PERFECT SOLIDITY: NATURAL LAWS AND THE PROBLEM OF MATTER IN DESCARTES' UNIVERSE

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In the *Principles of Philosophy,* Descartes attempts to explicate the well-known phenomena of varying bodily size through an appeal to the concept of "solidity," a notion that roughly corresponds to our present-day concept of density. Descartes' interest in these issues can be partially traced to the need to define clearly the role of matter in his natural laws, a problem particularly acute for the application of his conservation principle. Specifically, since Descartes insists that a body's "quantity of motion," defined as the product of its "size" and speed, is conserved in all material interactions, it is imperative that he explain how solidity influences the magnitude of this force. As a means of resolving this problem, Descartes postulated an idealized condition of "perfect solidity" which correlates a body's "agitation" force (a forerunner of Newton's concept of non-accelerating, or "inertial" motion) with the interplay of its volume, surface area, and composition of minute particles. This essay explores this often misunderstood aspect of Descartes' physics, as well as the special function of idealized conditions in his collision rules. Contrary to those commentators who regard "perfect solidity" as a stipulation on bodily impact, this notion, it will be argued, is primarily concerned with the internal composition of macroscopic bodies, and only indirectly with their collision characteristics. Along the way, many of Descartes' hypotheses will be shown to display a level of sophistication and intricacy that, despite their essential incompatibility, belie several of the common misconceptions of Cartesian science.

1. "Perfect Solidity" and the Natural Laws.

Among the ideal conditions that appear in his *Principles of Philosophy* (Pr II 37-52), Descartes remarks that his seven rules on the impact of material bodies "could easily be calculated . . . if [the two colliding bodies] were perfectly solid (*durus*). . ."(Pr II 45).[1] Although this stipulation was intended to assist in the application of the collision rules (as instances of his third natural law), the complex and puzzling concept of "perfect solidity," or "perfect hardness," is perhaps also notable for having generated several divergent interpretations among recent commentators. In some instances, "perfect solidity" has been translated into the modern dynamical locution "perfect elasticity," which denotes a class of material bodies that return to their original shape, volume, etc., after deforming under impact.[2] The translators of the first complete English edition of the *Principles,* V. R. Miller and R. P. Miller, for example, assume that Descartes had elasticity in mind in the passage quoted above.[3] While it is true that most commentators regard Cartesian bodies as "inelastically hard" (i.e., they do not alter their shape during collision), even a few of these scholars have been unable to completely resolve their doubts. For instance, R. S. Woolhouse has recently concluded that "since the 'before and after' of a perfectly hard collision is the same as the 'before and after' of a perfectly elastic one, there is some justification for the sometimes-made assumption that by 'hard' [solid] Descartes *really* means 'elastic'."[4] Notwithstanding the merits of this elastic/inelastic controversy, it would appear that such disputes overlook a more fundamental question that lies at the heart of the "perfect solidity" issue: Does Descartes' use of the "perfect solidity" concept encompass only the interactive, *collision* properties of bodies (i.e., how they behave under impact), or are other individual, non-interactive factors implicated as well, such as their internal composition and configuration of elementary particles? This essay presents the latter interpretation of perfect solidity, claiming that it constitutes the only means of correlating much of the information found in the latter portions of the *Principles* with the natural laws put forth in Part II. Despite its apparent connotations, "perfect solidity" pertains to the internal constitution of the basic particles that make up macroscopic bodies, and only indirectly the dynamic properties manifest in bodily collisions.

In order to demonstrate that Cartesian solidity is not merely a stipulation on bodily collisions, however, we will need to explore the interrelationship between the Cartesian conserved property of quantity of motion (often described as "size times speed") and the three bodily properties of volume, surface area, and quantity of matter. The specific function of these latter three properties is an issue that has received scant attention among Cartesian scholars, but it is crucial to a full understanding of Descartes' physics. For example, in the canonical presentation of the conservation law for the quantity of motion, Descartes' *Principles* specifically incorporates bodily surface area as a key factor in the determination of quantity of motion:

> We must however notice carefully at this time in what the force of each body to act against another or resist the action of that other consists: namely, in the single fact that each thing strives, as far as is in its power, to remain in the same state . . . . One which is at rest has some force to remain at rest, and consequently to resist everything which can change it; while a moving body has some force to continue its motion, i.e., to continue to move at the same speed and in the same direction. This force must be measured not only by the size of the body in which it is, and by the [area of the] surface which separates this body from those around it; but also by the speed and nature of its movement, and by the different ways in which bodies come in contact with one another. (Pr II 43)

Thus, the quantity conserved in the motion and impact of bodies, which Descartes refers to as "quantity of motion," is determined by three factors: size, surface area, and speed (where speed is conceived as a non-directional scalar property, unlike velocity). Although the role of surface area is not revealed at this stage in the *Principles,* the derivation of his natural laws in Part II apparently equates a body's size with its volume, and its quantity of motion with the product of its size and speed (e.g., "when one part of matter moves twice as fast as another twice as large, there is as much [quantity of] motion in the smaller as in the larger" Pr II 36.). Given his thesis that spatial extension (in three dimensions) constitutes the essential property of material substance (Pr II 11), it is probably not surprising that the alleged identification of Descartes' term 'size' with 'volume' is often accepted as an elementary fact of Cartesian science.[5] Nevertheless, it is also true that Descartes' conservation law implicates both size *and* surface area in the determination of quantity of motion, two properties whose exact interrelationships, and hence contribution to the conservation law, are never clearly detailed in the Cartesian theory of matter.

The problem of harmonizing Descartes' sporadic references to surface area, volume, and quantity of matter with his principal use of the term 'size' has long been a source of irritation among Cartesian scholars. Many who are aware of the ambiguous contribution of Cartesian matter to the natural laws have only mentioned this difficulty in passing, since their main research concerns others aspects of Descartes' physics. To illustrate, in an important essay on Cartesian force, M. Gueroult's only reference to these issues is the observation that "the notion of mass identified with volume remains very obscure in Descartes."[6] In other cases, scholars will draw attention to the imprecise meaning of Descartes' term by placing it in quotation marks (i.e., 'size'), as does A. Gabbey in the following: "for Descartes, . . . the force of motion of a body, . . . is the product 'size' × speed . . . ."[7] Overall, the sentiment of many of these Cartesian researchers is probably best captured by D. M. Clarke's remark that "the details of [Descartes'] theory are never sufficiently developed, so that one finds the same rather vague references to size, surface area, resisting media, and speed . . . ."[8]

Nevertheless, the details of Descartes' theory are less vague than most commentators have assumed. To demonstrate this point, Sections 1.1 through 1.4 will examine how the bodily properties of volume, surface area, and quantity of matter are integrated by Descartes' concept of perfect solidity. On the basis of this discussion, we will then return, in Section 1.5, to the question posed at the outset: i.e., Is "perfect solidity" chiefly a collision, or composition, property of macroscopic bodies? Perfect solidity may have other applications, however, especially with respect to the interactions of bodies that operate outside of the special conditions of the impact rules. Thus, Section 2 will examine various methods of utilizing the perfect solidity concept to resolve the problems that arise under these non-idealized conditions. The Cartesian concept of "rigidity", which will be introduced in Section 2.2, will also be seen to have important repercussions for our discussion in Section 1.5, providing further reasons for regarding perfect solidity as primarily a stipulation on bodily composition.

1.1. Rarefaction and Condensation. Descartes' definition of solidity first appears in Part II of the *Principles,* Articles 54: "Those bodies whose particles are all contiguous and at [relative] rest, are solid." In this passage, "contiguous" is the term requiring further elaboration, since the appeal to the "relative rest" of the particles pertains to Descartes' rejection of a binding force among the infinitely-divisible particles of matter (Pr II 20, 55). Essentially, Descartes treats the observed phenomenon of varying bodily density, or, as he phrases it, "solidity," through an appeal to the spaces between the particles of matter. With respect to those processes which either decrease or increase the size of material bodies (labeled, respectively, rarefaction and condensation), he states: "rarefied bodies are those with many spaces between their parts which are filled by other bodies. And rarefied bodies only become denser when their parts, by approaching one another, either diminish or completely eliminate these spaces; . . ." (Pr II 6). Thus, bodies whose particles are contiguous (i.e., they are not separated by an influx of foreign matter) are deemed "solid." Descartes evidently found these natural processes of varying density rather disturbing, for they "might lead one to doubt whether the true nature of body consists in extension alone," a remark that also explains their presence at so early a stage in the *Principles.* Yet, only in Part III, 48-52, are we first introduced to the hierarchy of material elements responsible for the swelling and shrinking of these large macroscopic bodies. Briefly, Descartes procures a threefold subdivision of matter in order to explicate the underlying mechanisms that operate his matter-filled, or plenum, world. These basic particles, largely differentiated by size and function, are: (i) the large, macroscopic third elements of matter, and (ii) the much smaller, globule-shaped second elements of matter; while the minute debris formed from the collisions of the second and third elements, known as (iii) the first elements of matter, serve to fill the lacunae manifest between these larger particles.

In Part III of the *Principles,* Descartes presents a somewhat more elaborate analysis of the problem of solidity. Quite possibly, he felt compelled to furnish a systematic explanation of this phenomenon after reflecting upon the variety of diverse behavior produced by identically-sized bodies in resisting or sustaining motion--behavior which today we would call inertial effects. That is, there often exists a disparity among bodies of the same spatial volume, such as two identically sized globes composed, respectively, of gold and wood, in resisting changes to their states of motion. (e.g., one is much harder to move than the other!) One of the principal motivating factors in the formulation of the Cartesian theory of solidity is the need to explicate the origin of these "inertial" effects (although Descartes would not have used this term). In fact, a discussion of the motions of celestial bodies occasions Descartes' next attempt at a definition: "the solidity of [a] star is the quantity of the matter of the third element, . . . in proportion to its volume and surface area" (Pr III 121). As defined, solidity is thus a function of three variables: quantity of third element matter, surface area, and volume. Since the distinction between these three quantities, and their role in affecting density, is often misunderstood, we shall examine this three-part interrelationship below.

1.2. Volume, Quantity of Matter, and the Agitation Force. At one point in the examination of solidity, Descartes utilizes his ratio of quantities to resolve the problem, just described, of divergent motions that originate from bodies of equal volume. He explains:

> Thus, here on earth, we see that, once moved, gold, lead, or other metals retain more agitation, or force to continue in their movement, than do pieces of wood or rocks of the same size and shape; and consequently metals are also thought to be more solid, or to contain more matter of the third element and smaller pores filled with the matter of the first and second elements. (Pr III 122)

Descartes' remarks contain an implicit conjecture on the origins of inertial effects (although, as previously noted, Descartes did not hold the modern concept of inertia):[9] A body's "force to continue in its movement," or "agitation," is directly proportional to its amount of third element matter. Therefore, provided two bodies of equal volume (and equal speed), the more solid object will possess the greater quantity of third element matter, and consequently produce a greater tendency to continue in its motion (or agitation). This interpretation of the passage is verified by his discussion of the motions of stars: "The force which [a star] acquires from its motion . . . to continue {to be thus transported or} to thus move, which I call agitation; must be estimated neither by the size of its surface area nor by the total quantity of matter {which composes it}, but only by the quantity of the third element matter . . ." (Pr III 121). In Part II, furthermore, a moving body's "quantity of motion" (or "size times speed") constitutes a measurement of its "force to continue its motion, i.e., to continue to move at the same speed in the same direction" (Pr II 43); which is the same description Descartes provides for his force of agitation in Part III (see Section 1). Quantity of motion is hence a gauge of agitation force, a conclusion that will later assume importance.[10]

At this juncture, one may begin to question the overall consistency of combining Descartes' agitation force hypothesis with his theory of matter. If, as Descartes believes, matter is mere spatial extension, then why should the agitation force of, say, a body entirely composed of tertiary matter differ from that of a body (of equal volume) containing only first element matter? Since all matter is extension, it would seem that both bodies should behave in exactly the same manner. Descartes, however, is quick to provide a rationale for this association. He reckons that, because the individual motions of a collection of elementary particles are not entirely unified, a volume of secondary globules cannot produce an agitation force equal to that of an identical volume of tertiary matter. In an insightful passage concerning stellar motion, he compares the agitation force of a star composed of third element matter against the force produced by an equal volume of secondary globules:

> Because these globules are separated from one another and have various {individual} movements; although their united force acts against the star, they cannot all unite their force simultaneously in such a way [as to ensure] that no part of their force is wasted. In contrast, all the matter of the third element . . . forms one

> single mass which is moved together as a whole, and thus all the force which it has to continue in its motion is applied in a single direction. (Pr III 124)

Put simply, the variably-directed motions of the individual globules of secondary matter (and, presumably, first element matter) lessen the total agitation force of the composite volume *in any single direction.* The motion of the star is not subject to these same effects, on the other hand, since all of the matter of third element constitutes a unified whole which moves in a single direction. Therefore, it is not the case that tertiary matter possesses some internal property that makes it qualitatively different from the other material elements; rather, it is simply the relative rest of the particles that comprise the third element which account for its role in determining the agitation force of *macroscopic* bodies, such as stars. Since the three types of Cartesian matter are identical in all respects except their relative size, Descartes' agitation theory is thus also applicable to particles entirely composed of primary and secondary matter. For instance, among secondary particles, the globules that possess more secondary matter will harbor the greater agitation forces (as will be evident in later sections).

Although Descartes' reasoning is rather ingenious, it does not entirely justify his correlation of a macroscopic body's agitation force with its quantity of third element matter. For, even if the individual motions of the first and second elements cannot simultaneously unite their forces in a single direction "in such a way [as to ensure] that no part of their force is wasted," it is still the case that they will contribute some force, albeit small, in that given direction. The force generated by these particles will probably be insignificant in comparison to the force provided by the tertiary matter, but it is still a distinct force, and thus it must make some contribution to the overall agitation force of the composite body (i.e., the star). In fact, Descartes seems to admit this interpretation of his theory: "As for the matter of the first or even the second element, it is continually leaving [a] star and being replaced by new matter. Consequently, this new matter approaching cannot retain the force of agitation acquired by the matter which has already left, which, in any case, was very small" (Pr III 121). If, as Descartes concedes, the departing first and second elements have acquired a force of agitation from the star, then this acquired force must have contributed something to the star's total quantity of motion, a force these particles obtained while inside the star. On this rendering of the evidence, it thus appears that a Cartesian body's agitation force should be equated with its total quantity of matter, and not just its quantity of third element matter.

It is unfortunate that Descartes did not adequately explain his reasons for disregarding the contribution of the primary and secondary elements to the agitation force of macroscopic bodies, but his brief statements may indicate two possible motives for this decision. First, as specifically mentioned in the quotation above, the primary and secondary elements of matter are continuously leaving a star and being replaced by new matter. Hence, these minute particles are best seen as foreign bodies that only temporarily occupy the macroscopic host body, and do not assist in the composition of that body. Of course, if these foreign particles do not qualify as constituent members, then the effects of their individual agitation forces on the host body can be ignored. This interpretation has the added bonus of nicely correlating with a claim made early in Part II: "whatever extension there is in the spaces between [a body's] parts must in no way be attributed to it, but to whatever other bodies fill those spaces" (Pr II 6). Nevertheless, it should be noted that this form of response does not sit well with Descartes' definition of an individual body; where "by *one body,* or *one part of matter,* I here understand everything which is simultaneously transported; even though this may be composed of many parts which have movements among themselves" (Pr II 25). As for the second possible rationale, since Descartes holds that the agitation force acquired by the first and second elements is "very small," he may believe that this minute quantity can be conveniently ignored when determining the body's overall agitation force. In other words, one can secure a fairly accurate approximation of a body's quantity of motion by simply taking the product of its total quantity of tertiary matter and speed. Regardless of which construal of Descartes' reasoning we adopt, his correlation of agitation force and third element matter in the case of macroscopic bodies remains somewhat obscure; but it will assume great importance below (in Section 2.2) when we discuss the impact of such composite bodies.

1.3. Surface Area and the Agitation Force. Besides quantity of tertiary matter, a body's agitation force is also substantially influenced by the magnitude of its surface area. On the whole, Descartes is well aware that the agitation force of a body can be modified by simply changing its shape. For instance, a golden sphere can assume shapes that will allow a less-dense wooden sphere to possess a "greater agitation; . . . if [the golden sphere] is drawn out into threads or {forged} into thin plates or hollowed out with numerous holes like a sponge, or if it in any other way acquires more surface area, in proportion to its matter and volume, than the wooden sphere" (Pr III 122). In this case, the magnitude of an object's surface area is clearly implicated in the resulting agitation force: The larger the proportion of surface area to third element matter, the smaller the resulting force. This formula likewise holds for the individual globules of secondary matter. In discussing the agitation force of various sized globules, he argues that "the smaller [globules] have {less force, because they have} more surface area {in proportion to the quantity of matter} . . . than the larger ones . . ." (Pr III 125).

Yet, the relationship between surface area and quantity of matter in Cartesian natural philosophy is a rather complex affair, and possibly uncertain. As quoted above, Descartes claims that a star's agitation force is a sole function of its quantity of third element matter, with surface area playing no role. In the very next article, though, he openly admits that a body's surface area can greatly change the magnitude of its agitation force. Contradictions of this sort bedevil much of the Cartesian theory of solidity and agitation, which provides a possible explanation for their lack of serious coordinated analysis. In essence, Descartes seems to desire a simple correlation between a body's agitation force and its quantity of second or third element matter; but, he also recognizes the important role of surface area in modifying this force, a variable he cannot completely ignore.

Descartes' uncertainty regarding the role of surface area is a dilemma that can be traced back to the definition of quantity of motion in Part II. On the whole, Descartes was cognizant of the plenum's capacity to retard the motions of bodies: "It is obvious, moreover, that [bodies] are always gradually slowed down, either by the air itself or by some other fluid body through which they are moving. . ." (Pr II 38). If we conjoin this observation with Descartes' comments on the motions of various shaped material objects (as noted above), then it seems plausible to infer that Cartesian surface area merely functions to *change* a body's existing agitation force, rather than assist in *constituting* that force. More carefully, a large surface area "slows down" a moving body by increasing the number of plenum particles it encounters along its path: the greater the magnitude of the body's surface area, the more particles it will confront (as opposed to a smaller shape), and hence the more quantity of motion it will lose or transfer to the surrounding plenum.[11] This interpretation of the role of surface area in Descartes' physics has been duly noted by R. S. Westfall: "Descartes asserted frequently that the quantity of surface on which other bodies can impinge modifies the force of a body to continue its motion" (*ibid.,* 70). This realization most likely prompted Descartes to incorporate surface area into his definition of the quantity of motion, the force that is conserved in all bodily collisions (as quoted in Section 1). "This force must be measured not only by the size of the body in which it is, and by the [area of the] surface which separates this body from those around it; but also by the speed and nature of its movement . . ." (Pr II 43).

Nevertheless, the prospects of "quantifying over" the retarding effects of surface area must have presented a serious obstacle to the formulation of the Cartesian conservation law. Prior to the analysis of the collision rules, and *just after* his definition of quantity of motion, Descartes strives to eliminate this extra variable by insisting that his "[colliding bodies are] separated from all others {both solid and fluid} in such a way that their movements would be neither impeded nor aided by any other surrounding bodies . . ." (Pr II 45). Accordingly, with the aid of this additional idealized condition, it is no longer necessary to take into account a body's particular shape when calculating its quantity of motion. In the presentation of the collision rules, the mitigating effect of surface area on the agitation force has been ruled out by definition, thus explaining its conspicuous absence in the derivation of these seven hypotheses.[12] Yet, Descartes' ideal conditions only prevail for the collisions depicted in this section of the *Principles.* In Part III, the reemergence of surface area as a factor in the motions of bodies clearly indicates that such conditions are no longer in effect. It is a general mistake, therefore, to disregard the influence of surface area when ascertaining a body's quantity of motion outside the context of the idealized conditions utilized in the collision rules.

1.4. Agitation and Solidity: Towards a Synthesis. In determining the factors involved in Descartes' agitation theory, we have examined thus far the functional relationship between surface area and quantity of matter, and between volume and quantity of matter; but the exact means by which all three quantities are integrated into a single concept or formula remains largely unexplained. Fortunately, in a discussion of the mitigating effects of surface area on a star's agitation force (relative to the agitation force of secondary globules), Descartes provides an outline of this three-part interrelationship:

> It can happen that [a star] has less solidity, or less ability to continue its movement, than the globules of the second element which surround it. . . . For these globules, in proportion to their size, are as solid as any body can be, because we understand that they contain no pores filled with other . . . matter; and because their figure is spherical; the sphere being the figure which has the least surface area in proportion to its volume. . . . (Pr III 123)

In this two-part analysis, Descartes essentially provides the clearest formulation of his theory of solidity, and of the means by which a body's agitation force is linked to its solidity. With respect to his first claim, a globule completely packed with (secondary) matter is more solid than any other globule of identical size; where, as exercised in this quotation, the notoriously obscure term "size" apparently denotes volume. That is, without pores filled with matter (presumably first element), these globules are fully condensed (as defined above), and thus possess the highest degree of solidity. Given the earlier reference to the solidity of stars, it is safe to assume that this account of solidity must also hold for bodies entirely composed of tertiary matter, rather than just the globules composed of secondary matter. As a result, we can translate the expression "contiguous" in Descartes' first definition of solidity with the degree of bodily porosity. For the second part of his definition, Descartes claims that a spherical body is more solid than any other *figure* of the same "size" (volume), since the sphere manifests the smallest proportion of surface area to volume.[13] Presumably, these identically-sized objects possess a similar quantity of second or third element matter; for, if they did not, a highly rarefied spherical body could conceivably retain more solidity than a fully condensed non-spherical body of similar volume, in direct violation of the first part of the definition. We can generalize this section of Descartes' hypothesis as follows: provided two bodies of identical volume and identical quantities of third element matter (or second, if it is a globule), the body possessing the smallest surface area will harbor the greatest agitation force. Therefore, inasmuch as agitation force is linked to solidity, our three quantities--volume, surface area, and quantity of matter--are essential ingredients in the magnitude of this force.

All told, at least one important lesson can be extracted from Descartes' complex and troublesome theory of solidity: Any attempt to simply identify a body's quantity of motion with its volume and speed, as seemingly implicated in the definition of quantity of motion, is inconsistent with the analysis of solidity offered in Part III. Given our analysis, it is thus evident that, *if used outside of the context of the idealized collision rules,* quantity of motion tacitly expresses an intricate relationship between a body's volume, surface area, and its total quantity of second or third element matter (besides speed). In various circumstances, these variable magnitudes determine the agitation force, and hence quantity of motion, of all physical bodies in the Cartesian plenum. In other words, inasmuch as agitation or quantity of motion are directly dependent upon our three quantities, the theory of solidity presented in Part III informs and governs the operation of the conservation law under the normal, non-idealized conditions that prevail in the Cartesian plenum. Most expositions of Descartes' laws of nature do not disclose or investigate this important aspect of the *Principles,* but it is crucial to a full understanding of Cartesian dynamics.[14] In fact, one can find in the Cartesian literature numerous attempts to isolate spatial volume, or quantity of matter, as the sole contribution of Cartesian matter to the conservation law (size times speed).[15] Yet, these readings of quantity of motion only hold for the highly idealized conditions assumed in the collision rules; where, as discussed in Section 1.3, the role of surface area in modifying the agitation force has been negated by Descartes' exclusion of disrupting plenum effects.

1.5. Perfect Solidity and the Natural Laws: A Proposal. We can now return to the analysis of the impact behavior of Descartes' "perfectly solid" bodies, which has been the motivating cause of our lengthy investigation. As we have seen, Descartes couples the agitation force to an intricate relationship among three different bodily quantities. Provided this theory, it would seem an almost impossible task to secure a systematic *quantitative* description of the inertial tendencies and conserved motions of material bodies. In order to produce such a law, one would need to determine the exact means by which the relative proportions of quantity of matter, volume, and surface area, contribute to the overall conserved motions of the colliding system. Yet, rather than undertake these potentially unrealizable determinations, Descartes circumvents the problem by (1) simply confining the scope of his collision laws to the impact of completely solid bodies, and (2) ignoring the plenum's capacity to retard bodily motion via bodily surface area. More precisely, if the globule or body is fully condensed (contains no pores), then it embodies as much second or third element matter (respectively) as its volume permits. No longer is it necessary to compute the ratios of quantity of matter to total bodily volume among the colliding bodies--under this requirement, all that is obligated is a measurement of their relative bodily volumes. In addition, without the need to consider surface area, a body's total quantity of second or third element matter, or volume (given perfect solidity), can now be conveniently equated with its agitation force (after taking its product with speed, of course). It is thus no longer necessary to calculate how different shapes affect the quantity of motion of bodies with identical volume.

As utilized in the collision rules (Pr II 46-52), the requirement for perfect solidity can be therefore largely viewed as a stipulation for completely dense, pore-less bodies. That is, provided the evidence of the entire *Principles,* Descartes' appeal to perfect solidity partially amounts to a restriction on the potential ratio of a colliding body's quantity of matter to its total volume. The motivation underlying this restriction stems from the obvious need for a simplified and manageable treatment of the variables influencing a body's agitation force, and hence quantity of motion. As mentioned at the outset, despite the strong dynamic connotations, perfect solidity is *not* a requirement exclusively allied with the impact behavior of material bodies--other non-interactive properties, such as internal composition, form an important part of this concept.

Put slightly differently, one can view perfect solidity as "an attempt to isolate behavior that can be regarded as fundamental given Descartes' metaphysics of matter," as J. Carriero has recently suggested.[16] Since Cartesian matter is pure extension, the perfect solidity criterion frees the collision rules from the internal complications--via elementary particles--that beset the interactions of most macroscopic bodies (see Section 1.2). Without the need to factor in these interfering effects, perfect solidity thus allows the collision rules to describe the "pure" or "actual" collision behavior of Descartes' extended bodies. However, even on this interpretation (which is complimentary to the one advanced here), perfect solidity still remains primarily a stipulation on bodily composition, and not a description of how bodies *behave* during impact (e.g., by losing and regaining their original shape, or completely reversing their direction of motion). Of course, there is a sense in which any specification of the composition of bodies is relevant to their collision properties, due to the simple fact that a body's constitution affects its interactions. But, bodily composition does not directly outline or detail such interaction behavior: rather, bodily composition, i.e., perfect solidity, is only indirectly concerned with the outcomes of bodily impact. In the next section, the analysis of the concept of "rigidity" will further substantiate this rendition of Descartes' perfect solidity concept.

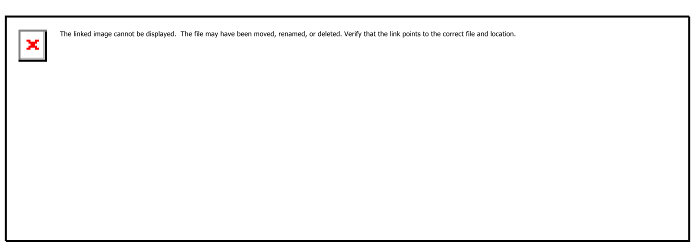
2. Non-Idealized Conditions and the Natural Laws.

This section examines the prospects of successfully applying the Cartesian conservation principle without relying on the highly idealized conditions presupposed in the collision rules; that is, without limiting the scope of the natural laws to the interactions of just poreless bodies. As will become evident, many facets of Descartes' theory of matter and motion can generate substantial difficulties for this undertaking. Nevertheless, we will consider various methods of utilizing the insights gained from our study of Cartesian solidity, and of our forthcoming analysis of Cartesian rigidity, to overcome these obstacles.

2.1. The Problem of Size Invariance. Besides definitional or computational simplicity, there may exist an aspect of Descartes' solidity hypothesis that specifically concerns the effects of bodily motion and impact. To demonstrate this point, we need to investigate the origins of Descartes' plenum universe. According to the Cartesian cosmological hypothesis, all space (matter) was initially divided into homogeneous parts of equal size, and impelled with a conserved quantity of motion: "God, in the beginning, divided all the matter of which He formed the visible world into parts as equal as possible and of medium size . . . . [Also] He endowed them collectively with exactly that amount of motion which is still in the world at present" (Pr III 46). Eventually, the collisions of these equally-sized spatial parts formed the three Cartesian elements of matter, as well as the vast diversity of material bodies comprised from these elements. The initial impact of Cartesian matter could not have been elastic, consequently, because of the absence of the fragmented particles necessary for the composition of porous bodies. Without pores, the original bodies in Descartes' universe were perfectly solid. Yet, these bodies were clearly not perfectly solid in the sense required for Descartes' collision rules, since their impact ultimately resulted in a loss of size through fragmentation. When objects disintegrate or shed particles in this manner, their total quantity of motion will inevitably decrease.[17] More specifically, macroscopic bodies that do not possess determinate volumes do not possess determinate quantities of motion, which is defined as the product of speed and size (or volume, recalling the interdependence of these concepts as discussed above). Of course, a Cartesian will probably insist that any lost quantity of observable bodily motion is merely transferred to the realm of the surrounding microscopic particles, thus preserving the total universal quantity of motion. This form of response, although possibly correct, does not remedy the plight of Descartes' collision rules, however. If the Cartesian collision rules are to be applied successfully *at the level* of macroscopic objects, it is necessary that they conserve the total *bodily* quantity of motion by maintaining an invariant magnitude of bodily size. Hence, besides the absence of bodily pores, Descartes' perfect solidity criterion may also be interpreted as sanctioning a property of unchanging bodily size (or volume).

Moreover, the union of the Cartesian theory of matter with a plenum universe poses major obstacles for the successful application of the Cartesian conservation law. This conflict probably stems from Descartes' denial of a material binding force; for when this theory, that relatively resting particles constitute solid bodies, is conjoined with any explication of bodily motion in a plenum, it becomes very difficult to accommodate the further contention that moving objects do not change volume over time. If the particles that comprise a solid object are at rest (relative to one another), the force exerted by the surrounding bodies and particles during motion, let alone impact, would seem quite sufficient to dislodge large numbers of them. Accordingly, over a given temporal interval it may be impossible to posit a determinate volume for any moving Cartesian body. Yet, solid bodies that persistently shed minute particles as they travel and collide will experience a decrease in their quantity of motion with their corresponding reduction in volume, once again presenting great difficulties for any attempt to apply the Cartesian law to the collisions of macroscopic bodies over any temporal span.

Evidence may exist, furthermore, to support the contention that the lack of a material binding force influenced Descartes' view of impact. In the illustrations of colliding bodies that accompany the letter to Clerselier, dated 17 February 1645, it is potentially significant that the cubes in one of Descartes' pictures possess equally sized rectangular surfaces on their sides of collision: i.e., the collision or contact surfaces of the two cubes are congruent (see the figure below).[18] Provided the Cartesian denial of binding forces, this congruence may amount to a logical or practical consideration, for there would seem no means of preventing larger cubes from breaking apart (on contact with smaller cubes) without equal contact surfaces.[19] For example, if the collision surface of a cube C extends beyond that of a second cube B, many particles on the periphery of C's surface will not encounter any opposing B particles upon impact, and thus continue their motion past the contact surface (resulting in C's disintegration). That the prospects for such incidents may have troubled Descartes is also disclosed in his letter to Clerselier. In picturing the collision of two unequally sized bodies, he merely increases the length of one of the cubes while preserving the congruence of their contact surfaces. If this interpretation of Descartes' illustrations is correct, then we can add a further stipulation to the ideal condition for perfect solidity; namely, that two bodies manifest identical impact surfaces.



Descartes' depiction of two unequally sized bodies in his letter to Clerselier (this is a slightly simplified version of the original). Note that both B and C are situated so as to collide upon sides possessing congruent surfaces.

This construal of the Clerselier illustrations faces problems, nonetheless. With respect to these same collisions (among objects of different size), a second drawing included in the Clerselier letter does not display identical contact surfaces, nor do any of illustrations contained in the *Principles* (see, Pr II 45-52). In addition, it is not clear just how congruent contact surfaces can prevent the disintegration of colliding Cartesian bodies. As previously discussed, if relatively resting particles are prone to separate when confronted by an external material agent, then the forces exerted by the numerous plenum particles would seem quite sufficient to disperse a Cartesian body even when it is not impacting. In essence, congruent contact surfaces will not compensate for the lack of a material binding force.

2.2. Rigidity. In order to further examine the application of Descartes' conservation law to the impact of plenum bodies, it will be necessary to explore the Cartesian concept of "rigidity," a notion that essentially constitutes a theory of elasticity. Towards the end of Part IV (on terrestrial phenomena), he states:

> Glass is rigid: that is to say, it can be somewhat bent by external force without breaking but afterwards springs back violently and reassumes its former figure, like a bow. . . . And the property of springing back in this way generally exists in all hard bodies whose particles are joined together by immediate contact rather than by the entwining of tiny branches. For, since they have innumerable pores through which some matter is constantly being moved . . . , and since the shapes of these pores are suited to offering free passage to this matter . . . , such bodies cannot be bent without the shapes of these pores being somewhat altered. As a result, the particles of matter accustomed to passing through these pores find there paths less convenient than usual and push vigorously against the walls of these pores in order to restore them to their former figure. (Pr IV 132)

On Descartes' estimation, a "rigid" body is capable of returning to its original configuration after impact due to the action of matter, presumably first element, contained within its pores. These primary elements of matter recover the body's initial shape by pressing against the walls of the pores during the contraction of impact.

Cartesian rigidity, the analogue of elasticity, thus stands in sharp contrast to the Cartesian definition of solidity: the former concept invokes pores or channels within the structure of material bodies, while the latter notion requires fully condensed bodies completely devoid of such conduits. To verify this material classification, one needs only to recall Descartes' synopsis of the completely solid secondary elements: "these globules, in proportion to their size, are as solid as any body can be, because we understand that they contain no pores filled with other . . . matter" (Pr III 123). Without pores, Cartesian bodies are thus incapable of changing and regaining their bodily shape under impact. Needless to say, these comments make it abundantly clear that, contrary to the suggestions of some commentators, Descartes did not intend perfect solidity to mean perfect elasticity.

In addition, by confidently asserting that rigidity "generally exists in all hard bodies," Descartes' exposition on elastic phenomena makes it clear that most physical bodies are not perfectly solid. One must exercise caution in interpreting this claim, however, since the standard explications of the terms "hard" and "rigid" somewhat overlap: Descartes is not claiming that all bodies, including the perfectly solid ones, are elastic (rigid); rather, he is merely pointing out the non-trivial fact that most *seemingly* perfect solid bodies are actually elastic. On a deeper level, one may in fact read into his statement a denial of the very existence of perfectly solid bodies. Descartes' observations thus reflect and corroborate the tacit assumption that perfect solidity is an *ideal condition* imposed on the domain of his conservation laws. Furthermore, as an historical aside, Descartes' analysis of rigidity belies the simplistic judgment that all Cartesian bodies are inelastically hard. On the contrary, his comments reveal an intuitive awareness of the fundamental elasticity of most, if not all, macroscopic objects; a conclusion closely akin to the later elastic theories of Leibniz and John Bernoulli.[20]

Returning to the problem of implementing the conservation law, it is important to note that the utilization of Descartes' hypothesis concerning tertiary matter and the agitation force, first introduced in Section 1.2, greatly assists in conserving the quantity of motion of colliding "rigid" bodies. As mentioned, rigid bodies possess pores or channels filled with elementary particles of matter, particles whose presence or absence occasions the phenomena of rarefaction and condensation. When these elastic bodies collide, consequently, it would seem that many of the small foreign particles housed in the objects, i.e., the primary and secondary matter, must be expelled or ejected during the contraction phase of the impact (when the distortion of the body during the brief instants after contact compresses its pores). Because of the Cartesian identification of matter and space, only by emitting matter can the body reduce the volume of space it occupies. Likewise, the primary and secondary elements of matter will somehow need to filter back into the object after the contortion phase of the impact to "puff" it back up to its original size. The exact manner by which this process takes place is decidedly unclear given Descartes' brief comments on the problem of elasticity--presumably, some sort of "hinge" mechanism on the surface of porous bodies could be invoked to meter the flow of particles both in and out of the channels. Nevertheless, if we adopt Descartes' correlation of tertiary matter and agitation force, then one aspect of "rigid" collisions is evidently clear: the reduction and increase in a body's overall volume during the temporal period spanned by an elastic collision will not vary its total quantity of motion, since all the particles ejected and recovered will not be third element matter. Descartes' stipulation of an "agitation" force, a theory that seemed somewhat unmotivated when first presented (in Section 1.2), thus proves invaluable in applying the conservation law to the collisions of the normal, non-idealized bodies that predominate in the Cartesian plenum (i.e., rigid bodies).

Conclusions

Finally, we should offer some concluding remarks on the success of Descartes' project. With the benefit of scientific hindsight, Descartes' decision to formulate his laws of motion upon the groundwork of his theory of matter was rather unfortunate. His conservation principle, which demands a firm material theory to operate effectively, is severely handicapped by the quirks of his physical program; most notably, by the interference of the plenum and the lack of a material binding force. In short, it seems impossible to integrate all of the Cartesian theories surveyed in this essay into a coherent system of dynamics. Yet, this conclusion should not be taken to invalidate or lessen the value of Cartesian natural philosophy, since the very influence of Descartes' work on the succeeding generations of scientists is enough to dispel this simplistic notion. The ideal condition of "perfect solidity," the main focus of our investigation, is a case in point: although the solidity thesis harbors various inconsistencies, it exhibits a striking awareness of the diverse factors involved in the inertial motion of bodies, as well as offering a sophisticated attempt at integrating these disparate elements into a single manageable formula. Despite its history of neglect, much can be learned from studying the intricacies and interrelationships of Descartes' theories of solidity, rigidity, and agitation.[21]

ENDNOTES

1. R. Descartes, *Principles of Philosophy,* trans. by V. R. Miller and R. P. Miller (Dordrecht: Kluwer Academic Publishers, 1983). Translations are based on the Miller and Miller but are checked against the Adam and Tannery edition of the *Oeuvres de Descartes* (Paris: Vrin/C.N.R.S., 1976). I will identify passages according to the standard convention: thus, Article 15, Part II, of the *Principles* will be labeled "Pr II 15." Passages in brackets, {}, indicate additions made to the French translation of 1644.

2. "The property of recovery of an original size and shape is the property that is termed *elasticity*." A. E. H. Love, *A Treatise on the Mathematical Theory of Elasticity* (New York: Dover, 1944), p. 92. However, the term 'elastic' receives various interpretations. E. J. Aiton reasons, for example, that a body is elastic only if it rebounds in the opposite direction (while presumably recovering its initial shape) "Descartes term 'hard' . . . cannot be equated with 'elastic,' since the hard bodies sometimes rebound and sometimes move together." E. J. Aiton, *The Vortex Theory of Planetary Motions* (MacDonald: London, 1972), p. 39. Consequently, Aiton interpretes "perfect solidity" as a dynamic, or collision, property of bodies.

3. See, Miller and Miller, in Descartes, 1983, p. 64, fn. 44.

4. He continues: "There is good reason for Descartes to have considered perfectly hard [solid], perfectly elastic bodies, for when soft and perfectly inelastic bodies collide the 'quantity of motion' is not conserved." R. S. Woolhouse, *Descartes, Spinoza, Leibniz: The Concept of Substance in Seventeenth Century Metaphysics* (London: Routledge, 1993), p. 114. For a recent example of an "inelastically hard" reading, see, D. Garber, *Descartes' Metaphysical Physics* (Chicago: Chicago University Press, 1992), p. 357.

5. Rohault's influential Cartesian text of the late seventeenth century provides a nice example of the contribution of bodily "volume" to quantity of motion: "If a body of *two Cubic Feet* runs through a Line *sixty Foot long,* it has twice as much Motion, as a Body of one *Cubic Foot,* which runs through the same Line . . . ." J. Rohault, *A System of Natural Philosophy, vol. 1* (written 1671), trans. J. Clarke and S. Clarke (1723), (New York: Johnson Reprint Corp., 1969), p. 43.

6. M. Gueroult, "The Metaphysics and Physics of Force in Descartes," in *Descartes: Philosophy, Mathematics, and Physics,* ed. S. Gaukroger (Sussex: Harvester Press, 1980), p. 228, fn. 100.

7. A. Gabbey, "Force and Inertia in the Seventeenth Century: Descartes and Newton," in *Descartes: Philosophy, Mathematics, and Physics,* ed. S. Gaukroger (Sussex: Harvester Press, 1980), p. 245.

8. D. M. Clarke, *Descartes' Philosophy of Science* (Manchester: Manchester University Press, 1982), p. 217. Clarke is certainly aware of the function of perfect solidity, however, as I will later comment.

9. For a thorough comparison of Descartes' and Newton's theories of inertial motion, see, for example; A. Gabbey, ibid., pp. 230-320.

10. As R. S. Westfall has pointed out, there is a great deal of ambiguity in Descartes' use of the term 'agitation.' Yet, Westfall agrees that this term seems to signify 'momentum' (or quantity of motion, if its a scalar property) in the articles on the motions of stars in Part III of the *Principles.* See, R. S. Westfall, *The Concept of Force in Newton's Physics* (London: MacDonald, 1971), pp. 61-62.

11. This interpretation also accords with the separate role that surface area plays as regards vortex motions. In Part III, surface area is responsible for the force exerted on an object while immersed within a circling mass of plenum particles, "because the larger [a body's] surface is, the greater the quantity of matter acting against the surface" (Pr III 121).

12. Descartes' account of "fluid" bodies seems also to confirm these conclusions. In Pr II 58, he remarks that many typical fluid bodies, such as air and water, put up great resistance to the rapid motions of bodies. Thus, in the collision rules, his ideal condition for the absence of plenum effects apparently translates into an appeal for a resistless "perfect fluid." E. J. Aiton, ibid., pp. 39-41, makes a similar observation.

13. Volume and surface area are two distinct quantities that can be easily confused as equivalent: For example, the volume of a sphere equals the cube of the radius times 4/3 π, while its area equals the square of the radius times 4π.

14. D. M. Clarke is a notable exception. (*ibid.,* 213-221), for he correctly points out both the function of solidity and the elimination of surface area via the idealized conditions. However, Clarke fails to mention the crucial passages in Part III of the *Principles*, especially Pr III 123 (see above). Thus he does not clearly draw together the three bodily properties of volume, quantity of matter, and surface area as utilized by Descartes.

15. For example, see: M. Jammer, *Concepts of Mass in Classical and Modern Physics* (Cambridge, Mass.: Harvard University Press, 1961), pp. 60-61; R. J. Blackwell, "Descartes' Concept of Matter", in *The Concept of Matter in Modern Philosophy* (Notre Dame: U. of Notre Dame Press, 1963), p. 69, fn. 18; J. B. Barbour, *Absolute or Relative Motion?*, vol. 1, *The Discovery of Dynamics* (Cambridge: Cambridge

University Press, 1989), p. 429; P. Damerow et al., *Exploring the Limits of Preclassical Mechanics* (New York: Springer-Verlag, 1992), p. 76.

16. J. Carriero, "Comments on E. Slowik's 'Perfect Solidity, Quantity of Motion, and the Problem of Matter in Descartes' Universe' (an earlier version of the present paper), which were both delivered at the American Philosophical Association Central Division Meeting, Spring 1995.

17. In general, the erosion and wear of a body's surface, which are rather complex phenomena, will result in a loss of kinetic energy. R. M. Brach, *Mechanical Impact Dynamics: Rigid Body Collisions* (New York: John Wiley & Sons), pp. 119-120.

18. R. Descartes, *The Philosophical Writings of Descartes, vol. 3, The Correspondence,* eds. and trans. J. Cottingham, et al. (Cambridge: Cambridge University Press, 1991), pp. 246-248. This point is raised by P. Damerow, et al., ibid., p. 102.

19. Leibniz was one of the first commentators to draw attention to this problem. See, G. W. Leibniz, "Critical Thoughts on the General Part of the Principles of Descartes," in *Leibniz: Philosophical Papers and Letters* (Dordrecht: D. Reidel, 1969), pp. 403-407. Moreover, a requirement for congruent contact surfaces can be found in work of another Cartesian, William Neile (1637-1670): "The whole square surface of the one [cube] meets in the same instant of time with the whole square surface of the other." See, W. Neile, "Hypothesis of Motion," in *The Correspondence of Henry Oldenburg, Vol. 5,* eds. A. R. Hall and M. B. Hall (Madison: University of Wisconsin Press, 1968), pp. 519-524. This is also noted by P. Damerow, et al., p. 102.

20. See, W. L. Scott, *The Conflict Between Atomism and Conservation Theory 1644-1860* (London: MacDonald, 1970), pp. 14, 23.

21. I would like to thank Calvin Normore, Ron Laymon, Mark Wilson, John Carriero, and an anonymous referee from the *History of Philosophy Quarterly,* for their helpful comments and suggestions on the various drafts of this paper.