker, New York, 2002.

[3] A. Figallo-Orellano and J. S. Slagter. An algebraic study of the first order intuitionistic fragment of 3-valued Lukasiewicz logic. Submitted, 2018.

[4] M. Osorio-Galindo, J. A. Navarro-Pérez, J. R. Arrazola-Ramírez, and V. Borja-Macías. Logics with common weak completions. *Journal of Logic and Computation*, 16(6):867–890, 2006.

[5] M. Osorio, J. R. A. Ramírez, J. L. Carballido, and O. Estrada. An axiomatization of $\mathbf{G}'_3$. In *LoLaCOM* (2006).

[6] M. Osorio-Galindo and J. L. Carballido-Carranza. Brief study of $\mathbf{G}'_3$ logic. *Journal of Applied Non-Classical Logics* 18, 4 (2008), 475–499.

Multialgebraic first-order structures for some logics of formal inconsistency

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The class of paraconsistent systems known as *Logics of Formal Inconsistency* (LFIs), was introduced in [4]. In these logics is possible allows to recover the explosion law in a controlled way. (The explosion law is $\alpha \wedge \neg\alpha \vdash \beta$ for some α and β.)

The logic **mbC** is the weakest system in the hierarchy of LFIs and the system **QmbC**, introduced in [3] (see also [1] and [2, Chapter 7]), is the extension of **mbC** to first-order language.

Multialgebras are algebras such that at least one of its operations returns, to each element of its domain, a non-empty set of elements instead of a single element (such operations are called *multioperations*). In 2016 Carnielli and Coniglio [2] (see also [5,6]) introduced a class of multialgebras called *swap structures*, as a semantic framework for dealing with several LFIs that cannot be semantically characterized by a single finite matrix. In particular, these LFIs are not algebraizable by the standard tools of abstract algebraic logic.

The goal of this talk is to introduce a multialgebraic semantics for **QmbC** and to extend this approach to several axiomatic first-order extensions of **mbC**. The multialgebraic semantics introduced here is based on swap structures and the non-deterministic matrices generated by them. As it will be shown, this semantics generalizes the approach to non-deterministic semantics for first-order LFIs proposed by Avron and his collaborators in [1]. From the algebraic point of view, these structures enable us to obtain properties of first-order logic LFIs.

References

[1] A. Avron and A. Zamansky. Many-valued non-deterministic semantics for first-order Logics of Formal (In)consistency. In: S. Aguzzoli, A. Ciabattoni, B. Gerla, C. Manara, and V. Marra, editors, *Algebraic and Proof theoretic Aspects of Non-classical Logics*, pp. 1–24. LNAI 4460, Springer, 2007.

[2] W. Carnielli and M. E. Coniglio. *Paraconsistent Logic: Consistency, Contradiction and Negation.* Springer, v. 40. Logic, Epistemology, and the Unity of Science, Switzerland, 2016.

[3] W. A. Carnielli, M. E. Coniglio, R. Podiacki, and T. Rodrigues. On the way to a wider model theory: Completeness theorems for first-order logics of formal inconsistency. *The Review of Symbolic Logic*, 7(3):548–78, 2014.

[4] W. A. Carnielli and J. Marcos. A taxonomy of C-systems. In: W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The Logical Way to the Inconsistent*, volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pages 1–94. Marcel Dekker, New York, 2002.

[5] M. E. Coniglio, A. Figallo-Orellano and A. C. Golzio. Non-deterministic algebraization of logics by swap structures. *Logic Journal of the IGPL*, to appear. First published online: November 29, 2018. DOI: 10.1093/jigpal/jzy072. Preprint available at arXiv:1708.08499 [math.LO]

[6] A. C. Golzio. *Non-deterministic matrices: theory and applications to algebraic semantics*. PhD thesis, IFCH, University of Campinas, 2017.

Extensionalist explanation and solution of Russell's Paradox

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In this paper, I propose a way out from the aporetical conclusion of the debate about the, so-called, explanation of Russell's paradox. In this debate, there are traditionally two main and incompatible positions: the "Cantorian" explanation and the "Predicativist" one. I briefly rehearse the reasons why both these positions can be neglected and propose a third, "Extensionalist", one.

The Extensionalist explanation identifies the key of Russell's Paradox in a proposition about the extensions: $\forall F \exists x(x = ext(F))$, which allows to derive, from the existence of Russell's concept, the existence of Russell's extension. This proposition is a theorem of classical logic whose derivation presupposes the classical treatment of identity (Law of identity) and quantification (Laws of Universal Specification, Universal and Existential Generalisation) 1 . So, we can explain Russell's paradox by the (inappropriate) classical correlation be- tween concepts and extensions: the flaw of this correlation does not consists (as in the Cantorian explanation) in the injective feature of the correlation but (as in the Predicativist explanation) in its domain, namely in the implicit assumption that the correlation is defined on the whole second order domain; however this result does not mean that, for restoring consistency, we have to restrict the whole second order domain (as in the Predicativist solution) but only the domain of the extensionality function.

The solution related to the Extensionalist explanation consists in a reformulation of Frege's theory, in which classical second order logic is replaced with a negative free logic to allow the derivation of Peano Arithmetic as a logical theory of extensions. We can analyse three different versions of this free fregean system.Their language L comprises two sorts of first order quantifiers (generalised Π, Σ and restricted $\forall$, $\exists$) respectively governed by classical and by negative free logic. From a syntactic point of view, the proposed systems share the logical part of the axiomatization (FL), consisting of the axioms of propositional classical logic, some specific axioms of predicative negative free logic and an impredicative comprehension's axioms schema. These systems differ each other only by the non logical axioms, which represent three different abstraction principles, obtained by the weakning of Basic Law V (with a generalised universal first order quantification): in the first theory, BLV is restricted to the existents abstracts (E-BLV): $\forall F \forall G(ext(F) = est(G) \leftrightarrow \exists x(x = ext(F)) \wedge \Pi x(Fx \leftrightarrow Gx))$; in the second theory, BLV is restricted to the abstracts obtained from predicative con- cepts (P-BLV): $\forall F \forall G(ext(F) = est(G) \leftrightarrow \varphi(F) \wedge \varphi(G) \wedge \Pi x(Fx \leftrightarrow Gx))$ — where φ means a concept specified by a predicative instance of comprehension's axioms schema; in the third theory, BLV is restricted to the abstracts obtained from "small" concepts (S-BLV): $\forall F \forall G(ext(F) = est(G) \leftrightarrow \varphi(F) \vee \varphi(G) \wedge \Pi x(Fx \leftrightarrow Gx))$ — where φ means a concept "small" (cfr. Boolos 1987:New V), namely a concept such that the universal set $[x : x = x]$

is not equinumerous to a subset of its extension. These versions of Basic Law V characterise the behaviour of the correlation denotated by "ext" as functional and injective only for a subset of second order domain, which excludes Russell's concept. Then, all these systems do not allow to derive Russell's Paradox and the third one allows to derive (in a negative free logic) Peano Arithmetic.

From a semantic point of view, the interpretation of these theories is provided by a model $M = \langle D, D_0, I\rangle$, in which D is the domain of restricted quantification (such that $D \subseteq D_0$), D_0 is the domain of generalised quantification and I is a total interpretation function on D_0. The symbol "ext" is interpreted as a partial injective function from a subset of the power set of D_0 in D.

References

[1] Antonelli A. and May R. (2005). Frege's Other Program, *Notre Dame Journal of Formal Logic*, Vol. 46, 1, 1-17.

[2] Boolos, G. (1986). Saving Frege from contradiction, *Aristotelian Society*, Supplementary Volume 87, 137–151.

[3] Cocchiarella, N. B. (1992). Cantor's power-set Theorem versus Frege's double correlation Thesis, *History and Philosophy of logic*, 13, 179-201.

[4] Uzquiano, G. (forthcoming). Impredicativity and Paradox.

Strong Normalization for Np-Systems via Mimp-Graphs*

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Theorem provers usually structure their proofs as trees. However, studies have shown that a significant reduction of time and space can be achieved when proofs is structured as a direct acyclic graph [1]. *Mimp-graphs* is a special type of direct graph whose nodes and vertices are labeled and can be used to represent logical proofs. In addition, two parts are separated in this graph, one representing the inferences of proof and another the formulas. The representation of a proof in *mimp-graph* requires less nodes than the tree representation and it is possible to represent any proof in natural deduction. Another important advantage of a compact graph representation is the deduction of structural properties, as an example, it is very easy to perceive an upper bound on the size of a reduction to obtain a normal proof by simply analyzing the amount of maximum formulas [2]. The system, called Np, is a natural deduction system that uses the Peirce rule instead of the classical absurd rule. The implicational fragment of the Np system is complete, normalizable and has a kind of principle of sub-formula, a proof of theorems can be implemented using only the rules of introduction and elimination of implication leaving the classical part exclusively at the end of deduction. However, the system does not have strong normalization [3,4]. There are studies that show the feasibility of unifying the Np system with the Mim-graphs structure [5]. In this work the gains from this unification will be presented, in particular, the achievement of strong normalization for Np-systems.

References

[1] L. Gordeev, E. Haeusler, and V. da Costa. Proof compressions with circuit-structured substitutions. *Journal of Mathematical Sciences*, 158:645–658, 2009. 10.1007/s10958-009-9405-3.

[2] M. Quispe-Cruz, E. Haeusler, and L. Gordeev. Proof-graphs for minimal implicational logic. In M. Ayala-Rincón, E. Bonelli, and I. Mackie, editors, *Proceedings 9th International Workshop on Developments in Computational Models, DCM 2013, Buenos Aires, Argentina, 26 August 2013*, volume 144 of EPTCS, pages 16–29, 2014.

[3] V. G. Costa, E. Hermann, L. C. Pereira, and W. Sanz. Revisiting peirce's rule in natural deduction. In *Volume of Abstracts of Logic Coloquium*, Bern, Suice, 2008.

[4] L. C. Pereira, E. Haeusler, V. G. Costa, and W. Sanz. A new normalization strategy for the implicational fragment of classical propositional logic. *Studia Logica*, 96(1):95–108, 2010.

[5] V. G Costa, E. Haeusleer, M. Quispe-Cruz, and J. B Santos. Np system and mimp-graph association. In *Proceedings 5th International Workshop on Universal Logic*, Unilog 2015, Istambul, Turkey, pages 346–347, 2015.

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MV-algebras as Sheaves of ℓ-Groups on a Fuzzy Topological Space

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We extend the concept of sheaf to fuzzy topological spaces, with particular emphasis to the class of MV-topologies [2]. Then we represent a class of MV-algebras as *MV-sheaves* of lattice-ordered Abelian groups. Our representation is strongly connected to Filipoiu and Georgescu's sheaf representation for MV-algebras [1], since we use essentially the same algebraic tool, that is, the fact that any MV-algebra A is subdirectly embeddable in the product of a family of local MV-algebras.

However, our representation differs from the one in [1] in the way the "information is encoded". In Filipoiu and Georgescu's representation, each MV-algebra is obtained as an algebra of global sections of a (classical) sheaf over the maximal spectrum of the algebra and whose stalks are local MV-algebras. So, grossly speaking, we can say that each element of the algebra is represented as an open set of maximal ideals, carrying just the Boolean information, with an element of a local MV-algebra attached to each of its points. In our representation, the base space is the maximal MV-spectrum (see [2]) and is in charge of encoding the whole semisimple skeleton of the given algebra, while the stalks only carry the non-semisimple (or infinitesimal) information of the elements of the algebra. Therefore, using the same description, each element of the algebra is a fuzzy open set along with ℓ-group elements attached to its (fuzzy) points; the fuzzy points of the open set form the semisimple part and the group elements represent exclusively the infinitesimal one.

References

[1] Filipoiu A., Georgescu G., Compact and Pierce representations of MV-algebras, *Rev. Roum. Math. Pures Appl*, 40:599–618,1995.

[2] Russo, C.; An extension of Stone duality to fuzzy topologies and MV-algebras. *Fuzzy Sets and Systems* 303:80–96, 2016.

Estruturalismo sem compromisso ontológico

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O objetivo deste trabalho é discutir o estruturalismo na filosofia da matemática, concentrando-se em dois de seus tipos, a saber, no estruturalismo eliminativista e no estruturalismo não-eliminativista. De acordo com a concepção estruturalista, a matemática é a ciência da estrutura, isto é, o objeto de estudo da matemática são as estruturas. Além disso, nessa concepção, os objetos matemáticos, como números, funções, etc, não são vistos como nada além de posições nessas estruturas, determinados apenas por suas relações uns com os outros no interior dessas estruturas. Sendo assim, as teorias matemáticas não tratam das qualidades (ou propriedades) dos objetos, elas apenas descrevem as propriedades estruturais de seus respectivos domínios. Encontramos na literatura várias abordagens estruturalistas. Entretanto, desconsideradas algumas particularidades, podemos considerá-las como abarcando dois tipos principais: o estruturalismo não-eliminativista (platonista), de acordo com o qual as estruturas matemáticas existem, são abstratas e seus lugares são objetos genuínos (abstratos); e o estruturalismo eliminativista (nominalista), segundo o qual as estruturas podem ser eliminadas sem qualquer prejuízo para as teorias matemáticas (as estruturas são concebidas de um ponto de vista modal, como estruturas apenas possíveis, e não como atuais). As concepções estruturalistas procuram responder ao problema da redução múltipla levantado por Paul Benacerraf em seu artigo "What numbers could not be" (1965). A inspiração para tais concepções também surge da proposta de Benacerraf, nesse artigo. Ao argumentar contra a ideia de que números são objetos, ele oferece uma explicação em que considera que os numerais não são de fato termos singulares e que, desse modo, não fazem referência a objetos abstratos. De acordo com ele, números são apenas lugares em estruturas. Desse modo, Benacerraf procura resolver o problema de explicar qual a natureza dos números naturais e, além disso, parece resolver o problema da redução múltipla dissolvendo-o, pois, uma vez que consideremos apenas as estruturas, basta que elas sejam isomórficas para que as consideremos como idênticas. Com isso, não importa qual redução seja adotada, se a de Zermelo ou a de von Neumann, pois ambas refletem a mesma estrutura. Os estruturalistas são motivados por essa aparente resolução do problema. Contudo, dado que há grandes diferenças entre as abordagens estruturalistas platonistas e nominalistas, o que pretendemos com esse trabalho é apontar em linhas gerais quais as vantagens e desvantagens de cada uma, indicando as razões pelas quais acreditamos que uma abordagem nominalista se apresenta como a alternativa mais viável para responder ao problema de Benacerraf. Faremos isso mostrando que, de fato, a partir da perspectiva da prática matemática tudo que importa são propriedades estruturais — e não objetos — mas que isso não implica que os matemáticos se comprometam ontologicamente com estruturas abstratas. Uma abordagem ontologicamente neutra parece mais promissora, pois evita transferir o problema da natureza dos objetos matemáticos abstratos para a natureza de estruturas abstratas. Para tratarmos de tudo isso, primeiro faremos uma breve exposição da ideia geral que fundamenta essas concepções. Em seguida, apresentaremos as principais objeções a cada uma e, por fim, os motivos pelos quais vemos o estruturalismo nominalista como mais adequado do que o plantonista para lidar com as principais questões em filosofia da matemática.

References

[1] Benacerraf, P. What Numbers Could Not Be (1965). In: Benacerraf, P. & Putnam, H. *Philosophy of Mathematics*. Cambridge: Cambridge University Press, 1983.

[2] Chihara, C. *Constructibility and Mathematical Existence*. N.Y.: OUP, 1990

[3] Hellmann, G. *Mathematics Without Numbers*. Oxford. OUP, 1989.

[4] Shapiro, S. *Philosophy of Mathematics: Structure and Ontology*. NY: OUP, 1997.

Galois pairs with the modal operators of paraconsistent logic J_3

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The objective of this paper consists in showing that the modal operators of the paraconsistent logic J_3 characterize as a Galois pair.

The paraconsistent logic J_3 was introduced, by D'Ottaviano and da Costa (1970), from a three-valued matrix semantics. It was conceived as a solution to a problem proposed by Jaśkowski (1948 and 1969), involving aspects of the recently created paraconsistent logics. This system, more than paraconsistent, is also many-valued and modal; and has been studied in the literature by several authors under distinct motivations and denominations.

In this paper, we emphasize the modal aspects of J_3. Instead of the operators $\bigtriangledown$ and $\bigtriangleup$ used in the original version of 1970, we use the operators with the alethic understanding for J_3, that is, the operators $\Box$ (necessary) and $\Diamond$ (possible).

We present the tableaux system for J_3, denoted by TJ_3, introduced by Silva, Feitosa and Cruz (2017), with the addition of new rules for formulas whose main operators are modal ones, which emphasize aspects of the modal character of J_3. This tableaux system allows us the characterization of the modal operators of J_3 as a Galois pair — we exhibit a Galois adjunction with the modal operators of the original version of J_3.

References

[1] D'Ottaviano, I. M. L.; da Costa, N. C. A. Sur un problème de Jáskowski. *Comptes Rendus de l'Académie de Sciences de Paris* (A-B), v. 270, p. 1349-1353, 1970.

[2] Jaśkowski, S. Propositional calculus for contradictory deductive systems. *Studia Logica*, v. XXIV, p. 143-157, 1969. (English version of Jaśkowski 1948).

[3] Jaśkowski, S. Rachunek zdán dla sustemów dedukcyjnych spr- zecznych. *Studia Societatis Scientiarum Torunensis*, Sec. A, I, n. 5, p. 55-57, 1948.

[4] Silva, H. G.; Feitosa, H. A.; Cruz, G. A. Um sistema de tableaux para a lógica paraconsistente J3 . *Kínesis*, v. 9, n. 20, p. 126-150, 2017.

Towards a Feminist Logic: Val Plumwood's Legacy and Beyond

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Val Plumwood's "Politics of Reason: Towards a Feminist Logic" makes a case for reevaluating the supposed neutrality of logical systems. For Plumwood, Classical Logic expresses and reinforces a fundamental pattern of Domination that underlies sexism — as well as other types of oppression such as, racism, classism, colonialism, heteronormativity and so on. Plumwood's bold and significant claim has gone under-appreciated. In this presentation I will set out three main facets of her critique: (1) Feminist and other Critical Theories working with a misinformed understanding of what Logic is stand in jeopardy of subtly reduplicating the patterns of reasoning they criticize; (2) Feminist and other Critical Theories working with misinformed understandings of Logic may fail to take advantage of formal resources that would support their viewpoints; and (3) Logical Systems can and should be evaluated with respect to their normative roles that coordinate with or counter and resist political hegemonies.

In the most controversial section of her paper (to date), Plumwood focuses on the role that Classical negation plays in what she terms "dualisms" intrinsic to the reasoning involved in Domination. Classical negation expresses of a type of Domination that centers and privileges one of the pair over the other, subordinated side, for example, the Male is centered against the subordinate background of the Female, Culture over and against a subordinated background of Nature, Whiteness against Blackness, the Colonizer against the Colonized, as so on. These dualisms are pervasive and operate to divide and conquer if and when we reason Classically. (It would be important to understand the circumstances in which we reason Classically, of course.) She suggests that negation in Relevance Logics avoids the Dualism and Centering found in Classical negation, making Relevance Logics preferable for not only technical reasons, but also political ones. I will discuss the reception of her proposal about Classical negation, and further her project through consideration of *gatekeeper functions* of Logics. It may not be necessary to attack the functions of operators as Plumwood suggests in order to recognize that systems of Logic have had gatekeeping functions historically, meaning they have acted as "the Masters Tools." Western culture's *centering* predates the development of Classical Logic. Entire cultures have been written off as "irrational" and "uncivilized" given "laws" like the "law of Non-Contradiction." To extent that logical principles and systems have been weaponized and served purposes of Domination, Plumwood was correct to insist that we need to carefully evaluate and select them for more than just technical advantageousness.

Bibliography

[1] Garavaso, Pieranna (2015). The Woman of Reason: On the Re-appropriation of Rationality and the Enjoyment of Philosophy in *Meta-Philosophical Reflection on Feminist Philosophies of Science*, Amoretti, M.C. and Vassallo, N. (eds), Springer, pp. 185-202

[2] Hyde, Dominic, (2014). *Eco-Logical Lives: the Philosophical Lives of Richard Routley/Sylvan and Val Routley /Plumwood*. White Horse Press.

[3] MacPherson, Brian (1999). Three Misrepresentations of Logic. *Informal Logic* 19 (2).

[4] Plumwood, Val (1993). The Politics of Reason: Towards a Feminist Logic. *Australasian Journal of Philosophy* 71 (4): 436–462.

[5] –Reprinted (2002). Plumwood, Val; Hart, Carroll Guen ; Olkowski, Dorothea ; Iselin, Marie-Genevieve ; Nelson, Lynn Hankinson ; Nelson, Jack ; Nye, Andrea & Oliver, Pam eds. *Representing Reason: Feminist Theory and Formal Logic*. Rowman & Littlefield Publishers.

[6] ——. (1994). *Feminism and the Mastery of Nature*. Routledge.

[7] ——. (1995) Human Vulnerability and the Experience of Being Prey, *Quadrant*, 29(3), March 1995, pp. 29–34

[8] ——. (2001). *Environmental Culture: The Ecological Crisis of Reason*. Routledge.

[9] ——. Feminism and the Logic of Alterity . Plumwood, Val; Hart, Carroll Guen ; Olkowski, Dorothea ; Iselin, Marie-Genevieve ; Nelson, Lynn Hankinson ; Nelson, Jack ; Nye, Andrea & Oliver, Pam eds. *Representing Reason: Feminist Theory and Formal Logic*. Rowman & Littlefield Publishers.

[10] Priest, Graham; Routley, Richard (1984). *On Paraconsistency*. Research Series in Logic and Metaphysics, RSSS ANU publication.

[11] Priest, Graham (1995). *Beyond the Limits of Thought*. Cambridge University Press.

[12] ——. (1998). What Is So Bad About Contradictions? *Journal of Philosophy* 95 (8): 410-426.

[13] ——. (2006). *In Contradiction: A Study of the Transconsistent*. Oxford University Press.

An algebraic model for the Standard Deontic Logic

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Since the Antiquity the alethic sense of 'necessarily' and 'possibly' has attracted the interest of logicians. These terms are used to qualify the truth of a proposition.

The pioneers of Modal Logic, in the beginning of XX century, investigated the formal behaviour of expressions as 'it is necessary that' and 'it is possible that' using formal propositional language with two modal operators for necessary and possibly. Nowadays we have used the symbols $\Box$ and $\Diamond$ for these two notions, respectively.

Now, the term 'modal logic' is broader and characterizes a family of logical systems, with several different modality. This family is always increasing, however it includes tense or temporal logics, epistemic logics, deductively aspects of logics, doxastic logics, among others, and the deontic logics.

We are particularly interested in a case of deontic logic, that analyses expressions as 'it is obligatory that', 'it is permitted that', and 'it is forbidden that'.

It is usual, in introductory texts on modal logics, to present a modal family constructed from a weak logic called **K**, in honour to Saul Kripke, that in decade of 1950 introduced the Kripke models for this family of logics (See [1], [2], [3] and [7]).

This family has as language the set $L = \{\neg, \wedge, \vee, \to, \Box\}$ such that the four first operators are the classical ones and the last one is the modal operator for necessary.

The operator for possibility must be defined from $\Box$ by $\Diamond\varphi = df \neg\Box\neg\varphi$.

The system **K** is obtained by adding the following two principles to classical propositional logic.

Necessitation Rule: If φ is a theorem of **K**, then $\Box\varphi$ also is a theorem of **K**.

Axiom K: $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$.

The axiom K is also known as Distributivity Axiom.

From the Necessitation Rule, any theorem of logic is necessary. The Axiom K says that if an implication $\varphi \to \psi$ is necessary, then always that φ is necessary, also ψ is necessary.

We are particularly interested in a case of deontic logic, the Standard Deontic Logic (**SDL**), that introduces the primitive symbol **O** for 'it is obligatory that', in the place of $\Box$. From the operator **O** we can define the operator P for 'it is permitted that' by $\mathbf{P}\varphi \Leftrightarrow \neg\mathbf{O}\neg\varphi$, and **F** for 'it is forbidden that' by $\mathbf{F}\varphi \Leftrightarrow \mathbf{O}\neg\varphi$.

The usual modal axiom (T): $\mathbf{O}\varphi \rightarrow \varphi$ is not appropriate for a deontic logic. Even if some action is obligatory, it is not always the case. But, the logic **SDL** admits the axiom (D): $\mathbf{O}\varphi \rightarrow \mathbf{P}\varphi$, that says if φ is obligatory, then φ is permissible. So, we start with the logical system **SDL** presented in different but equivalent axiom systems (as [4]).

Considering one of these presentations we introduce the **D**-algebras, that are planed as algebraic models for **SDL** (similar to [5], [9] and [10]).

Finally we show that the **D**-algebras, are completely adequate models for SDL.

References

[1] Blackburn, P.; Rijke, M.; Venema, Y. *Modal logic.* Cambridge: Cambridge University Press, 2001.

[2] Carnielli, W. A.; Pizzi, C. *Modalità e multimodalità.* Milano: Franco Angeli, 2001.

[3] Chagrov, A.; Zakharyaschev, M. *Modal logic.* Oxford: Clarendon Press, 1997.

[4] Chellas, B. *Modal Logic: an introduction.* Cambridge: Cambridge University Press, 1980.

[5] Dunn, J. M.; Hardegree, G. M. *Algebraic methods in philosophical logic.* Oxford: Oxford University Press, 2001.

[6] Ebbinghaus, H. D.; Flum, J.; Thomas, W. *Mathematical logic.* New York: Springer-Verlag, 1984.

[7] Fitting, M.; Mendelsohn, R. L. *First-order modal logic*?. Dordrecht: Kluwer, 1998.

[8] Mendelson, E. *Introduction to mathematical logic.* 3. ed. Monterey, CA: Wadsworth and Brooks / Cole Advanced Books and Software, 1987.

[9] Miraglia, F. *Cálculo proposicional: uma interação da álgebra e da lógica.* Campinas: UNICAMP/CLE, 1987. (Coleção CLE, v. 1)

[10] Rasiowa, H. *An algebraic approach to non-classical logics.* Amsterdam: North-Holland, 1974.

Steps towards the elucidation of the concept of relative expressiveness between logics

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The concept of expressive power, or strength, is very relevant and frequent in comparisons of logics. Despite its ubiquity, there are important issues shared by many works dealing with it. On the one hand, it is not uncommon to see the notion of expressiveness employed in comparisons of logics on imprecise and varying grounds. This creates confusion in the literature and hardens the process of building upon other's results. On the other hand, when care is taken to specify that the formal criterion of expressiveness being used is a certain E, there is generally no further comment on it, e.g. intuitive motivations or why E was chosen in the first place (E.g. in [5], [3], [1] and [4]).

It is clear in such papers that the term "expressiveness" as regards to logics is not arbitrarily used. It is assumed that there is an intuitive and obscure concept of relative expressiveness, and that E captures it formally. Therefore, it is taken for granted that the term at issue has been subjected to a conceptual elucidation and its clearest and best formal counterpart is E. However, to the best of our knowledge, no such work has been provided. Since there are prima facie plausible alternative criteria conflicting with E, one would have to show why these alternatives are not as good as E for capturing relative expressiveness.

Formal comparisons of expressiveness between logics can be traced back to [6], where a certain formal criterion for expressiveness (to be referred as $\preceq_{EC}$) is given. No conceptual discussion or motivations are offered for $\preceq_{EC}$, perhaps because it issues directly from Lindström's concept of logical system (a collection of elementary classes). In [2] there is a very brief discussion in which a pair of intuitions for expressiveness is given, and it is argued that one would be captured by $\preceq_{EC}$, and another by a new criterion $\preceq_{EQ}$. Shapiro questions the adequacy of $\preceq_{EC}$ in [7] due to its strictness and gives two broader criteria ($\preceq_{PC}$ and $\preceq_{RPC}$). One motivation for the latter is that, as opposed to $\preceq_{EC}$, they allow the introduction of new non-logical symbols in expressiveness comparisons. For example, in some logics the concept of infinitely many is embedded in a logical constant whereas in others, it must be "constructed" with the help of non-logical symbols. Thus, $\preceq_{PC}$ and $\preceq_{RPC}$ consider also the latent expressive power of a logic, so to speak. Up to now, four formal criteria of expressiveness were mentioned. When comparing logics, all of $\preceq_{EC}$, $\preceq_{PC}$ and $\preceq_{RPC}$ can be seen as mapping formulas in the source logic, to formulas in the target logic, with respective restrictions on the allowed mappings. This might be seen as too restrictive, as there are cases where a concept can be expressed in a logic but only using a (possibly infinite) set of formulas (e.g. the concept of infinity in first-order logic). If we allow that formulas in one logic be mapped to a (possibly infinite) set of formulas in the target logic, we get three new criteria for expressive power: $\preceq_{EC\delta}$, $\preceq_{PC\delta}$ and $\preceq_{RPC\delta}$.

Thus we have at least seven formal criteria for expressiveness, but in order to be able to choose between them, we need to select some intuitions for what it can mean for a logic to be more expressive than another. It will be seen that the seven criteria can be divided into two groups capturing each some basic intuition as regards expressiveness. In order to choose among the rival formal criteria intended to capture them, some adequacy criteria will be proposed, and the material adequacy of the formal criteria will be assessed

References

[1] Areces, C.; Figueira, D.; Figueira, S.; Mera, S. *The expressive power of memory logics*. The Review of Symbolic Logic, 4(2):290-318, 2011.

[2] Ebbinghaus, H. D. Extended logics: The general framework. In Jon Barwise and Solomon Feferman, editors, *Model-theoretic logics*, Perspectives in mathematical logic. Springer-Verlag, 1985.

[3] Kooi, B. Expressivity and completeness for public update logics via reduction axioms. *Journal of Applied Non-Classical Logics*, 17(2):231-253, 2007.

[4] Kuijer, L. B.. The expressivity of factual change in dynamic epistemic logic. *The Review of Symbolic Logic*, 7(2):208-221, 2014.

[5] Kracht, M.; Wolter, F. Normal monomodal logics can simulate all others. *Journal of Symbolic Logic*, 64(1):99-138, 1999.

[6] Lindström, P. On extensions of elementary logic. *Theoria*, 35(1):1-11, 1969.

[7] Shapiro, S. *Foundations without Foundationalism: A Case for Second-Order Logic*. Oxford Logic Guides. Clarendon Press, 1991

Preferencias, acciones y horizontes de preferencias bajo un modelo STIT

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La teoría lógica STIT, presentada por Belnap [1], ha resultado ser una poderosa herramienta de análisis a la hora de explorar y poner a prueba las intuiciones de filósofos y teóricos de la computación con respecto a las acciones y las interacciones de agentes. En particular provee un marco formal en el que se puede razonar acerca de las elecciones, habilidades y acciones de estos últimos. El núcleo de la teoría hace uso de sistemas modales que se conocen como “branching-time” o “branching space-times” y la modalidad stit o “seeing to it that” que dan cuenta de las acciones de los agentes como elecciones sobre historias que se ramifican.

En la actualidad se ha avanzado en unir STIT a otras teorías que dan cuenta de lo que los agentes conocen o creen cuando eligen, por ejemplo cuando un agente da una prueba como justificación [3]; o que acciones el agente considera que son obligatorias o prohibidas [2]. Como en el caso de lo que se conoce o se está obligado otro aspecto que hace a la racionalidad de los agentes es lo que se prefiere. Las preferencias son consecuencia de la comparación entre alternativas de distinto tipo: resultados, acciones, o situaciones. Estas comparaciones son normalmente asociadas con un orden en el que se indica que una alternativa es “mejor” que otra. Por ejemplo, cuando se juega al ajedrez u otros juegos, elegir una movida $\alpha 1$ en lugar de $\alpha 2$ es determinada en gran parte reflexionando sobre los resultados a los que llevan $\alpha 1$ y $\alpha 2$. En teoría de juegos y teoría de la decisión las preferencias individuales son usadas para predecir el comportamiento de agentes racionales. En este marco la lógica de las preferencias estudia las propiedades abstractas de las diferentes estructuras comparativas.

En von Wright [4] se presenta una lógica de la preferencia en la que se tiene en cuenta las preferencias de un agente bajo un conjunto acotado de alternativas o lo que se llama “horizonte de preferencias”, es decir, que el agente no prefiere sobre el conjunto total de alternativas. Por otro lado, se define intuitivamente a la preferencia como que el agente favorece un cambio hacia un estado por sobre otro en el que se dan ciertas alternativas. El objetivo de este trabajo es analizar y modelizar formalmente el concepto de preferencia, como “horizonte de preferencias”, en el marco de la teoría STIT.

Referencias

[1] Belnap, N.; M. Perloff; M. Xu. *Facing the future: Agents and choices in our indeterminist world*. Oxford: Oxford University Press, 2001.

[2] Horty, J. *Agency and deontic logic*. Oxford: Oxford University Press, 2001.

[3] Olkhovikov G.K.; Wansing, H., *Inference as doxastic agency. Part I: The basics of justification stit logic*, 2016, to appear in: Studia Logica.

[4] von Wright, G. H. *The logic of preference*. Edinburgh: Edinburgh University Press, 1963.

A note on many-valued functions

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By this work we challenge the idea, introduced and developed by Oliver's and Smiley's [1] and [2], that many-valued functions (as opposed to standard functions or now called single-valued functions) are not relations in a general and mathematically rigorous sense. I, first, recall the distinction between extensional and intensional views about functions [5]: the extensional view is based on the notion of ordered pair and in this sense it is a static conception of functions; on the other hand, the intensional view is based on the notion of transformation (or similar) and thus, as it appear, it is a dynamic conception of functions. Then I recall the set theoretic conflation between many-valued functions as relations [6] and functions [7], that constitute Oliver's and Smiley's point against set theoretic semantics. Oliver's and smiley's thesis is that many-valued functions are not relations (and/or single-valued functions). In order to prove this, they decline set theoretic semantics but not the extensional one. They are interested in introducing many-valued functions as a tool to provide a formal account of plural denotation: as to extend classical predicate logic to plural predicate logic. In this context, many-valued functional terms (plural terms, in general) can be introduced by (i) avoiding set theoretic domains, i.e., just (pluralities of) individuals, and (ii) introducing definite descriptions as genuine, primitive terms, that would allows to introduce plural definite descriptions too. All this, according to Oliver and Smiley, should be supported a) historically, by mathematicians speaking of many-valued functions as mathematical objects per sé and b) theoretically, by the grammatical distinctions between relations or n-ary predicates and functional terms. I argue against their reading of historical sources: I present evidences against their reading (some of them from the same mathematicians they quote [8] and [9]). Further, I show that the distinction between those syntactical categories is fundamental just in classical predicate logic and precisely for it was born in pair with extensional interpretations based on object ontologies. Following, instead, intensional interpretations of functions, i.e., that provided by application or operational systems [3], that distinction fall down, allowing to gather together functional terms and relational predicates under the same syntactical category [4]. In this sense, the claim that many-valued functions are relations find definitive justice.

Bibliografia

[1] Oliver A.; Smiley T.J. Plural descriptions and many-valued functions. *Mind* 114:1039–1068, 2005.

[2] Oliver A.; Smiley T.J. *Plural Logic*. Oxford University Press, 2013.

[3] Descles J.; Guibert G.; Sauzay B. *Calculus de signification par une logique d'operateurs*. Cepadues, 2016.

[4] Descles J.; Guibert G.; Sauzay B. *Logique combinatoire et λ-calcul: d'operateurs*. Cepadues, 2016.

[5] Gabbay R. Topoi. *A categorical analysis of logic.* Elsevier, 1984.

[6] Enderton H.B. *Elements of Set Theory.* Academic Press, 1977.

[7] Borges C.J.R. A study on many-valued functions. *Pacific Journal of Mathematics* 23:451–461, 1967.

[8] Euler, L. *Introductio to the Analysis of the Infinite.* Springer, 1988.

[9] Hardy G.H. *A Course of Pure Mathematics.* Cambridge University Press, third edition, 1921.

Sparse Models: a tractable fragment for SAT, MAXSAT and PSAT*

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Problems related to the satisfiability of propositional logic are usually NP-complete. In order to achieve tractability, one has to restrict the attention to a fragment of the propositional calculus. For example, the Horn-clause fragment and the 2-clause fragment. However, those fragments loose their tractability when extended to other NP-complete problems such as maximum satisfiability or probabilistic satisfiability. In this paper we describe a fragment of propositional calculus, which is called a sparse fragment, that is tractable and remains tractable when extended to maximum satisfiability and probabilistic satisfiability.

Consider n logical variables $\mathscr{P} = \{x_1, \ldots, x_n\}$ and let $\mathscr{L}$ be the set of all propositional formulas over those variables. A propositional valuation v is initially defined over propositional variables, $v : \mathscr{P} \to \{0,1\}$ and then is extended, as usual, to all formulas, $v : \mathscr{L} \to \{0,1\}$.

An *s-sparse model*, $0 \leq s \leq n$, is a proposition valuation v which assigns 1 to at most s variables. The class of *s-sparse models*, SPARSE_s, is defined as

$$\text{SPARSE}_s = \{v : |\{x_i : v(x_i) = 1\}| \leq s\}.$$

Lemma 1 *Given a class of s-sparse models over n variables, where s is fixed,* $0 \leq s \leq n$, *the size of the class is* $O(n^s)$ *and thus polynomial in n.*

Theorem 2 *The s-sparse SAT problem, the s-sparse weighted MAXSAT and the s-sparse Probabilistic Satisfiability (ss-PSAT) problems can be solved in poly- nomial time.*

This result is significant because most other tractable fragments of classical propositional logic do not generate tractable fragments of probabilistic satisfi- ability, namely 2-PSAT is NP-complete [3], and Horn PSAT is tractable only under very restricted conditions [1].

Furthermore, Theorem 2 allows us to polynomially approximate classical and probabilistic logic in the sense of [2].

References

[1] Andersen, K. and D. Pretolani (2001). Easy cases of probabilistic satisfiability. *Annals of Mathematics in Artificial Intelligence* 33 (1), 69–91.

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[2] Finger, M. and R. Wassermann (2003). The universe of approximations. In R. de Queiroz, E. Pimentel, and L. Figueiredo (Eds.), *Electronic Notes in Theoretical Computer Science*, Volume 84, pp. 1–14. Elsevier.

[3] Georgakopoulos, G., D. Kavvadias, and C. H. Papadimitriou (1988). Probabilistic satisfiability. *Journal of Complexity* 4 (1), 1–11.

Regrounding the Unworldly: Carnap's Politically Engaged Logical Pluralism

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The thought that we can be 'pluralist' about logic — that there are many formal systems, each of which has a claim at being 'logic' properly so called — has received quite a bit of recent attention. Throughout this attention has been the acknowledgement that, at least in some sense, this thought is also present in the very earliest days of analytic philosophy in the writing of Rudolf Carnap. Nonetheless, the exact nature of Carnap's pluralism remains obscure; clarifying just what it is that Carnap is pluralist about is the task of the first portion of this paper. There, an interpretation of Carnap resting on what I call the 'wide' interpretation of the principle of tolerance, which holds that he ought to be pluralist about (nearly) everything, will be offered. Because this reading of the principle is quite radical, substantial justification for reading it in this way is require. In section two I discuss the reasons Carnap has for his pluralism, and argue that they are based in the Vienna Circle's "Scientific World-Conception" — a platform of philosophical commitments which set the direction for the Circle's philosophical investigations as well as a program of social change. What emerges from this discussion is the often-ignored relationship between his logical pluralism and his political views. Finally in section three, I turn to the relationship between Carnap's pluralism and contemporary debates about logical pluralism. There, I argue that Carnap's is more interesting than it has been given credit for, due particularly to his motivations.

References

[1] Beall, J.C. and Restall, Greg (2006). *Logical Pluralism*, Oxford: Clarendon Press.

[2] Carnap, R. (1929). Wissenschaftliche Weltauffassung: Der Weiner Kreis (The Scientific Worldconception: The Vienna Circle). In Cohen, R. and Neurath, M. (eds.) *Empiricism and Sociology*, pp 299–318. Dordrecht: D. Reidel Publishing Company.

[3] Carnap, R. (1937). *The Logical Syntax of Language*. London: Kegan Paul, Trubner & Co.

[4] Carnap, R. (1959). The Elimination of Metaphysics Through the Logical Analysis of Language. In Ayer, A.J., editor, *Logical Positivism*, pp 60–81. New York: The Free Press

[5] Carus, A. (2005). *Carnap and Twentieth-Century Thought*. Oxford: Oxford University Press.

[6] Dutilh Novaes, C. (2018). Carnapian Explication and Ameliorative Analysis: A Systematic Comparison. *Synthese*, DOI: https://doi.org/10.1007/s11229-018-1732-9

[7] Friedman, M. (2000). *A Parting of the Ways: Carnap, Cassirer, and Heidegger*. Chicago: Open Court.

[8] Friedman, M. (2001). Tolerance and Analyticty in Carnap's Philosophy of Mathematics. In Floyd, J. and Shieh, S., editors, *Future Pasts: The Analytic Tradition in Twentieth-Century Philosophy*, pp 223–255. Oxford: Oxford University Press.

[9] Friedman-Biglin, N. (2015) Carnap's Tolerance and Friedman's Revenge. In Arazim, P. and Dancák, M., editors, *The Logica Yearbook 2014*, pp 109–125. Milton Keynes: College Publications.

[10] Ibarra, A. and Mormann, T. (2003). Engaged Scientific Philosophy in the Vienna Circle: The Case of Otto Neurath. *Technology in Society* Vol. 25 (2003): 235–247.

[11] Kissel, T. and Shapiro, S. (2017). Logical Pluralism and Normativity. *Inquiry*, DOI: `http://dx.doi.org/10.1080/0020174X.2017.1357495`.

[12] Loeb, I. (2013). Submodels in Carnap1s Early Axiomatics Reconsidered. *Erkenntnis*, DOI: `http://dx.doi.org/10.1007/s10670-013-9501-0`

[13] O'Neill, J. (2003). Unified Science as Political Philosophy: Positivism, Pluralism, and Liberalism. *Studies in History and Philosophy of Science* Vol. 34 (2003): 575–596.

[14] Restall, G. (2002). Carnap's Tolerance, Meaning, and Logical Pluralism. *The Journal of Philosophy* Vol. 99(8): 426-443.

[15] Richardson, S. (2008). The Left Vienna Circle, Part 1: Carnap, Neurath, and the Left Vienna Circle Thesis. *Studies in History and Philosophy of Science* Vol. 40 (2009): 14–24.

[16] Richardson, S. (2008a). The Left Vienna Circle, Part 2: The Left Vienna Circle, Disciplinary History, and Feminist Philosophy of Science. *Studies in History and Philosophy of Science* Vol. 40 (2009): 167–174.

[17] Romizi, D. (2012). The Vienna Circle's "Scientific World-conception": Philosophy of Science in the Political Arena. *HOPOS: The Journal of the International Society for the History and Philosophy of Science* Vol. 2 (2012): 205–242.

[18] Russell, G. (2017). Logic Isn't Normative. *Inquiry*, DOI: `http://dx.doi.org/10.1080/0020174X.2017.1372305.`

[19] Scheimer, G., Zach, R., and Reck, E. (2015). Carnap's Early Metatheory: Scope and Limits. *Synthese*, DOI: `http://dx.doi.org/10.1007/s11229-015-0877-z`.

[20] Steinberger, F. (2015) How Tolerant Can You Be? Carnap on Rationality. *Philosophy and Phenomenological Research* Vol. 92(3): 645–668.

[21] Steinberger, F. (2017) Frege and Carnap on the Normativity of Logic. *Synthese* Vol. 94: 143–162.

[22] Uebel, T. (2004). Carnap, the Left Vienna Circle and Neopositivist Antimetaphysics? In Awodey, S. and Klein, C., eds, *Carnap Brought Home: The View from Jena*, pp 247–277. Chicago: Open Court.

[23] Uebel, T. (2005). Political Philosophy of Science in Logical Empiricism: the Left Vienna Circle. *Studies in History and Philosophy of Science* Vol. 36: 754–773.

[24] Uebel, T. (2008). Writing a Revoloution: The Production and Early Reception of the Vienna Circle's Manifesto. *Perspectives on Science* Vol. 16(1): 70–102.

Towards the automated extraction of Hilbert calculi for fragments of classical logic

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From a universal-algebraic consequence-theoretic perspective, classical logic can be defined as the logic induced by the complete clone over 0,1. Up to isomorphism, any other 2-valued logic may then be seen as a sublogic / fragment of classical logic. In spite of the theoretical straightforwardness of such a presentation, one may argue that there is still very little common knowledge about the minimal combination of such fragments (cf. [1]), which in principle may be obtained by simply merging the corresponding Hilbert calculi. In 1941, Emil Post studied the lattice of all the 2-valued clones, (sets of finitary operations closed under projection and composition) ordered under inclusion (cf. [2]). This lattice - countably infinite yet constituted of finitely generated members - has constituted ever since an invaluable source of information and insights about the relationships among the sublogics of classical logic. In [3], Wolfgang Rautenberg explored Post's classification in proving that every 2-valued logic is strongly finitely axiomatizable; it is worth noting that this proof carries along an effective procedure for producing a Hilbert calculus to any fragment of classical logic. Thus, for each finite collection of 2-valued operations given as input, the only challenge concerning the implementation of Rautenberg's algorithm in a computational system would be to find the exact clone that it describes within Post's lattice. By an application of Kleene's Fixed-Point Theorem, any finite set of boolean functions may be closed under projection and finitary multiple composition in finite time, in a bottom-up fashion; as the thereby produced boolean clone must live in the corresponding lattice, by a clever top-down analysis of the latter one may then find its precise location. We have implemented both the latter tasks using the Haskell programming language and distributed the resulting system as a RESTful web service endpoint and as a web application to be freely used by the community. With that, the first step towards the automated extraction of a Hilbert calculus for any fragment of classical logic was made fully operational; the remainder of the work is to be based on Rautenberg's axiomatization procedure.

References

[1] Marcelino, S.; Caleiro, C.; Marcos, J. Combining fragments of classical logic: When are interaction principles needed? *Soft Computing*, in press, 2019. `https://doi.org/10.1007/s00500-018-3584-0`

[2] E. L. Post. The two-valued iterative systems of mathematical logic. *Annals of Mathematics studies*, No. 5, 1941.

[3] Rautenberg, W. 2-element matrices. *Studia Logica*, Vol. 40, Issue 4, pp. 315–353, 1981.

A formalization of the epistemological notion of the concept of surprise through game theory

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From the revision of the concept of surprise made in previous works*, the present paper aims to present a formalization of this concept using elements of game theory and information theory. This proposal of analysis of the formal structure of the concept of surprise differs from traditional proposals. An important starting point of this proposal is that there is no theoretical agreement in which surprise can be considered independently of an agent (epistemic or informative). In addition, it will be necessary to consider the relationship between the "Presupposition of Oniscience" about what is unknown and new phenomena in the agent environment. The presupposition of omniscience is defined from the iteration of the mode of knowledge of an agent. In this case, an agent knows that certain objects behave normally in an expected way and that, for cases where there is no such possibility, this will also be known. In other words, it is known when one can reasonably know the behavior of an object and when this is not the case. Finally, surprise will be defined as a certain type of outcome for bets made on facts in an informational system or context.

References

[1] Binmore, K. *Fun and games: A text on game theory.* [s.n.], 1991.

[2] Cover, T.; Thomas, J. *Elements of information theory.* [S.l.]: Wiley, 1991. (Wiley series in telecommunications).

[3] Davis, M. *Teoria dos Jogos. Uma Introdução não Técnica.* São Paulo: Editora Cultrix, 1973.

[4] Edwards, E. *Introdução à Teoria da Informação.* São Paulo: Cultrix, 1964.

[5] Jackson, M. O. A brief introduction to the basics of game theory. Electronic copy available at: http://ssrn.com/abstract=1968579, 2015.

[6] Laplace, P. S. *A Philosophical Essay on Probabilities.* [S.l.]: Chapman and Hall, 1902.

[7] Leyton-Brown, K.; Shoham, Y. *Essentials of Game Theory.* [S.l.]: Morgan and Claypool, 2008. (Synthesis Lectures on Artificial Intelligence and Machine Learning).

[8] Osborne, M. J.; Rubinstein, A. *A Course in Game Theory.* Cambridge, Massachussetts: MIT press, 1994.

[9] Plato. *Teeteto.* [S.l.: s.n.], 1973. (Em Diálogos. Tradução: Carlos Alberto Nunes).

[10] Rapoport , A. *Lutas, jogos e debates.* [S.l.]: Editora Universidade de Brasília, 1980.

*"On Surprise, Bewilderment and Information" at the Philosophy and Mathematics of Uncertainty and Vagueness (PMUV) (Campinas - SP 2012), "Sobre o conceito de surpresa" at the XXV Semana de Filosofia da UFRN (Natal - RN 2015), "O conceito de surpresa" at the XXVI Semana de Filosofia da UFRN (Natal - RN 2016), Gamble, Information and Knowledge em EBL 2019 (Pirenópolis - GO)

[11] Shannon, C. A. Mathematical theory of communication. Reimpresso com correções pelo *The Bell System Technical Journal*, v. 27, p. 379–423, 623–656, 1948.

[12] Smith, A. *The History of Astronomy*. Indianapolis: Liberty Fund, 1982. Vol. III of the Glasgow Edition of the Works and Correspondence of Adam Smith. (Essays on Philo- sophical Subjects, Vol. III of the Glasgow Edition of the Works and Correspondence of Adam Smith).

[13] Von Neumann, J.; Morgenstern, O. *The Theory of Game and Economic Behavior*. Princeton: Princeton University Press, 1953.

Compiling Reeo Certified Code*

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Critical systems require high reliability and are present in several domains. These are systems which failure may result in financial damage or even deaths [1]. Standard techniques of software engineering are not enough to ensure the absence of unacceptable failures and/or that critical requirements are fulfilled. Tests and diagrams are not enough to ensure that these requirements will always be satisfied. Formal methods provide a theoretical background which simplifies the task of mathematically guarantee that these systems will have their requirements met. Reo is a graphical modelling coordination language which focuses on model software construction in a compositional way [2]. Software interaction in Reo is modelled by taking advantage of natural properties in distributed systems, such as remote function calls and message passing. Constraint Automata are defined as the most basic formal semantic for Reo. Therefore Constraint Automata leads to the possibility of reasoning about the interaction of Reo connectors and providing certified code regarding the model.

This work describes the constructive formalization of Constraint Automata in Coq proof assistant, including a compositional operation tailored (but not limited) to model Reo connectors. Such formalization leads to a framework to define and reason about Reo connectors by means of Constraint Automata using Coq. Certified code extraction from the model regarding the compositions is also discussed, along with the implemented theory and usage examples. A front-end makes possible to a user not familiarized with Coq and logic formalisms to model using Reo connectors and compile it to certified code. The implementation is open-source and is available at https://github.com/simasgrilo/CACoq.

References

[1] Knight, John C.; Safety critical systems: challenges and directions. In: Proceedings of the 24th International Conference on Software Engineering, pp.547–550, ACM, 2002

[2] Arbab, Farhad; Reo: a channel-based coordination model for component composition. *Mathematical Structures in Computer Science*, 14:329–366, 2004.

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Applying Mathematics: Moving Beyond Wigner's Puzzle

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The *Applicability Problem* is the problem of explaining *why* mathematics is applicable to the empirical sciences. This problem is revived and reformulated by the physicist Eugene Wigner under the striking title "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" ([10], [2]). In this seminal work, Wigner argues that the applicability of mathematics is a *miracle*, "a wonderful gift which we neither understand nor deserve" (Wigner [10], [11], [9]). The reactions to this problem range from metaphysical claims about the mathematical structure of our universe to epistemic claims about the structure of our cognition and formalist claims about the nature of mathematics as a language (See e.g. [6], [5], [1], [8], [4], [7]).

On my view, the applicability of mathematics is limited and reasonable. More precisely, it is *limited to what is reasonable*([2]). Contrary to Wigner and other commentators, I argue that the applicability problem is a genuine philosophical problem the explanation for which is given on the basis of a detailed and case-by-case study. More fundamental than the why-question (why is mathematics applicable in the natural sciences) is the *how-question* (how is mathematics applicable in the natural sciences). By studying how mathematics is used in different eras and areas of natural sciences we begin to understand the relationship between mathematics and other sciences, and more importantly address questions such as what mathematics is as used and practiced. As a result of such inquiry, we realize that Wigner's applicability problem in its original formulation is a *pseudo problem*.

By inquiring into the origins of modern mathematical sciences we are in effect addressing the how-question and offering a solution to a modified version of the applicability problem. Our approach creates the space for the philosophical inquiry into the applicability of mathematics to itself as well as other (empirical and social) sciences ([3]).

References

[1] Ivor Grattan-Guinness. Solving wigner's mystery: The reasonable (though perhaps limited) effectiveness of mathematics in the natural sciences. *Mathematical Intelligencer*, pages 7–17, 2008.

[2] Arezoo Islami. A match not made in heaven: on the applicability of mathematics in physics. *Synthese*, 194(12):4839–4861, 2017.

[3] Jesper Lutzen. The physical origin of physically useful mathematics. *Interdisciplinary Science Reviews*, 36:229–43, 2011.

[4] Sundar Sarukkai. Revisiting the 'unreasonable effectiveness' of mathematics.*Current Science*, 88:415-423, 2005.

[5] Mark Steiner. *The Applicability of Mathematics as a Philosophical Problem.* Harvard University Press, Cambridge, MA, 1998.

[6] Max Tegmark. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality.* Knopf, New York, NY, 2014.

[7] K.V. Velupillai. The unreasonable ineffectiveness of mathematics in economics. *Cambridge Journal of Economics*, 29:849–872, 2005.

[8] EugeneWigner. Invariance in physical theory. *Symmetries and Reflections*, 1949.

[9] Eugene Wigner. The unreasonable effectiveness of mathematics in the natural sciences. *Communications in Pure and Applied Mathematics*, 13:1–14, 1960.

[10] Eugene Wigner. The role of invariance principles in natural philosophy. *Symmetries and Reflections*, 1963.

Conservative translations of four-valued logics in modal logic

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Following a proposal by Kooi and Tamminga (2013), we introduce a conservative Translation Manual for every four-valued truth-functional propositional logic into a modal logic. However, the application of this translation does not preserve the intuitive reading of the truth-values for every four-valued logic. In order to solve this problem, we modify the Translation Manual and prove its conservativity by exploiting the method of generalized truth-values.

References

[1] Kooi, B.; Tamminga, A. Three-valued Logics in Modal Logic. *Studia Logica* 101: 1061-1072, 2013.

Extensão de um provador automático de teoremas para lógicas confluentes

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Este trabalho objetiva estender o programa K_SP, um provador automático de teoremas para lógicas modais normais baseado em resolução, descrito em [3]. Um modelo para tais lógicas é uma tupla $\langle W; R_1, \ldots, R_n, \pi\rangle$, onde W é um conjunto não-vazio de *mundos possíveis*; cada R_a é uma relação binária $R_a \subseteq W \times W$, que define acessibilidade entre os mundos para cada agente $a \in A = \{1, \ldots, n\}$, um conjunto fixo e finito de índices; e $\pi \longrightarrow (W \longrightarrow P \longrightarrow \{true; false\}$ É uma função que associa com cada mundo em W e símbolo proposicional em P um valor de verdade [1].

A semântica da lógica K_n não impõe restrições sobre a relação de acessibilidade. Diferentes lógicas modais normais exigem propriedades específicas da relação de acessibilidade para que seus axiomas sejam satisfeitos. A lógica modal KT, por exemplo, exige que a relação seja refexiva; já $S5$ requer, além de reflexividade, simetria e transitividade.

O cálculo para estas diferentes lógicas depende do uso de uma ou mais regras de inferência derivadas das características destas relações. A implementação dessas regras é, portanto, necessária para que o software K_SP seja compatível com as respectivas lógicas modais normais. Até o momento foram implementadas com sucesso as regras para relações seriais, transitivas, euclidianas e simétricas, conforme definidas em [2]. A regra para refexividade havia sido implementada previamente. O provador é verdadeiramente multimodal, sendo tão somente necessário especificar quais operadores pertencem a quais lógicas, através dos arquivos de configuração da entrada. Por exemplo, na fórmula $\boxed{1}p \rightarrow \boxed{2}p$, o operador $\boxed{1}$ pode ser especificado como de conhecimento (na linguagem de $S5$) e o operador $\boxed{2}$ pode ser especificado como de crença, ou seja, na linguagem de $KD45$.

Como trabalho futuro, serão implementadas as regras de inferência descritas em [4], de modo que o K_SP possa ser usado para lidar com outras lógicas confluentes.

Referências

[1] Fitting and R. L. Mendelsohn. *First-Order Modal Logic.* Kluwer Academic Publishers, 1998.

[2] C. Nalon and C. Dixon. Clausal resolution for normal modal logics. *Journal of Algorithms*, 62(3):117-134, 2007.

[3] C. Nalon, U. Hustadt, and C. Dixon. K_S**P**: A resolution-based prover for multimodal K. In N. Olivetti and A. Tiwari, editors, *IJCAR 2016, Proceedings*, pages 406-415. Springer, 2016.

[4] C. Nalon, J. Marcos, and C. Dixon. Clausal resolution for modal logics of confluence. In S. Demri, D. Kapur, C. Weidenbach, (eds.), *IJCAR 2014, Proceedings*, p.322-36. Springer, 2014.

Lógica e sistemas implicativos

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Uma noção essencial da Lógica é a consequência lógica, que investiga sob que condições uma fórmula, pode ser obtida de uma coleção de outras fórmulas [4]. De modo simples, consideramos as fórmulas iniciais como as premissas e aquela obtida como a conclusão. Naturalmente temos uma relação de ordem nesta noção de consequência, as premissas que são anteriores, e a conclusão que é posterior ([2] e [3]). Contudo, as noções de consequência e ordem são imbricadas e buscamos explicitar algumas destas muitas relações íntimas. Desenvolveremos esta análise usando aspectos formais que baseiam estas noções.

Neste trabalho, tratamos dos conceitos de consequência lógica e ordem, primeiro em sistemas independentes, que procuram formalizar tais conceitos, e depois com uma lógica implicacional que ressalta o operador de implicação e internaliza o “se, então” num sistema formal específico dado por esta lógica.

O conceito de álgebra implicacional, para o qual usamos a expressão iálgebra, procura for- malizar uma noção lógica básica chamada implicação, ou condicional, no âmbito de uma estrutura algébrica [6]. Nesta apresentação, esquecemos que existem outros operadores lógicos importantes e nos concentramos na implicação.

A lógica implicacional [6], que denotamos por iL, é uma lógica proposicional com um único operador lógico para representar a implicação. Por portar um único operador, podemos destacar muitos aspectos lógicos da implicação. Veremos como este operador, com as propriedades aqui indicadas, conduzem naturalmente ao conceito de ordem.

A linguagem L de iL é construída a partir de um símbolo →, para a implicação, das variáveis proposicionais, cujo conjunto denotaremos por $Var(L) = \{p_1, p_2, p_3, \ldots\}$, e dos símbolos) e (. Em todos os sistemas tratados, destacamos o conceito de ordem.

A estrutura da lógica implicacional iL exigiu mais que somente a noção de ordem. Os axiomas da lógica implicacional conduzem para uma caracterização de ordem, mas só a ordem não é o suficiente para descrever o sistema tratado. De modo simplificado, podemos admitir que uma concepção usual de lógica exige, de maneira subjacente, um sistema de ordem, embora esta última noção não baste, em geral, para caracterizá-la.

Como uma contribuição adicional, apresentamos neste trabalho um modelo para a lógica iL, que acreditamos ser um teorema de representação das iálgebras, dado pelos espaços de clopens em espaços de Tarski.

Em trabalhos posteriores, sobre os quais já estamos tratando, devemos envolver estes desenvolvimentos com elementos das conexões de Galois ([1] e [5]).

References

[1] Feitosa, H. A.; Lázaro, C. A.; Nascimento, M. C. Pares de Galois e espaços de Tarski. Cognitio: Revista de Filosofia 19: 110–132, 2018.

[2] Feitosa, H. A.; Moreira, A. P. R.; Soares, M. R. Operadores de consequência e relações de consequência. Kínesis 8: 156–172, 2016.

[3] Feitosa, H. A.; Moreira, A. P. R.; Soares, M. R. Sobre relações de consequência com múltiplas conclusões. Cognitio: Revista de Filosofia, 17:73-82, 2016.

[4] Feitosa, H. A.; Paulovich, L. Um prelúdio à lógica. Editora UNESP, 2005.

[5] Lázaro, C. A.; Feitosa, H. A.; Nascimento, M. C.; Conexões de Galois e um exemplo vindo da lógica. In: Encontro Regional de Matemática Aplicada e Computacional (Ermac 2017), Bauru, Brasil, 2017. Caderno de trabalhos completos e resumos. pp. 94-100, UNESP-Bauru, 2017.

[6] Rasiowa, H. An algebraic approach to non-classical logics. Amsterdam: North-Holland, 1974.

A Lógica de Lewis Carroll no Ensino Médio

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O presente trabalho consiste na breve apresentação de uma unidade didática para o ensino médio sobre a silogística carrolliana, acompanhada do relato de sua aplicação e do exame de seus resultados, testando a proposta de Lindemann [3]. A principal motivação foi a busca por metodologias alternativas para o ensino de lógica, dado que a área é apresentada como um subtema nos livros didáticos brasileiros [5] e Carroll propôs uma metodologia original para o seu ensino [1].

A unidade foi aplicada na turma do segundo ano do Colégio Medianeira, em Candelária (RS), entre os dias 8 de outubro e 26 de novembro de 2018, totalizando oito encontros. A estrutura didática foi inspirada nas sugestões Gallo [3], almejando competências relativas ao conhecimento de noções lógicas e ao uso do método diagramático carrolliano.

A sensibilização do tema se deu pela apresentação de falácias com apelo cômico, problematizando aspectos dos argumentos com a introdução de noções lógicas. Após problematizar distintas propriedades lógicas, sugeri a investigação do método diagramático carrolliano enquanto uma teoria cujo conhecimento gera maior destreza lógica. A exposição teórica do método foi acompanhada por exercícios cuja complexidade aumentava paulatinamente. O penúltimo encontro consistiu em uma avaliação e o último foi usado para sanar dúvidas remanescente e fomentar uma conceituação, discutindo a importância e a utilidade da lógica.

Embora argumentos oriundos do cenário político atual possam ter uma capacidade de sensibilização maior, a unidade foi aplicada entre o primeiro e o segundo turno das eleições, com alunos votando pela primeira vez, por isso temi que a atenção se voltaria apenas para o conteúdo e não para a forma e optei por uma abordagem que, embora com menor potência sensibilizadora, cumpre melhor o seu papel.

O desenvolvimento seguiu tranquilo, recebendo um *feedback* negativo ao questionar por dúvidas. Mas o cenário mudou com a introdução dos exercícios. Diferente de algumas teorias filosóficas, aprender uma teoria lógica não se limita ao aprendizado de um conhecimento puramente teórico, mas incluí o aprendizado de uma técnica, possuindo um domínio prático relativo à proficiência de um fazer, cujas dificuldades só emergem a partir da própria prática. Daí a necessidade de exercícios assistidos em sala de aula, onde se pode esclarecer dúvidas com especificidades impossíveis de prever.

A distinção entre aspectos práticos e teóricos supracitada retém semelhança com os conceitos de *logica docens* e *logica utens* dos escolásticos. O termo *logica docens* refere-se à lógica pura ou científica, enquanto *logica utens* refere-se ao seu uso para tomadas de decisão [4]. O método carrolliano mostrou-se adequado ao ensino do domínio prático, pois possibilita que exercícios simples sejam feitos, sanando dúvidas básicas antes da introdução de exercícios complexos. Carroll [1] fornece exercícios úteis neste processo.

Referências

[1] Carroll, L. *The Game of Logic*. MacMillan and Co., 1886.

[2] Gallo, S. *Metodologia do ensino de filosofia: uma didática para o Ensino Médio*. Papirus, 2012.

[3] Lindemann, J. L. O Jogo da Lógica de Lewis Carroll: Uma alternativa para o ensino médio. In: *Refilo: Revista Digital de Ensino de Filosofia*. Vol. 3. 2:165-179, 2017.

[4] Newton, L. A. (Editor). *Medieval Commentaries on Aristotle's Categories*. Brill, 2008.

[5] Secco, G.D.; Pugliese, N. On how formal logic is presented to the Brazilian student: a critical analysis. In: Martínes, T. J. M. (Coord.). *Rutas didácticas y de investigación em lógica, argumentación y pensamiento crítico*. Chapter 3. Academia Mexicana de Lógica, pages 78-169, 2016.

Teaching Logic Historically

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We generally think that the history of logic is of merely historical interest, that it holds no important philosophical lessons. Our conception of logic as strictly formal, devoid of all content, reinforces the idea insofar as it suggests that, as Wittgenstein says in the *Tractatus*, there can be no surprises in logic, no significant logical advances. And yet, as the history of logic seems clearly to show, logic has been transformed over the course of its history, most recently, in the second half of the 19th century and on into the 20th. A philosophical investigation into the history of logic thus seems called for, one that will provide the critical distance on our conception of logic that is prerequisite to any adequate evaluation of that conception. Thinking historically, we find not only that logic begins with Aristotle's term logic, the study of syllogistic arguments, but also that it is subtly transformed by Kant. (Contrary to the received view, Kant did not think, and did not claim, that Aristotle's was the last word in logic.) Kant's logic, though it is monadic and not yet symbolic, is not a term logic but demonstrably the logic taken today to be classical—as Bertrand Russell clearly saw. Explicitly understanding the respects in which our (standard) logic is and is not Kantian, prepares the way for thinking about what might come after this logic. Does our logic reflect the revolutionary developments in the exact sciences of the 19th and 20th centuries as Kant's logic reflects the developments in the exact sciences in the 17th century? Certainly, our logic can seem to reflect those developments, for example, in our model-theoretic conception of language. Careful attention to the history of logic (and of mathematics) reveals that there are other lessons to be learned as well, lessons we would do well to teach our logic students. An adequate understanding of the science of logic as a vital and developing field requires familiarity with these different systems of logic in their historical contexts, as well as an understanding of the intellectual forces that led to their transformation one to another. Quite simply, if they are to understand logic, students need to learn the history of logic, And if students are to learn this history, we, in turn, must teach it historically.

References

[1] Macbeth, Danielle. *Frege's Logic.* Harvard University Press, 2005.

[2] Macbeth, Danielle. *Realizing Reason: A Narrative of Truth and Knowing.* Oxford University Press, 2014.

[3] Putnam, Hilary. *Philosophy of Logic.* George Allen and Unwin, 1971.

[4] Russell, Bertrand. *Principles of Mathematics.* Second edition. George Allen and Unwin, 1937.

[5] Sellars, Wilfrid. Empiricism and the Philosophy of Mind. In *Science, Perception, and Reality*, Routledge and Kegan Paul, 1963.

[6] Wittgenstein, Ludwig. *Tractatus Logico-Philosophicus.* Trans. D. F. Pears and B. F. McGuinness. Routledge and Kegan Paul, 1974.

Noções de Localidade baseadas em Categorias Modelo de Quillen sobre Estruturas Finitas

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Localidade é uma propriedade de lógicas, cujas origens se encontram nos trabalhos de Hanf [4] e Gaifman [7], tendo sua utilidade no contexto da teoria dos modelos finitos. Não existem dúvidas quanto a utilidade das noções de localidade, que são aplicáveis a um número enorme de situações. Entretanto, existe uma deficiência em tais noções: todas as suas versões se referem a isomorfismos de vizinhanças, que é uma propriedade bastante forte. Assim, a pergunta que imediatamente se coloca é: seria possível enfraquecer tal condição, e manter a Hanf/Gaifman-localidades? Em [1], Arenas, Barceló e Libkin estabelecem uma nova condição para as noções de localidade, enfraquecendo o requerimento de que vizinhanças sejam isomórficas, estabelecendo apenas a condição de que sejam indistinguíveis em uma dada lógica. Utilizando-se do fato de que equivalência lógica é frequentemente capturada por jogos de Ehrenfeucht–Fraïssé, os autores formulam um framework baseado em jogos no qual a localidade baseada em equivalência lógica pode ser definida. Assim, a noção definida pelos autores em [1] é a de localidade baseada em jogos. Apesar de bastante promissora, bem como fácil de aplicar, o framework baseado em jogos (utilizado para definir localidade sob equivalência lógica) tem o seguinte problema: a localidade sob equivalência lógica que utiliza o framework baseado em jogos é muito mais difícil de analisar do que a localidade sob isomorfismos. Por exemplo, se uma lógica L é Hanf/Gaifman-local sob isomorfismos, e L' é uma sublógica de L, então, L' também é Hanf/Gaifman-local sob isomorfismos. No entanto, para o caso de localidades sob equivalência lógica que se utilizam de frameworks baseados em jogos, as coisas não são tão simples assim. De fato, as propriedades de jogos que garantem a localidade não necessariamente são preservadas quando passamos para jogos mais fracos [1]. A questão que imediatamente se coloca é a seguinte: é possível manter a noção de localidade baseada em equivalência lógica, mas eliminar as dificuldades que surgem com frameworks baseados em jogos? Em outras palavras, é possível definir a noção de localidade sob equivalência lógica sem que, para isso, seja necessário recorrer a frameworks baseados em jogos? O ponto de partida motivacional da presente comunicação é dar o primeiro passo em direção a uma resposta para a questão acima. Para tal, eu irei apresentar o seguinte resultado: existe uma estrutura modelo de Quillen M sobre a categoria S de d-vizinhanças de tuplas de pontos de σ-estruturas e homomorfismos entre tais d-vizinhanças tal que para cada relação $\approx_k$ de equivalência k-homotópica, e cada relação de k-equivalência lógica $\equiv_k$, uma d-vizinhança $N\underline{a}$ de uma tupla $\underline{a}$ de pontos é k-homotopicamente indistinguível de uma d-vizinhança $N\underline{b}$ de uma tupla $\underline{b}$ de pontos, isto é, $N\underline{a} \approx_k N\underline{b}$ se, e somente se, $N\underline{a} \equiv_k N\underline{b}$, ou seja, em termos de k-equivalência lógica, $N\underline{a}$ e $N\underline{b}$ são indistinguíveis, para cada sentença primitiva-positiva com quantifier-rank k. Assim, diferente do que ocorre com a abordagem de Arenas, Barceló e Libkin, que partem de um framework baseado em jogos, eu proponho um framework baseado em estruturas modelo de Quillen, trazendo para o âmbito homotópico as questões sobre localidade sob equivalência lógica. Apesar de ser válido apenas para sentenças primitivas-positivas, o meu resultado mostra que tal abordagem é possível, e assim, temos indícios para seguir adiante. Isso é interessante não só por ser uma alternativa ao framework baseado em jogos, mas por abrir novas possibilidades para localidade sob equivalência lógica, a saber, o aparato técnico das estruturas modelo de Quillen.

References

[1] Arenas, M., Barcelo, P. & Libkin, L. *Game-based Notions of Locality over Finite Models*. Disponível em `https://homepages.inf.ed.ac.uk/libkin/papers/apal.pdf`, acessado em 01/02/2019.

[2] Droz, J-M. & Zakharevich, I. 2015, *Model Categories with Simple Homotopy Categories, Theory and Applications of Categories*, Vol. 30, No. 2, pp. 15-39.

[3] Gaifman, H. 1982, On local and non-local properties, *Logic Colloquium* '81, North Holland.

[4] Hanf, W. 1965, Model-theoretic methods in the study of elementary logic. In J.W. Addison et al., eds., *The Theory of Models*, North Holland, pp.132–145.

[5] Quillen, D.G.1967, *Homotopical Algebra*, SLNM 43, Springer, Berlin.

[6] Rossman, B. 2007, *Homomorphisms and First-Order Logic*. Disponível em `https://pdfs.semanticscholar.org/bc4e/0b69c02c916f419cbcde68e4f74c43ac394e.pdf`, acessado em 01/02/2019.

Should Schoolchildren Learn from Computers how to Construct Proofs?

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On October 2, 1956, Herbert Simon wrote to Bertrand Russell, reporting on the first computer proof found by the Logic Theorist (quotations are taken from [4]):

> Dear Earl Russell: Mr. Newell and I thought you might like to see the report of our work in simulating certain human problem-solving processes with the aid of an electronic computer. We took as our subject matter Chapter 2 of *Principia*, and sought to specify a program that would discover proofs for the theorems, similar to the proofs given there. We denied ourselves devices like the deduction theorem and systematic decision procedures of an algorithmic sort; for our aim was to simulate as closely as possible the processes employed by humans when systematic procedures are unavailable and the solution of the problem involves genuine "discovery". [. . .]

Russell replied:

> Dear Mr. Simon, Thank you for your letter of October 2 and for the interesting enclosure. I am delighted to know that *Principia Mathematica* can now be done by machinery. I wish Whitehead and I had known of this possibility before we wasted ten years doing it by hand. I am quite willing to believe that everything in deductive logic can be done by a machine.

A year later, Simon communicated to Russell the results of his and Newell's further experiments with the Logic Theorist:

> The proofs the *Logic Theorist* has discovered have generally been pretty close to those in Principia, but in one case it created a beautifully simple proof to replace a far more complex one in the book. [Explanation on LT's proof of proposition 2.85.] Since the machine's proof is both straightforward and unobvious, we were much struck by its virtuosity in this instance.

Simon continued with evidence that "the learned man and the wise man are not always the same person":

> the machine's problem solving is much more elegant when it works with a selected list of strategic theorems than when it tries to remember and use all the previous theorems in the book.

He concluded the letter with

> I am not sure that these facts should be made known to schoolboys.

Reply by Russell:

> Thank you very much for your letter of September 9, and for the enclosure. I am delighted by your example of the superiority of your machine to Whitehead and me. I quite appreciate your reasons for thinking that the facts should be concealed from schoolboys. How can one expect them to learn to do sums when they know that machines can do them better? I am also delighted by your exact demonstration of the old saw that wisdom is not the same thing as erudition.

In this paper, we shall report on an intelligent tutoring system for constructing proofs in natural deduction (Jaśkowksi-Fitch style). The system is based on a computer programme designed by Diderik Batens in the late 1980's, used ever since at Ghent University for teaching elementary logic, but recently extended and reimplemented into a web application by the authors. We shall show that the system relies on heuristic methods similar to those behind the *Logic Theorist*, but algorithmic in nature (though neither of the sort Simon referred to in his letter to Russell nor of the sort Simon and Newell had in mind when they introduced the concept of the *British Museum Algorithm* in [3]). We shall demonstrate that the system enables one to construct, in a "wise" and "elegant" way, proofs that share their high-level structure with informal proofs in logic and mathematics, and prove that (for decidable fragments) the heuristics form an algorithm for proofs in first-order predicate logic, thus extending the proofs from [2] to the predicative level. We shall show that the new implemenation satisfies the criteria from [5] and [1] for intelligent tutorings systems, argue that because of this it may contribute to the development of general problem solving skills, and conclude with an affirmative answer to the question in the title.

References

[1] S. Autexier, D. Dietrich, and M. Schiller. Towards an intelligent tutor for mathematical proofs. *arXiv preprint arXiv:1202.4828*, 2012.

[2] D. Batens. Natural heuristics for proof construction. Part I: Classical propositional logic. *Logique et Analyse*, 127–128:337–363, 1989. Appeared 1992.

[3] A. Newell, J. C. Shaw, and H. A. Simon. Elements of a theory of human problem solving. *Psychological review*, 65(3):151, 1958.

[4] H. A. Simon. *Models of my life.* MIT press, 1996.

[5] K. Vanlehn. The behavior of tutoring systems. *International journal of artificial intelligence in education*, 16(3):227–265, 2006.

Formalizando a Correspondência de Curry-Howard

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A Correspondência de Curry-Howard (CCH) é uma observação sobre a relação entre sistemas dedutivos usualmente entendidos como Lógicas e suas contrapartidas combinatoriais usualmente conhecidas como Teorias de Tipos. Apesar do uso difundido desse vocabulário no estudo de teorias de tipos (e.g. [4, §1.11]), evidências e argumentações são geralmente "folclóricas", indicadas vagamente de maneira intuitiva. Nao há, até onde pudemos verificar, nenhum tipo de formalização da CCH que permita que seus resultados possam ser enunciados de maneira formal fora dos dois ou três exemplos mais usuais. A consequência disso é que estender a CCH para outros sistemas é difícil.

Nosso objetivo é representar esses sistemas já conhecidos (como, por exemplo, a Lógica Proposicional Intuicionista PROP e a Teoria Simples de Tipos λPROP) em um framework unificado que permita a enunciação da CCH como um resultado formal. Não apenas os sistemas conhecidos, mas quaisquer outros que se encaixem nesse framework também pode ser explorados.

O framework escolhido é inspirado na ideia de sistemas dedutivos [2] como categorias enfraquecidas [1] — com a modificação do uso de multigrafos no lugar de grafos. Em uma formulação, **sistema dedutivo** é um multigrafo $(\mathsf{Obj}, \mathsf{Arr}, \mathsf{src}, \mathsf{trg})$ onde Obj e Arr são tipos arbitrários e

$$\mathsf{src}, \mathsf{trg} : \mathsf{Arr} \to \mathsf{Obj}$$

Um sistema com essas especificações também pode ser visto como o que é mais classicamente dito "uma lógica" — isto é, uma estrutura $(\Sigma, \vdash)$, onde$\vdash \subseteq \Sigma^{<\omega} \times \Sigma$. A construção é feita tomando $\Gamma \vdash \mathscr{J}$ se existe $f \in \mathsf{Arr}$ tal que $\mathsf{src} f = \Gamma$ e $\mathsf{trg} f = \mathscr{J}$.

O sistema de tipos $\overline{\mathscr{M}}$ correspondente à lógica representada pelo multigrafo $\mathscr{M}$ é construído a partir desses dados, como um novo sistema dedutivo.

Apresentamos resultados sobre essa construção que incluem relações funtoriais entre $\mathscr{M}$ e $\mathscr{M}$, que tanto explica a relação usual de "esquecimento" dada pela cch quanto a usual confusão sobre se a CCH é de fato uma bijeção ou não. Além do esquecimento, relações funtoriais tem como interpretação imediata a adequação dos sistemas de tipos a lógicas e vice-versa (resultados dessa natureza para alguns sistemas específicos podem ser encontrados em [3]). Finalmente, algumas das propriedades mais interessantes de sistemas de tipos, como propriedades de normalização de termos, podem ser extraídas diretamente da estrutura da lógica M e incluídas em $\overline{\mathscr{M}}$.

References

[1] J. Lambek e P. J. Scott. *Introduction to Higher-Order Categorical Logic.* Cambridge Studies in Advanced Mathematics. Cambridge University Press, 1988. isbn: 978-0-521-35653-4. (Acesso em 10/06/2018).

[2] nLab authors. deductive system. Em: *nLab*. Ago. de 2018. url: http://ncatlab.org/nlab/show/deductive+system (acesso em 25/08/2018).

[3] Morten Heine Sørensen e Paweł Urzyczyn. *Lectures on the Curry-Howard Isomorphism.* Studies in Logic and the Foundations of Mathematics 149. Elsevier Science, 2006. isbn: 978-0-444- 52077-7.

[4] The Univalent Foundations Program. *Homotopy Type Theory: Univalent Foundations of Mathematics.* Institute for Advanced Study: https://homotopytypetheory.org/book, 2013.

The vernacular as a source to dialetheism

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Dialetheism is the view that some contradictions are true. One of the main arguments for dialetheism originates from the allegation that natural languages, due their expressive resources, are able to derive true contradictions; one such prominent case would be the Liar paradox. So, it is claimed, natural language imposes dialetheism on us. In this paper we resist this argument from the vernacular to dialetheism. We argue that even if we can derive a contradiction using the resources of natural language, there is no obvious reason to believe that some contradictions are true. In fact, we argue that there are reasons (acceptable to dialetheists) to hold that contradictions cannot be true. To leave no room for worries, we also argue that one can resist the ensuing dialetheist argument for paraconsistency. According to the argument, facing contradictions without triviality in natural languages, a paraconsistent logic is mandatory. We argue that a classical consequence relation may be kept, even in the face of a contradiction. As a result, even in the face of contradictions in natural language, the classical picture according to which i) contradictions are false and ii) the consequence relation which is explosive, may be maintained.

Referências

[1] Michael, M. On a Most telling argument for paraconsistent logic. *Synthese.* pp. 3347–3362, 2016.

[2] Priest, G. Semantic closure. *Studia Logica.* 43: 117–29, 1984.

[3] Priest, G. *In Contradiction: a study of the transconsistent.* 2nd edition. Oxford: Oxford Un. Press, 2006.

Algebraizing Higher-Order Logics

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At first, the objective of this work is to show a way to extend (naturally) a category of propositional logics [1] to a category of logics with quantifiers, following some desired conditions:

- Generalize results of algebraizable logics, in particular for Blok-Pigozzi algebraizable logics [2] in a uniform way;
- Generalize some traditional proposals of algebrization of First-Order Logic (FOL), like Tarski's Cylindrical Algebra, Halmos's Polyadic Algebra, Quine's Predicative Functor Logic and Relation Algebra.
- Have connections with existing categorical logic approach ("quantifiers are adjoints") [3].
- Have definitions of abstract quantifier which are compatible with some of prescriptions with a quantifier *should be*, for example, for Frege, Russell or Quine.
- Generalize notions of "generalized quantifiers", for example, for Henkin, Mostowski, Curry and Lindström.

Based on Voutsadakis's approach of "sets with hierarchy" [4], [5], we propose definitions of formula functor and generalized quantifier:

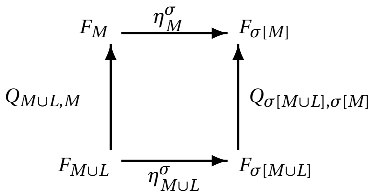
Lets **var** the category of variables $(Parts(V), \subseteq)$ and **clo** the category of (inflationary, increasing and idempotent) closure operators and morphisms that preserves indiscernibleness ($x \sim y \Leftrightarrow \forall A \subseteq X (x \in \overline{A} \Leftrightarrow y \in \overline{A})$). $F : \textbf{var} \to \textbf{clo}$ is a formula functor iff:

- (VA) F Preserves Intersections (Voutsadakis's Axiom)
- (CT) Images of monomorphisms are conservative translations (Conservative Translation)
- (PI) If $\sigma \in Aut(V)$ and $N \subseteq V$, then $F_N \cong F_{\sigma[N]}$ (Permutation Invariance)
- (DL) If $M \subseteq N$ and L is disjoint of M and N, then the inclusion $i : F_M \to F_N$ can be "lifted" to $\hat{i} : F_{M \cup L} \to F_{N \cup L}$ (Disjoint Lifting)

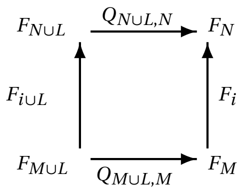
A family $\mathcal{Q}$ of quantifier instances is a family of **clo** morphisms, $Q_{N,M} : F_N \to F_M$, associated with inclusion morphisms $F_i : F_M \to F_N$, such that:

- (RD) $Q_{N,M} \circ F_{i_{M,N}} \dashv\vdash id_{F_M}$ (Deformation Retration)

- (PI) If $\sigma \in Aut(V)$ e $M, L \subseteq V$, so for all $Q_{M\cup L,M} : F_{M\cup L} \to F_M \in \mathcal{Q}$ exists $Q_{\sigma[M\cup L],\sigma[M]} : X_{\sigma[M\cup L]} \to F_{\sigma[M]} \in \mathcal{Q}$ such that the diagrams commutes: (Permutation Invariance)



- (DL) If $M \subseteq N$ and L are disjoint of M and N, the diagram commutes: (Disjoint Lifting)



A quantifier is a family of instances of quantifiers $\mathcal{Q} = \{A_{M,N} : M \subseteq N \subseteq V\}$ that is closed under composition.

A quantifier $\mathcal{A} = \{A_{M,N} : M \subseteq N \subseteq V\}$ is universal if:

$F_{i_{M,N}} \circ A_{N,M} \vdash id_{F_N}$ and for all $\mathcal{Q} = \{Q_{M,N} : M \subseteq N \subseteq V\}$, if $F_{i_{M,N}} \circ Q_{N,M} \vdash id_{F_N}$, then $Q_{N,M} \vdash A_{N,M}$ (Infimum)

A quantifier $\mathcal{E} = \{E_{M,N} : M \subseteq N \subseteq V\}$ is existential if:

$id_N \vdash i_{M,N} \circ E_{N,M}$ and for all $\mathcal{Q} = \{Q_{M,N} : M \subseteq N \subseteq V\}$, if $id_{F_N} \vdash F_{i_{M,N}} \circ Q_{N,M}$, then $E_{N,M} \vdash Q_{N,M}$ (Supremum)

Besides that, we want to show a covariant sheaf-like structure of algebras that works as a first-order algebraization of logics. In some cases, this structure can be extended to higher-order logics, generalizes the notions of topos, and have connections with monoidal categories and combinatory logic.

References

[1] Mariano, H. L.; Mendes, C. A. *Towards a good notion of categories of logics*, 2016. Availiable at: `https://arxiv.org/abs/1404.3780`

[2] Blok, W. J.; Pigozzi, D. *Algebraizable logics*, Memoirs of the AMS **396**, American Mathematical Society, Providence, USA, 1989.

[3] Lawvere, F. W. Adjointness in foundations *Dialectica* Vol. 23, No. 3/4, pp. 281-296. 1969.

[4] Voutsadakis, G. Categorical abstract algebraic logic categorical algebraization of first-order logic without terms. *Arch. Math. Logic* (2005) 44: 473.
`https://doi.org/10.1007/s00153-004-0266-7`

[5] Voutsadakis, G. On the Categorical Algebras of First-Order Logic. *Scientiae Mathematicae Japonicae*. 60. (2004) Avaliable at
`http://voutsadakis.com/RESEARCH/PUBLISHED/cylindric.pdf`

Tolerating Moral Conflicts in Paraconsistent Logics

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The study of moral conflicts has been studied extensively in the philosophy of moral reasoning and exclusively in the area of deontic logic. Moral conflicts are special kind of situations in which an agent ought to do each of a number of things but cannot do them all. On one hand, they seem common but on the other, core principles of deontic logic entail that they are impossible. This poses a dilemma for logics of normative propositions. Any system of logic that is supposed to apply to a broad range of normative discourse must somehow reconcile these two positions. In this paper I take two classic examples from famous Indian epic 'Mahabharat' where the protagonist Arjuna faces moral conflict in the battlefield of Kurukshetra. In the process of piecemeal analysis of Arjuna's dilemma, both the cases are intuitively characterized and logically examined. Since the conflicts are kind of inconsistencies and paraconsistent logics admit inconsistencies, even consider such situations as true. In classical logic, from a true conflict, the system becomes trivial; in the case of paraconsistent logics, the conflict does not necessarily trivialize the system. The conflict can be represented, operated, isolated, and the inference rules remain valid. The inquiry is to find an adequate set of principles to accommodate moral conflicts and to what extent should Krishna's intervention for resolving conflicts be logically valid. Meanwhile it is also interesting to relate Krishna's arguments for resolving Arjuna's conflict to paraconsistent approach of conflict tolerance.

References

[1] Almeida, M. J. (1990). Deontic logic and the possibility of moral conflict. *Erkenntnis*, 33(1), 57-71.

[2] Batens, D., Mortensen, C., Priest, G., & Van Bendegem, J. P. (2000). Frontiers in paraconsistent logic.

[3] Baskent, C. (2016). A Paraconsistent Logic for Contrary-to-Duty Imperatives.

[4] Brink, D. O. (1994). Moral conflict and its structure. The Philosophical Review, 103(2), 215-247.

[5] Da Costa, N. C., & Carnielli, W. (1986) On paraconsistent deontic logic. *Philosophia*, 16(3-4), 293-305.

[6] Da Costa, N. C., Krause, D., Bueno, O. (2007). Paraconsistent logics and paraconsistency. *Philosophy of logic*, 5, 655-781.

[7] De Haan, J. (2001). The definition of moral dilemmas: A logical problem. *Ethical Theory and Moral Practice*, 4(3), 267-284.

[8] Gabbay, D., Horty, J., Parent, X., van der Meyden, R., van der Torre, L. (2013). *Handbook of deontic logic and normative systems*.

[9] Goble, L. (2005). A logic for deontic dilemmas. *Journal of Applied Logic*, 3(3-4), 461-483.

[10] Goble, L. (2009). Normative conflicts and the logic of 'ought'. *Noûs*, 43(3), 450-489.

[11] Horty, J. F. (2003). Reasoning with moral conflicts. *Noûs*, 37(4), 557-605.

[12] Matilal, B. K. (2002). Moral dilemmas and religious dogmas. *The collected essays of Bimal Krishna Matilal's ethics and epics*, 6.

[13] Priest, G., Tanaka, K., Weber, Z. (1996). Paraconsistent logic.

[14] Priest, G. (2002). Paraconsistent logic. In *Handbook of philosophical logic* (pp. 287-393). Springer, Dordrecht.

[15] Radhakrishnan, S. (1963). *The Bhagavadgita*. With an Introductory Essay, Sanskrit Text, English Translation and Notes. 6th impression.

Boecio y el análisis de los indemostrables de los estoicos

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Una de las partes más interesantes del comentario de Boecio, *In Ciceronis Topica*, a los Tópicos de Cicerón es aquella que trata de lo que este último había escrito en su texto sobre los tópicos o lugares de argumentación (*loci argumentorum*) propios de los dialécticos. El análisis de Boecio está acompañado por una discusión de las nociones de antecedente, consecuente y cosas incompatibles (*repugnantia*) y de los llamados indemostrables de los estoicos. Éstos últimos son reglas de inferencia de lo que llamamos hoy la lógica proposicional estoica. Sexto Empírico, en su *Esbozo del pirronismo* (Sexto Empírico, 1993, p.190), cita cinco indemostrables y otras fuentes primarias para el estudio de los estoicos, como Diógenes Laercio, lo confirman. El *Modus Ponens* y el Tolens son el primero y el segundo indemostrables. El tercero puede simbolizarse así: $\neg(p \cdot q), p \Vdash \neg q$. Su premisa mayor es la negación de una conjunción, su premisa menor es la afirmación del primer miembro de la conjunción y la conclusión es la negación del otro miembro de la conjunción. En las palabras de Sexto: "No es verdad que sea de día y sea de noche; pero es de día; luego no es de noche" El cuarto indemostrable, según Sexto, tiene como premisa mayor una disyunción y como menor uno de sus miembros, y como conclusión, el otro. El ejemplo que da Sexto es: "O es de día o es de noche, pero es de día, luego no es de noche". Aparentemente se trataría de la disyunción exclusiva y se podría simbolizar así: $(p\mathrm{aut}q), p \Vdash \neg q$. Sin embargo, que se trate de la disyunción exclusiva ha sido colocado en duda no hace mucho tiempo (O' Toole y Jennings, 2004). El quinto indemostrable concluye a partir de la disyunción y de lo opuesto (*antikeímenon*) de uno de sus miembros, el otro. Por ejemplo, "O es de noche o es de día; pero no es de noche; luego es de día". En la notación contemporánea: $(p\mathrm{aut}q), \neg p \vdash q$. A estos cinco indemostrables Cicerón, en *Tópicos* §57, agrega dos indemostrables más, el sexto y el séptimo, cuya premisa mayor es la negación de una conjunción. El sexto: no esto y aquello, pero esto; luego no aquello (*non et hoc et illud, hoc autem; non igitur illud*). El séptimo: no esto y aquello, pero no esto ; luego aquello (*non hoc et illud; non autem hoc; illud igitur*). La introducción, por parte de Cicerón de éstos dos últimos indemostrables, es problemática. Por un lado el sexto indemostrable de la lista de Cicerón, parece ser una repetición del tercer indemostrable de la lista de Sexto y el séptimo es claramente una regla inválida puesto que del hecho de que una cosa no pueda ser un hombre y un perro al mismo tiempo, y no sea un hombre, no se sigue que sea un perro. Boecio, en In Ciceronis Topica modificó la lista de los indemostrables estoicos dada por Cicerón. Según Boecio, el tercer indemostrable tiene la forma siguiente: $\neg(p \to \neg q), p \vdash q$. Este indemostrable es el de las cosas incompatibles (repugantia). Para los estoicos dos estados p y q son incompatibles cuando no pueden coexistir y la expresión lingüística de esa incompatibilidad es: no es verdad esto y aquello $\neg(p \cdot q)$. Para Boecio, en cambio, la incompatibilidad es una relación entre proposiciones que surge de la siguiente forma: dadas dos proposiciones p, q tales que el condicional $p \to q$ sea verdadero, p y $\neg q$ son incompatibles y expresamos esa incompatibilidad en la forma $p \to \neg q$. En nuestro trabajo trataremos dos cuestiones. Primera, £por qué Boecio modificó en su comentario la lista de los indemostrables estoicos dada por Cicerón en sus Tópicos de la forma en que lo hizo? Segunda, llevando en cuenta esas modificaciones £cómo concibió Boecio, en su texto, la disyunción y el condicional estoico?

Bibliografia

[1] Boecio, A.M.S. *In Ciceronis Topica*. Traducción al inglés de Eleonore Stump, Ithaca: Cornell University Press, 2004.

Boecio, A.M.S *De topicis differentiis*. Traducción al inglés de Eleonore Stump, Ithaca:

[2] Cornell University Press, 2004.

[3] Cicerón, M.T *Topica*. Edición bilingüe con introducción anotación y traducción al inglés de Tobias Reinhardt. Oxford: Oxford University Press, 2003.

[4] Frede, M. *Die stoische Logik*. Göttingen: Vandenhoeck y Ruprecht, 1974.

[5] Kneale, W y Kneale M. *El desarrollo de la lógica*. Madrid: Tecnos, 1980

[6] O'Toole, R; Jennings, R. The megarians and the stoics. In: D.Gabbay y J. Woods. In: *Handbook of the history of Logic* Volume I , New York: Elsevier, 2004. Pp.397-522.

Neighbourhood semantics and modal logics with a necessity/impossibility operator

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In this paper I examine modal logics in which the modal operator $\Box$ can be read as necessity, or impossibility, or both. Consider classical modal logics, for example. They are the logics closed under the following rule of inference:

- $\alpha \leftrightarrow \beta \ / \ \Box\alpha \leftrightarrow \Box\beta$

Here the $\Box$ operator usually represents necessity. But it also can be read as possibility, impossibility, contingency, non-necessity, and even negation. On the other hand, if we consider the rule

- $\alpha \rightarrow \beta \ / \ \Box\beta \rightarrow \Box\alpha$,

$\Box$ can no longer be read as necessity, or possibility — but it makes sense to read it as impossibility or negation: if α implies β, and β is impossible, so is α.

I am interested in determining what conditions must be required on neighbourhood frames to force one of these different readings, and under what conditions the readings still remain neutral. A neighbourhood frame $\mathfrak{F}$ is a pair $\langle U, S\rangle$, where U is a nonempty set and S a function which assigns to every $u \in U$ a subset of the power set of U. Different conditions on the function S are required for the validity of different axioms or rules of inference. For example, for the second of the above rules, the condition is that if a set X belongs to a neighbourhood $S(u)$ of a point $u \in U$, then all subsets of X must also belong to $S(u)$.

In this paper I will consider only the necessity/impossibility readings. Several conditions of different strengths are identified, as well as the modal formulas corresponding to them. I also consider several logics obtained by adding one or more of these formulas as axioms, and prove soundness, completeness and decidability theorems for each of them.

Confluence of an Explicit Substitutions Calculus Formalized

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Rewriting theory is a well established model of computation equivalent to the Turing machines, and the best-known rewriting system is the λ-calculus, the theoretical foundation of the functional paradigm of programming. Confluence is an important property related to the determinism of the results given by a rewriting system. In this work, which is still in progress, we formalize the confluence property of an extension of the λ-calculus with explicit substitutions following the steps in [4,5]. Confluence is obtained through the Z property [3], and the first challenge of this work was to prove that an abstract rewriting system, i.e. a binary relation over an arbitrary set, that satisfies the Z property is confluent. The difficulty relies on the precise structure of the nested induction that needs to be done on the reflexive transitive closure generated by the divergence in the definition of confluence. The formalization is done in the Coq proof assistant [8], a system based on a constructive higher-order logic with a well developed extraction mechanism [6].

In the λ-calculus, terms that only differ by the name of its bound variables are considered equal. This notion is known as α-equivalence, which is a costly computational equivalence relation. Alternatives to α-equivalence include the so called De Bruijn indexes [2], where variables are represented by natural numbers. In De Bruijn notation terms have a unique representation, and hence there is no need of α-equivalence. Nevertheless, defining a reduction in De Bruijn notation requires a non-trivial algebra for referencing and updating indexes. The Locally Nameless Representation (LNR) [1] is a framework that takes the advantages of the two notations: bound variables are represented as De Bruijn indexes, while free variables uses names. The original framework uses classical logic and was built for representing pure λ-terms, therefore we decided to take some of its constructions (which are not based on classical logic) and extend it with a new operator for the substitution operation in such a way that our framework is constructive. This is important because one of the goals of this work is the generation of certified code via the extraction mechanism of Coq.

Our formalization is based on the paper [4], where the λex-calculus is defined. Another challenging step of this formalization is that the λex-calculus defines an equational theory based on the fact that reduction is done modulo permutation of independent substitutions. In order to avoid an explicit manipulation of permutation of independent substitutions, we use the generalized rewriting facilities of Coq [7]. Nevertheless, the generated equivalence relation needs to be defined over every expression generated by the LNR grammar, and not only over λ-terms with explicit substitutions.

In order to circumvent this problem, we proved that the reduction relation defined by the calculus in LNR modulo permutations of independent substitutions is restricted to λ-terms with explicit substitutions.

The formalization is available at `https://github.com/flaviodemoura/Zproperty.git` and is divided in two files:

1. The file `ZtoConfl.v` contains the proof that an abstract rewriting system R that satisfies the Z property is confluent;
2. The file `lex.v` contains the current status of the formalization showing that the calculus in LNR satisfies the Z-property, and hence is confluent.

References

[1] A. Charguéraud. The Locally Nameless Representation. *Journal of Automated Reasoning*, pages 1–46, 2011.

[2] N. G. de Bruijn. Lambda-Calculus Notation with Nameless Dummies, a Tool for Automatic Formula Manipulation, with Application to the Church-Rosser Theorem. *Indag. Mat.*, 34(5): 381– 392, 1972.

[3] B. Felgenhauer, J. Nagele, V. van Oostrom, and C. Sternagel. The Z property. *Archive of Formal Proofs*, 2016, 2016.

[4] D. Kesner. Perpetuality for full and safe composition (in a constructive setting). In Luca Aceto, Ivan Damgård, Leslie Ann Goldberg, Magnús M. Halldórsson, Anna Ingólfsdóttir, and Igor Walukiewicz, editors, *Automata, Languages and Programming, 35th International Colloquium, ICALP 2008, Reykjavik, Iceland, July 7-11, 2008, Proceedings, Part II - Track B: Logic, Seman- tics, and Theory of Programming & Track C: Security and Cryptography Foundations*, volume 5126 of *Lecture Notes in Computer Science*, pages 311–322. Springer, 2008.

[5] D. Kesner. A Theory of Explicit Substitutions with Safe and Full Composition. *Logical Methods in Computer Science*, 5(3:1):1–29, 2009.

[6] P. Letouzey. Coq Extraction, an Overview. In C. Dimitracopoulos A. Beckmann and B. Löwe, editors, *Logic and Theory of Algorithms, Fourth Conference on Computability in Europe, CiE* 2008, volume 5028 of *Lecture Notes in Computer Science*. Springer-Verlag, 2008.

[7] M. Sozeau. A new look at generalized rewriting in type theory. *J. Formalized Reasoning*, 2(1):41–62, 2009.

[8] The Coq Development Team. The coq proof assistant, version 8.7.2, February 2018.

The natural numbers as a yardstick for sets

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At the end of 'What numbers could not be' Paul Benaceraff sketches a conception of the natural numbers in which they serve "as a sort of yardstick we use to measure sets" [1, p. 292]. Following the approach to logical consequence laid out in [2], I present an account of elementary arithmetic in line with this conception. According to Etchemendy, we have defined the relation of logical consequence for a fixed language once we have a semantics that furnishes models for all and only those possibilities relevant to the truth of sentences in the language. If L is a language whose sentences concern the size of finite sets (and nothing else), then the possibilities at issue are precisely the natural numbers. The natural numbers thus provide a semantics in Etchemendy's sense for such a language L, and in doing so can be understood to fulfill the role Benaceraff attributes to them. What L amounts to in formal terms is natural and straightforward to formulate: it is the one variable language whose sentences are composed of numerical quantifiers and boolean combinations of 1-place predicates, termed C1 in [3]. From this perspective equations between arithmetical terms correspond not to sentences, but to rules of inference operating on sentences of $\mathscr{C}^1$. I consider how the resulting account of elementary arithmetic relates to standard formal accounts in which equations do correspond to sentences, and situate its picture of applied arithmetic within contemporary discussions of applied mathematics.

References

[1] Benacerraf, P. What numbers could not be. In: *Philosophy of mathematics: Selected readings* (P. Benacerraf and H. Putnam, editors). pp. 272–297, Cambridge University Press, 1998.

[2] J. Etchemendy. Reflections on consequence. In *New Essays on Tarski and Philosophy* (D. Patterson, editor), pages 263-299. Oxford University Press, Oxford, 2008.

[3] I. Pratt-Hartmann. On the Computational Complexity of the Numerically Definite Syllogistic and Related Logics. *Bulletin of Symbolic Logic.* 14: 1-28, 2008.

The Epistemic Account of Logic

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This talk is about the twin concepts of logical consequence and logical truth. For simplicity's sake only the former will be mentioned here. The picture presented here is purely epistemic; it shows that the concept of logical consequence is captured nicely on epistemic grounds only. Possible worlds and necessities, model-theoretic abstract entities, as well as proof-theoretic abstract entities, are not needed.

The picture to be presented starts with familiar informal definitions and examples:

Argument 1

If John is in London, is in Europe.
Alas, he is not in Europe.
Therefore, he is not in London.

This is a standard example of a valid argument. The familiar explanations of its validity count also as informal definitions of that concept: the above argument is valid because it is not possible for its conclusion to be false if all its premises are true; or because the class of possible worlds in which the premisses are true is a subset of the class of possible worlds in which the conclusion is true; ditto for models. Or it is valid because if one holds the logical constants in place, all sentences that result from replacing any other terms will have these feature: if the first two are true, so is the third. And so on and so forth.

These and other familiar explanations and informal definitions point sometimes in the same direction, sometimes in different directions. The proposed picture gets down to the philosophically basic and builds from there. It starts by asking why is the above argument valid, but not the following two:

Argument 2

London is a British city.
Therefore, snow is white.

Argument 3

John is looking at Venus.
Therefore, he is looking at Hesperus.

One may be tempted to say that Argument 2 is not valid because although both statements are actually true, it is possible for the conclusion to be false even if the premise is true. However, Argument 3 shows that this is not the whole story — indeed, it is not even part of the story under the epistemic picture. For although Argument 3 is not valid, it is not possible for the conclusion to be false if the premise is true (or so it is arguable after Kripke). Someone who doubted the truth of the conclusion although accepting the truth of the premise would not be making a reasoning mistake; it would be that person's geography that was to blame, not that person's reasoning powers (Edgington 2004).

The epistemic picture presents validity very simply as follows:

An argument is valid if on the basis of the truth conditions of its statements alone it is known that it does not have only true premisses and a false conclusion.

This definition will be explained in the talk, and some difficulties will be considered. All in all, this surprisingly simple picture looks promising although it is spartan in its simplicity. One of its most surprising consequences is that it allows for the complete elimination of alethic modal concepts like necessity and possibility. I call that modal eliminativism.

References

[1] Edgington, D. (2004) Two Kinds of Possibility. *Aristotelian Society Supplementary Volume* 78:1–22.

Implementação de um Tableaux para Lógica Modal Proposicional

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A lógica modal proposicional [2] é uma extensão da lógica clássica com operadores $\boxed{a}$ e $\langle a \rangle$, cujos significados estão usualmente associados às noções de necessidade e possibilidade, respectivamente, de acordo com o ponto de vista de um agente $a \in A = \{1, \ldots, n\}$, um conjunto fixo e finito de índices. Fórmulas modais são avaliadas em *estruturas de Kripke* $(W, R_1, \ldots, R_n, \pi)$, onde W é um conjunto não-vazio (de mundos), $R_i \subseteq W \times W$, $1 \leq i \leq n$ e $\pi : W \times P \to \{\mathbf{true}, \mathbf{false}\}$ é uma função que associa a cada mundo e símbolo proposicional em P um valor de verdade. Dada uma estrutura de Kripke M e um mundo $w \in W$ desta estrutura, a semântica do operador $\boxed{a}$ é definida por: $(M, w) \models \boxed{a}\varphi$, se e somente se $\forall w', (w, w') \in R_a, (M, w') \models \varphi$, onde $\models$ denota a relação de satisfatibilidade. O operador $\langle a \rangle$ é o dual do operador $\boxed{a}$, ou seja, $\langle a \rangle\varphi \leftrightarrow \neg\boxed{a}\neg\varphi$.

Este trabalho descreve a implementação de um tableaux modal rotulado [3], onde as regras de inferância são aplicadas a fórmulas rotuladas na forma normal negada.

Na implementação, foi utilizada entrada preprocessada pelo provador descrito em [5]. O preprocessamento coloca a fórmula da entrada na forma normal negada e aplica técnicas de eliminação de redundância, como simplificação, eliminação de literais puros e propagação de constantes são aplicadas. A construção do tableaux é feita em profundidade. Uma tabela de *hash* é utilizada para detecção eficiente de fórmulas já constantes em um ramo proposicional (ou seja, com um mesmo rótulo w).

A performance da implementação do cálculo melhora significativamente com a utilização da técnica de *caching* [4], que consiste em guardar informações sobre conjuntos de fórmulas sabidamente satisfatíveis ou não. Para tais conjuntos, não é necessário repetir a aplicação das regras de inferência. Avaliação sobre o *Logic Workbench* [1] mostrou um aumento de 20% no número de fórmulas para as quais o provador apresentou solução.

Referências

[1] P. Balsiger, A. Heuerding, and S. Schwendimann. A benchmark method for the propositional modal logics k, kt, s4. *Journal of Automated Reasoning*, 24(3):297-317, Apr 2000.

[2] B. F. Chellas. *Modal Logic: An Introduction*. Cambridge University Press, 1980.

[3] M. Fitting and R. L. Mendelsohn. *First-Order Modal Logic*. Kluwer Academic Pub., 1998.

[4] R. Goré, K. Olesen, and J. Thomson. Implementing tableau calculi using bdds: Bddtab system description. In S. Demri, D. Kapur, and C. Weidenbach, editors, *IJCAR 2014, Proceedings*, pages 337-343. Springer, 2014.

[5] C. Nalon, U. Hustadt, and C. Dixon. K_SP: A resolution-based prover for multimodal K. In N. Olivetti and A. Tiwari, editors, *IJCAR 2016, Proceedings*, pages 406-415. Springer, 2016.

A Resolution-Based Calculus for Preferential Reasoning: Abstract

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Preferential logics are part of a family of conditional logics intended for counterfactual reasoning [5], allowing to infer and to withdraw conclusions in the presence of new facts. Sequent or tableau calculi for such logics are notoriously hard to construct, and often require additional syntactic structure. Various conditional logics require nested sequents, labelled sequents or special transition formulae, together with non-trivial proofs of either semantic completeness or cut elimination [3, 4, 7, 8].

Recently [6], we have developed a resolution-based calculus for the preferential logic S [1]. The language of the preferential logic S extends the propositional logic with the conditional implication ($\Rightarrow$). In finite models, the modal formula $\varphi \Rightarrow \psi$ can be interpreted at a world w by stipulating that every φ-minimal world, according to a preference relation over the set of worlds, satisfies ψ. The dual of $\Rightarrow$ is $\not\Rightarrow$, that is, $\varphi \not\Rightarrow \psi$ is an abbreviation for $\neg(\neg\varphi \Rightarrow \neg\psi)$. The satisfiability problem for S is PSPACE-complete [2].

The calculus is clausal: a formula to be tested for satisfiability is firstly translated into an equisatisfiable set of clauses (via rewriting and renaming). There are two set of rules: one set of rules resembles propagation of formulae in the scope of the conditional implication and are very closely related to the axioms of S; the other set of inference rules are resolution-based [9], relying on the fact that monotonicity holds on the right-hand side of the conditional operator. The calculus is sound, complete, and terminating. Moreover, its pure syntactic nature makes it well suited for automation.

References

[1] J. P. Burgess. Quick completeness proofs for some logics of conditionals*Notre Dame Journal of Formal Logic*, 22(1):76-84, 1981.

[2] N. Friedman and J. Y. Halpern. On the complexity of conditional logics. In J. Doyle, E. Sandewall, and P. Torasso, editors, *Proc. of KR'94*, pages 202-213. M. Kaufmann, 1994.

[3] L. Giordano, V. Gliozzi, N. Olivetti, and G. L. Pozzato. Analytic tableaux calculi for KLM logics of nonmonotonic reasoning. *ACM Trans. Comput. Log*, 10(3):18:1-18:47, Apr. 2009.

[4] L. Giordano, V. Gliozzi, N. Olivetti, and C. Schwind. Tableau calculus for preference-based conditional logics: PCL and its extensions. *ACM Trans. Comput. Log.*, 10(3), 2009.

[5] D. Lewis. *Counterfactuals*. Harvard University Press, 1973.

[6] C. Nalon and D. Pattinson. A resolution-based calculus for preferential logics. In D. Galmiche, S. Schulz, and R. Sebastiani, editors, *Automated Reasoning - 9th International Joint Conference on Automated Reasoning, IJCAR 2018, held as Part of the Federated Logic Conference, FLoC 2018, Oxford, UK, July 14-17, 2018, Proceedings*, volume 10900 of *Lecture Notes in Computer Science*, pages 498–515. Springer, 2018.

[7] N. Olivetti and G. L. Pozzato. Nested sequent calculi and theorem proving for normal conditional logics: The theorem prover NESCOND. *Intelligenza Artificiale*, 9(2):109-125, 2015.

[8] N. Olivetti, G. L. Pozzato, and C. Schwind. A sequent calculus and a theorem prover for standard conditional logics. *ACM Trans. Comput. Log.*, 8(4), 2007.

[9] J. A. Robinson. A Machine–Oriented Logic Based on the Resolution Principle. *J. ACM*, 12(1): 23-41, Jan. 1965.

Ecumenismo Lógico

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Há mais de uma forma de se falar sobre a mesma coisa, e é plenamente possível que longas discussões sejam travadas sem que os debatedores discordem substancialmente sobre o objeto da controvérsia. Wittgenstein deixou seus contemporâneos em choque ao sugerir que muitas controvérsias filosóficas poderiam não passar de confusões linguísticas, e os reflexos desta sugestão se fazem sentir na filosofia analítica até hoje.

Passou-se a admitir que confusões linguísticas poderiam ter se infiltrado na filosofia, bem como que a aparente profundidade de diversos debates não nos ofereceria nenhuma garantia de que a discussão não envolve algum engano fundamental em suas bases.

Diante desta perspectiva, é de se perguntar: se as confusões linguísticas podem assolar a filosofia num geral, seria possível que também assolassem as discussões sobre Lógica? Em caso positivo, como se poderia proceder a um "esclarecimento de conceitos"? De que modo dois lógicos poderiam saber se estão discordando legitimamente ou, ao revés, se sua discordância não passa de um mal-entendido?

A dissertação que será apresentada, "Ecumenismo Lógico", investiga esta possibilidade a partir da análise de um caso emblemático na história da Lógica Matemática: o debate fundacional entre lógicos clássicos e intuicionistas.

A história recente da Lógica Matemática foi marcada por conflitos entre diferentes correntes filosóficas, cada uma buscando contextualizar, na tentativa de conquistar para si mesma o pódio fundacional das Ciências Formais, a atividade matemática a partir de seu próprio prisma analítico. Neste contexto, a dissertação realiza uma descrição da emergente literatura de propostas integrativas entre diferentes sistemas lógicos (apelidadas por Dag Prawitz de "ecumenismo lógico"), além de investigar alguns impactos que mudanças formais poderiam ocasionar nas concepções filosóficas de certas teorias matemáticas.

São investigadas, em especial, possíveis definições de operadores com comportamento clássico nas lógicas intuicionista e minimal, bem como versões alternativas e equivalentes destas mesmas regras. A estratégia ecumênica já empregada por Prawitz é estendida para sistemas ecumênicos que envolvam as lógicas clássica, intuicionista e minimal, e uma estratégia ecumênica inteiramente nova é apresentada (juntamente com uma discussão sobre suas vantagens e desvantagens).

Tais considerações são importantes porque, se tomadas a sério, sanariam diretamente algumas objeções de autores tão proeminente quanto Dummet e Heyting à lógica clássica, calcadas essencialmente em uma suposta ininteligibilidade desta. Como exemplo, podemos citar a crítica feita por Heyting no livro *"Intuitionism: an introduction"*:

"Indeed, the only positive contention in the foundation of mathematics which I oppose is that classical mathematics has a clear sense; I must confess that I do not understand that."

Portanto, se aceitarmos que estas novas fórmulas possuem o mesmo "sentido" que as fórmulas clássicas (em função da possibilidade de transformação de quaisquer derivações clássicas em derivações intuicionistas destas mesmas fórmulas, por exemplo), isto significa que o debate entre clássicos e intuicionistas deve ser revisitado, já que — e este parece ser o objetivo de Prawitz em seu artigo — os conceitos clássicos poderão ser redefinidos em termos intuicionistas, ainda que as fórmulas clássicas, é óbvio, passem a possuir um status semântico diferente das fórmulas intuicionistas.

Referências

[1] Nascimento, V. L. B.; *Ecumenismo Lógico.* Dissertação de Mestrado, PUC-RIO, 2018.

[2] Heyting, A.; *Intuitionism: An Introduction.* Studies in Logic and the Foundations of Mathematics, 17. Elsevier Science, 1956.

[3] Prawitz, D.; Classical versus intuitionistic logic. In: Haeusler, E.; Sanz, W.; Lopes, B. (Editors), *Why is this a proof?: Festschrift for Luiz Carlos Pereira*, número 27, Tributes, p. 15–32. 2015.

[4] Van Dalen, D.; *Brouwer's Cambridge Lectures on Intuitionism.* 1981.

Quine: Epistemologia, Ontologia e Linguagem

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Neste trabalho apresentaremos um breve estudo da epistemologia e ontologia de Quine, cujo objetivo é explicitar os esquemas conceituais ou sistemas coordenados utilizados para definição do estatuto ontológico de uma teoria ou objeto. Para isso, discutiremos o critério do compromisso ontológico, as teses da indeterminação da tradução e da inescrutabilidade da referência. O nosso referencial teórico será o ensaio Relatividade Ontológica e a obra Palavra e Objeto. No ensaio, verificaremos o problema da inescrutabilidade da referência, e a solução em uma perspectiva holística, ou seja, o objeto poderá ter sua explicação ou definição ontológica baseada nos esquemas conceituais, os quais possibilitam a construção de teorias. Já a partir do livro Palavra e Objeto discutiremos a tese indeterminação da tradução e a sua relação com o contexto da evidência. A relevância deste trabalho consiste em expor e discutir as teses da indeterminação da tradução e da inescrutabilidade da referência, as quais nos permitem discutir vários aspectos sintáticos e semânticos de sistemas linguísticos.

Crítica ao Logicismo, do *Tractatus Logico-Philosophicus* ao Período Intermediário

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O objetivo desta apresentação será o de argumentar em favor da hipótese de que a crítica de Wittgenstein ao logicismo presente nas *Philosophical Remarks* (PR) é desenvolvida pelo filósofo com base em argumentos que podem ser extraídos de sua obra de juventude, o *Tractatus Logico-Philosophicus* (TLP). Isso trará consequências interpretativas importantes. Relativamente às PR, podem-se discernir dois argumentos-chave que seriam desferidos contra a tese de que a aritmética se reduz à lógica, os assim chamados "*surveyability argument*" e o argumento modal. Mathieu Marion e Mitsuhiro Okada (2014) analisam ambos; porém, segundo sua leitura, tanto o *surveyability argument* quanto o argumento modal seriam postos contra o logicismo contendo nada além de premissas de caráter eminentemente epistemológico.

> The'modality argument' is directed at the Frege/Russell-definition of numbers in terms of one-one correlations. According to this argument, it is only when the F's and G's are few in number that one can know that they can be one-one correlated without knowing their numbers. Wittgenstein's 'surveyability argument' purports to show that only a limited portion of arithmetic can actually be proven within Principia Mathematica. For proof-constructions within this system quickly become unsurveyable and thereby loose their cogency. ([2])

Marion & Okada resguardam-se de avaliar que força teriam aqueles argumentos, assim postos, contra as teses de Frege e Russell. Não obstante está claro que, se tais objeções forem creditáveis a Wittgenstein, seremos forçados a admitir que o filósofo austríaco errou o alvo, procedendo muito semelhantemente ao matemático francês Henri Poincaré, que, ao se voltar contra o logicismo — tal como este lhe fora apresentado por Couturat — cometeu, segundo Goldfarb (1988), os enganos de confundir a noção de clareza com a de familiaridade, pretendendo introduzir investigações psicológicas na busca pelo discernimento do conteúdo objetivo de um juízo possível; em particular, na busca pelo discernimento do conteúdo objetivo fundamental de uma atribuição numérica. Possivelmente, Wittgenstein cometeu os mesmos enganos; algo que seria, entretanto, em alguma medida surpreendente, tendo em vista o explícito comprometimento daquele filósofo com o anti-psicologismo ditado por Frege (TLP 4.1121). Assim, será defendida nesta apresentação a hipótese de que, se corretamente entendidos, os argumentos apresentados especialmente nos §§99-121 das PR corroboram uma concepção daquilo que se expressa na linguagem, na lógica e na aritmética que induz ao reconhecimento de que é: tanto possível fazer espelhar-se a estrutura de qualquer equação na estrutura de uma tautologia, quanto necessário distinguir a aritmética da lógica; não, porém, com base em como conhecemos o que se expressa em equações e tautologias, mas antes com base na natureza daquilo que se expressa nelas. Sobre os mesmos parágrafos de que Marion & Okada se valem para reconstruir os argumentos de Wittgenstein como foi acima delineado, nos debruçaremos para entrever a possibilidade de se projetar ali algumas das ideias centrais do TLP acerca da lógica e da aritmética, apostando que assim se poderá obter um melhor entendimento da posição de Wittgenstein nos anos 1930, e da continuidade de sua crítica ao logicismo desde sua primeira incursão à filosofia.

Bibliografia

[1] Goldfarb, W. Poincaré Against The Logicists. *History and Philosophy of Modern Mathematics*, v.11:61–81, 1988.

[2] Marion, M. & Okada, M. Wittgenstein on Equinumerosity and Surveyability. Em *Themes From Wittgenstein and Quine, Grazer Philosophische Studien*, v.89, EUA, 2014.

[3] Wittgenstein, L. *Philosophical Remarks*. RU: Basil Blackwell, 1975.

[4] Wittgenstein, L. *Tractatus Logico-Philosophicus* Brasil: Edusp, 2001.

Five applications of the "Logic for Children" project to Category Theory

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Category Theory is usually presented in a way that is too abstract, with concrete examples of each given structure being mentioned briefly, if at all. One of the themes of the "Logic for Children" workshop, held in the UNILOG 2018, was a set of tools and techniques for drawing diagrams of categorical concepts in a canonical shape, and for drawing diagrams of particular cases of those concepts in essentialy the same shape as the general case; these diagrams for a general and a particular case can be draw side by side "in parallel" in a way that lets us transfer knowledge from the particular case to the general, and back.

In this talk we will present briefly five applications of these techniques: 1) a way to visualize planar, finite Heyting Algebras — we call them "ZHAs" — and to develop a feeling for how the logic connectives in a ZHA work; 2) a way to build a topos with a given logic (when that "logic" is a ZHA); 3) a way to represent a closure operator on a ZHA by a "slashing on that ZHA by diagonal cuts with no cuts stopping midway"; 4) a way to extend a slashing on a ZHA H to a "notion of sheafness" on the associated topos; 5) a way to start from a certain very abstract factorization of geometric morphisms between toposes, described in Peter Johnstone's "Sketches of an Elephant", and derive some intuitive meaning for what that factorization "means": basically, we draw the diagrams, plug in it some very simple geometric morphisms, and check which ones the factorization classifies as "surjections", "inclusions", "closed", and "dense".

Verificação do Protocolo de Autorização Dinâmica usando o Método Indutivo

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O Protocolo de Autorização Dinâmica [1] foi desenvolvido com o objetivo de garantir autenticidade e integridade para transações bancárias em um dispositivo inseguro. O protocolo utiliza um *smartphone* como um verificador confiável, manuseado por um usuário, para validar cada transação. O protocolo é a primeira versão de uma aplicação utilizada por uma base considerável de clientes, porém não é formalmente verificado.

A proposta deste trabalho é analisar formalmente o protocolo, utilizando o Método Indutivo, o qual já foi utilizado em outras análises interessantes. A teoria do método se baseia em construir um modelo definido indutivamente, representando os passos do protocolo, avaliando se as propriedades do modelo condizem com as especificações propostas pelo protocolo.

A formalização do canal definido pelo *smartphone* e considerável parte do protocolo foi implementada no assistente de prova Isabelle, usando técnicas similares de outras formalizações. Resta agora a descrição das propriedades e avaliação do modelo que define o protocolo, gerando uma prova formal da correção ou falha do protocolo em quaisquer características prometidas.

References

[1] L. P. Melo. DAP (Dynamic Authorization Protocol): Uma Abordagem Segura Out-of-Band para E-bank com um Segundo Fator de Autenticação. 2012. xv. 118 f. Tese (Doutorado em Engenharia Elétrica)–Universidade de Brasília, Brasília, 2012.

[2] L. C. Paulson. The Inductive Approach to Verifying Cryptographic Protocols. *J. Computer Security*, 85–128 1998.

[3] G. Bella. Formal Correctness of Security Protocols. *Information Security and Cryptography*, Springer, 2007.

[4] L. C. Paulson and G. Bella. Accountability Protocols: Formalized and Verified. In *ACM Transactions on Information and System Security*. 9(2):138–161,2006.

Grounding the Brazilian Consumer Protection Code on Unified Foundational Ontology: An Axiomatization in Description Logics

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The Brazilian Consumer Protection Code (BCPC) is a regulation that presents complex features: it embodies different aspects of law, like rules and principles, procedural law and substantive law, civil and criminal rules, and it is full of vague terms. Also, consumer complaints are one of the most frequent cases in Brazilian Law. A proper formalization of BCPC is not only a relevant challenge for formal methods, but also of practical importance, because of its applications, such as information integration, information retrieval, semantically enhanced content management and reasoning services (including with expert systems). Taking advantage of Semantic Web technologies, we construct Description Logic axioms for a significant part of BCPC, the respective OWL model, some study cases with possible uses in legal domain, and we discuss the limitations of this genre of formalization of law. Looking for a top level ontology as a foundational for our representation, we work upon Unified Foundational Ontology (UFO), which provides a profile with support for ontological well-founded models.

References

[1] Casanovas, P.; Sartor, G.; Biasiotti, M.A.; Fernández-Barrera, M. (2011) Introduction: Theory and Methodology in Legal Ontology Engineering: Experiences and Future Directions. In: Sartor G., Casanovas P., Biasiotti M., Fernández-Barrera M. (eds) *Approaches to Legal Ontologies: Theories, Domains, Methodologies*. Springer Netherlands, Dordrecht, pp.1–14, 2011.

[2] Grimm, S., Hitzler, P. Knowledge Representation and Ontologies. In: *Semantic Web Services: Concepts, Technologies, and Applications*, 51-106, 2007.

[3] Love N., Genesereth M. Computational Law. In: *Proceedings of the 10th International Conference on Artificial Intelligence and Law*. ACM, New York, NY, USA, pp.205–209, 2005.

[4] Rodrigues, C. M. de O.; de Freitas F. L. G. ; Oliveira I. J. da S. An Ontological Approach to the Three-Phase Method of Imposing Penalties in the Brazilian Criminal Code. In: *2017 Brazilian Conference on Intelligent Systems (BRACIS)*. IEEE, Uberlandia, pp.414–419, 2017.

[5] Valente, A. Types and Roles of Legal Ontologies. In: Benjamins V. R.; Casanovas P.; Breuker J.; Gangemi A. (eds) *Law and the Semantic Web: Legal Ontologies, Methodologies, Legal Information Retrieval, and Applications*. Springer Berlin Heidelberg, Berlin, Heidelberg, pp.65–76, 2005.

A dynamic logic for blind faith

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Over the last decades, the combination of epistemic logic with resources from dynamic logic has been used to represent changes of epistemic states in groups of agents, triggered by several epistemic actions [2]. Among those formulations for Dynamic Epistemic Logic (DEL), the systems for Public Announcement Logic (PAL) have stood out by their pioneering and simpler presentation. PAL systems deal only with the formalization of epistemic changes triggered by a simultaneous and universal disclosure (an "announcement") of a true information for all epistemic agents [3,4]. Among the semantic treatments for PAL, the standard approach has been a combination of well-known relational semantics (Kripkean style) and update semantics. Many important results and open problems can be found in specialized literature [1,5]. However, an interesting possibility for research has been neglected so far: the formulation of PAL systems that admit false formulas as announcement contents (that is, *within* modal operators of public announcements). Once formally implemented, this possibility is expected to become a generalization for standard formulations for PAL, and it should also provide a dynamic doxastic logic. This logic should formally represent changes in doxastic states in groups of agents who would (blindly) believe in public announcements (regardless of their truth or falsity). Besides, it should be an interesting contribution to a formal epistemology of belief. The development of this research program is not trivial in any sense, because the admissibility of false information within public announcements demands some radical reformulations, both in syntax and semantics, of standard PAL. As a matter of fact, our treatment for this dynamic logic for (blind) belief has revealed itself to be, not a general case for PAL (as initially intended), but a parallel research program. In our presentation, we shall propose a proper semantic treatment, as well as a sound and complete axiom system, for this dynamic doxastic logic.

References

[1] Benthem, J. van. *Logical Dynamics of Information and Interaction.* Cambridge University Press, 2011.

[2] Ditmarsch, H. van; W. van der Hoek, W. van der; Kooi, B. *Dynamic Epistemic Logic (Synthese Library).* Springer, 2007.

[3] Gerbrandy, J.; Groeneveld, W. Reasoning about information change. *Journal of Logic, Language and Information* 6(2):147–169, 1997.

[4] Plaza, J. Logic of public communications. *Synthese* 158:165–179, 2007.

[5] Wang, Y.; Q. Cao, Q. On axiomatizations of public announcement logic. *Synthese*, 190(1): 103–134, 2013.

Four-valued Non-deterministic Semantics for Free Logics

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Contrary to the classical logic, in the so-called free logics the singular terms do not necessarily refer to elements of the domain of a structure. According to Nolt [4], free logics are divided semantically into three major groups: positive, negative and neutral.

Neutral semantics has two main accounts: Lehman's truth-value gaps [3] and van Frassen's supervaluation [4]. Lehman's idea is that formulas in which empty terms occur should have no truth-value. The first problem is, as Nolt emphasizes, that the resulting semantics not only invalidates the Law of the Excluded Middle, but also many other standard logical principles. Besides, if the antecedent of an implication is true and the consequence is truth-valueless, should the implication as a hole be false or truth-valueless?

Van Frassen's supervaluation begins with a structure with a single (possibly empty) domain. Then, the set of completions of the structure is constructed. Each new structure that belongs to the set of completions will be based on positive semantics. However, as pointed out by Bencivenga in [1], this semantic consequence relation is too restrictive, since it does not validate the Deduction Metatheorem.

The positive and the negative structures have two domains: an inner domain, that is possibly empty, and a non-empty outer domain, that provides reference to all terms that have no referent in the inner domain. The positive and the negative approaches differ in the way they deal with sentences having terms that do not refer to individuals of the inner domain, that is, empty terms. In the positive case, an atomic formula with an empty term is always true. In the negative case, it is always false. Any alternative, however, seems arbitrary and faces difficulties in dealing with fictional names.

Fictional names are interpreted, in general, as empty terms in a free logic. The problem of a positive semantics is that the sentence

(A) Sherlock Holmes exists

is true. For the other hand, in a negative approach, (A) is false but

(B) Sherlock Holmes is a detective

is also false. In order to overlap those difficulties, Orlando suggests in [5] two kinds of truth-values: fictive and metafictive. In this perspective, it can be found a model in which (A) is metafictionally false but (B) is fictionally true.

An easy way in order to guarantee the Deduction Metatheorem and many other standard logical principles is a non-deterministic approach of implication respecting the classical conditions of implication. In this non-determinstic four-valued approach, if the antecedent of an implication is designated and the consequence of the implication is non-designated, then the implication as a whole is non-designated; otherwise, it is designated. Besides, Lehmann's concept of truth-valueless could be understood as the set of values:

{fictionally true, fictionally false}

Finally, we introduce as primitive the operator $\circ$, that makes easier to obtain the completeness result. In Da Costa's paraconsistent systems presented in [2], the operator $\circ$ marks the formulas that behave classically. Analogously, in this four-valued free-logic semantics, this operator marks the formulas that have a metafictional value.

References

[1] Bencivenga, E. Free Logic. In Gabbay, D. and Guenthner, F., editors, *Handbook of Philosophical Logic, vol. III: Alternatives to Classical Logic*, 373–425. 1986.

[2] Da Costa, N. C. A. *Sistemas Formais Inconsistentes.* UFPR, Curitiba, 1963.

[3] Lehmann, S. Strict Fregean Free Logic. *Journal of Philosophical Logic*, 23(3):307–336, 1994.

[4] Nolt, J. Free Logic. In Zalta, E. N. (Editor), *The Stanford Encyclopedia of Philosophy.* Metaphysics Research Lab, Stanford University, 2018.

[5] Orlando, L. Fictional Names without Fictional Objects. *Revista Hispanoamericana de Filosofía*, 40(120):111–127. 2008.

[6] van Fraassen, B. C. Singular Terms, Truth-Value Gaps, and Free Logic. *Journal of Philosophy*, 63(17):481–495, 1966.

F_{at} is undecidable

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System F, introduced in Girard [1], is a typed lambda-calculus which allows one to formalize second-order intuitionistic arithmetics. The universal quantifier elimination rule of system F

$$\frac{\forall X\, A}{A[B/X]}\ \forall\mathrm{E}$$

allows one to deduce all possible instantiations of a proposition A from a proof of its universal quantification. Due to the fact that the formula B, called the witness, can have arbitrary logical complexity, this rule is often called *impredicative*. *Predicative* subsystems of F are obtained by restricting the possible witnesses of the ∀E rule. One of the simplest systems in this family is System F_{at}, or atomic System F, which is the system obtained from System F by restricting the ∀E rule to atomic instances (i.e. witnesses have to be atomic formulas).

In a recent series of papers, Ferreira & Ferreira advocated the view that F_{at} should be conceived as the correct framework to investigate intuitionistic natural deduction, as it allows to escape the impredicativity of system F (see, e.g., [2], [3]). They introduced a technique they call instantiation overflow allowing to transform derivations in System F into derivations in Fat which satisfy a weak notion of the subformula property. In this setting, it seems rather natural to ask whether derivability in F_{at} is decidable or not.

We show that the answer is negative by providing the first proof of the undecidability of F_{at}. More precisely, given a system S over a language $\mathscr{L}$, let the S-derivability problem be the problem of deciding, given a formula $A \in \mathscr{L}$, whether there exists a closed S-derivation of conclusion A. We show that the F_{at}-derivability problem is undecidable. Undecidability is proved by showing that F_{at} is equivalent to a fragment of first-order intuitionistic logic with at most binary predicates which, in turn, is proved to be undecidable. This equivalence is interesting in itself as it shows that atomic second-order quantification can be faithfully simulated by first-order quantification. In conclusion, we argue that our result casts some doubts on Ferreira & Ferreira proposal to consider F_{at} as the correct framework to investigate intuitionistic natural deduction, as the former, unlike the latter, is decidable.

References

[1] Girard, J.-Y. Interprétation fonctionnelle et élimination des coupures de l'arithmétique d'ordre supérieur. Thése de Doctorat d'Etat, Université Paris VII, Paris, 1972.

[2] Ferreira F.; Ferreira, G. Commuting conversions vs. the standard conversions of the "good" connectives. *Studia Logica*, 92(1):63-84, 2009.

[3] Ferreira F.; Ferreira, G. Atomic polymorphism. *Journal of Symbolic Logic*, 78(1):260-274, 2013.

Aspectos do platonismo na matemática de Gödel

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Em seus textos filosóficos, publicados em vida ou postumamente, Gödel apresenta um platonismo na matemática forte. Sua filosofia sugere a existência de objetos matemáticos em um mundo extra-sensorial. Objetos como conceitos e conjuntos são acessados por um tipo de intuição matemática que justifica a introdução de axiomas da teoria de conjuntos a partir de uma exploração mais acurada dos conceitos envolvidos. Discutiremos as críticas a esse platonismo forte realizadas pela teoria causal do conhecimento. Apesar de ser uma filosofia da matemática baseada nos resultados de Gödel, esse platonismo não constitui um método de produção de teorias matemáticas praticável, mas pode ser visto a partir de uma interpretação metafórica do desenvolvimento de teorias em termos da prática matemática conforme mostraremos.

Referências

[1] Benacerraf, P. Mathematical Truth. *The Journal of Philosophy*. 70: 661-679. 1973.

[2] Chihara, C. S. *Ontology and Vicious-Circle Principle*. Cornell University Press, 1973.

[3] Gödel, K. What is Cantor's Continuum Problem?. 1947. In: Gödel, K. *Collected Works*. Vol. II, Oxford University Press, pages 176-188, 1990.

[4] Gödel, K. What is Cantor's Continuum Problem?. 1964. In: Gödel, K. *Collected Works*. Vol. II, Oxford University Press, pages 254-270, 1990.

[5] Gödel, K. Russell's Mathematical Logic. 1944. In: Gödel, K. *Collected Works*. Vol. II, Oxford University Press, pages 119-143, 1990.

[6] Gödel, K. Some basic theorems on the foundations of mathematics and their implications. *1951 Gödel, K. *Collected Works*. Vol. III. Oxford University Press, pages 304-323, 1995.

[7] Gödel, K. The Modern Development of the Foundations of Mathematics in the Light of Philosophy. In: Gödel, K. *Collected Works*. Vol. III. Oxford University Press, pages 374-387, 1995.

[8] Maddy, P. *Second Philosophy: A Naturalistic Method*. Oxford University Press, 2007.

[9] Parsons, C. *Mathematics in Philosophy: Selected Essays*. Ithaca, 1983.

[10] Steiner, M. Platonism and the Causal Theory of Knowledge. *The Journal of Philosophy*. 70.3: 57-66, 1973.

[11] Wang, H. *A Logical Journey: From Gödel to philosophy*. MIT Press, 1996.

Representing Rational McNaughton Functions via MODSAT Relativisation*

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Łukasiewicz Infinitely-valued Logic ($Ł_\infty$) is one of the most studied many-valued logics [5]. It has continuous truth-functional semantics, classical logic as a limit case and it possesses well developed proof-theoretical and algebraic presentations. A McNaughton function is a piecewise linear function with integer coefficients, and the semantics of $Ł_\infty$-functions represent all McNaughton functions and only those [2,3]. A rational McNaughton function is as a McNaughton function where its linear pieces have rational coefficients instead of the integer ones. There are some attempts to define logics based in $Ł_\infty$ with semantics representing rational McNaughton functions; these logics either have expensive computational complexity for deciding satisfiability [4,6] or extend the $Ł_\infty$-language [1]. In this work we investigate the possibility of implicitly representing a rational McNaughton function in a $Ł_\infty$-theory, which we call the $Ł_\infty$ MODSAT Φ logic, where Φ is a set of $Ł_\infty$-formulas that always have value 1 for the $Ł_\infty$-semantics.

References

[1] Amato, P.; Di Nola, A.; Gerla, B. Neural networks and rational Lukasiewicz logic. In: Annual Meeting of the North American Fuzzy Information Processing Society (NAFIPS-FLINT 2002), New Orleans, USA, 2002. *2002 Annual Meeting of the North American Fuzzy Information Processing Society Proceedings.* pp. 506–510, IEEE, 2002.

[2] McNaughton, R. A theorem about infinite-valued sentential logic. *The Journal of Symbolic Logic* 16:1–13, 1951.

[3] Mundici, D. A constructive proof of McNaughton's theorem in infinite-valued logic. *The Journal of Symbolic Logic* 59:596–602, 1994.

[4] Amato, P.; Porto, M. An algorithm for the automatic generation of a logical formula representing a control law. *Neural Network World*, 10:777–786, 2000.

[5]] Cignoli, R. L. O.; D'Ottaviano I. M. L.; Mundici D. *Algebraic foundations of many-valued reasoning.* Kluwer Academic Publishers, 2000.

[6] Aguzzoli, S.; Mundici, D. Weierstrass approximation theorem and Łukasiewicz formulas with one quantified variable. In: M. Fitting; E. Or lowska (Editors), *Beyond Two: Theory and Applications of Multiple-Valued Logic.* Springer, Studies in Fuzziness and Soft Computing, Volume 114, chapter 14, pages 315–335, 2003.

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Alguns apontamentos sobre Multi-Álgebra Universal

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Estruturas algébricas constituem boa parte da linguagem fundamental para modelar objetos da lógica e da matemáticos. A perspectiva aqui é a da Álgebra Universal, que estuda estruturas formadas por um conjunto X (geralmente não-vazio) munido de uma coleção de operações finitárias (geralmente de aridades 0, 1 ou 2) $* : X^n \to X$, $n \geq 0$, isto é, podemos pensar em $*$ como sendo uma "máquina" que recebe uma sequência finita de elementos de X, $\bar{x} = (x_0, \cdots, x_{n-1})$, e retorna um resultado $*(x_0, \cdots, x_{n-1} \in X$, que também é um elemento de X.

Em meados dos anos 1930, essa visão foi expandida para o que viria a ser uma multi-álgebra, que nada mais é do que um conjunto munido de uma multi-operação $* : X^n \to \mathbb{P}(X) \setminus \{\emptyset\}$, $n \geq 0$. A filosofia aqui, é a de que $*$ é uma nova "máquina" que recebe uma n-upla de elementos de X, $\bar{x}$, e retorna um resultado $*(\bar{x}) \subseteq X$, que é um subconjunto não-vazio de X. Evidentemente, toda álgebra determina uma multi-algebra com operações univaloradas. Claro que esta ilustração da noção de multi-algebra não conta parte importante da história, que é o fato das multioperações poderem ser descritas em linguagens relacionais por lógica de primeira ordem.

Essa nova abordagem traz algumas aplicações em Lógica, Teoria Álgebrica de Formas Quadráticas, Teoria dos Números e Geometria Tropical (conforme [3], [5], [6], [7], [4] e [1]). Indo mais além, as semelhanças e diferenças entre álgebras e multi-álgebras, bem como os aspectos lógicos envolvidos constituem um novo e interessante objeto de pesquisa.

Tendo isso em vista, começamos a fazer uma análise desta abrangente teoria do ponto de vista da teoria dos modelos e da álgebra universal. Mais especificamente, estamos em busca de uma compreensão de como adaptar os teoremas clássicos da álgebra universal (como por exemplo, os de Birkhoff [2]) no caso das multi-álgebras. A perspectiva inicial é considerar as diversas noções possíveis de (semi)identidade já que cada termo formal de aridade $n \in \mathbb{N}$ determina (recursivamente) uma multi-operação n-ária em uma multi-algebra X.

Referências

[1] Peter Arndt and Hugo Luiz Mariano. The von neumann-regular hull of (preordered) rings and quadratic forms. *South America Journal of Logic*, 2(2):201–244, 2016.

[2] Stanley Burris and Hanamantagida Pandappa Sankappanavar. *A Course in Universal Algebra - With 36 Illustrations.* 2006.

[3] Marcelo E Coniglio, Aldo Figallo-Orellano, and Ana C Golzio. Non-deterministic algebraization of logics by swap structures. *arXiv preprint arXiv:1708.08499*, 2017.

[4] Jaiung Jun. Algebraic geometry over hyperrings. *arXiv preprint arXiv:1512.04837*, 2015.

[5] Murray Marshall. Real reduced multirings and multifields. *Journal of Pure and Applied Algebra*, 205(2):452–468, 2006.

[6] Hugo Rafael Ribeiro, Kaique Matias de Andrade Roberto, and Hugo Luiz Mariano. Functorial relationship between multirings and the various abstract theories of quadratic forms. *arXiv preprint arXiv:1610.00816*, 2016.

[7] Oleg Viro. Hyperfields for tropical geometry i. hyperfields and dequantization. *arXiv preprint arXiv:1006.3034*, 2010.

Paraconsistency, evidence, and information*

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In [2], two paraconsistent formal systems, the basic logic of evidence (*BLE*, equivalent to Nelson's logic *N4* interpreted in terms of evidence) and the logic of evidence and truth (LET_J) were proposed. LET_J is a extension of *BLE* that recovers classical logic by means of a classicality operator $\circ$. The notion of evidence for a proposition A was explained in [2, Sec. 2] as 'reasons for believing and/or accepting A'. Evidence, when conclusive, gives support to the truth of a proposition A, and thus it has to do with the justification of A. But evidence can be non-conclusive, and so there may be conflicting evidence for A. Besides being weaker than truth, evidence does not imply belief: there may be evidence for A, an agent may be aware of such evidence but still not believe in A.

In [3, p. 589], Dunn characterizes a 'bare-boned' notion of information as "what is left from knowledge when you subtract, justification, truth, belief, and any other ingredients such as reliability that relate to justification." Information so understood is a pure propositional content, indeed similar to a Fregean thought but without its platonic ingredient. If we add some degree of non-conclusive justification to the bare-boned notion of information characterized above, we get precisely the notion of non-conclusive evidence presented in [2]. Note, however, that non-conclusive justification can be wrong, so it may end up not being a justification at all. We call this idea of a non-conclusive and maybe wrong justification a 'quasi-justification'. So, non-conclusive evidence for A is a quasi-justification for A. Situations in which we have something that may be or may not be a justification for some proposition A are quite common. So understood, the notion of information is more general than evidence, and the notion of evidence has an epistemic ingredient that the notion of bare-boned information lacks.

The logic LET_J, in particular, can be seen as a further development of Belnap's proposal in [1]. He remarks that his suggestion for the utility of a non-classical logic is local one – the 'global' logic is still the two-valued classical logic. In [1], the idea is that a computer receives information that can be contradictory and has to compute the values of complex propositions and draw inferences from that information. With LET_J, the computer would be able to perform these computations in two different ways, according to whether the information/evidence is reliable or not: unreliable information/evidence requires *BLE*, while reliable information/evidence requires classical logic.

The aim of this talk is to present and discuss this characterization of non-conclusive evidence as information plus a justification that can be non-conclusive, and to argue that both notions of evidence and information are fit to the non-dialetheist interpretation of the logics *BLE* and LET_J presented in [2].

*Joint work with Walter Carnielli.

References

[1] N. D. Belnap. How a computer should think, in *Contemporary Aspects of Philosophy* (ed. G. Ryle), Oriel Press, 1977.

[2] W. Carnielli and A. Rodrigues. An epistemic approach to paraconsistency: a logic of evidence and truth, *Synthese*, 2017, 10.1007/s11229-017-1621-7, preprint in `http://bit.ly/syntletj`.

[3] J. M. Dunn. Information in computer science, in *Philosophy of Information (Volume 8 of the Handbook of the Philosophy of Science)*, Elsevier, 2008.

Defeasible and ampliative reasoning according to C. S. Peirce

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According to C. S. Peirce, all valid reasoning is of one of three kinds – deduction, induction and abduction – or a combination of them three [CP 2.588, 1902] [1]. The way Peirce deals with these forms of reasoning gives much to think about non-monotonicity and defeasibility. Since very early, Peirce distinguished two kinds of deduction, necessary or strict deduction, and probable deduction, which is deduction upon probabilities. Neither forms are ampliative, that is, the conclusion does not enlarge the universe of discourse stated in the premisses. In the beginning, Peirce thought that induction and hypothesis were kinds of ampliative reasoning. But later on in his intellectual development he came to regard abduction as the very process of devising hypothesis, with an explanatory role in the economy of scientific research, and induction with the testing of hypothesis. Then, only abduction can be regarded as a form of ampliative reasoning. In his mature thought, Peirce strongly claimed to give to induction what is proper to induction and leave to abduction what only abduction can ascertain. This claim is supported by the distinction of at least three forms of induction – raw, quantitative, and qualitative – and the interpretation of abduction in terms of the known fallacy of affirming the consequent. He came to consider these latter forms of reasoning as the most important ones, for despite of not being deductively valid, they are rationally compelling. Raw induction is identified with induction by simple enumeration, the most basic form of inductive reasoning. Quantitative induction is statistical inference, which determines the value of a mathematical value of a whole class by sampling, that is, a real probability. Qualitative induction gathers evidence to make the hypothesis stronger, and so it is testing based on sampling of the possible predictions of the hypothesis. So understood, induction is a self-corrective method that in the long run would probably lead to the truth. Abduction is identified with the very process of introducing an explanatory hypothesis. As such, it introduces a counterfactual that opens up the range of possibilities without the smallest degree of necessity in the passage from premisses to conclusion, nonetheless its high heuristic power. Thus, despite being rationally compelling, abduction is not deductively valid. In this presentation the differences between the logical forms of these three kinds of reasoning will be presented in quasi-syllogistic terms, as Peirce often preferred. This will be understood upon the background of Peirce's general definition of reasoning as a general habit or self-controlled method of passing from the recognition of the truth of a proposition to the recognition of the truth of another one. For Peirce, the passage from premisses to conclusion in deduction would always or almost always lead to the truth. Induction means that if this kind of reasoning is consistently adhered to, it would eventually approximate indefinitely to the truth. And in abduction, the adoption of the premisses would be generally conducive to the ascertainment of truth, supposing there be any ascertainable truth. This is useful to see how in the end abduction and induction meet from the point of view of non-monotonicity, if their role in inferring general explanatory laws is considered.

References

[1] Peirce, Charles S. *The Collected Papers of Charles Sanders Peirce*. Ed. by: C. Hartshorne & P. Weiss (vols. 1-6); A. Burks (vols. 7-8). Cambridge, MA: Harvard University Press, 1931-58. 8 vols.

[2] Peirce, Charles S. *The New Elements of Mathematics*. Ed. by Carolyn Eisele. The Hague; Paris: Mouton Publishers; Atlantic Highlands, NJ: Humanities Press, 1976, 4 vols in 5.

[3] Peirce, Charles S. *Writings of Charles Sanders Peirce*: A Chronological Edition. Ed. by "The Peirce Edition Project". Bloomington; Indianapolis: Indiana University Press, 1982-2010. 7 vols.

Towards a Tableaux System for Propositional Minimal Implicational Logic

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In [3] we introduced the sequent calculus $\mathbf{LMT}^{\rightarrow}$ for Propositional Minimal Implicational Logic ($\mathbf{M}^{\rightarrow}$). $\mathbf{LMT}^{\rightarrow}$ has the properties to be terminating, sound and complete for $\mathbf{M}^{\rightarrow}$ and it is aimed to be used for proof search in a bottom-up approach. Termination of the calculus is guaranteed by a strategy of rule application that forces an ordered way to search for proofs such that all possible combinations are stressed. For an initial formula α, proofs in $\mathbf{LMT}^{\rightarrow}$ has an upper bound of $|\alpha| \cdot 2^{|\alpha|+1+2\mathring{u}log_2|\alpha|}$, which together with the system strategy ensure decidability. System rules are conceived to deal with the necessity of hypothesis repetition and the context-splitting nature of $\rightarrow$-left, avoiding the occurrence of loops and the usage of backtracking.

Therefore, $\mathbf{LMT}^{\rightarrow}$ steers the proof search always in a forward, deterministic manner. The system has the property to allow extractability of counter-models from failed proof searches (bicompleteness), i.e., the attempt proof tree of an expanded branch produces a Kripke model that falsifies the initial formula. Counter-model generation (using Kripke semantics) is achieved as a consequence of the completeness of the system.

In this article, we propose a Tableaux System for the same Logic. We compare our calculus with other known tableaux systems, especially with a variation of Fitting's Tableaux ([2]) adapted to the specific case with $\mathbf{M}^{\rightarrow}$. To do this comparison we use the criteria proposed in [1]. Finally, we present a transformation procedure that works in the rules of $\mathbf{LMT}^{\rightarrow}$ to produce a tableaux version of it, keeping all the properties aforementioned. We also evaluate the benefits of this version of our system in the implementation of automatic theorem provers for $\mathbf{M}^{\rightarrow}$.

References

[1] Dyckhoff, R. Intuitionistic decision procedures since gentzen. In *Advances in Proof Theory*. Springer, 2016, pp. 245–267.

[2] Fitting, M. C. *Intuitionistic logic, model theory and forcing*. North-Holland Amsterdam, 1969.

[3] Santos, J. B., Vieira, B. L., and Haeusler, E. H. A unified procedure for provability and counter-model generation in minimal implicational logic. *Electronic Notes in Theoretical Computer Science* 324 (2016), 165–179.

Intuitionistic Logic: Proof Search x Tableau Rules

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The objective in this presentation is to examine critically the relation between Analytic Tableau Rules for Intuitionistic Logic and its background in Kripke Semantics. This semantics makes some assumptions that seem far from being a direct consequence of the intuitionist position. Not that this position is clear and forever settled. In the end, the notion embedded in the syntactical calculus is a better guideline for formulating the rules. In fact, Gentzen's was one of the first to consider the possession of a decision procedure for Propositional Intuitionistic Logic, and what he had was a general proof search. His idea does not depend on any kind of semantical assumption unfit for satisfying some of the main intuitionistic tenets as we believe is the Kripke semantics.

References

[1] Sanz, W. *Cut as a semantical principle*, draft version. http://www.academia.edu/30059074/Cut_as_a_Semantical_Principle 18/02/2019.

As diferenças entre os pensamentos de Peirce e Russell sobre Filosofia, Matemática e Lógica

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O objetivo desta comunicação consistirá em, sob um determinado recorte, expor e comparar os pensamentos de Peirce (Charles Sanders Peirce) e Russell (Bertrand Russell): procurar-se-á (nesta comunicação) expor e comparar as concepções de Peirce e Russell sobre filosofia, matemática e lógica. Essa exposição comparativa terá o objetivo de mostrar e ressaltar as diferenças dos dois autores estudados (Peirce e Russell). Tendo-se em vista esse objetivo, a presente exposição será dividida em dois momentos: no primeiro momento, tratar-se-á exclusivamente de Peirce, e far-se-á uma breve exposição de suas concepções de filosofia, matemática e alguns elementos de lógica; no segundo momento, tratar-se-á de Russell nos temas de filosofia matemática e lógica, notando-se que, ainda neste segundo momento, efetuar-se-á, para cada um dos temas tratados, uma comparação entre as ideias de Peirce e de Russell.

Referências

[1] Anellis, I, H. *Peirce rustled, Russell pierced: how Charles Peirce and Bertrand Russell viewed each other's work in logic, and an assessment of Russell's accuracy and rôle in the historiography of logic.* Disponível em: https://projecteuclid.org/euclid.rml/1204835530. Consulta realizada dia 01/04/2018.

[2] Brady, G. From the algebra of relations to the logic of quantifiers. In Houser, N (org) e Roberts, D, D (org) e Van Evra, J (org). *Studies in the logic of Charles Sanders Peirce.* Indiana: Indiana University Press, 1997.

[3] Eames, E. R. *Bertrand Russell's dialogue with his contemporaries philosophical explorations.* Carbondale: Southern Illinois University Press, 1989.

[4] Hilpnen, R. e Queiroz, J. Uma introdução aos sistemas alfa e gama de grafos existenciais de C. S. Peirce. In Queiroz, J (org) Moraes, L (org). *A lógica de diagramas de C. S. Peirce.* Juiz de fora: Editora UFJF, 2013.

[5] Ibri, I, A. *Kósmos Noētos: a arquitetura metafísica de Charles S. Peirce.* São Paulo: Editora Perspectiva e Editora Hólon, 1992.

[6] Moore, M, E. Introduction. In Peirce, C, S. *Philosophy of Mathematics: selectd writings.* Indiana: Indiana University Press, 2010.

[7] Peirce, C, S. The logic of relatives. In *The monist.* Vol VII, nº: 2, Janeiro de 1897.

[8] Pinto, P, R, M. *Iniciação ao silêncio: análise do Tractaus de Wittgenstein.* São Paulo: Edições Loyola, 1998.

[9] Queiroz, J e Moraes, L. *Grafos existenciais de C. S. Peirce: uma introdução ao sistema alfa.* Disponível em: `https://revistas.pucsp.br/index.php/cognitiofilosofia/article/viewFile/13484/9994.` Consulta realizada em 01/05/2018.

[10] Russell, B. *Introduction to mathematical philosophy.* Londres: George Allen & Unwin LTD, 1920.

[11] Russell, B. *Mysticism and logic and other essays.* Londres: George Allen & Unwin LTD, 1959.

Sobre a não-admissibilidade de uma linguagem universal de acordo com a Teoria de Tarski

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Neste ensaio estou considerando uma observação elaborada por Tarski [3] concernente à não-admissibilidade de uma teoria semântica generalizada. Com base na crítica de Tarski, apresento uma *hipótese* fundamentada nos seguintes resultados:

I) *prova* de Gentzen [2] da *consistência* da aritmética recursiva em primeira ordem $\mathbf{AR^1}$ [via indução transfinita];

II) teorema de Shoenfield-Feferman [1]: 'todas as sentenças verdadeiras da $\mathbf{AR^1}$ são demonstráveis a partir de uma progressão transfinita recursiva de sistemas axiomáticos' [via princípios de reflexão e regra-ω restrita (ou recursiva)];

III) metateorema de Tarski [3] sobre a *indefinibilidade* da *verdade* para linguagens de ordem infinita.

Admitindo-se essa hipótese é possível mostrar que, de fato, não há um sistema formal linguístico *único* que contenha todas as demonstrações de todas as sentenças aritméticas verdadeiras. Assim, nenhuma linguagem pode conter toda a teoria dos números naturais e, por conseguinte, a aritmética pressupõe uma sequência transfinita *ilimitada* de linguagens de ordem superior.

References

[1] Feferman, S., Transfinite Recursive Progressions of Axiomatic Theories, *The Journal of Symbolic Logic*, vol. 27 (1962), pp. 256-316.

[2] Gentzen, G., Die Widerpruchsfreiheit, *Math. Ann.*, vol. 112 (1936), pp. 493-565.

[3] Tarski, A., *Logic, Semantics, Mathematics*, Ed. Hackett Publishing Company (Second edition, J. Corcoran ed.), 1983.

Diagrams and computers in the first proof of the Four-Color Theorem

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The use of diagrams and the use of computers are two significant themes within the philosophy of mathematical practice. Although case studies concerning the former are abundant — from the notorious case of Euclidean geometry to the uses of diagrams within arithmetic, analysis, topology, knot theory, and even Frege's *Begriffschrift* —, the latter has received less attention in the field.

I show in my talk how the two themes can be investigated simultaneously via an analysis of the famous case of the Four-Color Theorem (4CT). Whenever the use of computers in mathematical practice is considered, the computer-assisted proof of the 4CT is mentioned. Philosophical discussion of the proof has centered mostly on Tymoczko's argument for the introduction of experimentation in mathematics via 4CT — notably made in [1]. (See [2] for a recent version of this position.)

In previous work, I revised central leitmotifs in rejoinders presented against Tymoczko's claims, arguing from a Wittgensteinian perspective that the 4CT is relevant to contemporary discussions on the use of computers in mathematics (especially in [3]). Aiming a discussion about the criteria for the identity of computer-assisted proofs through an examination of the various proofs of the 4CT, in my talk, I will show the main lines of articulation between the more than 3000 diagrams and the computational machinery mobilized in the construction and the verification of Appel and Haken first version of the proof.

After presenting the way diagrams and computers participate in the proof, dealing with the passage from topology to combinatorics operated in it, my primary strategy consists in projecting the methodological contribution recently suggested by De Toffoli — namely, the three criteria she proposes as tools for evaluating the effectiveness of mathematical notations (expressiveness, calculability, and transparency; cf. [4]) — into the case of Appel and Haken's proof of the 4CT. In so doing, I will specify the ways in which the diagrams of this case study can be considered a perspicuous mathematical notation, as well as to propose some questions regarding the way this notation is related to the computational devices indispensable to the proof.

References

[1] Tymoczko, T. The Four-Color Problem and its Philosophical Significance. *The Journal of Philosophy* 27-2:57-83, 1979.

[2] Johannsen, M. W.; Misfeldt, M. Computers as a Source of A Posteriori Knowledge in Mathematics. *International Studies in the Philosophy of Science* 30-2:111-127, 2017.

[3] Secco, G. D.; Pereira, L. C. Proofs *Versus* Experiments: Wittgensteinian Themes Surrounding the Four- Color Theorem. In: M. Silva (Editor), *How Colours Matter to Philosophy*. Springer, Studies in Epistemology, Logic, Methodology, and Philosophy of Science, Volume 388, chapter 17, pages 289-308, 2017.

[4] De Toffoli, S. 'Chasing' The Diagram–the Use of Visualizations in Algebraic Reasoning. *The Review of Symbolic Logic* 10-1:158-186, 2017.

Construção Semântica para Categoria de Hilbert

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O objetivo principal deste estudo é o de fornecer as bases para a elaboração de uma semântica para a "Categoria dos Espaços de Hilbert" (Hilb), o que requer, a nosso ver, o recurso a uma álgebra, necessária para interpretar certas propriedades dos bifuntores e dos isomorfismos, específicas em (Hilb). Concebida, inicialmente, para tratar certos problemas característicos da Computação Quântica, essa categoria mostrou-se, posteriormente, também eficaz para lidar com determinados objetos da Mecânica Quântica. Assim, além das suas notórias possibilidades praticas, essa mesma categoria possui um enorme potencial teórico que, nos parece, merece ser amplamente explorado, sobretudo na medida em que isto permite integrar sintaxe, semântica e pragmática de uma categoria através da qual se pode fácil e eficazmente transitar tanto pela Matemática e pela Física, quanto pela Logica. Preliminarmente, expomos a sintaxe de (Hilb) com base nos trabalhos de Baez (2004), Coecke (2008) e Heunen (2009); em seguida faremos uma interpretação de determinadas propriedades de (Hilb) e ao final apresentaremos um esboço da semântica que estamos construindo para essa categoria.

Referências

[1] Baez, J. C. A category-theoretic perspective. *math.CT*, p. 21, Feb 2004.

[2] Coecke, B. Introducing categories to the practicing physicist. *quant-ph*, p. 29, Aug 2008.

[3] Heunen, C. J. M. An embedding theorem for Hilbert categories. *Theory and Applications of Categories*, v. 22, n. 13, p. 321–344, 2009.

Relações Lógicas entre Princípios de Indução para os Números Naturais

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Na literatura matemática, principalmente em livros introdutórios e artigos básicos sobre os números naturais, um tipo comum de asserção – frequentemente deixada como exercício para o leitor – é que certas formas de indução em $\mathbb{N}$ (regular/ordinária, completa/forte) são equivalentes umas às outras e ao princípio da boa ordem, entre outros [1, pp. 3, 62], [7, p. 11], [3, p. 17], [4, pp. 124-125], [6, Chapter 4, Problem 3], [2, p. 17], [8, p. 107]. Isto significa que se P1 e P2 são dois destes princípios, então, sob todas os outros princípios usualmente considerados para o conjunto dos números naturais (e.g., axiomas de Peano, exceto axioma de indução [9]), P1 implica P2 e vice-versa. Neste trabalho, mostramos que, para uma formalização razoável, algumas das implicações alegadas entre esses princípios valem somente mediante uma condição adicional, a saber: todo número natural não nulo é um sucessor. Esta condição é uma consequência do chamado princípio de indução regular (**RI**) (axioma de indução na aritmética de Peano), mas não de outros, como o princípio de indução completa (**CI**), que diz o seguinte: "Suponha que uma propriedade P dos números naturais é tal que para todo número natural n, se P vale para todos números naturais menores que n, então P vale para n. Conclusão: P vale para todos números naturais." [6]. Na verdade, **RI** implica **CI**, mas, em geral, **CI** não implica **RI**: um contraexemplo para a implicação **CI** $\Rightarrow$ **RI** é o número ordinal transfinito $\omega + \omega$. Por outro lado, se tomarmos como axioma que todo natural não nulo é um sucessor, então a implicação **CI** $\Rightarrow$ **RI** torna-se válida. Diante disto, faz-se necessária uma revisão de todas as implicações usualmente aceitas entre princípios de indução em $\mathbb{N}$ e afirmações similares. A partir de uma lista de 9 propriedades, consideradas princípios de indução, ou similares, indentificamos todas as implicações válidas e todas as implicações "quase válidas" entre elas. Refutamos as implicações inválidas por meio de contraexemplos que consistem em estruturas semelhantes a modelos de Peano [5, p. 323], mas que possuem uma relação de ordem e não necessariamente satisfazem o axioma de indução de Peano (**RI**). Ter uma relação de ordem, aqui, é importante porque alguns princípios de indução sobre os naturais fazem referência a uma (e.g., **CI**). Tomamos cuidado para não supor muitas coisas sobre tal relação de ordem, mas percebemos que, para que as implicações estudadas sejam válidas ou "quase válidas", é preciso definir, de certa forma, como a ordem se comporta com respeito ao zero e aos sucessores. Também avaliamos quais os efeitos de enfraquecer as hipóteses sobre essa relação de ordem.

Referências

[1] Baldoni, M. W.; Ciliberto, C.; Cattaneo, G. M. P. *Elementary Number Theory, Cryptography and Codes*, eISBN 978-3-540-69200-3. Springer-Verlag, 2008.

[2] Cameron, P. J. *Introduction to Algebra*, ISBN 0-19-850195-1. Oxford University Press, 1998.

[3] Childs, L. N. *A Concrete Introduction to Higher Algebra*, 3rd ed., ISBN: 978-0-387-74527-5. Springer, 2008.

[4] Ernest, P. Mathematical induction: a recurring theme. *The Mathematical Gazette* 66(436): 120-125, 1982. `https://doi.org/10.2307/3617747`

[5] Henkin, L. On Mathematical Induction. *The American Mathematical Monthly* 67(4): 323-338, 1960. `https://www.jstor.org/stable/2308975`

[6] Smullyan, R. M. *A Beginner's Guide to Mathematical Logic*, eISBN-13 978-0-486-78297-3. Dover, 2014.

[7] Sohrab, H. H. *Basic Real Analysis*, ISBN-13: 978-1-4612-6503-0. Birkhäuser, 2012.

[8] Weintraub, S. H. *The Induction Book*, ISBN-13: 978-0-486-81199-4. Dover, 2017.

[9] Weisstein, E. W. *MathWorld – A Wolfram Web Resource*. Peano's Axioms. `http://mathworld.wolfram.com/PeanosAxioms.html` (Acessado em 6 de março de 2019.)

Revision of Logic: a pragmatist proposal

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How can we rationally justify our logical principles if the very possibility of rational justification presupposes them? Which rational argument can be used to convince an opponent that a set of basic rules is the correct one if any argument has to be, from the beginning, based on a set of already accepted inferential rules? How can we use reason to ground the most basic principles of reason without circularity or an infinite regress? These questions address an important and still unresolved epistemological topic in the philosophy of logic. Today, this old problem about justification turns out to be even more complicated, because we observe the existence of several well-established and legitimate non-classical systems that fruitfully challenge classical logical principles. In this contemporary scenario, a traditional way to ground principles of logic using classical laws that, arguably, are necessary, universal, and self-evident seems to be simply an unbearable metaphysical myth.

The aim of this paper is to apply Wittgensteinean epistemology, the so-called hinge epistemology (Moyal-Sharrock 2004 and 2005) developed mainly from his remarks posthumously published as *On Certainty* (OC), within a discussion about the rationality of logical principles. Some remarks held by Wittgenstein about conversion (*Bekehrung*) and persuasion (*Überredung*) in conflicts among different world-pictures (*Weltbilder*) may offer a seminal treatment of contemporary foundational discussions in the philosophy of logic, especially in addressing challenges to logical pluralism as the conventionalism and radical incommensurability among rival logics. Accordingly, we have to instructively acknowledge that logical principles behave as Moorean propositions: as they form the very normative basis of our convictions, we would not reject them in any (rational) confrontation, because they define, delimitate, and orientate what rationality for us is, or better, they define what we would take as rational behavior and as correct reasoning in our inferential practices.

My discussion will particularly focus on conflicts between realists and anti-realists concerning the nature of logic. Logical realism holds that logic represents the ultimate structure of reality, even though it could be an inconsistent one. On the other hand, an anti-realist maintains that logic expresses only possible ways of describing reality. For my proposal, I assume that Wittgenstein is an anti-realist concerning the nature of logic and use some of his arguments and ideas against a broad realist view on logic. Actually, logical rules, for him, describe regularities neither of the world nor of any other independent structure but rather that logical principles should express some ways we fix methods, or forms of representations, to solve certain problems. As a result, logical principles are to be viewed not as revealing any deep structure of reality or of mind but rather as instructions for actions and operations for individuals in communities.

A well-known Wittgensteinean position is that we can only talk about justification and the production of evidence within logical systems and theories and not outside them or among them. It is, of course, possible to ask *locally* if a certain proposition or formula is correct in a certain system, if a logical consequence relation holds (or not) between some strings of symbols, or, even, whether a rule can be derived or not from other rules in a system. Our methods of justification, validity, and argumentation presuppose a system where they must be necessarily embedded. Hence, we cannot demand justification for the *whole* system. Justification cannot go beyond the system that gives life to it. Wittgenstein seems to agree that we cannot justify some procedures *outside* a system; we can only justify things *inside* a system. However, for him, to deny the very possibility of a system is nonsensical, since doubt makes sense only inside it.

Those former aspects of Wittgenstein scholarship are advanced by other authors (as Hacker, 1986 and Engelmann, 2014). My proposal here is to appropriate and integrate those tenets into the discussion on rival logics, that is, when there is a radical divergence on the correctness of some logical principles; particularly, some of Wittgenstein's remarks about heretical activities and some radical conflicts of principles (especially in OC 92, 610, 611, and 612) can be illuminating.

Reductions between certain incidence problems and the Continuum Hypothesis

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In this work, we consider two types of incidence problems, $\mathscr{C}_1$ and $\mathscr{C}_2$, which are related to real numbers and countable subsets of the real line. Problems of $\mathscr{C}_1$ are as follows: given a real number x, pick randomly a countable set of reals A hoping that $x \in A$, whereas problems in $\mathscr{C}_2$ are as follows: given a countable set of reals A, pick randomly a real number x hoping that $x \notin A$. One could arguably defend that, at least intuitively, problems in $\mathscr{C}_2$ are more simple to be solved than problems in $\mathscr{C}_1$.

After some suitable formalization, we prove (within **ZFC**) that, indeed, problems in $\mathscr{C}_2$ are at least as simple to be solved as problems in $\mathscr{C}_1$. On the other hand, the statement "Problems in $\mathscr{C}_1$ have the exact same complexity of problems in $\mathscr{C}_2$" is shown to be an equivalent of the Continuum Hypothesis.

The suitable formalization for the notion of comparison of complexities between problems will be given by *reductions*: for instance, problems in $\mathscr{C}_2$ will be shown to be *simpler* (or *not more complicated*) than the ones in $\mathscr{C}_1$ because we give a **ZFC** proof that a solution of a problem in $\mathscr{C}_2$ may be reduced to the solution of a problem in $\mathscr{C}_1$. Those reductions will be given in terms of morphisms between objects of the category $\mathscr{PV}$, which is the dual of the simplest case of the Dialectica Categories introduced by Valeria de Paiva ([4], [5]); such morphisms are also known as *Galois-Tukey connections*, which is a terminology due to Peter Vojtáš ([9]). Several connections between such category and Set Theory have been extensively studied by Andreas Blass in the 90's (see e.g. [1], [2]), and, more recently, have also been investigated by the author ([7], [8]).

And, as randomly taken countable subsets of the reals may be seen (in some guided, thought experiment) as *the set of punctures of a countable set of darts thrown at the real line*, the proof of our announced equivalence for the Continuum Hypothesis is, in fact, pretty similar to the one presented by Freiling in [3] – whose mathematical content, however, is due to Sierpiński, in his classical monograph on **CH** [6]. However, in our context Freiling's polemical assumption of symmetry seems unnecessary, and our approach lead us, apparently, to an even more dramatic discussion – if one considers the following question:

- Before being given a countable set A of reals and a real number x, both to be randomly taken, should one say that it will be *easier* (or it will be *more likely*) that, eventually, this real number x *will miss* the countable set A? Or should one say that, under the very same conditions and interpretations, *will hit it*?

References

[1] Blass, A. Questions and Answers – A Category Arising in Linear Logic, Complexity Theory, and Set Theory, in: J.-Y. Girard, Y. Lafont, and L. Regnier (Editors), *Advances in Linear Logic*, London Math. Soc. Lecture Notes 222, 61–81, 1995.

[2] Blass, A. Propositional Connectives and the Set Theory of the Continuum, *CWI Quarterly* 9:25–30, 1996.

[3] Freiling, C. Axioms of Symmetry: throwing darts at the real number line, *Journal of Symbolic Logic*, 51(1):190–200, 1986.

[4] de Paiva, V. A dialectica-like model of linear logic, in: Pitt, D., Rydeheard, D., Dybjer, P., Pitts, A. and Poigne, A. (Editors), *Category Theory and Computer Science*, Springer, 341–356, 1989.

[5] Sierpiński, W. *Hypothèse du Continu*. Monografie Matematyczne, 1 ère ed. PWN, Varsóvia, 1934, v + 192 pp.

[6] da Silva, S. G.; de Paiva, V. Dialectica categories, cardinalities of the continuum and combinatorics of ideals, *Logic Journal of the IGPL* 25(4):585-603, 2017.

[7] da Silva, S. G. The Axiom of Choice and the Partition Principle from Dialectica Categories, 2018. Submitted.

[8] Vojtáš, P. Generalized Galois-Tukey-connections between explicit relations on classical objects of real analysis, in: Judah, H. (Editor), *Set theory of the reals* (Ramat Gan, 1991), Bar-Ilan Univ., Ramat Gan, Israel Math. Conf. Proc. 6, 619-643, 1993.

Formalization in Philosophy and the Ontological Argument: Anselm, Gaunilo, Descartes, Leibniz and Kant

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The ontological argument is one of the most famous arguments (or family of arguments, to be more precise) in the history of philosophy. It was proposed in full-fledged form for the first time by Anselm of Canterbury, and either analyzed or reformulated by philosophers such as Descartes, Spinoza, Leibniz, Hume and Kant. It is also perhaps the argument that has most attracted the attention of formal philosophers. Attempts to formally analyze the arguments attributed to Anselm, for instance, are abundant [1, p. 49-57] [2] [3] [4] [5, p. 60-65] [6] [7]. Although there have been new formulations of the ontological argument directly embedded in formal and "semi-formal" frameworks [8] [9], the most common enterprise is still the formal analyses of traditional (and non-formal) versions of the ontological argument.

As far as formal analysis of existing philosophical arguments is concerned, some steps might be identified. First, there must be some sort of previous, informal analysis of the argument, meant to say, for example, what the premises and conclusion of the argument are, whether or not there are subsidiary arguments and hidden premises, etc. Second, there must be a formal language in which premises and conclusion are represented. Third, there might be an attempt to reconstruct the inferential steps of the original argument, possibly inside a specific theory of inference, be it proof theoretical or semantical or both.

In a sense, the whole thing can be seen from the viewpoint of Carnap's project of conceptual explanation [10, p. 1-18). On one side, we have an argument, in general a prose text, whose relevant aspects — premises and conclusion, presuppositions, structure, etc. — are obscure and ambiguous. This would correspond to Carnap's notion of explicandum. On the other hand, we have the outcome of the analysis: a representation of the argument, possibly accompanied by a derivation, embedded in a formal framework, which is supposed to be a reconstruction, or to use Carnap's terminology, an explanation of the original argument. This is the explicatum.

Due to its exactness or formal feature, let us say, the explicatum is supposed not to have those obscure features of the explicandum. In particular, it must be evident in the explicatum the exact meaning of premises, conclusion and hidden presuppositions, the structure of the argument, and whether or not it is valid. The explicatum is also supposed to help in the evaluation of the reasonableness of the premises. This has to do with Carnap's second requirement: that the explicatum must be fruitful. Due to this, as well as to the very nature of formal reconstructions (Carnap would probably say their exactness) and the obscurity and incompleteness of informal arguments, the explicatum shall most probably have many features not shared by the original argument.

However, this must not cause it to depart too much from the original argument, otherwise the former cannot be said to be an explanation of the latter. In Carnap's words [10, p.5], "the explicatum must be as close to or as similar with the explicandum as the latter's vagueness permits." To these three requirements — exactness, fruitfulness and similarity — I will add a fourth one: that the explicatum should not be troublemaker, by which I mean that the explicatum or formal reconstruction should neither produce problems, confusing questions and unfruitful issues which are not already present in the explicandum nor obscure important and otherwise clear aspects of it. (There still is a fourth requirement in Carnap's theory of conceptual explanation: simplicity.)

The formal analysis of existing philosophical arguments can be categorized inside the umbrella of formalization in philosophy. As a methodology, the use of formal tools in philosophy has been the object of much debate in recent years [11] [12] [13] Among other issues is the relation between formal philosophy and non-formal philosophy. Sven Hansson [12, p. 162;173] has rather dramatically put this as follows:

> Few issues in philosophical style and methodology are so controversial among philosophers as formalization. Some philosophers consider texts that make use of logical or mathematical notation as nonphilosophical and not worth reading, whereas others consider non-formal treatments as—at best—useful preparations for the real work to be done in a formal language. [...] This is unfortunate, since the value—or disvalue—of formalized methods is an important metaphilosophical issue that is worth systematic treatment. [...] It is urgently needed to revitalize formal philosophy and increase its interaction with non-formal philosophy. Technical developments should be focused on problems that have connections with philosophical issues.

He correctly points out, although not that explicitly, that in order to revitalize formal philosophy and increase its interaction with non-formal philosophy, there must be a very clear understanding of the dangers and exaggerations of formalization [12, p. 168-170).

The purpose of this paper is twofold. First, it aims at introducing the ontological argument through the analysis of five historical developments: Anselm's argument found in the second chapter of his Proslogion, Gaunilo's criticism of it, Descartes' version of the ontological argument found in his Meditations on First Philosophy, Leibniz contribution to the debate on the ontological argument and his demonstration of the possibility of God, and Kant's famous criticisms against the (cartesian) ontological argument.

Second, it intends to critically examine the enterprise of formally analyzing philosophical arguments and, as such, contribute in a small degree to the debate on the role of formalization in philosophy. For this purpose, in my presentation of Anselm's argument and Gaunilo's criticism I shall refer to Robert Adam's [2] pioneer work on the formalization of the ontological argument. Descartes' argument shall be introduced with the help of Howard Sobel's [5, p. 31-40) analysis; as far as Leibniz's argument is concerned, I shall refer to Graham Oppy's [14, p. 24-26] analysis, which, albeit not being a formal one, shall be useful as an instance of the first step in the task of formally analyzing an argument which I have mentioned above. My focus will be mainly on the drawbacks and limitations of these approaches as attempts to analyze existing philosophical arguments; as a guideline, I shall strongly refer to the Carnapian (or Carnapian-like) theory of argument analysis sketched above, specially its similarity and non-troublesome criteria.

References

[1] Hartshorne, C. *The Logic of Perfection*. Open Court, 1962.

[2] Adams, R. The Logical Structure of Anselm's Arguments. *The Philosophical Review* 80: 28-54, 1971.

[3] Oppenheimer, P.; Zalta, E. On the Logic of the Ontological Argument. In: James Tomblin (Editor), *Philosophical Perspectives 5: The Philosophy of Religion*. Ridgview Press, pages 509-529, 1991.

[4] Klima, G. Saint Anselm's Proof: A Problem of Reference, Intentional Identity and Mutual Understanding. In: Ghita Holmström-Hintikka (Editor), *Medieval Philosophy and Modern Times*. Kluwer, pages 69-87, 2000.

[5] Sobel, J. *Logic and Theism*. Cambridge University Press, 2004.

[6] Maydole, R. E. The Ontological Argument. In: W. L. Craig; J. P. Moreland (Editors), *The Blackwell Companion to Natural Theology*, Blackwell, pages 553-592, 2009.

[7] Eder, G.; Ramharte, E. Formal Reconstructions of St. Anselm's Ontological Argument. *Synthese 192*: 2791-2825. 2015.

[8] Gödel, K.*Kurt Gödel: Collected Works*, vol. 3. Oxford University Press, 1995.

[9] Plantinga, A. *The Nature of Necessity*. Oxford University Press, 1974.

[10] Carnap, R. *Logical Foundations of Probability* (2nd edition). University of Chicago Press, 1962.

[11] Horsten, L.; Douven, I. Formal Methods in the Philosophy of Science. *Studia Logica* 89: 151-162, 2008.

[12] Hansson, S. Formalization in Philosophy. *The Bulletin of Symbolic Logic 6*: 162-175, 2000.

[13] Engel, P. Formal Methods in Philosophy. In: T. Czarnecki (Editor), *Proceedings of the 6th European Congress of Analytic Philosophy*, College Publications, 2010.

[14] Oppy, G. *Ontological Arguments and Belief in God*. Cambridge University Press, 1995.

On order algebraic relations and logical deductions

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Through an analysis of formal methods, we investigate additional elements in order to highlight the intrinsic relation between order algebraic relations and logical deduction relations. Among the scope of logic, it is found the analysis of consequence relations. Why does the collection of some given information, the assumptions, support a conclusion? Naturally, we have an order relation, for the assumptions come first and the conclusion comes after. The formal methods explicit these relations that are highlighted here. In this context of formalization, we present a logic of deductibility, which introduces the notions of consequence in the logical environment. We will show that we have an order relation inside the other order, and how the partial order structure serves as a model to a logic of deductibility.

References

[1] de Souza, E. G. Lindenbaumologia I: a teoria geral. *Cognitio: Revista de Filosofia*, n. 2, p. 213-219, 2001.

[2] Feitosa, H. A.; Rodrigues, A. P.; Soares, M. R. Operadores de consequência e relações de consequência. *Kínesis*, v. 8, p. 156-172, 2016a.

[3] Feitosa, H. A.; Nascimento, M. C. Logic of deduction: models of pre-order and maximal theories. *South American Journal of Logic*, v. 1, p. 283-297, 2015.

[4] Font, J. M.; Jansana, R.; Pigozzi, D. A survey of abstract algebraic logic. *Studia Logica*, v. 74, p. 13-97, 2003.

[5] Kracht, M. Modal consequence relations. In: van Benthem, J.; Venema, Y.; Wolter F. (eds.). *Handbook of Modal Logic*. Elsevier, p. 497-549, 2006.

[6] Tarski, A. *Logic, semantics, metamathematics*. 2nd. ed. J. Corcoran (ed.). Indianapolis: Hackett Publishing Company, 1983.

Algebraic Monteiro's notion of maximal consistent theory for tarskian logics

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Antonio Monteiro developed several thechniques for studying some algebraic systems. One of the most important may be is his characterization of congruences by deductive systems. Indeed, in his book about lattice theory one can find the notion of *Systèmes deductifs liés à "a"* which is an element of a given algebra. In the case when the lattice is boolean, this notion of deductive system characterizes the maximal congruences. Using this fact he gave a technique to prove that the algebraic variety is semisimple. In this technique an implication with the language operations is defined, and the deductive systems characterize the congruence. Monteiro and many other authors used this techniques in some algebraic systems such as 3-valued Nelson algebras, Łukasiewicz-Moisil algebras, Tetravalent modal algebras, n-valued Wajsberg algebras (or MV_n-algebras), n-valued Heyting algebras, Tarski algebras (or semisimple Hilbert algebras), n-valued Łukasiewicz implication algebras, M_3-Lattices, n-valued Hilbert algebras, etc.

On the other hand, MV-algebras are semantic for Łukasiewicz logic. Furthermore, MV-algebras generated for finite chain are Heyting algebras where the Ǵodel implication can be written in terms of De Morgan and Moisil's modal operators. In our work, a fragment of Łukasiewicz logic is studied in the n-valued case, the implication allow us to use Monterio's technique mentioned above. The propositional and first order logic are presented. The maximal consistent theories is studied as Monteiro's maximal deductive system of the Lindenbaum-Tarski algebra in both cases. Using the same homomorphism that we considered to determine the generating algebras of the variety associated to this fragment of logic, we prove the strong adequacy theorem with respect to the suitable algebraic structure. Our algebraic strong completeness theorem does not need a negation in the language, in this sense Rasiowa's work is improved. A general presentation of these ideas will be expose.

References

[1] FRS1 M. Canals Frau and A.V. Figallo, *(n + 1)-valued Hilbert modal algebras*, Notas de la Sociedad Matematica de Chile, vol.X, 1(1991), 143-149.

[2] FS A. Figallo Orellano and J. S. Slagter, *An algebraic study of the first order intuitionistic fragment of 3-valued Łukasiewicz logic*, submitted.

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[†]The author had the support of a post-doctoral grant from FAPESP, Brazil (2016/21928-0)

[3] AM A. Monteiro, *Sur les algèbres de Heyting simetriques*, Portugaliae Math., 39, 1-4 (1980), 1–237.

[4] RA H. Rasiowa, *An algebraic approach to non-clasical logics*, Studies in logic and the foundations of mathematics, vol. 78. North-Holland Publishing Company, Amsterdam and London, and American Elsevier Publishing Company, Inc., New York, 1974.

A game-theoretical analysis of the role of information in implementing non-determinism in algorithms

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Automata theory has already been employed to provide upper and lower bounds for the computational complexity in solving games. Algorithmic game theory [1] is one of the main areas whose works analyze the computational complexity of games' solutions. Several other results have already been discussed [2] These works so far have been using automata theory to provide new ways of thinking about game solving. In our research, we propose an approach that looks at computer science from a game theoretical perspective.

When implementing a (possibly) nondeterministic automaton, in a computer deterministic program, some decisions have to be considered. These decisions have to be optimal from a rational perspective. Taking into account strategic games as a basic model of rationality, we aim to provide a game theoretical analysis of decisions made by a program developer.

This is initially established by a proposition that we show for the case of finite automata. Namely, given a finite automaton A, there is a class G_A of two-person zero-sum games such that the set of winning strategies for the second player of these games are in 1 : 1 correspondence to the words accepted by A, or $L(A)$. Additionally, if the automaton is deterministic, the game is of perfect information. If not, the game is an imperfect information game. This is an step towards a relationship between models of rationality described by rational games and decision-making at the pragmatic level when developing computational artifacts based on automata. We discuss how any extension of this proposition to more powerful automata (Stack automata, bound-memory automata, and Turing machines) can be used to explain the decisions usually made by algorithm developers when they implement non-deterministic procedures in deterministic machines. The analysis of the degree of concurrency obtained in any of these implementations regarded on the initial specifications may be improved by taking into account the information-based analysis derived from our game-theoretical approach.

These results may be applied to many areas, such as concurrent and distributed computation, and also impacts the learning of Computer Science [3].

References

[1] Tim Roughgarden. Algorithmic game theory. *Communications of the ACM*, 53(7):78, 2010.

[2] Tim Roughgarden, Noam Nisan, Eva Tardos, and Vijay V. Vazirani, editors. *Algorithmic Game Theory*. Cambridge University Press, 1 edition, 2007.

[3] Cleyton Slaviero and Edward H. Haeusler. Computational Thinking Tools: Anayzing concurrency and its representations. *SBC Journal on Interactive Systems*, 9(1):40–52, 2018.

Conexões de Galois e álgebras modais

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Este trabalho está na tradição da lógica algébrica. Um conjunto parcialmente ordenado (poset) A é aquele sobre o qual temos definida uma relação binária $\leq$ que é reflexiva, anti-simétrica e transitiva. Usualmente indicamos um poset por $\langle A, \leq \rangle$. A partir de generalizações de elementos teóricos desenvolvidos por Galois, tornou-se fundamental o conceito hoje conhecido na literatura por conexão de Galois. Assim é que, dados dois posets $\langle A, \leq_A \rangle$ e $\langle B, \leq_B \rangle$ e duas funções $f : A \longrightarrow B$ e $g : B \longrightarrow A$, dizemos que o par de funções (f, g) é uma conexão de Galois se, para todos $a \in A$ e $b \in B$ tenhamos que $a \leq_A g(b) \Leftrightarrow b \leq_B f(a)$. O propósito original para esta definição residia na relação entre extensões de corpos e grupos solúveis associados a estas extensões. Neste contexto algébrico e genérico, ao fazermos pequenas variações na definição de conexão de Galois, alterando as ordens das desigualdades, podemos produzir outras três situações que, juntas, formam o que chamamos de pares de Galois (ver [2], [8] ou [12]). Outro conceito fundamental na lógica algébrica é o de reticulado. Este pode ser definido, axiomaticamente, como um conjunto, não vazio, L sobre o qual temos definidas duas operações $\wedge$ e $\vee$, que satisfazem três propriedades específicas: associatividade, comutatividade e absorção (ver [10] ou [2]). A partir de incrementos nas propriedades que devem ser obedecidas por estas duas operações, podemos obter estruturas mais elaboradas (reticulados mais específicos), como as bem conhecidas álgebras de Boole e de Heyting, que são, respectivamente, modelos algébricos para o cálculo proposicional clássico e o cálculo intuicionista [7]. Todo reticulado é um poset, na verdade, podemos definir um reticulado, de maneira equivalente, a partir de um poset $\langle L, \leq \rangle$ no qual seja válido que, para dois elementos quaisquer $x, y \in L$, existe sempre o supremo e o ínfimo do conjunto $\{x, y\}$. Os conceitos de pseudo-complemento e complemento, uma vez definidos em reticulados, vinculam-se às noções de negação nas estruturas lógicas que têm tais reticulados como seu modelo algébrico. Dessa forma, o complemento em uma álgebra de Boole se vincula à negação no cálculo proposicional clássico, assim como o pseudo-complemento em uma álgebra de Heyting à negação na lógica intuicionista [7]. Em reticulados específicos temos definidos os operadores modais de necessidade ($\Box$) e possibilidade ($\Diamond$), como em [1]. Identificamos então o par $(\Diamond, \Box)$ como um tipo especial de par de Galois, uma adjunção. Derivado deste fato, decorrem propriedades sobre tais reticulados. Além disso, $\triangledown a = \Box\Diamond a$ e $\vartriangle a = \Diamond\Box a$ definem, respectivamente, um operador de fecho (ou de Tarski) e um operador de interior sobre o reticulado investigado em [1]. No contexto ampliado dos pares de Galois, pudemos definir outros operadores motivados pelos operadores $\Box$ e $\Diamond$ e também mostrar mais propriedades sobre os anteriores e os novos operadores.

References

[1] Castiglioni, J. L.; Ertola-Biraben, R. Modal operators for meet-complemented lattices. *Logic Journal of the IGPL*, Vol. 25. Num. 4. pp. 465-495, 2017.

[2] Dunn, J. M.; Hardegree, G. M. *Algebraic methods in philosophical logic.* Oxford: Oxford University Press, 2001.

[3] Ebbinghaus, H. D.; Flum, J.; Thomas, W. *Mathematical logic.* New York: Springer-Verlag. 1984.

[4] Enderton, H. B. *A mathematical introduction to logic.* San Diego: Academic Press. 1972.

[5] Herrlich, H.; Husek, M. Galois connections categorically. *Journal of Pure and Applied Algebra*, Vol. 68, pp. 165-180, 1990.

[6] Mendelson, E. *Introduction to mathematical logic.* Princeton: D. Van Nostrand. 1964.

[7] Miraglia, F. *Cálculo proposicional: uma interação da álgebra e da lógica.* Campinas: Unicamp/CLE. 1987.

[8] Ore, O. Galois connections. *Transactions of the American Mathematical Society*, Vol. 55, pp. 493-513, 1944.

[9] Orlowska, E.; Rewitzky, I. Algebras for Galois-style connections and their discrete duality. *Fuzzy Sets and Systems*, Vol. 161, pp. 1325-1342, 2010.

[10] Rasiowa, H.; Sikorski, R. *The mathematics of metamathematics.* 2. ed. Waszawa: PWN - Polish Scientific Publishers. 1968.

[11] Rasiowa, H. *An algebraic approach to non-classical logics.* Amsterdam: North-Holland, 1974.

[12] Smith, P. *The Galois connection between syntax and semantics.* Technical report. Cambridge: University of Cambridge. 2010.

Referencia singular y representación espacial

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De acuerdo con Campbell (1997, 2002) el núcleo del problema sobre la referencia a objetos consiste en explicar cómo con base en la percepción de un objeto es posible referir a este por medio del uso o del pensamiento de un demostrativo, *i.e.*, explicar la relación entre contenidos conceptuales judicativos del tipo *esto es F* y contenidos pictóricos o no-conceptuales que estructuran la percepción de un agente cognitivo en su entorno. Para Campbell no existe mayor misterio en aceptar la distinción entre contenidos conceptuales característicos del juicio y del pensamiento y contenidos no-conceptuales característicos de nuestra experiencia sensorio-motora, lo importante es proveer una explicación que vincule los dos tipos de contenido.

Estas ideas derivan en buena parte de la propuesta de Evans (1982, 1985) de acuerdo con la cual seguir el rastro de los objetos es precondición para la concepción de pensamientos demostrativos. Dicho de otro modo, la normatividad de los contenidos sensorio-motores que caracterizan el rastreo de objetos en nuestra experiencia como agentes es precondición para la concepción de contenidos veritativo-condicionales característicos de la referencia a objetos en pensamientos del estilo *esto es F*.

Aunque esta concepción ha ganado terreno, no parece que haya sido desarrollada suficientemente. Buena parte del problema consiste en especificar en qué consiste seguir el rastro de objetos, *i.e.*, especificar qué tipo de estructura tiene el contenido no-conceptual disponible en nuestra experiencia. ¿Es suficiente la caracterización en términos de redes sensorio-motoras que determinan actual o disposicionalmente nuestra habilidad de percibir y actuar sobre el entorno? Alva Nöe (2012), por ejemplo, considera que la explicación enactiva y disposicional es suficiente (ver también O'Keefe, 1980).

De acuerdo con Strawson (1971) la singularidad de un sistema de representación espacio-temporal es requerida como precondición para la referencia objetiva. Una explicación de cómo objetivad y singularidad son propiedades de la habilidad de referir requiere postular que la identificación y reidentificación son posibles a pesar de los defectos de nuestras capacidades perceptuales.

Siguiendo a Strawson, Evans sostiene que antes de entender los casos de referencia demostrativa, es necesario entender en qué consiste nuestra habilidad de identificar lugares. Parte de su propuesta es, en acuerdo con la explicación disposicional enactiva, que nuestra habilidad de seguir el rastro de lugares depende fundamentalmente de la capacidad de representar egocéntricamente las relaciones espaciales de una forma tal que da sentido a nuestra percepción y acción. Relaciones espaciales que se estructuran egocéntricamente como una red disposicional sensorio-motora. Otra buena parte sin embargo consiste en reconocer que la identificación de lugares no se reduce al rastreo sensorio-motor. Depende también de la concepción de estructuras espaciales alocéntricas que permiten localizar lugares en su relación con otros independientemente de nuestros estados. Rastrear objetos no se reduce a tener acceso a su presencia disposicionalemente requiere además la concepción de una estructura unificada del entorno espacial que permite guiar nuestras acciones independientemente de lo que es o puede ser percibido.

Desde su formulación en Tolman (1948) y su reaparición en O'Keefe y Nadel (1978), la postulación de mapas cognitivos como representaciones tipo que sustentan la habilidad de navegación y guía de algunos mamíferos ha ganado sustento y reconocimiento progresivamente Gallistel (1990, 2008, 2011), Epstein (2017), Redish (1999), Kitchin (1994). El propósito de esta ponencia es argumentar a favor de la propuesta neokantiana consistente en postular la representación de un sistema espacio-temporal unificado como condición de posibilidad de la referencia a objetos. Así como trazar la diferencia estructural entre contenidos característicos de la experiencia tipo mapa y contenidos característicos del juicio tipo sentencias, presentada en Evans (1982, 1985), Cussins (1990, 1992, 2003) y revaluada recientemente en Casati (1999), Flombaum (2009), Heck (2007), Raftopoulos (2009), Camp (2007) y Rescorla (2009, 2017).

Referencias

[1] Acredolo, L.. *Behavioral approaches to spatial orientation in infancy*. Annals of the New York Academy of Sciences 608 (1), 596-612, 1990.

[2] Camp, E. *Thinking with maps*. Philosophical perspectives 21 (1), 145-182, 2007.

[3] Campbell, J. *Past, space, and self.* MIT Press, 1995.

[4] Campbell, J. *Reference and consciousness*. Clarendon Press , 2002.

[5] Carey, S.; Xu, F. *Infants' knowledge of objects: Beyond objects and object tracking*. Cognition 80 (1-2), 179-213, 2001.

[6] Casati, R., Varzi, A. *Parts and places: The structures of spatial representation*. Mit Press, 1999.

[7] Cussins, A. The connectionist construction of concepts. In: Margaret A. Boden (Editor), *The Philosophy of Artificial Intelligence*. Oxford University Press CSLI, 1990.

[8] Cussins, A. *Content, embodiment and objectivity: the theory of cognitive trails*. Mind 101 (404), 651-688, 1992.

[9] Cussins, A. *Experience, thought and activity*. In: York Gunther (Editor), *Essays on nonconceptual content*, chapter 6, pages 133-163, 2003.

[10] Eilan, N., McCarthy, R., Brewer, B. (Editors) *Spatial representation: Problems in philosophy and psychology*. Oxford University Press, 1993.

[11] Epstein, R. A., Patai, E. Z., Julian, J. B., Spiers, H. J. The cognitive map in humans: spatial navigation and beyond. *Nature neuroscience* 20 (11), 1504, 2017.

[12] Evans, G. *The Varieties of Reference*. Oxford University Press., 1982.

[13] Evans, G. *Collected Papers*. Oxford University Press., 1985.

[14] Flombaum, J. I., Scholl, B. J., Santos, L. R. Spatiotemporal priority as a fundamental principle of object persistence. In: Bruce M. Hood and Laurie R. Santos (Editors) *The origins of object knowledge*. pages 135-164, 2009.

[15] Gallistel, C. R. *The organization of learning*. The MIT Press, 1990.

[16] Gallistel, C.R. Dead reckoning, cognitive maps, animal navigation and the representation of space: An introduction. In: Jefferies M.E., Yeap WK (Editors) *Robotics and cognitive approaches to spatial mapping*. pages 137-143, Springer, 2008.

[17] Gallistel, C. R.; Matzel, L. The Neuroscience of Learning: Beyond the Hebbian Synapse. *Annual Review of Psychology* 64, pages 169-200, 2011.

[18] Grush, R. Self, world and space: The meaning and mechanisms of ego and allocentric spatial representation. *Brain and Mind* 1 (1), 59-92, 2000.

[19] Gunther, Y. (Editor) *Essays on Nonconceptual Content*. The MIT press, 2003.

[20] Heck, R. Are there different kinds of content? In: Brian P. McLaughlin; Jonathan D. Cohen (Editors), *Contemporary Debates in Philosophy of Mind*. pages 117-138, Blackwell, 2007.

[21] Kitchin, R. M. Cognitive maps: What are they and why study them? *Journal of environmental psychology* 14 (1), 1-19, 1994.

[22] Nöe, A. *Varieties of presence*. Harvard University Press Cambridge, MA., 2012.

[23] O'Keefe, J., Nadel, L. *The hippocampus as a cognitive map*. Oxford: Clarendon Press, 1978.

[24] O'Shaughnessy, B. *The will: A dual aspect theory*. Cambridge University Press, 1980.

[25] Pylyshyn, Z. W. *Things and places: How the mind connects with the world*. MIT press, 2007.

[26] Raftopoulos, A. *Cognition and perception: How do psychology and neural science inform philosophy?* Mit Press, 2009.

[27] Redish, A. D. *Beyond the cognitive map: from place cells to episodic memory*. MIT press., 1999.

[28] Rensnik, R. A. The dynamic representation of scenes. *Visual cognition* 7 (1-3), 17-42, 2000.

[29] Rescorla, M. Cognitive maps and the language of thought. *The British Journal for the Philosophy of Science* 60 (2), 377-407, 2009.

[30] Rescorla, M. Predication and cartographic representation. *Synthese* 169 (1), 175-200, 2009.

[31] Rescorla, M. Maps in the head. In: Kristin Andrews; Jacob Beck (Editors) *The Routledge Handbook of Philosophy of Animal Minds*, chapter 3, pages 34-45, Routledge, 2017.

[32] Strawson, P. F. *Individuals*. Routledge, 1971.

[33] Tolman, E. C.Cognitive maps in rats and men. *Psychological review* 55 (4), 189., 1948.

[34] Spelke, E. S.; Wang, R. F. Human spatial representation: Insights from animals. *Trends in cognitive sciences* 6 (9), 376-382, 2002.

Thought Experiments in (teaching) Logic?

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The philosophy of thought experiments (TEs) focuses mostly on philosophy of science. Few (e.g. Jim Brown, Irina Starikova and Marcus Giaquinto, Jean-Paul Van Bendegem) tackle the question of TEs in mathematics (MTEs), where, as they point out that non-trivial TEs can be found. I have not yet came across literature on TEs in logic (LTEs).

Jean-Paul Van Bendegem, former opponent of MTEs in (1998), now accepts and even defends the possibility of (specific types) mathematical experiments (2003), by showing several cases in which a highly abstract mathematical result is the outcome of research that has a concrete empirical origin. In recent discussion (PhilMath Intersem 2018, Paris) he makes an observation that the thinking in most examples of MTEs is primarily aimed at a better understanding and more detailed consideration of abstract constructions than at real experimentation; so he stresses the function of thought experiments as "an aid to proof" or "evidential mediators".

Starikova and Giaquinto accept that the expression "thought experiment" can be applied to a broad range of mathematical cases but focus on the most interesting creative candidates from mathematical practice, namely, those in which (a) experimental thinking goes beyond the application of mathematically prescribed rules, and (b) uses sensory imagination (as a way to drawing on the benefits of past sensory experience) to grasp and mentally transform visualizable representations of abstract mathematical objects, such as knots, graphs and groups. I will consider TEs with these characteristics in my discussion.

TEs are a rare combination of historical, philosophical, cognitive and social practices and a very special way of extracting new knowledge. Also, research in education provides evidence for usefulness of TEs in teaching: not only do TEs help to examine causal and correlational relationships in academic content (e.g. in physics) but also to develop problem-solving, logical thinking skills, conceptual understanding and scientific creativity among other skills (for a review see e.g. Tortop, 2016).

Using examples from mathematics I will discuss the principal stages of constructing MTEs, whether these stages can be reapplied in another context or problem and seen as a systematic tool of mathematical practice, and asses possible constrains constraints. I will especially dwell on how sensory imagination enters a symbolically represented context. I will finally address my question whether thought experiments can be used in (teaching) logic.

References

[1] Brown, James R., 2007, "Thought experiments in science, philosophy, and mathematics", *Croatian Journal of Philosophy*, VII: 3–27.

[2] ——. 2011, "On Mathematical Thought Experiments", *Epistemologia: Italian Journal for Philosophy of Science*, XXXIV: 61–88.

[3] Van Bendegem, Jean Paul, 1998, "What, if anything, is an experiment in mathematics?", in Dionysios Anapolitanos, Aristides Baltas & Stavroula Tsinorema (eds.), *Philosophy and the Many Faces of Science*, (CPS Puňblications in the Philosophy of Science). London: Rowman & Littlefield, 172-182.

[4] ——. 2003, "Thought Experiments in Mathematics: Anything But Proof". *Philosophica*, 72, (date of publication: 2005), 9-33.

[5] Starikova, Irina & Giaquinto, Marcus, 2018, "Thought Experiments in Mathematics", in Michael T. Stuart, Yiftach Fehige & James Robert Brown (eds.), *The Routledge Companion to Thought Experiments*. London: Routledge, 257-278.

[6] Tortop, Hasan Said, 2016, "Why Thought Experiments Should Be Used as an Educational Tool to Develop Problem-Solving Skills and Creativity of the Gifted Students?", *Journal of Gifted Education and Creativity*u, 3(3), 35-48.

What do we see through a Magic Glass?
Well-Adapted Bases of Eigen-Solutions and Scientific Production of Objective Realities

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La vie est fascinante: il faut seulement la regarder avec les bonnes lunettes.
Life is fascinating: one just has to look at it using the right lenses.
Alexandre Dumas fils (1824–1895).

Science: The Sharper Image

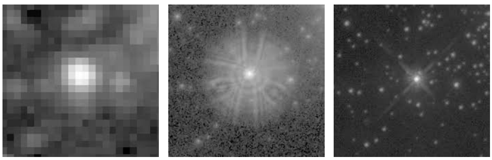

Figure 1: Blurry vs. Sharp (less aberration) images of star Melnick-34

Telescopes work by producing images of distant objects at higher magnification and better resolution than the observer's naked eye is capable of. The magnification factor specifies how many times larger the observer sees an object, while resolution specifies the observer's ability to distinguish apart (resolve) two nearby objects, that is, resolution refers the image's sharpness or amount of fine detail it makes available to the observer. Figure 1 illustrates these concepts with three images of the same region of space, centered at Melnick-34 – a distant star in the Tarantula Nebula. Figures 1a,b,c were produced by: (a) The ground based telescope of the European Southern Observatory; The Hubble's wide field planetary camera, (b) before and (c) after the space shuttle Endeavor 1993 mission to correct a spherical aberration problem. While the magnification factor of these three images are the same, their resolution range from low(er) to high(er), from left to right. Images to the left are impaired by stronger aberration or distortion effects that make them more confused, blurry or indistinct, when compared to the sharper image to the right. Moreover images to the left are impaired by the occurrence of artifacts – spurious effects like pixelation, replications of bright points by dimmer copies around it, or hallo-like effects around larger sources of light.

This article explores the metaphor of Science as provider of "sharp images" of our environment. As in the telescope example, the images we can possibly see obviously depend on the instruments we have the ability to build and, perhaps less obviously, also depend on the characteristics of our inborn (biological) sense of vision. Similarly, we argue that our ability to interpret those images is constrained by our cognitive abilities, by our theoretical and philosophical frameworks, etc. In this sense, the epistemological framework developed in this article belongs to the philosophical tradition of Cognitive Constructivism.

Nevertheless, the epistemological framework developed in this article has a distinctly "objective" character that sets it apart from many alternatives in the constructivist tradition. Using once more our optical analogy, the quality of the images provided by a telescope can be characterized by tailor-made measures used to quantify magnification, resolution, and specific aberration effects. Similarly, we claim that the quality of scientific representations can be quantitatively accessed and precisely measured. Such measures are the technical touch-stones needed to lay down the mathematical foundations of the *Objective Cognitive Constructivism* epistemological framework.

In previous works we define a statistical measure tailor-made for the aforementioned purpose, namely, $\text{ev}(H \mid X)$, the e-value, or epistemic value of hypothesis H given observational data X. The reference section lists previous articles of this author giving: (a) The definition and theoretical properties of this statistical significance measure; (b) Some interesting statistical applications highlighting good properties of this measure; (c) Formal analyses of the logical properties of the e-value; and (d) Some previous philosophical considerations on the *Objective Cognitive Constructivism* epistemological framework.

The statistical, mathematical and logical properties of the e-value are perfectly adapted to support and to work in tandem with the *Objective Cognitive Constructivism* epistemological framework. Nevertheless, this article does not focus on formal analyses of such mathematical constructs. Rather, its goal is to develop the optical metaphor introduced in this section in order to explain, in an easy and intuitive way, some of the basic ideas and key insights used in this framework. In accordance with this goal, epistemological arguments are supported by figures illustrating analogies or providing visual context. In order to better develop our arguments we use, as historical background, the science of optics as seen trough the work of Giambattista della Porta (1535-1615), Galileo Galilei (1564-1642), Johannes Kepler (1571-1630), René Descartes (1596-1650), and Pierre de Fermat (1607-1665); see references for the original works, some historical analyses and well-designed didactic materials.

References

[1] Borges, Wagner, and Julio Michael Stern (2007). The Rules of Logic Composition for the Bayesian Epistemic E-Values. *Logic Journal of the IGPL*, 15, 5/6, 401-420.

[2] P. Brunet (1938). *Étude Historique sur le Principe de la Moindre Action*. Paris: Hermann.

[3] Esteves, Luis Gustavo; Izbicki, Rafael; Stern, Julio Michael; Stern, Rafael Bassi (2016). The Logical Consistency of Simultaneous Agnostic Hypothesis Tests. *Entropy*, 18, 7, 256.

[4] Fermat, Pierre de (1894). *Oeuvres*, V.2, Correspondance. Gauthier-Villars, Paris, 1894. Including: (a) Letter of Fermat to de la Chambre of August 1657, LXXXVI, p.354-359. (b) Letter of Clerselier to Fermat of May 6th, 1662, CXIII, 464-472.

[5] Fröhlich, Wilhelm (1923). Anleitung zum Gebrauch des Kosmos-Baukasten Optik: Versuche aus der Lehre vom Licht. Kosmos Gesellschaft der Naturfreunde/ Franckh'sche Verlagshandlung, Stuttgart.

[6] Le Opere di Galileo Galilei. Edited by Antonio Favaro. Edizione Nazionale. 20 vols. Firenze: G. Barbèra, 1892-1904. Vol. X (1900), Carteggio 1574–1642.

[7] Goldstine Herman H. (1980). *A History of the Calculus of Variations from the 17th through the 19th Century*. Springer-Verlag, New York.

[8] Heeffer, Albrecht (2017). Using Invariances in Geometrical Diagrams: Della Porta, Kepler and Descartes on Refraction. Ch.7, p.145-168 in Borrelli (2017).

[9] Kepler, Johannes (1604, 2000). *Ad Vitellionem Paralipomena, quibus Astronomiae pars Optica*. In his *Gesammelte Werke* (), 2, 181-xxx. Translated by William H. Donahue as *Optics: Paralipomena to Witelo and Optical Part of Astronomy*. Lion Press, Santa Fe.

[10] Kepler, Johannes (1611). *Dioptrice*. In his *Gesammelte Werke* (1937), 4, 355-414.

[11] Kepler, Johannes (1610, 1965). *Dissertatio, cum Nuncio Sidereo, nuper ad mortales misso a Galilaeo Galilaeo, Mathematico Patauino*. translated by Edward Rosen as *Conversation with Galileo's Sideral Messenger*. Johnson Reprint Co., NY.

[12] Madruga, M. Regina; Esteves, Luis G.; Wechsler, Sergio (2001). On the Bayesianity of Pereira-Stern tests. *Test* 10, 2, 291-299.

[13] Molesini, Giuseppe (2011). Early advances on rays and refraction: a review through selected illustrations. *Optical Engineering*, 50, 121704, 1-6.

[14] Nakano, Hideya; Martins, Roberto Andrade; Krasylchik, Myriam ed. (1972). *Os Cientistas*, v.19 - Descartes: A Trajetória dos Raios Luminosos. FUNBEC/ Abril S.A., São Paulo.

[15] Pereira C. A. B.; Stern, J. M. (2008). Can a Signicance Test be Genuinely Bayesian? *Bayesian Analysis*, 3, 1, 79–100.

[16] Planck, Max (1915). Das Prinzip der kleinsten Wirkung. *Kultur der Gegenwart*.

[17] Porta, Giovanni Baptista Della (1658). On Strange Glasses – Wherein are propounded Burning-glasses, and the wonderful sights to be seen by them. Book XVII, p.355-381, in *Natural Magick: In XX Bookes by John Baptist Porta, a Neopolitane*. Thomas Young & Samuel Speed, London. Translation from the original *Magiae Naturalis sive de Miraculis Rerum Naturalium*, 2nd ed., 1589.

[18] Rottman, Gerald (2008). *The Geometry of Light: Galileo's Telescope, Kepler's Optics*. Gerald Rottman, Baltimore.

[19] Stern, Julio Michael (2007a). Cognitive Constructivism, Eigen-Solutions, and Sharp Statistical Hypotheses. *Cybernetics & Human Knowing*, 14, 1, 9-36.

[20] Stern, Julio Michael (2007b). Language and the Self-Reference Paradox. *Cybernetics & Human Knowing*, 14, 4, 71-92.

[21] Stern, Julio Michael (2008). Decoupling, Sparsity, Randomization, and Objective Bayesian Inference. *Cybernetics & Human Knowing*, 15, 2, 49-68.

[22] Stern, Julio Michael (2011). Symmetry, Invariance and Ontology in Physics and Statistics. *Symmetry*, 3, 3, 611-635.

[23] Stern, Julio Michael (2014). Jacob's Ladder and Scientific Ontologies. *Cybernetics & Human Knowing*, 21, 3, 9-43.

[24] Stern, Julio Michael (2015). Cognitive-Constructivism, Quine, Dogmas of Empiricism, and the Münchhausen's Trilemma. In: *Springer Proceedings in Mathematics & Statistics*, 118, 55-68.

[25] Stern, Julio Michael (2017a). Continuous versions of Haack's Puzzles: Equilibria, Eigen-States and Ontologies. *Logic Journal of the IGPL*, 25, 4, 604-631.

[26] Stern, Julio Michael (2017b). Jacob's Ladder: Logics of Magic, Metaphor and Metaphysics: Narratives of the Unconscious, the Self, and the Assembly. *Sophia*, published Online First on June 7, 2017, `doi:10.1007/s11841-017-0592-y`.

[27] Stern, Julio Michael (2017c). Karl Pearson on Causes and Inverse Probabilities: Renouncing the Bride, Inverted Spinozism and Goodness-of-Fit. Submitted for publication.

[28] Stern, Julio Michael; Izbicki, Rafael; Esteves, Luis Gustavo; Stern, Rafael Bassi (2017). Logically-consistent hypothesis testing and the hexagon of oppositions. *Logic Journal of IGPL*, 25, 5, 741-757.

[29] Vasconcellos, D.F. (1960). *Poyiopticon*. Instructions for kit PO-M1. Sao Paulo: D.F. Vasconcellos.

[30] Zik, Yaakov; Giora Hon (2017). History of science and science combined: solving a historical problem in optics: The case of Galileo and his telescope. *Archive for History of Exact Sciences*, 71, 4, 337-344.

Verificação de modelos Reo com nuXmv*

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A validação de sistemas é crucial em sistemas críticos. Garantir que requisitos são atendidos numa especificação é vital quando há possibilidades de grandes perdas ou até mesmo mortes em caso de falhas. Entretanto, a modelagem em um sistema formal usualmente é uma tarefa árdua no processo de desenvolvimento de software. Ao se usar ferramentas, parte desta complexidade é reduzida, além de agilizar e simplificar esse processo. Dentre as categorias de ferramentas disponíveis, *model checkers* são amplamente utilizados. Eles consistem em ferramentas capazes de verificar se propriedades (usualmente modeladas através de alguma lógica) estão presentes em modelos dados como entrada.

Reo [1] é uma linguagem gráfica baseada em coordenações usada na modelagem de sistemas, com foco em modelos de sistemas distribuídos. Sua construção desenvolveu-se para que características como chamadas remotas de métodos e transferência de mensagens sejam nativas e intuitivas. *Constraint Automata* [2] é um formalismo criado para denotar a semântica formal de Reo; além das definições básicas ele possui uma operação de produto para composição de modelos. O produto possibilita a composição colapsando os pontos em comum, de forma a evitar explosão de estados no modelo.

Este trabalho consiste no desenvolvimento de um algoritmo para converter um modelo Reo (i.e. um *Constraint Automata*) em um modelo para o *model checker* nuXmv†, ferramenta amplamente utilizada na indústria. A operação de produto pode ser efetuada tanto no autômato quanto no modelo gerado no *model checker*. Esta última opção provê um modelo consideravelmente maior, entretanto simplifica a rastreabilidade no caso de detecção de erros. A implementação é de código aberto e está disponível em `https://github.com/Daniel-02/Reo2nuXmv`.

References

[1] Arbab, Farhad; Reo: a channel-based coordination model for component composition. *Mathematical Structures in Computer Science*, 14: 329–366, 2004.

[2] Arbab, Farhad; Coordination for Component Composition. *Electronic Notes in Theoretical Computer Science*, 160: 15–40, 2006.

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†`https://nuXmv.fbk.eu`

Semânticas não-determinísticas para lógicas não-clássicas: uma abordagem da perspectiva de Teoria de Modelos e de Álgebra Universal

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Nesta apresentação, analisaremos alguns tópicos no estudo formal de semânticas não-determinísticas para sistemas lógicos não-algebrizáveis utilizando ferramentas de álgebra universal, teoria de modelos e teoria de categorias. Especificamente, pretendemos abordar algumas questões em aberto propostas em trabalhos recentes sobre semânticas não-determinísticas e algebrização não-determinística de sistemas lógicos através de estruturas swap e de Fidel.

Uma das maiores dificuldades no estudo das chamadas lógicas não-clássicas, incluindo as paraconsistentes, é a de que estes sistemas frequentemente não podem ser caracterizados por semânticas verofuncionais. Em particular, os sistemas de maior interesse filosófico não são em geral algebrizáveis pelos métodos usuais, tais como os de Blok e Pigozzi.

Atualmente, uma das classes de lógicas paraconsistentes mais estudadas é a das chamadas Lógicas da Inconsistência Formal (LFI), introduzidas por Carnielli e Marcos no ano 2000 [1]. Em [2], Coniglio, Figallo-Orellano e Golzio realizaram um estudo, da perspectiva da álgebra universal e da teoria de categorias, das classes de estruturas swap para diversas LFI's, começando por **mbC**. Foi obtido, dentre outros importantes resultados, um teorema de representação de tipo Birkhoff para cada classe de estruturas swap.

Um problema importante a ser abordado é o desenvolvimento de uma teoria de equações em multiálgebras, de modo a definir formalmente variedades. Isto permitiria caracterizar hiperálgebras de maneira intrínseca, tornando este tópico mais próximo da disciplina de álgebras universais e da de teoria de modelos.

Começaremos esta comunicação com uma breve análise das lógicas da inconsistência formal, mostrando a seguir como são construídas as estruturas swap e como estas servem de contraparte algébrica a LFI's como **mbC**.

Mostraremos então como as estruturas swap podem ser tratadas como multiálgebras, justificando nossa passagem ao estudo abstrato de multiálgebras. Discutiremos sua definição, assim como possíveis definições de homomorfismo, subálgebra, produto, termo e identidade, enfatizando aqui não haverem ainda definições preferíveis neste contexto para cada um destes conceitos. Finalmente usaremos a abordagem de invariantes por operadores de classes para mostrar algumas generalizações possíveis do teorema de Tarski para álgebras universais. Dado que haja tempo, abordaremos também nossa pesquisa recente em multiálgebras livremente geradas.

References

[1] W. A. Carnielli and J. Marcos. A taxonomy of C-systems. Em: W. A. Carnielli, M. E. Coniglio, e I. M. L. D'Ottaviano, editores, *Paraconsistency: The Logical Way to the Inconsistent*, volume 228 de *Lecture Notes in Pure and Applied Mathematics*, páginas 1 a 94.

[2] M. E. Coniglio, A. Figallo-Orellano and A. C. Golzio. Non-deterministic algebraization of logics by swap structures. Aceito para publicação no *Logic Journal of the IGPL*. Versão preliminar disponível em arXiv:1708.08499 [math.LO], 2017.

Infinite forcing and the generic multiverse

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In this article we present a technique for selecting models of set theory that are arbitrary in a forcing-generic sense. Specifically, we will apply Robinson infinite forcing to the collections of models of ZFC obtained by Cohen forcing. This technique will be used to suggest a unified perspective on generic absoluteness principles.

References

[1] Venturi, G. Infinite forcing and the generic multiverse, *Studia Logica*. To appear.

Back to Da Costa: a comparative study of the C-systems and the V-hierarchy

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In this talk we will introduce a hierarchy of non-classical logics that we will call the $\mathfrak{V}-$logics. Moreover, this $\mathfrak{V}-$ logics are obtained from paraconsistent valued models of set theory. Therefore, we will give a brief introduction into the topic of boolean valued models for set theory and explain several extensions of this work to more complicated lattices. We will focus on a particular model introduced by [5], called $\mathbf{V}^{(PS_3,*)}$ and an infinite subclass of linear algebras that contains this particular model as instance.

We present an uniform axiomatization for all finite $\mathfrak{V}-$logics and proof the completeness and soundness for this axiomatization. We also present an alternative but equivalent axiomatization that allows for the consistency operator in our formal language [2], introduced normally in logics of inconsistency (LFI's) [1]. Then we consider an axiomatization of some of the infinite $\mathfrak{V}-$ logics and show that this logics are highly paraconsistent.We conclude by comparing this $\mathfrak{V}-$hierarchy to da Costas $C-$systems (see [4], [3]).

This means that on a philosophical side we will discuss whether the $\mathfrak{V}-$logics are faithfull to da Costas criteria for paraconsistent logics and on a more technical side we can provide inference rules and more generally a sintactic framework for paraconsistent models of Zermelo-Fraenkel (ZF) set theory. We finnish up, by claiming that we have fullfiled da Costas dream and present several promising open problem for future work!

References

[1] W. Carnielli and M. E. Coniglio. *Paraconsistent Logic: Consistency, Contradiction and Negation.* Basel, Switzerland: Springer International Publishing, 2016.

[2] M. E. Coniglio and L. H. D. C. Silvestrini. An alternative approach for quasi-truth. *Logic Journal of the IGPL*, 22(2):387–410, 2014.

[3] N. C. A. D. Costa and E. H. Alves. A semantical analysis of the calculi C_n. *Notre Dame Journal of Formal Logic*, 18(4):621–630, 1977.

[4] N. C. da Costa. On paraconsistent set theory. *Logique et Analyse*, 115(361):71, 1986.

[5] B. Löwe and S. Tarafder. Generalized algebra-valued models of set theory. *Review of Symbolic Logic*, 8(1):192–205, 2015.

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The Brazilian Logic Conferences (EBL) is one of the most traditional logical conferences in South America. The 19th edition of EBL takes place in the northeast coast of Brazil, in the city of João Pessoa, Paraíba, from May 6-10, 2019. It is conjointly organized by Federal University of Paraíba, Federal University of Campina Grande and the Brazilian Logic Society. This volume contains the abstracts of keynote talks and contributed talks of this 19th Brazilian Logic Conference. It also contains the abstracts of the tutorials of the Logic School which runs parallel to the conference.

O Encontro Brasileiro de Lógica (EBL) é uma das conferências de lógica mais tradicionais da América do Sul. A 19º edição do EBL ocorre no nordeste do Brasil, na cidade de João Pessoa, Paraíba, de 6 a 10 de maio de 2019. É organizada conjuntamente pela Universidade Federal da Paraíba, Universidade Federal de Campina Grande e Sociedade Brasileira de Lógica. Este volume contém os resumos das palestras deste 19º Encontro Brasileiro de Lógica. Ele também contém os resumos dos tutoriais da Escola de Lógica que ocorre paralelamente à conferência.

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