Update Semantics for Weak Necessity Modals

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Abstract

This paper develops an update semantics for weak necessity modals like 'ought' and 'should'. I start with the basic approach to the weak/strong necessity modal distinction developed in Silk 2018: Strong necessity modals are given their familiar semantics of necessity, predicating the necessity of the prejacent of the actual world (evaluation world). The apparent "weakness" of weak necessity modals derives from their bracketing the assumption that the relevant worlds in which the prejacent is necessary (deontically, epistemically, etc.) need be candidates for actuality. 'Should ϕ' can be accepted without needing to settle that the relevant considerations (norms, preferences, expectations, etc.) that actually apply, given the facts, verify the necessity of ϕ. I formalize these ideas within an Update with Centering framework. The meaning of 'Should ϕ' is explained, fundamentally, in terms of how its use updates attention toward possibilities in which ϕ is necessary. The semantics is also extended to deontic conditionals. The proposed analyses capture various contrasting logical properties and discourse properties of 'should' and 'must' — e.g., in sensitivities to standing contextual assumptions, entailingness, and force — and provide an improved treatment of largely neglected data concerning information-sensitivity.

Keywords: Modals, weak/strong necessity modals, update semantics.

This paper develops an update semantics for English weak necessity modals such as 'ought' and 'should'. I start with a basic approach to the weak/strong necessity modal distinction developed in Silk 2018 (§1). The central idea, on this view, is that the apparent "weakness" of weak necessity modals derives from their bracketing the assumption that the necessity of the prejacent need be verified in the actual world. 'Should ϕ' can be accepted without accepting that the actual circumstances verify the necessity of ϕ. Weak necessity modals afford a means of coordinating on the implications of our values, norms, etc. without having to settle precisely how they weigh against one another in particular circumstances, and while remaining open to new evidence about how they apply. I formalize these ideas within an Update with Centering framework, adapted from Bittner (2011) and Murray (2014) (§2). The conventional meaning of 'Should ϕ' is explained, fundamentally, not in terms of truth conditions,

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but in terms of how its use updates attention toward possibilities in which ϕ is necessary. The semantics is also extended to deontic conditionals. The proposed analyses capture various contrasting logical and discourse properties of 'should' and 'must' — e.g., in sensitivities to standing contextual assumptions, entailingness, and force — and provides an improved treatment of largely neglected data concerning information-sensitivity (Charlow 2013, Silk 2013). I close by considering several alternative static and dynamic implementations of the general approach to the weak/strong necessity distinction from §1 (§3).

A word on terminology: Following common practice, I label modals such as 'should'/'ought' "weak necessity" modals, and modals such as 'must'/'have to' "strong necessity" modals. The expressions in each family pattern with one another in linguistically distinctive ways. In invoking these labels I am not assuming that uses of the former modals invariably convey a weaker conversational force, nor am I assuming a particular type of theoretical analysis. For instance, I am not assuming that the modals express different "kinds" of necessity, that they have a structurally analogous semantics, that they comprise a scale of quantificational strength, or even that they stand in an entailment relation. Indeed we will see reasons for questioning each of these claims.

1 Weak and strong necessity modals in context

This section presents the general approach to the weak/strong necessity modal distinction which I will assume in this paper, as developed in Silk 2018. I won't be able to fully defend the approach here. The aim is simply to present certain core data to motivate the paper's primary constructive project in §2.

There is robust evidence supporting a distinction in strength between modals such as 'should' and 'ought', on the one hand, and 'must', 'have to', and '(have) got to', on the other.[1] For instance, even holding the readings of the modals fixed, 'Should ϕ' can be followed by 'Must ϕ', but not vice versa, as reflected in (1). Similarly, (2a) is consistent in a way that (2b) is not.

(1) a. I should help the poor. In fact, I must.
b. I must help the poor. #In fact, I should.

(2) a. I should help the poor, but I don't have to.
b. #I must help the poor, but it's not as if I should.

There are also important conversational differences. The relative felicity of 'should' and 'must' depends on standing assumptions in the context. It is this

[1] E.g. Sloman 1970, Horn 1972, Wertheimer 1972, Woisetschlaeger 1977, Coates 1983, McNamara 1990, Palmer 1990, 2001, Myhill 1995, Myhill & Smith 1995, Huddleston & Pullum 2002, von Fintel & Iatridou 2008, Lassiter 2011, Rubinstein 2012, Silk 2012, 2013, 2018, Portner & Rubinstein 2016, Swanson 2016b, Yalcin 2016. I use 'should'/'must' as my representative weak/strong necessity modals. These modals are typically used "subjectively" (Lyons 1977), in the sense that the speaker is typically presented as endorsing the considerations with respect to which the modal is interpreted; non-endorsing uses (more common with e.g. 'have to', 'supposed to') introduce complications that would be distracting here (see Silk 2018).

feature of weak and strong necessity modals that I focus on here. [2]

Start with an epistemic case. Suppose we are working on an art project, and I ask where the colored pencils are. Normally you put them in the drawer, but sometimes you accidentally leave them on the shelf. In this context it is more appropriate for you to use 'should' in responding to my question, as in (3).

(3) *Me:* Do you know where the colored pencils are?
You: They should be in the drawer with the crayons.

Suppose, alternatively, that we are looking for the colored pencils together, and you indicate that you have seen something that leads you to conclude that they are in the drawer. Perhaps you noticed that they weren't on the shelf, and this is the only other place you think they could be. In this context it is more natural for you to use 'must', as in (4).

(4) *Me:* Do you know where the colored pencils are?
You: They must be in the drawer with the crayons.

Its following from our evidence (knowledge, information) that the colored pencils are in the drawer depends on today not being one of the atypical days when you accidentally put the colored pencils on the shelf. Using the strong necessity modal 'must' is preferred if, and only if, you know that conditions are indeed normal in this way. What is illuminating is that you can use 'should' even if you aren't in a position to judge that they are. Accepting your 'should' claim doesn't require us to presuppose that your evidence is indefeasible.

Similarly, consider a deontic case (cf. Rubinstein 2012, Silk 2012, 2018). Suppose I am considering whether to fight in the Resistance or take care of my ailing mother. I mention the importance of the value of family, and you agree. But the issue is complex, and we haven't settled whether there might be more important competing values. Sensitive to this, you may find it more appropriate to express your advice that I help my mother by using 'should', as in (5).

(5) *Me:* Family is very important.
You: I agree. You should take care of your mother.

But if we settle that family is of primary importance, as in (6), it can become more natural to use 'must' and for us to accept that I have to help my mother.

(6) *Me:* Family is most important — more important than country.
You: I agree. You must take care of your mother.

My having an obligation to help my mother depends on the value of family being more important (or at least not less important [3]) in my situation than

[2] See Woisetschlaeger 1977, McNamara 1990 for prescient early discussion. See Rubinstein 2012, Silk 2012, 2013, 2018, Swanson 2016b for extensive recent discussion.

[3] Hereafter I bracket complications concerning incomparabilities and irresolvable dilemmas.

any competing value. Using 'must' is appropriate if it is settled that this condition obtains. Parallel to (3), what is illuminating is that you can express your advice that I help my mother using 'should', advice which I may accept, even if it isn't settled that this precondition for my having a genuine obligation is satisfied. Accepting your 'should' claim needn't require us to presuppose that the value of family is more important than other potentially competing values.

These cases highlight what I regard as the fundamental difference between the class of weak necessity modals and the class of strong necessity modals. It is typical to gloss epistemic notions of necessity as concerning what follows from one's evidence (knowledge, information), and deontic notions of necessity as concerning what is obligatory.[4] Whether 'should' or 'must' is preferred depends on context in the sense of depending on whether certain preconditions for the prejacent to be necessary, in the above sense, are accepted. If these preconditions are accepted, using 'must' can be appropriate. Even if they aren't, we can still use 'should'. We can accept your epistemic 'should' claim in (3) without settling that conditions are normal in the relevant respects, and thus without accepting that our evidence actually implies that the colored pencils are in the drawer; and we can accept your deontic 'should' claim in (5) without settling that family is the most important relevant value, and thus without accepting that I have an actual obligation to help my mother. Accepting 'Should ϕ' doesn't conventionally commit one to accepting that ϕ is necessary (in the relevant sense, i.e. deontically, epistemically, etc.[5]).

In Silk 2018 I develop these points in what I call a *modal-past approach* to the weak/strong necessity modal distinction. The core of this approach is as follows. Strong necessity modals are given their familiar semantics of necessity: 'Must ϕ' is true iff ϕ is necessary (deontically, epistemically, etc.), and uses of 'Must ϕ' predicate the necessity of ϕ of the actual world — just as 'May ϕ' is true iff ϕ is possible, and uses of 'May ϕ' predicate the possibility of ϕ of the actual world. The apparent "weakness" of weak necessity modals derives from their bracketing the assumption that the relevant worlds in which the prejacent is a necessity need be candidates for actuality. 'Should ϕ' can be accepted without committing that ϕ follows from what the relevant considerations (norms, etc.) imply given the facts, or that the necessity of ϕ is verified throughout the context set (the set of worlds compatible with what is accepted for purposes of conversation (Stalnaker 1978)).

This feature of 'should' certainly doesn't mark the only dimension along which necessity modals differ. However, I claim that it does distinguish the *class* of weak necessity modals from the *class* of strong necessity modals. There are various ways of implementing the difference in the formal semantics and pragmatics. Yet even at the present level of abstraction, we can see several respects in which the above approach to weak necessity modals differs from the other main approaches in the literature. (See Silk 2018 for detailed comparisons.)

[4] E.g., Lyons 1977, Coates 1983, Palmer 1990, 2001, Huddleston & Pullum 2002, a.m.o.

[5] I will often omit this parenthetical in what follows, but it should be understood.

First, I couched the main conclusion above by saying that accepting 'Should ϕ' doesn't conventionally commit one to accepting that ϕ is necessary. This doesn't amount to the trivial claim that 'should' is weaker than 'must', or that accepting 'Should ϕ' doesn't commit one to accepting 'Must ϕ'. The typical approach in the literature is to posit a concept of a distinctive *kind* of necessity, and to explain accepting 'Should ϕ' as accepting that this (logically weaker) kind of necessity holds of ϕ.[6] One might posit a concept of weak epistemic necessity, and formalization of this concept, such that accepting in (3) that it is a weak epistemic necessity that the colored pencils are in the drawer doesn't require accepting that today is relevantly normal and the evidence actually implies that the colored pencils are in the drawer; and one might posit a concept of weak deontic necessity (weak obligation), and formalization of this concept, such that accepting in (5) that I have a weak obligation to help my mother doesn't require accepting that the value of family isn't ultimately outweighed or defeated. But an alternative approach is to stick with the single familiar notions of necessity — e.g., understanding epistemic necessity in terms of following from a body of evidence, and deontic necessity in terms of being genuinely obligatory and following from a body of norms (n. 4) — and provide an account of the meaning and use of 'should' that captures how 'Should ϕ' can be accepted without accepting that ϕ is necessary — period. The aim of this paper is to pursue one way of developing this latter approach.

Second, the approach takes seriously the effects of standing contextual assumptions on the relative felicity of 'should' and 'must'. Explanatory mechanisms for capturing this effect of context on uses of 'should' vs. 'must' are often lacking in existing accounts (e.g., von Fintel & Iatridou 2008, Finlay 2009, 2010, Lassiter 2011, Swanson 2011; see n. 2 for notable exceptions).

Third, existing accounts of weak necessity modals are often developed by considering a limited range of modal flavors. Extensions to other readings, to the extent that they are discussed at all, are often strained (e.g., Copley 2006, Swanson 2011, Rubinstein 2012, Charlow 2013, Ridge 2014, Portner & Rubinstein 2016, Yalcin 2016). The approach pursued here, by contrast, generalizes across readings of the modals (epistemic, deontic, etc.).

2 Update semantics for weak/strong necessity modals

This section develops one way of implementing the general approach to the weak/strong necessity modal distinction from §1 in an update semantics. The arguments in §1 are by no means conclusive (see Silk 2018), though I hope

[6] Very roughly: On comparative possibility/probability analyses (Finlay 2009, 2010, Lassiter 2011), ϕ is a weak necessity if ϕ is more likely (more desirable, better) than any relevant alternative to ϕ. On domain restriction analyses (Copley 2006, von Fintel & Iatridou 2008, Swanson 2011, Rubinstein 2012), ϕ is a weak necessity if ϕ is true throughout a certain set S of worlds, where S is a subdomain of the set of worlds quantified over by 'must'. Although Yalcin's (2016) normality-based semantics denies that the domains for 'should' and 'must' are logically related, 'Should ϕ' is still interpreted by evaluating the truth of ϕ throughout a set of minimal worlds relative to the actual world (evaluation world).

they may suffice to motivate the positive project pursued here. I develop the semantics in an Update with Centering framework, adapting Bittner 2011. Alternatives are of course possible.[7] (Hereafter I couch the discussion in terms of deontic readings, though the points generalize to other readings.)

2.1 UC_ω background

Update with Centering is a dynamic system that represents how informational (Veltman 1996) and attentional (Grosz et al. 1995) states develop in discourse. Update with *Modal* Centering, UC_ω, includes typed discourse referents not only for individuals δ, but also for worlds ω and propositions Ω (sets of worlds ωt) (Bittner 2011; cf. Stone 1999). The meanings of sentences are given in terms of how they update contexts, conceived as informational-attentional states. Such states are represented with sets of sequences of discourse referents. The discourse referents in each sequence are divided between those currently in the center of attention, or *topical* ($\top$), and those currently *backgrounded* ($\bot$). The bottom sublist $\bot$ can be utilized in analyzing grammatical centering, negation, questions, and, I will suggest, modal remoteness/weakness. The discourse referents in each sublist, $\top, \bot$, are *ranked* according to their relative salience or attentional prominence. The column $||$ picks out the set of discourse referents from a given list. For instance, $\top\Omega_1$ is the most salient (leftmost) proposition in the top sublist, and $\top\omega_1||$ is the set of worlds in the most salient world column in the top sublist. (I write $\top a, \bot a$ as short for $\top a_1, \bot a_1$.) Each $\top\bot$-list, i.e. pair $\langle \top, \bot \rangle$ of sublists of discourse referents, is a semantic object of type s, though not a discourse referent. A *context* is a set of $\top\bot$-lists (type st). The *context set* is the topical proposition $\top\Omega = \top\omega||$.

In UC_ω *all* sentences are treated as introducing a possibility, or modal topic, being talked about. (I treat possibilities as propositions.) With simple indicative sentences the possibility commented on is the context set, typically the most salient possibility in the discourse (cf. Stalnaker 1975, Iatridou 2000,). Modifying Bittner 2011, I propose the UC_ω representation of (7) in (8). (I simplify by translating 'gives to charity' via the simple ($\omega\delta t$-)predicate **donates**.)

(7) Alice gives to charity.

(8) $^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{donates_w}\langle \top\delta \rangle]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||];$
$[\mathbf{w} \mid \mathbf{w} = \bot\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

Boxes without variables, $[\dots]$, are information updates, or tests, which eliminate sequences in the context that don't satisfy the constraint '$\dots$'. Boxes with variables, $^{\top}[\mathbf{d} \mid \dots \mathbf{d} \dots]$ or $[\mathbf{d} \mid \dots \mathbf{d} \dots]$, are recentering updates which introduce a discourse referent satisfying '$\dots \mathbf{d} \dots$' into the most prominent spot in the center of attention or background, respectively. Following Murray 2014 I use the top sequence in representing the context set, and the bottom sequence for keeping track of possibilities we are considering but not yet committed to.

Suppose our model contains three worlds w_0, w_1, w_2; Alice gives to charity

[7] See Silk 2018 for alternative developments in a static framework. See also §3.

only in w_1 and w_2; and the input context c_0 consists of two $\top\bot$-lists each of which includes a discourse referent p_0 for the initial context set and some world $w_0, w_1 \in {}^{\{\}}p_0$. Output contexts for the sequence of updates in (8) are as follows, as specified in the subsequent simplified derivation. [8]

(9) $c_0 = {}^{\chi}\{\langle\langle w, p_0\rangle, \langle\rangle\rangle \mid w \in {}^{\{\}}p_0\} = {}^{\chi}\{\langle\langle w_0, p_0\rangle, \langle\rangle\rangle, \langle\langle w_1, p_0\rangle, \langle\rangle\rangle\}$

c_1	c_2	c_3	c_4
$\langle\langle a, w_0, p_0\rangle, \langle\rangle\rangle$	$\langle\langle a, w_0, p_0\rangle, \langle w_1\rangle\rangle$	$\langle\langle a, w_0, p_0\rangle, \langle q, w_1\rangle\rangle$	$\langle\langle a, w_0, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$
	$\langle\langle a, w_0, p_0\rangle, \langle w_2\rangle\rangle$	$\langle\langle a, w_0, p_0\rangle, \langle q, w_2\rangle\rangle$	$\langle\langle a, w_0, p_0\rangle, \langle w_2, q, w_2\rangle\rangle$
$\langle\langle a, w_1, p_0\rangle, \langle\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle w_1\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle q, w_1\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$
	$\langle\langle a, w_1, p_0\rangle, \langle w_2\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle q, w_2\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle w_2, q, w_2\rangle\rangle$

c_5	c_6	c_7
$\langle\langle a, w_0, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$		
$\langle\langle a, w_1, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$	$\langle\langle a, w_1, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$	$\langle\langle p_1, a, w_1, p_0\rangle, \langle w_1, q, w_1\rangle\rangle$

$c_0[\![^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{A}]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \exists\mathbf{x}\exists\mathbf{i}(\mathbf{j} = (\mathbf{x}\, {}^{\top}\!\oplus\, \mathbf{i}) \wedge \mathbf{I}\mathbf{i} \wedge \mathbf{x} = \mathbf{A})]\!]^g(c_0)$
$= {}^{\chi}\{\langle\langle a, w, p_0\rangle, \langle\rangle\rangle \mid w \in {}^{\{\}}p_0 \;\&\; a = [\![\mathbf{A}]\!]\} = c_1$

$c_1[\![[\mathbf{w} \mid \mathbf{gen}_{\mathbf{w}}\langle\top\delta\rangle]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \exists\mathbf{w}\exists\mathbf{i}(\mathbf{j} = (\mathbf{w}\, {}^{\bot}\!\oplus\, \mathbf{i}) \wedge \mathbf{I}\mathbf{i} \wedge \mathbf{gen}(\mathbf{w}, \top\delta_{\mathbf{1}}\mathbf{i})]\!]^g(c_1)$
$= {}^{\chi}\{\langle\langle a, w, p_0\rangle, \langle w'\rangle\rangle \mid w \in {}^{\{\}}p_0 \;\&\; a = [\![\mathbf{A}]\!] \;\&\; a \in {}^{\{\}}[\![\mathbf{gen}]\!](w')\} = c_2$

$c_2[\![[\mathbf{p} \mid \mathbf{p} = \bot\omega||]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \exists\mathbf{p}\exists\mathbf{i}(\mathbf{j} = (\mathbf{p}\, {}^{\bot}\!\oplus\, \mathbf{i}) \wedge \mathbf{I}\mathbf{i} \wedge \mathbf{p} = \bot\omega_{\mathbf{1}}\{\mathbf{I}\})]\!]^g(c_2) = c_3$

$c_3[\![[\mathbf{w} \mid \mathbf{w} = \bot\omega]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \exists\mathbf{w}\exists\mathbf{i}(\mathbf{j} = (\mathbf{w}\, {}^{\bot}\!\oplus\, \mathbf{i}) \wedge \mathbf{I}\mathbf{i} \wedge \mathbf{w} = \bot\omega_{\mathbf{1}}\mathbf{i})]\!]^g(c_3) = c_4$

$c_4[\![[\bot\omega \in \top\omega||]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \mathbf{I}\mathbf{j} \wedge \bot\omega_{\mathbf{1}}\mathbf{j} \in \top\omega_{\mathbf{1}}\{\mathbf{I}\}]\!]^g(c_4) = c_5$

$c_5[\![[\top\omega = \bot\omega]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \mathbf{I}\mathbf{j} \wedge \top\omega_{\mathbf{1}}\mathbf{j} = \bot\omega_{\mathbf{1}}\mathbf{j}]\!]^g(c_5) = c_6$

$c_6[\![^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]]\!]^g := [\![\lambda\mathbf{I}\lambda\mathbf{j}.\, \exists\mathbf{p}\exists\mathbf{i}(\mathbf{j} = (\mathbf{p}\, {}^{\top}\!\oplus\, \mathbf{i}) \wedge \mathbf{I}\mathbf{i} \wedge \mathbf{p} = \top\omega_{\mathbf{1}}\{\mathbf{I}\})]\!]^g(c_6) = c_7$

The first update introduces into each top sequence $\top$ an individual discourse referent a for Alice, yielding c_1. The second update introduces the worlds where Alice donates, w_1 and w_2, into the bottom sequence, yielding c_2. The worlds added to the bottom sequence at this step needn't be in the current context set. The third update introduces a propositional discourse referent q for this set of most prominent worlds in the bottom sequence $\bot\omega|| = {}^{\{\}}q = \{w_1, w_2\}$, yielding c_3. However, the context set isn't yet restricted; the update is a pure attention update. The fourth update represents a commitment to this possibility, by reintroducing the worlds in which it is true into the bottom sequence, yielding c_4. The fifth update represents the proposal to update with the proposition that Alice gives to charity, by restricting the set of worlds introduced in the fourth update to the worlds in the context set, yielding c_5. The sixth update represents acceptance of the assertion, by checking for each world $\top\omega$ in the context set that it is identical to the most prominent world $\bot\omega$ in its row. The first sequence is ruled out and the context set is restricted to $\{w_1\}$, yielding c_6.

[8] The superscript $^{\chi}$ indicates the characteristic function, and $^{\{\}}$ the characteristic set; variables $\mathbf{i}$, $\mathbf{j}$ are for $\top\bot$-lists (type s), $\mathbf{I}$ for a set of lists (type st), i.e. a context. See the Appendix for relevant definitions and DRT-style abbreviations.

The final update centers attention on the new context set by introducing into the top sequence a propositional discourse referent p_1 for it, yielding c_7.

The main features of this sequence of updates are these: First, updates 1–3 introduce the at-issue proposition that Alice gives to charity into the bottom sequence. This represents that the proposition is under consideration, or on the conversational table, though no attitude has yet been taken toward it. Second, I treat all (declarative) sentences as involving a commitment update such as update 4. Though the commitment update may seem trivial in (8), its importance will become apparent below. This update is distinctive of the version of UC_ω developed here. Third, updates 5–6 occur with all assertions (cf. Murray 2014). The proposal update reflects how in assertions the worlds being talked about are typically the worlds treated as live for purposes of the conversation. The success of the assertion registers an attitude of acceptance toward the proposed possibility. Asserting (7) thus both updates information, reflected in the reduction of the context set, and updates attention, reflected in the introduction of a new modal referent as the primary topic.

2.2 Modals in root clauses

Turning to modal sentences, I follow standard ordering semantics in treating modals as contributing a preorder frame $\lesssim^{?}$, or function from worlds to preorders, where the resolution of ? is tied to the reading of the modal (Lewis 1973, 1981, Kratzer 1981, 1991). The "ideal" of a preordered set, written $\text{MIN}(Q, \lesssim_w)$, is the set of $\lesssim_w$-minimal elements of a modal base Q, the set of Q-worlds that aren't $\lesssim_w$-bettered by any other Q-world. For instance, $\text{MIN}(Q, \lesssim^d_w)$ is the set of worlds in Q that best satisfy certain relevant norms $\lesssim^d$ in w.[9]

Start with (10) with the strong necessity modal 'must'. Like with (7), the meaning of (10) is given in terms of how it updates the default modal topic, or context set. The distinctive dynamic contribution of the modal is that it itself introduces a topical possibility — here, the possibility that Alice gives to charity — and comments on it (cf. Stone 1999, Stone & Hardt 1999, Kaufmann 2000, Brasoveanu 2010).[10] I propose the UC_ω representation of (10) in (11). As the reader can verify, the input and output updates for (11) are as in (12). (As above, assume an input context c_0 with context set ${}^{\{\}}p_0 = \{w_0, w_1\}$. And assume a model with three worlds w_0, w_1, w_2, such that Alice donates only in w_1 and w_2, and Alice's donating is deontically necessary only at w_0 and w_2.)

(10) Alice must give to charity.

(11) $^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{donates}_{\mathbf{w}}\langle\top\delta\rangle]; [\mathbf{w} \mid \text{MIN}\{\top\omega||, \lesssim^d_{\mathbf{w}}\} \subseteq \bot\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||]; [\mathbf{w} \mid \mathbf{w} = \bot\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

[9] The preorder could be determined from a premise set (Kratzerian ordering source) in the usual way: $u \lesssim_{P(w)} v := \forall p \in P(w)\colon v \in p \Rightarrow u \in p$. For simplicity I make the limit assumption (Lewis 1973: 19–20) and assume that MIN is well-defined.

[10] Interestingly, it has been argued that a principle use in the development of modals diachronically involves encouraging the hearer to "focus mentally" on the embedded proposition (Van Linden 2012: chs. 6, 8).

(12) $c_0 = {}^{\chi}\{\langle\langle w_0, p_0\rangle, \langle\rangle\rangle, \langle\langle w_1, p_0\rangle, \langle\rangle\rangle\}$ $\quad c_8 = {}^{\chi}\{\langle\langle p_1, a, w_0, p_0\rangle, \langle w_0, q, w_0, w_1\rangle\rangle, \langle\langle p_1, a, w_0, p_0\rangle, \langle w_0, q, w_0, w_2\rangle\rangle\}$

As with (8), the first update introduces into each top sequence an individual discourse referent a for Alice, and the second update introduces into each bottom sequence the worlds where the topical individual $\top\delta$ (=Alice) gives to charity, i.e. w_1 and w_2. The third update reflects the modal's evaluation of this possibility $\bot\omega|| = \{w_1, w_2\}$. The update introduces into the bottom sequence the worlds $\mathbf{w}$ such that every $\lesssim^d_{\mathbf{w}}$-minimal world in the topical modality (=the context set $\top\omega||$) is a world where Alice donates, i.e. w_0 and w_2. The fourth update introduces a propositional discourse referent q for this set of worlds, $\bot\omega|| = {}^{\{\}}q = \{w_0, w_2\}$. This attentional update represents the necessity claim being put on the conversational table. The fifth update represents the speaker's commitment to this possibility, and the sixth update represents the proposal to update the context set with it. The seventh update represents the acceptance of the necessity claim. This update eliminates sequences in which w_1 is the (local) topical world, restricting the context set $\top\omega||$ to $\{w_0\}$. The final update recenters attention on the new context set by introducing a propositional discourse referent $p_1 = {}^{\chi}\{w_0\}$, yielding c_8.

Two remarks: First, as in (8), the update $[\mathbf{w} \mid \mathbf{w} = \bot\omega]$ represents a commitment to the proposition placed on the conversational table, here the deontic necessity claim. Accepting (10) requires that the deontic necessity of Alice's giving to charity is verified throughout the context set. Asserting (10) updates both information, reflected in the reduction of the context set, and attention, reflected in the introduction of the modal referent p_1 as the primary modal topic. Second, with 'must' the modal base for the relevant norms is resolved to the topical modality $\top\omega||$. This reflects the indicative presupposition that the worlds being talked about are in the context set. We will return to this.

In §1 I argued, following Silk 2018, that accepting 'Should ϕ' needn't commit one to accepting that ϕ is actually necessary (epistemically, deontically, etc.)—that all preconditions for ϕ to be a genuine obligation are satisfied, that one's evidence for ϕ isn't misleading or defeated, and so on. One way of capturing this is to treat 'should' as having an ordinary semantics of necessity, but as canceling the usual implication that the speaker is committed to the at-issue proposition. 'Should ϕ' is interpreted with respect to the same body of considerations as 'Must ϕ' (given a relevant flavor of modality), and "introduces" (intuitively speaking) the proposition that ϕ is a necessity relative to these considerations; yet the attitude taken toward this proposition needn't be acceptance. I propose (14) as a first-pass UC_ω representation of (13).

(13) Alice should give to charity.

(14) ${}^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{donates}_{\mathbf{w}}\langle\top\delta\rangle]; [\mathbf{w} \mid \textsc{min}\{?\omega||, \lesssim^d_{\mathbf{w}}\} \subseteq \bot\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||]; [\mathbf{w} \mid \mathbf{w} = \top\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

The first four updates are (nearly) the same as in (11): the deontic necessity claim is placed on the conversational table, as represented by the introduction of

a propositional discourse referent q for this possibility into the bottom sequence. (I return to the semantically unspecified modal base $?\omega||$ in update 3 shortly.) The crucial contrast is in the fifth update: The worlds in the context set, rather than the worlds in the at-issue proposition q, are introduced into the bottom sequence. This update recommits to the topical modality $\top\omega||$, rather than to the necessity claim q. The subsequent updates, which are associated with any assertion, have no effect: There is no restriction of the context set to worlds where it is deontically necessary that Alice give to charity, and the output context set p_2 is ${}^{\chi}\{w_0, w_1\} = p_0$, as reflected in (15).

(15) $$\begin{aligned} c_8 = {}^{\chi}\{&\langle\langle p_2, a, w_0, p_0\rangle, \langle w_0, q, w_0, w_1\rangle\rangle,\ \langle\langle p_2, a, w_0, p_0\rangle, \langle w_0, q, w_2, w_1\rangle\rangle,\\ &\langle\langle p_2, a, w_0, p_0\rangle, \langle w_0, q, w_0, w_2\rangle\rangle,\ \langle\langle p_2, a, w_0, p_0\rangle, \langle w_0, q, w_2, w_2\rangle\rangle,\\ &\langle\langle p_2, a, w_1, p_0\rangle, \langle w_1, q, w_0, w_1\rangle\rangle,\ \langle\langle p_2, a, w_1, p_0\rangle, \langle w_1, q, w_2, w_1\rangle\rangle,\\ &\langle\langle p_2, a, w_1, p_0\rangle, \langle w_1, q, w_0, w_2\rangle\rangle,\ \langle\langle p_2, a, w_1, p_0\rangle, \langle w_1, q, w_2, w_2\rangle\rangle\} \end{aligned}$$

Although the updates in (14) don't directly restrict the context set, they don't have no conversational import. Both (11) and (14) introduce a modal topic for consideration — the possibility q that Alice's donating is deontically necessary. Updating with (11), with 'must', requires committing to this possibility; it requires committing that norms of charity take precedence in Alice's situation over other potentially competing considerations, and that Alice's donating follows from what the relevant norms enjoin given the facts. We might not be prepared to restrict the future course of the conversation in this way. After all, there is typically a range of interests, values, and norms potentially relevant for determining what to do. How the normatively relevant factors interact — how they weigh against one another, under what conditions they apply, etc. — is often quite complex. (Witness: deontic logic, normative ethics, etc.) Even given a specific set of normative factors, there may be uncertainty about empirical facts relevant to determining which apply in the given case. We may be unsure about the details of Alice's financial or family situation, how donations are used, the short- and long-term impact on those in need, etc. Using deontic 'must' may thus be inapt.

Deontic 'should' affords a means of guiding our deliberations and plans while remaining open to new evidence about what values are at stake and how they interact with one another and the non-normative facts. The conventional role of weak necessity modals, on the proposed semantics, isn't to update information; it is to place a necessity claim on the conversational table and center attention on it.[11] Updating with (14) centers attention on the set of worlds at which Alice's giving to charity is deontically necessary, but doesn't explicitly require committing that the actual world is among them. So, in accepting (13) with 'should' we can leave open the possibility that norms of charity might ultimately be outweighed or defeated in Alice's situation; we needn't commit that values

[11] This contrasts with non-assertive discourse moves like questions, which introduce into the bottom sequence discourse referents for each answer, inducing a partition on the context set.

of charity are more important than other competing values we accept or may come to accept. We can capture a crucial role for 'should'-claims in discourse without treating them as conventionally constraining the context set.

The above analyses give precise expression to the informal intuition that 'should' is "weaker" and more tentative than 'must'. In uttering 'Should ϕ' the speaker introduces a claim about the necessity of ϕ but fails to mark her utterance as being about worlds that are candidates for actuality. Yet, as Stalnaker notes, "normally a speaker is concerned only with possible worlds within the context set, since this set is defined as the set of possible worlds among which the speaker wishes to distinguish" (1975: 69). So, using 'should' implicates that one isn't in a position to commit to the prejacent's being necessary throughout the context set. The basis of the scale between 'should' and 'must' isn't fundamentally logical but epistemic strength; 'Should ϕ' and 'Must ϕ' are ordered not in terms of (e.g.) subset/superset relations in their domains of quantification, but in terms of epistemic attitude regarding the proposition that ϕ is a necessity.[12] This aspect of the account marks an important contrast with domain restriction analyses of the weak/strong necessity modal distinction (§1).

Since 'should' is weaker than 'must' in the above way, Grice's first quantity maxim — "Make your contribution as informative as is required" (Grice 1989: 26) — can generate a familiar upper-bounding implicature (Horn 1972; cf. n. 12). Using 'should' implicates that for all one knows — more accurately, for all one is willing to presuppose in the conversation — 'Must ϕ' is false. This implicature has the usual properties of implicatures. For instance, it is cancelable and reinforceable, as in (1a)–(2a). In (1a) the speaker first places the deontic necessity claim on the conversational table, with 'should', and then positively commits to it, with 'must', by proposing to restrict the context set to worlds in which I have a genuine obligation to help the poor.

I noted above that the modal base for 'must' is resolved to the topical modality $\top\omega||$, reflecting the indicative presupposition that the worlds being talked about are in the context set. 'Should' lacks this restriction, as reflected in the semantically unspecified modal base $?\omega||$ in (14). This difference in modal bases helps capture another attested contrast between 'should' and 'must', in entailingness. Uttering 'Should ϕ' is compatible with denying 'ϕ' ((16)); when used with the perfect it even implicates $\neg\phi$ ((17)). Uses of '(Must ϕ) $\wedge$ $\neg\phi$', by contrast, are generally anomalous. There is robust evidence that this holds not only with epistemic readings ((18a)) but also with deontic readings: when one wishes to convey that one thinks a given obligation won't be satisfied, one typically uses 'should'/'ought' rather than 'must' ((18b)).[13]

[12] Cf. Verstraete 2005, 2006, Van Linden & Verstraete 2008.

[13] See, a.o., Wertheimer 1972, Harman 1973, Lyons 1977, Coates 1983, Palmer 1990, Myhill 1996, Huddleston & Pullum 2002, Werner 2003, Ninan 2005, Portner 2009, Close & Aarts 2010, Campbell 2014. The point about 'must' holds only for "subjective" uses (Lyons 1977), i.e. uses which present the speaker as accepting the norms (information, etc.) with respect to which the modal is interpreted; see Silk 2018: §5 for discussion.

(16) a. Alice should be here by now, but she isn't.
b. You should help your mother, but you won't.

(17) We should have given to Oxfam. (*Implicates:* we didn't)

(18) a. #Alice must be here by now, but she isn't.
b. ??You must help your mother, but you won't.

Of course obligations can go unfulfilled. What is interesting is that speakers appear to assume otherwise, at least for purposes of conversation, when expressing obligations with 'must'.

One way of adapting common definitions of truth in dynamic semantics for UC_ω is as follows. Definition 2.1 says that a sentence K is true at w iff given perfect information about w, i.e. an initial context set $\{w\}$, updating with K doesn't lead to the absurd state (cf. van Benthem et al. 1997: 594). [14]

Definition 2.1 (truth, v1). For an $(st)st$ term K and world w:

- Let C_w be the set of contexts c such that $\{(\top j)_1 \mid j \in {}^{\{\}}c\} \neq \{^{\chi}\{w\}\}$ and $\{((\top j)_{\omega t})_1 \mid j \in {}^{\{\}}c\} = \{^{\chi}\{w\}\}$

i. K is true at w iff for any $c \in C_w$, $\{(\top j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \{^{\chi}\{w\}\}$
ii. K is false at w iff for any $c \in C_w$, $\{(\top j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \emptyset$

This definition predicts that '(Must ϕ)$\wedge\neg\phi$' is necessarily false: there is no $\neg\phi$-world at which 'Must ϕ' is true, hence no world at which '(Must ϕ)$\wedge\neg\phi$' is true. Although Definition 2.1 doesn't assign a truth value to (14) (since (14) doesn't recenter the primary modal topic), it could be revised to assign truth values to terms that update the primary background item $(\bot j)_1$ to a specific proposition. Replacing $\top$ with $\bot$ throughout Definition 2.1 would predict the possible truth and consistency of '(Should ϕ) $\wedge\neg\phi$'. Note that the semantics does allow for consistent updates with 'Must ϕ ... $\neg\phi$' (in that order); observe that w_0 in (12) is a $\neg\phi$-world. Yet such sequences are still *incoherent*, in the sense that no non-empty information state is a fixed-point of an update with it: [15]

Definition 2.2 (coherence). K is coherent iff for some c, $\exists p \in D_{\omega t}\colon {}^{\{\}}p \neq \emptyset$ and $\{((\top j)_{\omega t})_1 \mid j \in {}^{\{\}}c\} = \{((\top i)_{\omega t})_1 \mid \exists g\colon i \in {}^{\{\}}[\![K]\!]^g(c)\} = \{p\}$

Unlike many previous accounts, we can capture the (non-)entailingness data with 'should' and 'must' without ad hoc stipulations in the semantics or pragmatics — e.g. fundamental appeals to differences in the modals' performative properties, separate imperatival entries for root deontic 'must', etc. (cf. Ninan 2005, Swanson 2008, 2016a, Portner 2009). The data falls out of the general discourse framework and semantics for the modals.

Finally, the account in this section sheds light on common informal observations about the illocutionary force of deontic 'should'/'ought' vs. 'must'.

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[14] K has a truth value iff it updates the primary topic to a proposition. For a list j, $(\top j)_1$ is the first element in the top sublist, and $((\top j)_{\omega t})_1$ the first type ωt element in the top sublist.

[15] I leave open how this definition might be revised to generalize to non-declarative sentences.

> To say that one ought to take a certain option is merely to provide a nudge in that direction. Its typical uses are to offer guidance, a word to the wise ("counsel of wisdom"), to recommend, advise or prescribe a course of action... In contrast, to say that one must take a certain option is to be quite forceful. Its typical uses are to command, decree, enact, exhort, entreat, require, regulate, legislate, delegate, or warn. Its directive force is quite strong. (McNamara 1990: 156)

We have seen that accepting 'Must ϕ' commits one to accepting that ϕ is necessary, and is incompatible with denying 'ϕ'. So, if the truth of 'ϕ' is assumed to be under the control of some relevant subject, updating with deontic 'Must ϕ' will commit that subject to seeing to it that ϕ. So, it is no surprise that deontic 'Must ϕ' is often thought to conventionally perform a directive speech act (n. 13). By contrast, accepting 'Should ϕ' needn't commit one to accepting that ϕ is necessary, and is compatible with denying ϕ. Updating with deontic 'Should ϕ' needn't commit anyone to seeing to it that ϕ. So, even if deontic 'should' can be used to perform a directive speech act, it doesn't do so as a matter of its conventional meaning. Uttering 'Should ϕ' can convey one's preference that 'ϕ' be accepted, but without imposing ϕ (or the deontic necessity of ϕ) on the common ground. Deontic 'should' thus typically provides a less face-threatening impression to deontic 'must', in particular in contexts where the speaker might be construed as imposing on the addressee or other relevant subject. [16]

2.3 Conditionals and information-sensitivity

This section describes one way of extending the above semantics for 'should' and 'must' in root clauses to deontic conditionals. Simple indicative sentences comment on the topical modality $\top\Omega$, the input context set. Indicative conditionals introduce a *sub*domain of this modality — the set of context-set worlds where the antecedent is realized — which is commented on by the consequent. I offer (20) as a UC_ω representation of an indicative conditional such as (19) ($K\,{}^{\top};\ K'$ is a topic-comment sequence).

(19) If Alice has a job, she will give to charity.

(20) $({}^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{job}_{\mathbf{w}}\langle\top\delta\rangle]; [\bot\omega \in \top\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||])$
${}^{\top}; ([\mathbf{donates}_{\bot\omega}\langle\top\delta\rangle]; [\mathbf{w} \mid \text{MIN}\langle\bot\Omega, \lesssim^{e}_{\mathbf{w}}\rangle \subseteq \bot\omega||]);$
$[\mathbf{w} \mid \mathbf{w} = \bot\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

The 'if'-clause introduces a propositional discourse referent into the bottom sequence for the set of worlds in the context set $\top\omega||$ in which Alice has a job, as reflected in the first line of (20). This topical subdomain $\bot\Omega$ forms the modal base of an expectational modal comment in the consequent clause, as reflected in the second line: the first update restricts the topical subdomain to

[16] It isn't implausible that the drastic decline in frequency of deontic 'must' is due in part to the above features of its meaning and use (cf. Myhill 1995, 1996, Smith 2003, Leech et al. 2009, Close & Aarts 2010, Goddard 2014).

worlds in which Alice donates (modal anaphora via $\bot\omega$), and the next update introduces worlds $\mathbf{w}$ in which the most $\mathbf{w}$-expected ($\lesssim^e_{\mathbf{w}}$-minimal) worlds in the modal base $\bot\Omega$ are worlds in which this possibility ($\bot\omega||$) is realized. The now-familiar updates in the third line represent commitment to this possibility $\bot\omega||$, the proposal to update with it, the acceptance of this proposal, and the recentering of attention on the new topical modality $\top\Omega$.

Our analyses of 'should' and 'must' in root clauses can be integrated into this general treatment of indicative conditionals:

(21) If Alice has a job, she must give to charity.

(22) $(^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{job}_{\mathbf{w}}\langle\top\delta\rangle]; [\bot\omega \in \top\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||])$
$^{\top}; ([\mathbf{w} \mid \mathbf{donates}_{\mathbf{w}}\langle\top\delta\rangle]; [\textsc{min}\{\bot\omega_2||, \lesssim^d_{\bot\omega_2}\} \subseteq \bot\omega||];$
$[\mathbf{w} \mid \textsc{min}\langle\bot\Omega, \lesssim^e_{\mathbf{w}}\rangle \subseteq \bot\omega_2||]);$
$[\mathbf{w} \mid \mathbf{w} = \bot\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

(23) If Alice has a job, she should give to charity.

(24) $(^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{job}_{\mathbf{w}}\langle\top\delta\rangle]; [\bot\omega \in \top\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||])$
$^{\top}; ([\mathbf{w} \mid \mathbf{donates}_{\mathbf{w}}\langle\top\delta\rangle]; [\textsc{min}\{?\omega||, \lesssim^d_{\bot\omega_2}\} \subseteq \bot\omega||];$
$[\mathbf{w} \mid \textsc{min}\langle\bot\Omega, \lesssim^e_{\mathbf{w}}\rangle \subseteq \bot\omega_2||]);$
$[\mathbf{w} \mid \mathbf{w} = \top\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

As in (20), the 'if'-clauses introduce the set of worlds in the context set $\top\omega||$ in which Alice has a job. In both (22)/(24) this subdomain is further restricted to worlds in which Alice's donating is deontically necessary (modal anaphora via $\bot\omega_2$). The comment is that these worlds are the most expected worlds in the subdomain $\bot\Omega$. The proposition "that the most expected context-set worlds where Alice has a job are worlds in which Alice's donating is deontically necessary" is introduced into the bottom sequence. As with root assertions, the crucial contrast between the 'should' and 'must' conditionals concerns what attitude is taken toward this possibility, as reflected in the first update of the fourth lines. Updating with (24) places on the table the possibility that Alice's donating is deontically necessary conditional on her having a job, yet there is no conventional restriction of the context set. In accepting (23) with 'should' one can leave open the possibility that norms of charity may be outweighed or defeated in Alice's situation even if she has a job. Commitment to the conditional obligation isn't required by the conventional meaning of (23).

This account helps capture an observed, but little discussed, contrast between 'should' and 'must' conditionals in information-sensitivity. [17] Consider the Miners Puzzle:

> Ten miners are trapped either in shaft A or in shaft B, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one shaft, but not both. If we block one shaft, all the water will go

[17] I use 'information-sensitivity' pretheoretically for phenomena where a modal in the consequent of a conditional seems to be interpreted as if the information in the antecedent is available. How to capture this intuitive idea is contentious (see below).

> into the other shaft, killing any miners inside it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed. (Kolodny & MacFarlane 2010: 115–116)

As has been extensively discussed, there are readings of (25)–(27) on which they appear jointly consistent with (28). (25) seems true, since we don't know which shaft the miners are in, and the consequences will be disastrous if we choose the wrong shaft. (26)/(27) are also natural to accept, since, given that the miners are in shaft A/B, blocking shaft A/B will save all the miners.

(25) We should block neither shaft.

(26) If the miners are in shaft A, we should block shaft A.

(27) If the miners are in shaft B, we should block shaft B.

(28) The miners are in shaft A, or the miners are in shaft B.

A wrinkle in the discussions of information-sensitivity is that nearly all examples use weak necessity modals, and little attention is paid to how context affects speakers' judgments. Several authors have observed that using 'must' in the conditionals is generally dispreferred (Charlow 2013, Silk 2013).

(29) ?If the miners are in shaft A, we must block shaft A.

(30) ?If the miners are in shaft B, we must block shaft B.

Intuitively, the 'should' conditionals say what is best on a condition: given that the miners are in shaft A/B, our blocking shaft A/B is the expectably best action. (26)–(27) don't impose obligations on us conditional on how the world happens to be, unbeknownst to us. By contrast, (29)–(30) do seem to impose such obligations. This is likely part of why many speakers find using 'must' in the conditionals to be dispreferred to using 'should'.

Elsewhere I have argued that deontic conditionals with 'must' (and also with 'may') don't give rise to the same apparent modus ponens violations as deontic 'should' conditionals, and that the puzzles raised by cases like the Miners Case turn on features peculiar to weak necessity modals (Silk 2013). Here I only wish to observe how the account in this section sheds light on apparent differences in interpretation and acceptability between examples with 'should' and 'must'. That said, the data about the broader spectrum of examples is admittedly less robust than would be desired. More careful assessment of the predicted contrasts among deontic conditionals must be left for future research.

As with (21), updating with (29), using 'must', requires committing that our blocking shaft A is deontically necessary at every world in the context set in which the miners are in shaft A. Yet in some of these worlds we don't know that the miners are in shaft A. So, updating with (29) requires accepting that we have an obligation to block shaft A conditional on the miners being in shaft A, independent of whether we learn that they are. Though perhaps one could imagine accepting this in a particularly urgent context, at least for purposes of conversation, doing so would typically be inapt. Hence the general

anomalousness of (29). (Likewise for (30).)

Correspondingly, compare the variant of (25) with 'must' in (31).

(31) We must block neither shaft.

Suppose we accept information-dependent norms which obligate us to block neither shaft in scenarios where we don't learn the miners' location, and obligate us to block shaft A/B in scenarios where we learn that the miners are in shaft A/B (formally, e.g. a preorder frame $\lesssim$ such that the $\lesssim_N$-minimal relevant worlds are worlds where we block neither shaft, and the $\lesssim_{A/B}$-minimal relevant worlds are worlds where we block shaft A/B, for relevant worlds N where we remain ignorant of the miners' location and worlds A/B where we learn they are in shaft A/B). Given these norms, accepting (31) would require committing that we won't get new evidence about the miners' location within the interval circumscribing our deliberation. Though we may wish to proceed as if this is the case, we might prefer not to explicitly register this commitment, at least for purposes of the conversation.

Accepting (25)–(27) with 'should' avoids requiring the above commitments associated with (29)–(31). On the one hand, updating with (25) allows us to entertain the possibility that we won't learn where the miners are and hence will have an obligation to block neither shaft. In accepting (25) we can coordinate on a plan for the likely scenario in which we remain ignorant of the miners' location — namely, to block neither shaft — but without needing to settle decisively that no new relevant evidence will come in, as we would need to do if we accepted (31). On the other hand, updating with (26)/(27) places on the conversational table the possibility that we will be obligated to block shaft A/B conditional on the miners' being in shaft A/B, and hence — given the information-dependent norms we accept — the possibility that we learn that the miners are in shaft A/B. In accepting (26)–(27) with 'should' we can remain open to the possibility, however slight, that we might learn which shaft the miners are in, and coordinate on plans for such contingencies. It is weak necessity modals like 'should', in contrast to 'must', that play this complex role in conversation and contingency planning.

As with the examples in §1, the account captures how preferences for using 'should' vs. 'must' depend on standing assumptions in the context — specifically, on the speakers' (un)willingness to commit to certain preconditions for the prejacent to be necessary. Our treatment of the contrast between (25) and (31) correctly predicts that using 'must' will become more appropriate to the extent that we are willing to settle that we won't learn where the miners are, as in (32).

(32) *A:* Well, it looks like we're never going to be able to find out which shaft the miners are in.
B: So what should I tell the team?
A: That we must block neither shaft.

We also predict that in the latter sorts of contexts it should become harder to hear the 'should' conditionals as felicitous. This prediction appears to be borne out as well: *A*'s response to *B* in (33) is marked.

(33) *A:* Well, it looks like we're never going to be able to find out which shaft the miners are in.
B: So what should I tell the team?
A: We should/must block neither shaft. ?But if they're in A, we should block A, and if they're in B, we should block B.

Unlike many previous accounts, we can capture the above phenomena regarding the broader spectrum of examples without treating the 'if'-clauses as explicitly reinterpreted as 'if ϕ and we learn it' (as in von Fintel 2012), or introducing general revisions to the semantics of modals or conditionals (information-dependent preorder frames, selection functions, etc., as in Kolodny & MacFarlane 2010, Cariani et al. 2013, Charlow 2013, Silk 2014; cf. Willer 2010). This provides a notable contribution to the literature's growing understanding of possible approaches to information-sensitivity.

3 Alternatives: Static and dynamic

The accounts in §§1–2 carve out crucial roles for weak necessity modals in discourse and deliberation. Weak necessity modals afford a means of coordinating our values, norms, etc. without having to settle precisely how they weigh against one another in particular circumstances, and while remaining open to new evidence about how they apply. In §2 we saw how the proposed formal implementation helps capture certain logical properties such as the non-entailingness of 'should' and the joint acceptability of (25)–(28) in the Miners Case. Yet one might worry that this feature of the account actually points to a defect in the semantics: If using 'should' merely centers attention on a necessity claim, why can't *any* set of 'should' sentences be coherently accepted? I won't attempt to resolve this question here. I simply wish to raise several strategies of reply, so as to introduce certain of the critical empirical and theoretical issues.

'Should ϕ', on the proposed update semantics, places the possibility that ϕ is necessary on the conversational table and centers attention on this possibility. One option is to maintain this as an account of the conventional meaning, and to capture ideas about the logic of 'should' sentences in an *extra*-semantic account of rationality constraints on this kind of discourse move. We might view work in deontic logic on prima facie obligations, weights and priorities, dilemmas, etc. as addressing precisely this issue. Settling on controversial issues about the logic and metaethics — e.g., concerning the possibility of irresolvable moral dilemmas, the proper order in which to apply defaults, etc. — arguably isn't required for semantic competence with modals. This line provides a way of situating respective work in logic and linguistic semantics in an overall theory of modality and modal language. [18]

[18] For discussion of related methodological issues, see Forrester 1989, Silk 2016, 2017.

An alternative response is to treat updates with 'Should ϕ' as restricting the context set, but revise what proposition is conventionally placed on the conversational table. We have seen how using 'should' allows us to consider the necessity of ϕ as holding, not necessarily in the current context, but in a "preferred" (normal, desirable) continuation or minimal revision of the context, whatever that might turn out to be. One move would be to encode this apparent discourse role more directly into the semantics — e.g., by treating 'Should ϕ' as predicating the necessity of ϕ of a set of worlds that satisfy some (possibly counterfactual) condition (cf. Wertheimer 1972, Silk 2012), or as predicating the necessity of ϕ of a set of (possibly counterfactual) worlds that are minimal in some contextually relevant sense (most desirable, normal, etc.) (cf. Silk 2018: Def. 4). Updating with (34) restricts the context set to worlds $\mathbf{w}$ such that the relevant $\mathbf{w}$-accessible worlds in $\textsc{pref}_{\mathbf{w}}$, which may not themselves be in the context set, are worlds at which Alice's donating is deontically necessary.

(34) $\quad {}^{\top}[\mathbf{x} \mid \mathbf{x} = \mathbf{Alice}]; [\mathbf{w} \mid \mathbf{donates}_{\mathbf{w}}\langle\top\delta\rangle]; [\mathbf{w} \mid \textsc{min}\{?\omega||, \lesssim^{d}_{\mathbf{w}}\} \subseteq \bot\omega||]; [\mathbf{w} \mid \textsc{pref}^{?}_{\mathbf{w}} \subseteq \bot\omega||]; [\mathbf{p} \mid \mathbf{p} = \bot\omega||]; [\mathbf{w} \mid \mathbf{w} = \bot\omega]; [\bot\omega \in \top\omega||]; [\top\omega = \bot\omega]; {}^{\top}[\mathbf{p} \mid \mathbf{p} = \top\omega||]$

(34) explicitly represents an attitudinal comment about the (deontic) necessity claim. The logic of 'should' sentences could then be captured via the logic of the relevant notion of minimality (preference, normality).

(34) treats 'should' assertions as updating context like any other assertion. Although UC_{ω} isn't merely an eliminative informational update system, the general kind of analysis in (34) could be implemented in a static framework which provides straightforward truth conditions for 'Should ϕ'; compare (34) with (35) (for modal base function f; cf. Silk 2018: Def. 4).

(35) $\quad [\![\text{Should } \phi]\!]^{c,w} = 1 \text{ iff } \forall w' \in \textsc{pref}_{w}\colon \textsc{min}(f(w'), \lesssim_{w'}) \subseteq [\![\phi]\!]^{c}$

The analyses in (34)–(35) both capture the core ideas from §1: both avoid analyzing accepting 'Should ϕ' in terms of ϕ being a necessity (in any posited sense) at every candidate for the actual world; and, insofar as the worlds in $\textsc{pref}_{w}$ (i.e., the minimal worlds at which the necessity of ϕ is evaluated) needn't be in the context set, both capture how 'Should ϕ' brackets whether ϕ is actually necessary (epistemically, deontically, etc.). Yet even if informal ideas about the contrasting discourse properties of 'should' and 'must' *could* be implemented in a static or dynamic semantics, this leaves open whether the ideas are best explained in terms of truth. Thorough investigation of grammatical and discourse differences among necessity modals (nn. 1, 16), as well as general reflection on the theoretical significance and explanatory power of alternative static vs. dynamic frameworks (Starr 2010, Rothschild & Yalcin 2016), is needed.

4 Conclusion

Let's recap. Following Silk 2018 I argued that the common semantic core of weak necessity modals — what makes them intuitively "weak" — is that they

bracket the assumption that the relevant worlds in which the prejacent is necessary (deontically, epistemically, etc.) need be candidates for actuality. 'Should ϕ' can be accepted without accepting that ϕ is necessary, and without needing to settle that the relevant considerations (norms, values, expectations, etc.) that actually apply, given the facts, verify the necessity of ϕ. To implement this idea I developed an update semantics for 'should' and 'must'. The contrast between updates with 'Must ϕ' and 'Should ϕ' isn't explained, fundamentally, in terms of a different *kind* of necessity ("strong" vs. "weak") being predicated of ϕ at (every candidate for) the actual world; rather it is explained in terms of a difference in attitude conventionally conveyed about the necessity of ϕ, in the same relevant sense of necessity. The conventional discourse function of 'Should ϕ' isn't to update information, but to update attention toward possibilities in which ϕ is necessary and place the necessity claim on the conversational table. An account of deontic conditionals was also integrated into a more general update semantics for conditionals. These analyses carve out important roles for expressions of weak necessity in discourse, deliberation, and planning. Weak necessity modals afford a means of coordinating on the implications of our norms, values, expectations, etc. without having to decisively settle how they apply and weigh against one another in particular circumstances.

The data considered here certainly aren't the only data that must be explained by an overall theory of necessity modals. For instance, first, there are also contrasts between weak and strong necessity modals in data involving incomparabilities, comparatives, quantifiers, modifiers, and neg-raising, among others (n. 1). Second, though I focused on what distinguishes the *classes* of weak vs. strong necessity modals, I bracketed differences among weak necessity modals and among strong necessity modals (see Silk 2015, 2018 and references therein). Third, elsewhere (Silk 2012, 2018) I argue that the general approach to the weak/strong necessity modal distinction from §1 sheds light on the morphosyntactic properties of expressions of weak necessity in other languages. Though I focused here on lexicalized weak necessity modals in English, it is cross-linguistically common to mark the weak/strong necessity distinction *morphologically*, by using the form of a strong necessity modal used in counterfactuals (schematically: *STRONG+CF*) (e.g., Palmer 2001, von Fintel & Iatridou 2008). The cross-linguistic data raises difficult diachronic and synchronic questions about the relations among the meanings of the grammaticalized forms, weak necessity interpretations of *STRONG+CF*, and literal counterfactual interpretations of *STRONG+CF* (cf. '(If ψ,) would have to ϕ') — e.g., about how weak necessity interpretations might come to be associated with uses of *STRONG+CF*, and how, post-grammaticalization, the meanings of the dedicated lexical items is related, if at all, to the literal counterfactual interpretations. (See Silk 2018: §4 for preliminary discussion.) Examining more general connections among various types of "modal remoteness" phenomena across languages — e.g., counterfactuality, weakness/tentativeness, evidential hedges, futurity, negation — promises fruitful avenues to explore (cf. Stone & Hardt 1999, Bittner 2010, 2011, Murray 2010, 2014). Moreover our discussion high-

lighted how phenomena with weak and strong necessity modals interact with general issues concerning context-sensitivity, assertion, the roles of truth and discourse function in linguistic theorizing, and relations among logic, semantics, and pragmatics in an overall theory of modals. These interactions afford rich possibilities for future research.

Appendix

The following are several relevant definitions and abbreviations. For additional definitions and syntactic and compositional semantic details, see Bittner 2010, 2011, Murray 2010; cf. Muskens 1996, Stone 1999.

Definition A.1 (lists, contexts). Given a non-empty set D of objects:

- $D^{n,m} = D^n \times D^m$ is the set of $\top\bot$-lists of n $\top$-objects and m $\bot$-objects.
- For any $\top\bot$-list $i \in D^{n,m}$, $\top i := i_1$ and $\bot i := i_2$. Thus, $i = \langle \top i, \bot i \rangle$.
- An (n,m)-context is any subset of $D^{n,m}$. $\emptyset$ is the absurd context.

Definition A.2 (UC_ω types). The set of UC_ω types Θ is the smallest set such that (i) $\delta, \omega, t, s \in \Theta$, and (ii) $(ab) \in \Theta$ if $a, b \in \Theta$. s is the type of $\top\bot$-lists. The subset $\textsc{dr}(\Theta) = \{\delta, \omega, \omega t\}$ is the set of discourse referent types in Θ. Propositional discourse referents are defined as type $\Omega := \omega t$.

Definition A.3 (UC_ω frames). A UC_ω frame is a set $\{D_a \,|\, a \in \Theta\}$ of non-empty pairwise disjoint sets D_a such that

i. $D_t = \{1, 0\}$,
ii. $D_{ab} = \{\, f \mid \emptyset \subseteq \mathrm{Dom}\, f \subseteq D_a \wedge \mathrm{Ran}\, f \subseteq D_b\}$, and
iii. $D_s = \bigcup\{D^{n,m} \mid 0 \leq n \wedge 0 \leq m\}$,

where $D = \bigcup\{D_a \mid a \in \textsc{dr}(\Theta)\}$.

Definition A.4 (UC_ω models). A UC_ω model $M = \langle \{D_a \mid a \in \Theta\}, [\![\cdot]\!]^g \rangle$ is a pair of a UC_ω frame and interpretation function $[\![\cdot]\!]$ s.t. $\forall A \in Con_a \colon [\![A]\!] \in D_a$.

Definition A.5 (UC_ω semantics). The following are several relevant semantic clauses. ($^{\chi}$ and $^{\{\}}$ indicate the characteristic function and characteristic set, respectively.) For any model M and assignment function g:

i. $[\![A]\!]^g = [\![A]\!]$, if $A \in Con_a$
$\quad = g(A)$, if $A \in Var_a$
ii. $[\![\lambda u_a(B)]\!]^g(d) \doteq [\![B]\!]^{g[u/d]}$, if $d \in D_a$
iii. $[\![BA]\!]^g \doteq [\![B]\!]^g([\![A]\!]^g)$
iv. $[\![A = B]\!]^g = 1$ iff $[\![A]\!]^g, [\![B]\!]^g \in D_a \wedge [\![A]\!]^g = [\![B]\!]^g$
v. $[\![{u_a}^{\top}\!\oplus B]\!]^g \doteq \langle (g(u_a) \oplus \top [\![B]\!]^g), \bot [\![B]\!]^g \rangle$
$[\![{u_a}^{\bot}\!\oplus B]\!]^g \doteq \langle \top [\![B]\!]^g, (g(u_a) \oplus \bot [\![B]\!]^g \rangle$
vi. $[\![\top a_n]\!]^g(i) \doteq ((\top i)_a)_n$, if $i \in D_s$
$[\![\bot a_n]\!]^g(i) \doteq ((\bot i)_a)_n$, if $i \in D_s$
vii. $[\![A\{B\}]\!]^g \doteq {}^{\chi}\{[\![A]\!]^g(j) \mid j \in {}^{\{\}}[\![B]\!]^g\}$
viii. $c[\![A; B]\!]^g \doteq c[\![A]\!]^g[\![B]\!]^g$

In the centering rule (v), $d \oplus x := \langle d, x_1, \ldots, x_n \rangle$, for an object d and sequence x. (v) says that $g(u_a)$ is added to the top of the specified sublist of the input

$\top\bot$-list $[\![B]\!]^g$. In (vi), $(x)_a$ is the subsequence of type a coordinates of x, and $(x)_n$ is the nth coordinate of x. (vii) treats $A\{B\}$ as denoting the set of all A-objects on the B-lists.

Definition A.6 (context sets, defaults).

- The context set is a non-empty set of worlds p. By default p is the topical proposition $\top\Omega$; it sets the default modal topic.
- The initial context set p_0 determines the default context ${}^{st}p_0 = {}^{\chi}\{\langle\langle w, p_0\rangle, \langle\rangle\rangle \mid w \in {}^{\{\}}p_0\}$.

Definition A.7 (truth, v1, v2). For an $(st)st$ term K:
version 1:

- For a world w, let C_w be the set of contexts c such that $\{(\top j)_1 \mid j \in {}^{\{\}}c\} \neq \{{}^{\chi}\{w\}\}$ and $\{((\top j)_{\omega t})_1 \mid j \in {}^{\{\}}c\} = \{{}^{\chi}\{w\}\}$

i. K is true at w iff $\forall c \in C_w$, $\{(\top j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \{{}^{\chi}\{w\}\}$
ii. K is false at w iff $\forall c \in C_w$, $\{(\top j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \emptyset$

version 2:

- For a world w, let C^*_w be the set of contexts c^* such that $\{(\bot j)_1 \mid j \in {}^{\{\}}c\} \neq \{{}^{\chi}\{w\}\}$ and $\{((\bot j)_{\omega t})_1 \mid j \in {}^{\{\}}c\} = \{{}^{\chi}\{w\}\}$

i. K is true at w iff $\forall c^* \in C^*_w$, $\{(\bot j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \{{}^{\chi}\{w\}\}$
ii. K is false at w iff $\forall c^* \in C^*_w$, $\{(\bot j)_1 \mid \forall g\colon j \in {}^{\{\}}[\![K]\!]^g(c)\} = \emptyset$

Definition A.8 (coherence). K is coherent iff for some c, $\exists p \in D_{\omega t}\colon {}^{\{\}}p \neq \emptyset$ and $\{((\top j)_{\omega t})_1 \mid j \in {}^{\{\}}c\} = \{((\top i)_{\omega t})_1 \mid \exists g\colon i \in {}^{\{\}}[\![K]\!]^g(c)\} = \{p\}$

Abbreviations ($a \in \textsc{dr}(\Theta), \mathtt{R} \in \{=, \in, \notin, \subseteq\}$)

- **Static relations**

$A_a \in B_{at}$	for	BA
$A_a \notin B_{at}$	for	$\neg BA$
$A_{at} \subseteq B_{at}$	for	$\forall u_a\colon u \in A \to u \in B$
$B(A_1, \ldots, A_n)$	for	$BA_1 \ldots A_n$

- **Local projections, conditions, updates**

$\top a, \bot a$	for	$\top a_1, \bot a_1$
A°_a, A°_{sa}	for	$\lambda i_s.A, \lambda i_s.Ai$
$B\,\mathtt{R}_i\,A$	for	$\lambda i_s.B^\circ i\ \mathtt{R}\ A^\circ i$
$B_W\langle A_1, \ldots, A_n\rangle$	for	$\lambda i_s.B(W^\circ i, A^\circ_1 i, \ldots, A^\circ_n i)$
(C_1, C_2)	for	$\lambda i_s.C_1 i \wedge C_2 i$
$[C]$	for	$\lambda I_{st}.\lambda j_s.Ij \wedge Cj$
${}^{\top}[u_a \ldots u_n \mid C]$	for	$\lambda I_{st}.\lambda j_s.\exists u_a \ldots u_n \exists i_s\colon j = ({u_1}^{\top}\oplus \ldots ({u_n}^{\top}\oplus i)) \wedge Ii \wedge Ci$
$[u_a \ldots u_n \mid C]$	for	$\lambda I_{st}.\lambda j_s.\exists u_a \ldots u_n \exists i_s\colon j = ({u_1}^{\bot}\oplus \ldots ({u_n}^{\bot}\oplus i)) \wedge Ii \wedge Ci$
$\textsc{min}\,(P, \lesssim_w)$	for	$\lambda u.u \in P \wedge \forall v\,(v \in P \to u \lesssim_w v)$
$\textsc{min}\langle A, \lesssim_W\rangle$	for	$\lambda I_{st}.\lambda j_s.\textsc{min}\,(A^\circ j, \lesssim_{W^\circ j})$
$\textsc{min}\,\{A\|, \lesssim_W\}$	for	$\lambda I_{st}.\lambda j_s.\textsc{min}\,(A\{I\}, \lesssim_{W^\circ j})$

- **Global updates**

$[A \texttt{ R } B\|\|]$	for	$\lambda I_{st}.\lambda j_s.Ij \wedge Aj \texttt{ R } B\{I\}$
$[A\|\| \texttt{ R } B\|\|]$	for	$\lambda I_{st}.\lambda j_s.Ij \wedge A\{I\} \texttt{ R } B\{I\}$
${}^{\top}[u_a \mid u \texttt{ R } A\|\|]$	for	$\lambda I_{st}.\lambda j_s.\exists u_a \exists i_s\colon j = (u^{\top} \oplus i) \wedge Ii \wedge u \texttt{ R } A\{I\}$
$[u_a \mid u \texttt{ R } A\|\|]$	for	$\lambda I_{st}.\lambda j_s.\exists u_a \exists i_s\colon j = (u^{\perp} \oplus i) \wedge Ii \wedge u \texttt{ R } A\{I\}$
$(K_{(st)st}; K'_{(st)st})$	for	$\lambda I_{st}.\lambda j_s.(K'(KI))j$

Derivations of the updates described in §§2–3 may proceed accordingly.

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