Abstract

Inflationary cosmology has been widely accepted due to its successful predictions: for a "generic" initial state, inflation produces a homogeneous, flat, bubble with an appropriate spectrum of density perturbations. However, the discovery that inflation is "generically eternal," leading to a vast multiverse of inflationary bubbles with different low-energy physics, threatens to undermine this account. There is a "predictability crisis" in eternal inflation, because extracting predictions apparently requires a well-defined measure over the multiverse. This has led to discussions of anthropic predictions based on a measure over the multiverse, and an assumption that we are typical observers. I will give a pessimistic assessment of attempts to make predictions in this sense, emphasizing in particular problems that arise even if a unique measure can be found.

Predictability Crisis in Early Universe Cosmology

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1 Introduction

Contemporary presentations of the history of the universe treat inflationary cosmology as a well-entrenched part of the standard hot big bang model of cosmology. According to inflation, the early universe underwent a brief phase of exponential expansion. This growth spurt sets the stage for subsequent big bang evolution, yielding a large, flat region with the right balance of overall uniformity and slight wrinkles needed to seed structure formation. From an observational point of view, the basic account offered by inflation has held up to scrutiny through three decades of ever more precise observations of the cosmic background radiation (CBR) and other aspects of the universe. Yet many of the theorists working on inflation assert that it leads to a more radical account of the global structure of the universe. The basic mechanism driving inflationary expansion naturally produces, on this view, a baroque global structure fitting for a Borgesian fiction. Inflationary expansion continues until arbitrarily late times in some regions, leading to a complex global structure at large scales – a "multiverse," with "pocket universes" continually forming as inflation comes to a halt locally, while continuing in other regions. The process of forming these pocket universes is expected to lead to variation in the local, low-energy physics relevant to what transpires in each pocket. Rather than treating inflationary expansion as simply modifying the earliest phase of evolution in the big bang model, "eternal inflation," as this view is known, predicts that the universe has an intricate worlds-within-worlds structure. Many aspects of what was once taken as fundamental physics are demoted, on this view, to contingent, parochial properties of our pocket universe.

How can we tell whether we inhabit the extravagant multiverse of eternal inflation? As Guth (2007) puts it, "In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times." How can we test such a theory, which seems far too promiscuous to have the virtue of clear empirical content? Advocates of eternal inflation have argued that despite this embarrassment of riches, the theory does make predictions regarding what a "typical observer" in the multiverse observes for the value of, for example, a given fundamental physical constant α_i.[1] Making such a prediction requires two distinct ingredients: (1) a probability

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[1] I will follow the common practice of calling these "constants," although "parameters" may be more appropriate. I am treating "fundamental constants" broadly, to include those appearing in the Standard Model of particle physics and in cosmological models: dimensionless coupling constants characterizing the strength of the fundamental forces, mass scales, scales related to the vacuum and to phase transitions, and cosmological parameters such as the baryon-to-photon ratio. Hence the original predictions of inflation can be treated as predictions of fundamental constants such as the

distribution $P_i(\alpha_i)$ specifying the variation of α_i across the ensemble of pocket universes, and (2) a treatment of the selection effect imposed by restricting consideration to pocket universes with observers and then choosing a "typical" observer. An empirical case for eternal inflation could then be based on successful predictions of this kind, which I will call "anthropic predictions" (because of the second ingredient), for a number of fundamental constants. The most famous anthropic prediction regards the value of the cosmological constant Λ (discussed in §3).

Yet obstacles have frustrated efforts to carry out anthropic predictions. Vanchurin et al. (2000) describe the situation as a "predictability crisis" (the source of my title), a phrase which is still apt a decade later. Further reflection on both ingredients readily reveals the sources of the crisis.

Regarding (1), assigning a probability distribution to an infinite ensemble of pocket universes leads to difficulties well-illustrated in a simple example. What is the probability that a randomly chosen positive integer is an even number? Since the set $\mathbb{N}$ is infinite, a measure that assigns to each subset of $\mathbb{N}$ the number of elements of that subset is not sufficient to fix this probability. The intuitive answer ($\frac{1}{2}$) can be obtained by taking the limit of the probability for finite sequences of the form $S_n = \{1, 2, 3, ..., n\}$ as $n \to \infty$. Choosing other nested sequences of subsets of $\mathbb{N}$, however, leads to different answers, and there is nothing "wrong" with other choices. Analogously, making predictions in eternal inflation requires some choice of an additional structure to define probabilities, such as the distribution $P_i(\alpha_i)$, over an infinite ensemble. What is the appropriate additional structure, and on what grounds can it be justified? This is known as the "measure problem."

The second ingredient (2) is no less problematic. The assumption of "typicality" is a form of the principle of indifference: we should treat all possible observers as equally probable. Just as with other applications of the principle of indifference, this assignment of equal probabilities implicitly singles out one preferred reference class. Probabilities that are uniform with respect to one reference class will, in general, not be uniform with respect to another class. What, then, justifies a particular treatment of a "typical observer"?

In response to the "predictability crisis," physicists have sought a well-motivated measure that yields finite, reasonable anthropic predictions. There is little dispute that finding such a measure is a difficult technical problem, but optimists regard it as one that may be solved based on further insights into quantum gravity. Candidate measures motivated by hints from quantum gravity are assessed "phenomenologically" by calculating anthropic predictions for some parameters. Below I offer a brief overview of these discussions in §4, intended as a primer for philosophers. I then challenge the idea that a solution of the measure problem in this sense would be sufficient to insure that anthropic predictions can be used to justify cosmological theories. In doing so, I aim to give a sharp formulation of skepticism regarding multiverse theories similar in spirit to that expressed by many others (in particular, Ellis and Smolin). In §5, I argue that two ways of justifying probabilities once a measure has been found both fail. Furthermore, anthropic predictions lack the power to discriminate among competing theories, and are in this regard quite different than other scientific predictions.[2] §6 considers the evidential value of anthropic predictions. Rather than providing detailed tests of a specific multiverse theory, I argue that these predictions are only sufficient to

amplitude of the density perturbations in the early universe, initial value of spatial curvature, and so on. (I will call these cosmological parameters rather than constants to avoid confusion with the cosmological constant, Λ.)

[2]Despite reservations, I will nonetheless follow the physics literature in calling these predictions. My concern does not stem from the probabilistic nature of anthropic predictions – any account of confirmation theory should be able to handle confirmation by successful probabilistic predictions. Rather, the contrast concerns how probative anthropic predictions can be regarding the details of the theory being tested.

establish the compatibility between the general idea of a multiverse and observations. Compatibility may promote a sense of progress if all other proposals fail to clear even this low bar, but does not provide compelling positive evidence. In §7, I briefly consider alternative arguments in favor of EI that may take the place of anthropic predictions, before arguing in §8 that the style of reasoning adopted in EI undermines the conventional case in favor of inflation.

2 From Inflation to Eternal Inflation

The central idea of inflationary cosmology is that the early universe underwent a transient phase of accelerated expansion. There are a number of different proposals regarding the physical source of this inflationary phase, but one of the simplest involves a scalar field ϕ (the "inflaton" field) displaced from the true minima of its effective potential $V(\phi)$. If the field is approximately homogeneous and in a false vacuum state, the inflaton field effectively mimics a cosmological constant term.[3] This term in isolation leads to exponential expansion $R(t) \propto e^{\xi t}$, where $\xi^2 = \frac{8\pi G}{3}V(\phi)$. By way of contrast, "ordinary" matter and radiation decelerates cosmic expansion, $\ddot{R} < 0$, reflecting the fact that gravity is a force of attraction. Accelerating expansion requires an unusual type of matter with, roughly speaking, negative energy.[4] In addition, the energy density of the false vacuum state remains *constant* during expansion rather than diluting away like other types of matter. This suggests the "cosmic no-hair conjecture": the vacuum energy should come to dominate the cosmic evolution as everything else dilutes away, regardless of the initial mix of fields present at the onset of inflation.

The addition of a transient inflationary phase apparently resolves two puzzles regarding the initial conditions of the standard cosmological model. For a Friedman-Lemaître-Robertson-Walker (FLRW) model with $\ddot{R} < 0$, the particle horizon at recombination is much, much smaller than the scales at which the CBR is observed to have uniform temperature.[5] This is extremely puzzling if one expects the physical properties in causally disjoint regions (those separated by a distance greater than the particle horizon) to be uncorrelated. A second puzzle regards the initial value of the curvature. The flat FLRW model, with $\Omega = 1$ is an unstable fixed point under dynamical evolution.[6]

[3] $R(t)$ is the scale factor for the FLRW models, a function of the cosmic time t. The stress-energy tensor for a scalar field is given by $T_{ab} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}g^{cd}\nabla_c\nabla_d\phi - g_{ab}V(\phi)$. If the derivative terms are negligible, $T_{ab} \approx -g_{ab}V(\phi)$.

[4] More precisely, the strong energy condition (SEC) fails; this condition holds if the stresses in matter are not so large as to produce negative energy densities. Formally, SEC requires that $T_{ab}\xi^a\xi^b \geq \frac{1}{2}\mathrm{Tr}(T_{ab})$ for every unit timelike ξ^a; for a perfect fluid, for example, this implies that $\rho + 3p \geq 0$, where ρ is the energy density and p is the pressure. Accelerating expansion requires violation of the SEC on the assumption that general relativity holds, and that there is no large-scale net rotation of matter (as in the FLRW models); it is also possible to modify the gravitational theory.

[5] The particle horizon is defined as the integral:

$$d = R(t_0)\int_{t_e}^{t_0} \frac{dt}{R(t)}. \tag{1}$$

in the limit $t_e \to 0$. This is a finite distance when $\ddot{R} < 0$, corresponding to roughly 2° of angular separation on the surface of last scattering.

[6] The density parameter is defined as $\Omega =: \frac{\rho}{\rho_c}$, where the critical density is $\rho_c = \frac{3}{8\pi}\left(H^2 - \frac{\Lambda}{3}\right)$, where $H = \frac{\dot{R}}{R}$ is the Hubble parameter and Λ is the cosmological constant. It follows from the FLRW dynamics that $\frac{|\Omega - 1|}{\Omega} \propto R^{3\gamma - 2}(t)$. $\gamma > 2/3$ if the strong energy condition holds, and in that case an initial value of Ω not equal to 1 is driven rapidly away from 1.

This aspect of the dynamics makes it extremely puzzling to find that the observed universe is still close to a flat model, apparently requiring an extraordinarily finely-tuned choice of $\Omega(t_i)$ as an initial condition.

Both puzzles reflect the fact that $\ddot{R} < 0$ in the FLRW models, but this is reversed during an inflationary phase. For $\mathscr{N}$ e-foldings of expansion the horizon distance d_h is multiplied by $e^{\mathscr{N}}$; with $\mathscr{N} > 60$ the horizon distance, while still finite, encompasses the observed universe. Second, Ω is driven towards 1 during inflation.[7] An inflationary stage long enough to solve the horizon problem drives a large range of pre-inflationary values of $\Omega(t_i)$ close enough to 1 by the end of inflation to be compatible with observations. Rather than requiring an initial state with glorious pre-established harmony, inflation makes a wide variety of pre-inflationary initial states compatible with the observed universe. The recognition of these consequences, by Guth (1981) and others, and Guth's ensuing argument that inflationary cosmology should be pursued because it leads to a theory that does not require special initial conditions, has shaped the field since that time.

Inflation would probably not have inspired so much interest had it not also provided a compelling answer to a third puzzle. Research on the formation of large scale structures in the 1970s led to a hypothesis regarding what initial density perturbations would be needed to produce the observed distribution of galaxies. Harrison, Peebles, and Zel'dovich independently proposed that the initial density perturbations should be small amplitude, adiabatic, approximately scale invariant, and Gaussian. Although this proposal was phenomenologically successful it was mysterious how such perturbations could be produced. Inflation provides an elegant account of how these density perturbations arose: they result from vacuum fluctuations of the inflaton field. During inflation a given Fourier mode ϕ_k of the inflaton field scales with the exponential expansion, whereas the Hubble parameter H is approximately constant. As a given mode "exits the horizon" (i.e., when $\frac{k}{R}$ becomes $> H$) the equation governing its evolution becomes like that of an over-damped oscillator. As a result the mode is "frozen in" at a given fluctuation amplitude; after inflation ends, the scaling behavior of the proper wavelength of the mode compared to the Hubble parameter reverses and it eventually "re-enters" the horizon (when $\frac{k}{R}$ becomes $< H$). More importantly, the account of structure formation has provided the main avenue for bringing observational results to bear on inflation. Observations of temperature variations in the CBR have provided ongoing tests of the details of inflation, and have ruled out competing models of structure formation.

In brief outline, the early history of the universe according to inflationary cosmology starts with a pre-inflationary patch in the appropriate state to initiate inflation. If the inflaton field is displaced from the minimum of the potential and homogeneous in a region larger than the horizon scale, it will contribute an effective Λ term to Einstein's field equations (EFE) and trigger inflationary expansion. Provided that the effective potential $V(\phi)$ is sufficiently flat, as the inflaton field "rolls down" the potential, it will continue to act as an effective Λ term, driving a stage of exponential expansion. $\mathscr{N} > 60$ e-foldings suffice to solve the horizon and flatness problems as described above, and the vacuum fluctuations of ϕ are amplified to produce a spectrum of scale-invariant density perturbations. Finally, since any pre-inflationary matter and energy have been diluted during inflation, the universe needs to be re-populated with matter and energy in a process (misleadingly) called "re-heating." This process results from the inflaton field reaching the end of the "slow-roll" regime with a field value ϕ_{rh}; for $\phi < \phi_{rh}$ it oscillates around its true minimum and decays into other fields. The spacelike hypersurface where the field attains the value ϕ_{rh} is called the "re-

[7] In the equation in footnote (6), $\gamma = 0$ during inflation, driving Ω towards 1.

heating" surface. Inflation promises to yield an appropriate state on this surface to serve as the "initial conditions" for subsequent evolution according to the standard big bang model.

Shortly after the first models of inflation were proposed, theorists turned to questions regarding the pre-inflationary initial state and the large-scale implications of inflation. Vilenkin, Linde and others argued that *globally* inflation should continue indefinitely, even as some regions make the transition to a more sedate expansion rate. There are two defining features of what is now called eternal inflation (EI): (i) inflationary expansion continues indefinitely, (ii) regions where inflation ends can be treated as independent "pocket universes." I will briefly review two arguments in favor of eternality, based on stochastic evolution of the inflaton field and tunneling from a false vacuum state, that have been developed for different versions of inflation.[8]

One account of EI is based on effects of quantum fluctuations in the field ϕ during its "slow roll" down a nearly flat potential $V(\phi)$. Classically the field rolls down the potential, described by a solution ϕ_{sr}. Stochastic inflation treats the evolution of ϕ semi-classically, with quantum fluctuations leading to random jumps superimposed on the classical trajectory. In many inflationary models, there is a range of field values within which the random jumps are sufficiently large to dominate over the slow-roll trajectory ϕ_{sr}. Roughly speaking, this will be the case if the typical amplitude of a "jump" is larger than the variation in the field along the slow-roll trajectory (for a given time scale); then the jumps "up" the potential are large enough to counteract classical slow-rolling down the potential. The effect of such a fluctuation-dominated regime is to extend the inflationary phase. Since the jumps in spatially distant regions are presumably uncorrelated, the duration of the inflationary stage will vary across different regions. This stochastic evolution of the inflaton field yields a reheating surface that consists of disconnected components separated by regions with ongoing inflation. "Pocket universes" correspond to non-compact, connected components of the reheating surface, each of which may have infinite spatial volume despite originating from a finite spatial region.

EI may also arise via quantum tunneling from a false vacuum state. Rather than a flat potential, this approach assumes that $V(\phi)$ includes at least one barrier sufficient to classically confine ϕ in a false vacuum state. Quantum effects may allow the field to evolve out of this state, due either to thermal fluctuations or to quantum tunneling. This leads to the formation of a bubble with a new value of ϕ, with dynamics and a bubble nucleation rate described by Coleman and De Luccia (1980). For typical scenarios, the bubble nucleation rate is sufficiently low that the bubbles do not merge into a single domain; instead, each bubble is treated as a "pocket universe" — the bubble's interior is an infinite, homogeneous open universe. For a sufficiently low nucleation rate inflation will be eternal, in that the global volume of spacetime undergoing inflation increases exponentially, even though any given region eventually tunnels out of the false vacuum state and stops inflating.This approach to EI has been connected in recent work with string theory, with tunneling taking place among the vast number of possible metastable vacua constituting the string theory "landscape."

Even this brief discussion illustrates that the potential $V(\phi)$ and the initial field values must satisfy some constraints in order for EI to arise. Advocates of EI typically argue that it is "generic," in the sense that the most "natural" models of inflation satisfy these constraints. Yet these claims (as noted by, e.g., Guth 2007 and Aguirre 2007b) have not been made precise. This would require delimiting the space of allowed potentials and initial field values and introducing a measure over this

[8] This is not an exhaustive list, and it excludes in particular "topological inflation," in which topological defects seed eternal inflation. See Aguirre (2007b) for an overview of these different approaches, which I draw on here, and further references.

space, in order to assess whether the conditions for EI are satisfied within "most" models compatible with observations. In place of a more thorough understanding of the space of allowed models and how many of these lead to EI, one has qualitative arguments focusing on the stochastic nature of the end of inflation. On either of the approaches described above, an incredibly unlikely correlation between physical properties in distant regions would be required for inflation to come to an end globally.

These arguments in favor of EI are contentious, for reasons that I will discuss briefly in §8. Here my main aim is to assess the evidential value of anthropic predictions, so for the bulk of the paper I will bracket objections to EI and focus on making predictions in the multiverse. I will also bracket problems with inflation itself, such as the question of how the quantum fluctuations in the vacuum state become classical density perturbations and the plausibility of introducing a scalar field with the appropriate properties to drive inflation. These and other problems have motivated the study of alternatives to inflation, as discussed, e.g., in Brandenberger's contribution to this issue.

3 Anthropic Prediction of Λ

Weinberg's (1987) prediction of the value of Λ exemplifies successful anthropic prediction, and is routinely cited as powerful evidence in favor of EI. I will use this case to introduce some of the features of anthropic predictions, before turning to an examination of the rationale for such predictions in the next section.

The background to Weinberg's calculation is our humbling inability to understand the value of Λ in terms of fundamental physics. The calculation of the value of Λ within QFT is often cited as one of the worst theoretical failures in physics: the result is 120 orders of magnitude larger than the value of the cosmological constant inferred from observational cosmology. The calculation assumes that the vacuum energy of quantum fields couples to gravitation as an effective Λ-term.[9] The vacuum energy is given by integrating the zero-point contributions to the total energy, $\frac{1}{2}\hbar\omega(k)$ per oscillation mode, familiar from the quantum harmonic oscillator, over momentum (k). Evaluating this quartically divergent quantity by introducing a physical cutoff at the Planck scale leads to the huge discrepancy.[10] Theorists once sought a new dynamical principle or symmetry that would force Λ to be exactly zero. With new observational evidence in favor of dark energy in the late 90s, the question is now why Λ is not *exactly* zero but is still very small relative to the vacuum energy density calculated in QFT.

Weinberg (1987) predicted a non-zero value of Λ at a time when it was routinely assumed to be zero (in spite of the QFT calculations). The predicted value fell within two orders of magnitude of currently observed values. The first step of the argument elucidates constraints on Λ, which cannot

[9] Several philosophers of physics have questioned this assumption, given that the coupling of vacuum energy to gravity arguably plays no role in the empirical success of QFT; see, in particular, Rugh and Zinkernagel (2002) for a thorough discussion, as well as Earman (2001) and Saunders (2002).

[10] The relevant integral is

$$\rho_v = \int_0^{\ell} \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+m^2}}{2} \approx \frac{\ell^4}{16\pi^2}$$

For a Planck scale cut-off, $\ell_p \approx 1.6 \times 10^{-35}\mathrm{m}$, the vacuum energy density is then given by $\rho_v \approx 2 \times 10^{110}\mathrm{erg/cm^3}$, compared to observational constraints on the cosmological constant — $\rho_\Lambda \approx 2 \times 10^{-10}\mathrm{erg/cm^3}$. Choosing a much lower cut-off scale, such as the electroweak scale $\ell_{ew} \approx 10^{-18}\mathrm{m}$, is not enough to eliminate the huge discrepancy (still 55 orders of magnitude).

be an entirely free parameter within the standard big bang model of cosmology. Because a Λ term does not dilute with expansion, a cosmological model with $\Lambda > 0$ will eventually transition from matter-dominated to vacuum-dominated expansion. Weinberg showed that structure formation via gravitational enhancement of initial inhomogeneities stops in the vacuum-dominated stage. Large, gravitationally bound systems (large enough to lead to the formation of stars) are a plausible proxy for observers. The existence of such systems imposes an upper bound on possible values of Λ, assuming one keeps other aspects of the Standard Model fixed.[11] There is also a lower bound. A negative Λ term contributes to EFE in the same way as normal matter and energy. So adding a large negative Λ term leads to a model that recollapses, in a graviational big crunch, before observers arrive on the scene.

The anthropic element of Weinberg's argument appears in the transition from noting these constraints to predicting what a typical observer should measure. Grant, as a hypothesis, that we inhabit a multiverse in which the value of Λ varies across different regions. (Weinberg (1987) did not endorse a particular multiverse hypothesis, instead listing four proposals that would suffice for his argument.) An anthropic prediction requires specifying, first, the probability distribution for values of Λ in distinct regions of the multiverse. In principle, the prior probability distribution $P_i(\Lambda)$ should be calculable within a given multiverse proposal; in practice, one often makes do with plausibility arguments. For example, there is a general argument in favor of assigning a uniform probability distribution over a constant α_i, within the anthropic bounds: the probability distribution can be treated as uniform if the constant varies in the multiverse over a range that is much larger than the anthropically allowed interval. This argument has been used to justify a uniform probability $P_i(\Lambda)$. The second step takes the presence of observers into account. Suppose we take the existence of galaxies as a proxy for observers, and are able to calculate how this varies as a function of Λ, obtaining $N(\Lambda)$ — the number density of galaxies as a function of Λ, normalized to unity. A "randomly chosen" observer should then assign the following probability: $P(\Lambda) = N(\Lambda)P_i(\Lambda)$.[12]

This is, schematically, Weinberg's line of argument; subsequent refinements assign a probability of 5%, or 12 % (depending on other assumptions regarding the cosmological model), to the currently measured value. At first blush this seems too inaccurate to support a research program. Yet this is a striking success relative to the failure of other proposals for understanding the value of Λ.

The next sections will take up problems related to $P_i(\Lambda)$ and "typical" observers. For the sake of completeness, I will briefly mention other problems orthogonal to these (cf. Aguirre 2007a). Like many other anthropic predictions, Weinberg's focuses on a single constant among the many that could vary. Broadening the scope to include variation in a multi-dimensional space may undermine several aspects of the argument. The anthropic constraints on Λ implicitly keep a number of other factors fixed. For example, the effect of Λ on structure formation can be counteracted by increasing the amplitude of the initial density perturbations. The arguments for assigning a uniform

[11] More precisely, the upper bound relates the Λ term to the total energy density in matter at the time when most galaxies formed; the upper bound on Λ is ≈ 200 times the contemporary matter density. Considering variation of multiple parameters may undermine this bound; larger values of Λ can be tolerated if one increases the amplitude of the initial spectrum of density perturbations, for example. This illustrates a more general problem with anthropic arguments, namely the limitations of treating constraints on a single parameter instead of constraints on a set of free parameters; see Aguirre (2007a) for further discussion.

[12] Note that anthropic predictions do not require a uniform probability. Suppose instead that $P_i(\alpha_i)$ monotonically increases in one direction, and that $N(\alpha_i)$ is sharply peaked near the observed value. Then the observed value should lie near the corresponding edge of the allowed range rather than near the mean; this has been called the "principle of living dangerously."

probability to $P_i(\Lambda)$, and estimates of $N(\Lambda)$, do not directly carry over to the multi-constant case. It is often quite difficult to consider the combined effects of variation in a multi-dimensional space, so cosmologists have focused on cases which seem "clean" in the sense that there are physical reasons to expect the value of a given constant to be relatively unconnected with other constants. Aguirre (2007a) emphasizes the possibility of a quite different kind of anthropic prediction. If there are a large number of constants that vary in the multiverse, one would expect physically unrelated constants to have "coincidental" values due to the selection effect.

4 The Measure Problem

The multiverse of eternal inflation consists of an infinite ensemble of pocket universes.[13] The "measure problem" refers to two related obstacles to introducing probabilities characterizing the variation of observable quantities across the multiverse. First, one needs to characterize the ensemble or sample space Σ, equipped with a measure μ. This is difficult due to incomplete knowledge regarding the sample space Σ, and the lack of a unique, physically motivated measure μ. Many of the measures that have been considered are non-normalizable, i.e. $\mu(\Sigma) = \infty$. This leads to the second obstacle to defining a probability distribution over the physical properties of pocket universes. Suppose that the measure μ is connected to probability by a form of the principle of indifference: subsets of equal μ-measure are assigned equal probability. For a normalizable μ, probabilities can be introduced by the ratio of favorable cases (falling within a measurable subset $S \subseteq \Sigma$) to the whole space: $P(S) = \frac{\mu(S)}{\mu(\Sigma)}$. Yet if μ is non-normalizable, probabilities introduced in this way will typically be undefined (see, e.g., Hollands and Wald 2002a,b). The probability (for a measurable subset S) will be well-defined in two cases: if $\mu(S) < \infty$, then $P(S) = 0$; and if $\mu(\Sigma \setminus S) < \infty$ (where $\Sigma \setminus S$ is the complement of S), then $P(S) = 1$. If $\mu(S) = \infty$ and $\mu(\Sigma \setminus S) = \infty$, on the other hand, the probability is not well-defined. Nearly all of the probabilities one would like to compute fall into this third case.

Probabilities can be introduced for the third case via a "regularization procedure." Returning to the example mentioned in the introduction (also discussed in Guth 2007, Schiffrin and Wald 2012), the natural answer that a random natural number has a $1/2$ probability of being even can be defended by defining the probability as follows:

$$P(S) = \lim_{n \to \infty} \frac{\mu(S \cap \sigma_n)}{\mu(\sigma_n)}, \tag{2}$$

where S is the property of being even, $\sigma_n = \{1, 2, 3, ..., n\}$, and μ is the counting measure. Different sequences of finite subsets, such as $\sigma'_n = \{1, 3, 5, 2, 7, ..., n\}$, lead to a different probability. The strategy in this simple case — defining probability as a limit, using a nested sequence of subsets of Σ — applies more generally. But this trick relies on introducing additional structure, namely the choice of a particular sequence of nested subsets. The probabilities so defined are only as well justified as the choice of this further structure. Recent discussions of the measure problem have focused on finding a regularization procedure in order to extract probabilities, but as in this simple case they require introducing further structure.

There are two quite different contexts in which the "measure problem" has been discussed in cosmology. In the first, the ensemble consists of a set of solutions to EFE and the measure in question is the canonical phase space measure. The need for additional structure to extract probabilities

[13]See Ellis et al. (2004) for a general, systematic discussion of multiverse ensembles and the measure problem.

from a non-normalizable space has been particularly clear in debates regarding the probability of inflation. The second case is that of eternal inflation, in which the ensemble consists of a collection of observers (or some other type of object) occupying a single, connected multiverse.

4.1 Phase Space Measures

Attempts to estimate the "probability of inflation" nicely illustrate the difficulties associated with introducing probabilities in cosmology. Gibbons et al. (1987) addressed the question of how likely it is for inflation to occur, based on defining a measure μ_{GHS} on the space of homogeneous and isotropic solutions to EFE coupled to a scalar field ϕ.[14] The measure they introduced follows naturally from the Hamiltonian structure of general relativity, and they argued that it gives a high probability to $\mathscr{N} >> 1$ e-foldings of inflation in the early universe. (I will return to the connection between the phase space measure and probability in §5.)

Yet this use of the canonical measure involves a subtle ambiguity (first noted in Hawking and Page (1988), cf. Ashtekar and Sloan (2011); Schiffrin and Wald (2012)). The probability of inflation falls into the third case described above: it is not well-defined without some further structure. The canonical measure is non-normalizable, but this can be dealt with by shifting from the original phase space Σ to an alternative Σ', obtained by identifying points in Σ that lie on the same dynamical trajectory. One requires additional structure to define the canonical measure on Σ' based on μ_{GHS} — namely, the choice of a value of the Hubble parameter. The measure on Σ' depends explicitly on this choice. A well-defined probability can then be obtained by regularizing the phase space for this value of the Hubble parameter; that is, considering the finite subset of models with a scale factor smaller than some cut-off a_c and then taking the limit $a_c \to \infty$. This regularization procedure depends, however, upon the initial choice of a Hubble parameter (equivalently, choice of a particular cosmic time). Carroll and Tam (2010) estimate the probability for 60 e-foldings of inflation to be near one, based on choosing an "early time" (pre-inflation) compared with the probability of 10^{-80} calculated by Gibbons and Turok (2008), given a "late time" (post-inflation). Here my purpose is not to argue for the legitimacy of choosing an "early" or "late" time, but to emphasize that the apparent conflict reflects the choice of an additional structure that is required to obtain a well-defined probability.

4.2 Observer-based Measures

The second type of measure is introduced to answer questions regarding correlations between particular structures and other properties in the multiverse. For example, what is the probability of observing a value of Λ within a particular interval, given the existence of galaxies? As noted above, the probability for a "randomly chosen" galaxy is taken to be: $P_G(\Lambda) = N_G(\Lambda)P_i(\Lambda)$, where $P_i(\Lambda)$ is the prior probability distribution and $N_G(\Lambda)$ is the normed number density of galaxies. I have followed Winitzki in referring to this as an "observer-based" measure, although one can choose to conditionalize on other structures instead, from "spacetime volume" to "pocket universes" to "Hubble volume identical to the observed universe." (Although my focus will be on "observers," similar problems arise for other choices.) The ensemble Σ consists of the collection of observers in the

[14] To be clear, the measure is defined over this space of solutions, each regarded as an independent cosmological model, rather than over a single connected spacetime consisting of distinct pocket universes (as in eternal inflation).

multiverse, a single causally connected spacetime. Calculating the posterior requires a measure μ over the chosen ensemble, as well as a way of regularizing the measure if $\mu(\Sigma) = \infty$.

It is challenging to find an appropriate measure, and regularization procedure, because the EI multiverse lacks symmetries or other structures that could be exploited for these purposes. Cosmologists have pursued a number of different measure proposals, and most of these fall into two types: global (or volume-based) and local (or worldline-based).[15] Global measures take the ensemble to consist of *all* observers in the multiverse, whereas local measures define the ensemble as the collection of observers falling within some (possibly finite) region — e.g., those "close" to a given timelike curve.

Several proposed global measures consider the collection of observers lying to the future of some finite region of spacetime cut-off at some time t, and then take the limit as $t \to \infty$. Fix, for example, a finite spatial hypersurface Σ in a region undergoing inflation, and consider a congruence of timelike geodesics orthogonal to Σ. The spacetime region traced out by this collection of curves after some "time" τ_0, after inflation comes to an end in local pockets, should include only a finite number of observers. We can then define a finite ensemble of observers, with a measure μ assigning equal weights to equal volumes of the initial hypersurface Σ. In other words, the weight of a given pocket universe (and all of its inhabitants) will be determined by the fraction of the curves emanating from Σ that occupy it. The probabilities assigned in the limit should be insensitive to the initial choice of Σ. Several proposals share this basic starting point, and diverge in choosing an appropriate "time" to use in defining equal volumes and taking the limit.

A natural first proposal is to consider the proper time elapsed along the curves emanating from Σ. This proposal has been rejected due to the "youngness paradox." The measure μ_{PT} assigns probability proportionally to the spacetime volume at some elapsed proper-time τ_0, in the limit as $\tau_0 \to \infty$. During inflation, spacetime expands exponentially; in stochastic models of inflation, the number of pocket universes is proportional to volume, so this number also increases exponentially. As Guth (2007) puts it, this leads to a "youth-dominated society" of pocket universes: at any given time, "youthful" pocket universes will far outnumber more mature ones. Probabilistic predictions based on μ_{PT} are thus strongly weighted towards youth, predicting (for example) that observers in a universe 13 Gyr old should vastly outnumber those in a universe 13.7 Gyr old.[16] Subsequent global proposals have used a different time-coordinate in defining the measure, such as the "scale-factor" measure obtained by replacing proper time with a time function that depends on the local expansion rate. This measure assigns equal weight to co-moving volumes, and as a result avoids the youngness paradox.

Local proposals single out a particular worldline and an associated collection of observers, such as those falling within the "causal diamond" determined by the worldline, its apparent horizon, or a fixed spacetime distance. This leads to a finite collection of observers provided that the worldline has finite length, so there is no need for a regularization procedure. The results of the calculation will, however, depend upon the specific choice of a worldline. Thus, by contrast with the global measures, the local measures do not reflect the "attractor" behavior of inflation: namely, that EI will

[15]Here I will not go into the details involved in implementing these general ideas in different approaches to EI, or explore the connection with properties of the string theory landscape of allowed vacua. For more systematic reviews of various proposed measures, including ideas that do not fall into either of these two camps, see, for example, Vilenkin (2007); Aguirre et al. (2007); Winitzki (2009); De Simone et al. (2010); Freivogel (2011).

[16]More concretely, as Tegmark (2005) showed, this measure assigns a very low probability to the observed temperature of the CBR.

lead to something like an equilibrium distribution of different properties in the multiverse at late times. The local proposals need to be supplemented with a specification of initial probabilities over the different worldlines to yield general results.

Currently the measure problems is unresolved, with no agreement on the appropriate measure to use in EI. The debate also seems to reflect basic disagreements about what desiderata a measure should fulfill. One part of the debate is relatively uncontroversial. Candidate measures are evaluated phenomenologically by assessing their predictions. As described briefly above, the proper time measure has been rejected because it assigns a very low probability to an "old" universe such as ours, and similar results have been obtained for other measures. But the debate also focuses on how a measure choice relates to fundamental physics. The terms for doing so have shifted along with the theoretical context in which EI has been studied. In the 90s, Vilenkin and several collaborators studied the measure problem in the context of stochastic inflation, whereas more recent studies by Bousso, Freivogel, and others have treated the problem within the context of string theory. It is not clear whether current theories are sufficient to determine a unique measure choice for EI. Rather than venturing into this debate, in the following I will consider further problems that arise even if one grants that this difficult problem has been solved.

5 Typicality

Grant, for the sake of argument, that we have a well-motivated "pocket-counting" measure μ^* that allows one to count elements of the ensemble. This measure presumably reflects the physical mechanism that generates the pocket universes and variations in the constants α_i among them. It seems natural to take the probability that the value of a constant α_i falls within a specified range $(\bar{\alpha}_i \pm \delta)$, for example, to be $P(\bar{\alpha}_i) = \frac{\mu^*(S_{\bar{\alpha}})}{\mu^*(\Sigma)}$, where $S_{\bar{\alpha}}$ denotes the subset of pocket universes for which this condition holds. Suppose we further grant that the measure yields a finite result rather than a ratio of infinities. The probability then represents the chance of finding the value of α_i within the specified range at a "randomly chosen" pocket universe within the multiverse. (The choice is "random" with respect to μ^*: subsets assigned equal μ^*-measure are equally probable.) Following Weinberg's treatment of the cosmological constant (§3), we can also take selection effects into account by estimating the normalized number density $N(\alpha_i)$ of observers in each pocket universe, as a function of the constant α_i. If we further assume that a "typical" observer is chosen at random from among all observers, then the probability distribution for such an observer's measurement of α_i is $P(\alpha_i) = N(\alpha_i)P_i(\alpha_i)$, as in §3.

This line of thought leads directly from the measure μ^* to anthropic predictions for α_i. Yet I will now argue that an ensemble equipped with a measure is *not* sufficient to justify physical probabilities. The assignment of probabilities depends on "randomly choosing" among the elements of the ensemble or the class of observers. We *cannot* just equate the chance or objective probability that a given pocket universe has a value of α_i within some range with the proportion of the elements of the ensemble that do so, for to do so, we need to invoke a *distinct* idea of chance – the equal chance that each μ^*-equal subset of the ensemble, or each observer, has of being picked out. This second notion of chance makes a substantive claim regarding how an element of the ensemble is chosen. For the phase-space measure, I will argue that a claim to this effect has to be regarded as an independent, and hitherto unmotivated, assumption. I will further argue that an alternative that associates the "random choice" with the epistemic situation of a cosmologist, who should reason as if she is a typical member of an appropriate reference class, fares no better.

5.1 Physical Probabilities

In the case of the phase-space measure, such as μ_{GHS} discussed in §4.1, there is a suggestive analogy with probabilities in statistical mechanics. Textbook presentations of statistical mechanics often invoke the principle of indifference in justifying probabilities, along similar lines to the argument above. Consider, for example, an isolated classical system with conserved total energy. The phase space for the system is equipped with an invariant measure μ that naturally induces the micro-canonical measure μ_E on constant energy hyper-surfaces. It is plausible to define the probability of finding the system in a region of phase space S, a subset of the energy hypersurface, at a "randomly chosen" time, to be $\mu_E(S)$. This seems to depend only upon the principle of indifference, suggesting a defense of cosmological probabilities based on μ_{GHS}.

This account of the justification of probabilities in statistical mechanics is, however, quite misleading. It ignores the dynamics! The status of physical probabilities in statistical mechanics is one of the most contentious topics in the foundations of physics, but it will be useful to briefly sketch the role of dynamics in the ergodic approach. Two aspects of the dynamics are particularly important. If a system's trajectory remained confined to a region S_1 of the energy hyper-surface, and did not intersect S_2 (with $\mu_E(S_1) \neq 0$ and $\mu_E(S_2) \neq 0$), the suggestion above would assign incorrect probabilities. A dynamical system is called *metrically transitive* if it is impossible to find subsets like S_1, S_2 — non-overlapping, positive measure, and closed under dynamical evolution. This property is sufficient to prove a version of the ergodic theorem, which (roughly speaking) establishes the equality between time averages and phase averages with respect to μ and justifies the familiar application of probabilities to the properties of a system in equilibrium. Second, the appeal to ergodicity only justifies using probabilities in a specific context. The dynamics for a given system will determine the time scale on which the system relaxes to an equilibrium state. At shorter time scales, the system will still bear the imprint of the initial state, rendering equilibrium probabilities incorrect. Thus the equilibrium probabilities can only be justified for time scales larger than the dynamical relaxation time, such that the "memory" of the initial state will be washed out.

This way of justifying probabilities does not extend to classical cosmology. The mathematical apparatus used in stating the ergodic theorem is a poor fit for general relativity. What would it mean for a single system to "sample" the phase space, consisting of different cosmological models? Furthermore, the phase space is non-compact with infinite measure, even in subsets of the phase space obtained by imposing symmetries (such as minisuperspace). Finally, there is not a straightforward analog of time-translation. Even if we could circumvent these problems, the timescale for dynamical relaxation for large-scale gravitational degrees of freedom is of the same order of magnitude as the current age of the universe. Schiffrin and Wald (2012) argue persuasively, based on these and other considerations,[17] that the strategy used to justify equilibrium probabilities in statistical physics sketched above does not apply to classical relativistic cosmology. The association between μ_{GHS} and probability has to be regarded as a bald posit — a posit about as well supported as the conceit of a Creator choosing an initial state by throwing darts at a board with measures of areas given by μ_{GHS}.

[17]They discuss a further more subtle contrast: introducing an analog of time evolution for minisuperspace models leads to a time-dependent Hamiltonian. This poses a further obstacle to equilibriation, since the Hamiltonian (and hence energy) of the system will depend on its initial state and subsequent history.

Shifting venues to semi-classical or quantum gravity may improve the prospects for basing cosmological probabilities on something like ergodicity. Several authors have argued for a form of ergodicity provided by the mechanism for creating pocket universes — in slogan form, EI "populates the string theory landscape" (see, e.g., Clifton et al. 2007, Brown and Dahlen 2011). These arguments have been used to bolster the idea that EI will produce an equilibrium ensemble of pocket universes, if the multiverse-generating mechanism provides a (sequence of) allowed transitions connecting any two vacua in the landscape. These suggestions are still very speculative, but it is at least clear that detailed study of the dynamics will be needed to establish whether an analog of ergodicity holds and to estimate the relaxation time. If quantum gravity justifies probabilities where classical general relativity does not, it will be due to the dynamics of the theory rather than an appeal to the indifference principle.

5.2 Epistemic Probabilities

A different approach treats the probabilities associated with the measure as subjective. Probabilities would then be justified on the basis of the "principle of mediocrity" (PM, following Vilenkin 1995): we should reason as if we are a *typical* member of the reference class of observers.[18] The "random choice" is associated with the epistemic situation of a cosmologist evaluating a theory. We imagine that the cosmologist has, in a sense, third-person information about the distribution of physical systems that qualify as "observers" (or some other chosen reference class) throughout the multiverse. Yet she lacks first-person or indexical information singling out one of these systems as "herself." She should then, according to the advocates of PM, reason as if she is a "randomly chosen member" of the reference class of observers. The resulting sense of probability introduced is obviously epistemic, and this proposal does not require (as Bostrom 2002 emphasizes) a physical process such as a cosmic lottery determining where in the universe one's soul is imparted. What the proposal does require, however, is an argument that the epistemic situation of a cosmologist demands using this new type of reasoning, along with a clear characterization of the rules she should follow to do so.

Philosophers have imagined situations (such as Elga (2000)'s Sleeping Beauty problem) in which it is plausible that one should reason as an agent who has all relevent non-indexical information, but lacks relevant indexical information. Yet the arguments as to why cosmologists should also reason in this way are not nearly as compelling. When this question is addressed at all, cosmologists often invoke the infinity of the multiverse. Tegmark (2003)'s calculation that a nearly identical copy of you, who differs only in deciding to stop reading this article on the previous sentence, is probably found within a distance of $10^{10^{28}}$ meters, is fairly typical.[19] All evidence available up to a moment ago does not distinguish between you and such a multiversal doppelgänger (one of many), and it supposedly follows that you should reason as if you were a random choice drawn from this reference class.[20] Advocates of the PM treat this as an obvious step, and try to shift the burden of

[18] Vilenkin (1995)'s formulation used "civilization" as the reference class: we should reason as if our civilization is "randomly chosen" among all the civilizations in the multiverse. Bostrom (2002) formulates a similar principle, his "Self Sampling Assumption": One should reason as if one were a random sample from the set of all observers in one's reference class. He later discusses a modification which takes the appropriate reference class to be "observer-moments."

[19] Precisely what distance is being estimated is not entirely clear, given that the multiverse may lack the structures necessary to identify a sensible notion of spatial distance. Tegmark's argument is also typical in not acknowledging that infinity alone is not sufficient to ground such claims; one must also assume that there is some source of appropriate variations, a point emphasized in Mosterín (2005) and Zinkernagel (2011).

[20] This reference class would be maximally fine-grained, consisting of individuals with a full data set as close as

proof to an opponent who advocates something other than random choice. Yet this fails to address why the proposed "random sampling" and the associated epistemic probability is even relevant to evidential reasoning in cosmology.[21] The debate here is not between randomly picking an observer from among a given reference class or some other unmotivated, biased weighting, but between different accounts of how to assess evidence. Why should the existence of multiversal doppelgängers with the same data affect our cosmologist's assessment of a theory based on her data? In other words, why should the cosmologist's method depend on the infinity of the multiverse?

Even more problematic is the lack of a clear account of what the cosmologist willing to reason in line with the PM is actually supposed to do. Advocates and critics of the PM have, for the most part, conducted the debate within the context of a Bayesian approach.[22] There is then a clear contrast between different proposals: BU (Bayesian Updating) – following the standard Bayesian updating rule for adjusting one's credences in light of new evidence, or AR (Anthropic Reasoning) – a specific modification of the Bayesian rule, or alternative account of inductive methodology, to be applied in anthropic reasoning. There is not a single, canonical candidate for AR. But versions of AR typically require the choice of a reference class, from which "you" are randomly chosen.

The difference between BU and AR can be illustrated using the case of Λ. The main contrast concerns how the prediction of vast regions of the multiverse unfit for the presence of observers is handled. Consider the evaluation of a multiverse theory T_M in which Λ varies in different pocket universes, equipped with a measure μ^* that successfully regulates infinities, in comparison to a single-universe theory T_1 which treats the value of Λ as a free parameter. We will take the range of values of Λ compatible with all available evidence to be Δ (and assume that this is not empty). In applying BU, assuming that we assign the theories equal prior probabilities, the difference in the credence assigned to them will depend on two numbers: the fraction of pocket universes in which $\Lambda \in \Delta$ (call this $S_{\bar{\Lambda}}$), $P_M = \frac{\mu^*(S_{\bar{\Lambda}})}{\mu^*(\Sigma)}$, according to T_M; and P_1, the probability assigned to $S_{\bar{\Lambda}}$ by T_1. Given evidence that Λ falls with Δ, the odds ratio for T_M to T_1 will be $\frac{P_M}{P_1}$.[23] Within the standard Bayesian approach, then, the evaluation of the two theories reflects the probability they assign to a value of Λ within Δ. T_M suffers in comparison with T_1 if it predicts a large number of pockets in which Λ falls outside of Δ, as P_M will then be lower. The assessment also does not depend on considering how Δ compares to the range of values of Λ, call it Δ', compatible with "observers" (or some other reference class defined by AR, such as "civilizations" or "galaxies"). In contrast, the effect of AR is to replace P_M with $P'_M = \frac{\mu^*(S_{\bar{\Lambda}})}{\mu^*(\mathscr{A})}$, disregarding the pocket universes that fall outside the "anthropic" subset $\mathscr{A}$, for which $\Lambda \in \Delta'$. The theory T_M is no longer punished for predicting

possible to yours up to the point of divergence.

[21]The problem I am addressing is not the problem of "freak observers" (in Bostrom's terminology) or "Boltzmann Brains" (physics terminology), which consider a reference class — moments of observations or brain states — such that most members of the class have non-veridical observations. By contrast, I am considering here a class of observers whose entire Hubble volume is as close as one may care to specify to that of a given observer, and whose observations are (mostly) veridical.

[22]Norton (2010) criticizes anthropic reasoning as part of a general criticism of Bayesianism, based on the claim that probability distributions do not correctly represent evidential neutrality. The PM supports only an ascription of neutrality, and hence on Norton's view not assignment of probabilities.

[23]The odds for a hypothesis H are defined as $O(H) = \frac{P(H)}{P(\neg H)}$, and the odds ratio is the ratio by which the prior odds must be multiplied given new evidence E to obtain the conditional odds $O(H|E)$. Bayes' Theorem implies that the odds ratio given evidence E is $\frac{P(E|H)}{P(E|\neg H)}$. In considering the odds ratio between two competing theories, the factor corresponding to the so-called "catchall hypothesis" (the negation of theories under consideration, in this case $\neg(T_1 \vee T_M)$) cancels out.

observer-less pocket universes with incorrect values of Λ. But the assessment depends on a choice of the appropriate reference class: $\mu^*(\mathscr{A})$ depends on whether we consider observers, the existence of galaxies, etc.

It is unappealing to have the assessment of evidence vary depending on the choice of reference class. For if we consider "observers" as the appropriate reference class, it is difficult to assess whether observers can arise in physical conditions significantly different than those of our universe. Yet the advocate of AR has to address this difficulty in order to count the number of pocket universes with observers, and then evaluate $\mu^*(\mathscr{A})$. Furthermore, the simplest version of AR leads to paradoxical conclusions. The Doomsday Argument, for example, follows from applying PM to one's place in human history. Imagine the (in)numerable caravan of humanity, encompassing the entire history of mankind ordered chronologically by birth, lumbering toward that mysterious realm. If our place in the caravan is "typical," there should be roughly as many ahead of us as behind. For this to be true, given current population growth rates, there must be a rapid drop in the growth rate of the human population — "doomsday" — in the near future. This striking conclusion seems absurd as it depends on almost no empirical input.[24]

The Doomsday Argument and other paradoxical conclusions drawn from the most straightforward versions of AR have led to other proposals. Bostrom (2002) considers combining PM with a second principle he calls the "Self Indication Assumption" (SIA), according to which the prior probabilities of theories are proportional to the number of possible observers they predict. In the case of the Doomsday argument, accepting SIA blocks the paradoxical conclusion. The priors are adjusted precisely to cancel the effect of introducing the reference class and the PM, leading to the same conclusion that would have been obtained by applying BU directly.[25] It is unclear what is accomplished by this indirect route to the same conclusion. Furthermore, the SIA is unacceptable as a general principle (as Bostrom 2002 notes) for the following reason. Given two theories differing in their yield of predicted observers by a factor of, say, 10^{20}, it would be nearly impossible for empirical results to sway credences in favor of the theory with fewer observers. The ratio of priors required by the SIA dominates the assessment of posterior probabilities for the two theories.

5.3 Summary

Summing up this line of argument, two existing proposals for how to relate a measure to probability fall short of their mark. One strategy for basing physical probability on the measure (§5.1), by appealing to ergodicity, does not extend to cosmology. The second proposal (§5.2), which introduces a new inductive methodology appropriate for cosmology in an infinite universe, would treat the probabilities as epistemic, associated with an observer's lack of indexical information regarding her location in the multiverse. I have argued that the reasons for adopting a new methodology are not compelling, and existing proposals for the revised updating rule lead to paradoxical conclusions.

In both cases, my objections may indicate a failing of current discussions of these issues rather than a basic failing of the proposal. This is certainly not an exhaustive discussion, and at best I hope to have formulated challenges to introducing probabilities. Recent work in the foundations of physics on probabilities in statistical and quantum physics suggests a number of other strategies

[24] One man's *reductio* is another man's proof, and the Doomsday argument has its defenders, including Gott (1993); Leslie (1992); see Bostrom (2002) for a review of this literature, and Dieks (2007); Norton (2010) for critical assessments.

[25] This result has been noted and discussed before; see, e.g., Bartha and Hitchcock (1999), Bostrom (2002), and Neal (2006).

for justifying physical probabilities that may be fruitfully applied here. There are also specific proposals tying EI to the Everettian interpretation of quantum physics, and perhaps a decision-theoretic approach to understanding the nature of probabilities in the Everettian interpretation can be employed in EI as well.

6 Evidence from Anthropic Predictions

Lack of clarity about the nature of probabilities involved in anthropic predictions might not undercut their evidential value. Foundational questions about the nature of probabilities in equilibrium statistical mechanics remain unresolved, for example, yet there is little dispute regarding the theory's empirical success.[26] So now, for the sake of argument, I will set aside the objections raised in the previous section regarding the propriety of probabilities in the multiverse and grant that one can make sense of the usual account of anthropic predictions. I will argue that, even so, such predictions have little evidential value.

Consider, as an illustration, choosing between two theories T_1, T_2 in light of anthropic predictions regarding the value of constants (assuming that T_1 is a multiverse theory).[27] I will set aside other considerations bearing on the evaluation of these theories in order to isolate the impact of anthropic predictions. What would have to be the case for anthropic predictions alone to render a decisive verdict?

This is not a lead-in to the shopworn criticism that multiverse theories are not falsifiable because they posit pocket universes that are in principle unobservable. My point is rather to draw a contrast between the probative value of predictions in other cases and anthropic predictions. Evidence in favor of our most well-entrenched physical theories tightly constrains their structure, so much so that it is hard to imagine an alternative theory governing the same domain with equal success. I will argue that anthropic predictions provide, by contrast, nearly no constraints on the physics governing the formation of the multiverse.

Extracting anthropic predictions from T_1 (and T_2, if it is also a multiverse theory) requires specifying: (i) the multiverse ensemble Σ, (ii) the measure μ used to weight elements of the ensemble and to regulate infinities, and (iii) the anthropically allowed subset of the ensemble, $\mathscr{A}$. Anthropic predictions then take the form of a probability distribution over a collection of physical constants, $P(\alpha_i)$, and (other things being equal) the theory assigning a higher probability to the observed values of the physical constants is favored by the evidence. The obstacles to utilizing anthropic predictions reflect each of these three aspects of a multiverse theory.

Regarding (i), the ensemble produced by T_1 reflects the dynamics generating the multiverse. The ensemble should differ from a "kinematic" multiverse ensemble $\aleph$, consisting of pocket universes realizing all possible combinations of the fundamental constants, without dynamical constraints imposed by an account of how the ensemble is generated.[28] One might hope that the results would

[26] I will concede this point, for the sake of argument, although I remain doubtful whether one can make sense of anthropic predictions without defending the use of probability. I do not claim that the situation in statistical mechanics is closely analogous to that in cosmology; while the correct way of grounding probabilities in statistical mechanics is still contentious, the competing approaches are much more well developed than in the case of cosmology.

[27] See, for concrete examples, Smolin's (2013) comparison of EI, his proposal of cosmological natural selection, and cyclic cosmologies, and Steinhardt's (2011) discussion of EI compared to cyclic cosmologies.

[28] This point is emphasized by Smolin (2007). I will assume that a kinematic ensemble comes equipped with a measure $\mu_{\aleph}$, and is defined in light of dynamical constraints from GR and the Standard Model of particle physics, but without

differ quite strongly from the ensemble $\aleph$, in that some combinations of fundamental constants either violate the laws of the theory or are inaccessible given the allowed transitions among pocket universes. But Smolin (2007) notes that this hope runs counter to the decoupling of different energy scales in renormalizable QFTs. Dynamics at the energy scale of grand unification or higher is responsible for the formation of pocket universes in EI, yet the constants discussed in connection with anthropic predictions are related to energy scales several orders of magnitude lower. He thus argues that EI should lead to an essentially random distribution of low-energy constants, that is, an ensemble indistinguishable from $\aleph$. Responding to Smolin's criticism requires showing how T_1 leaves an imprint on parameters characterizing low energy physics, yet many discussions of anthropic predictions are vague on precisely this point. More generally, it is clear that EI advocates need to specify the ensembles generated by particular multiverse theories. If this is not done, what is tested is the general idea that the constants vary across different pocket universes, and not a specific physical theory governing creation of a particular ensemble.

Turning to (ii), even if a measure that regulates infinities is found, it may not be unique. If the theory T_1 naturally dictates a unique measure to be used in anthropic predictions, then the success or failure of the predictions bears directly on T_1. If there are multiple candidate measures, by contrast, then what is tested is instead the combination $\langle T_1, \mu_j \rangle$. Many discussions of the measure problem exploit the freedom in choosing μ_j: the aim is not to find a unique μ dictated by basic theoretical principles, but rather to find some μ_j yielding anthropic predictions compatible with what is observed. The obvious worry is that T_1 can be saved by jerrymandering. Furthermore, the freedom to choose a measure threatens our ability to draw a meaningful contrast between competing theories. In comparing T_1 to T_2, the question is not whether T_1 produces an ensemble that leads to distinctive anthropic predictions given a unique choice of measure. Instead, the assessment would then regard the collections $\langle T_1, \{\mu_j\} \rangle$ and $\langle T_2, \{\mu_k\} \rangle$, where $\{\mu_j\}$ and $\{\mu_k\}$ are sets of "reasonable" measures; we have then tested measures rather than theories. Even if there is an empirical contrast between theories given a specific, unique measure, it seems unlikely to survive as a clear contrast if we consider an entire collection of possible measures.

Finally, there are two challenges posed by (iii) above, the choice of an anthropic subset. The first was mentioned previously: how do we delineate the set $\mathscr{A}$? Suppose we choose to consider a specific reference class, such as "observers" (with some specific physical characterization in mind). What combinations of physical constants realized in a pocket universe will support observers? Aguirre (2001) argues that the number of solar-mass, metal-rich stars as a function of various cosmological parameters has multiple peaks, some quite far from the values for the observed universe. Taking these stars as plausible proxies for "observers," this indicates that the normed number density of observers $N(\alpha_i)$ will be non-zero in widely separated regions of parameter space – leading to a very complex anthropic subset $\mathscr{A}$.[29] The effect of extending the anthropic subset is to lower the probability of a universe like ours, assuming that the prior probability $P_i(\alpha_i)$ assigned to such pockets is non-zero. The verdict in favor of one of the theories can be reversed by discovering new regions of parameter space that fall within its anthropic subset.

taking a position on the physics relevant to generating the multiverse (other than to assume that there is *some* mechanism generating variation among pocket universes). It is unclear whether there is a unique or maximal ensemble satisfying this description, but this point does not seem to be crucial for the argument in the text.

[29] Aguirre explores the consequences of varying cosmological parameters. Considering the physics of a universe with different constants appearing in the Standard Model of particle physics would be much more challenging, but it is plausible that at least one combination of constants quite far from those of the observed universe could support the complexity required for the existence of observers.

A clear contrast between two theories may disappear when they are viewed through the anthropic filter, so to speak. This is the second challenge posed by the choice of an anthropic subset. Anthropic reasoning can convert an apparently disastrous prediction — in the form of a very low prior probability assigned to the observed values of some constants — into a success. For example, take T_2 to be standard hot big bang cosmology restricted to models with infinite spatial sections, regarding the gravitational degrees of freedom at some specified initial time as uncorrelated in spacelike separated regions. This leads to a plain vanilla multiverse Σ_2, with variation only in cosmological parameters, such as the amplitude of density perturbations and the curvature of finite spatial hypersurfaces. The original motivations for inflation can be re-iterated to make an argument in favor of T_1 (a specific implementation of EI) over T_2: surely T_2 assigns a vanishingly small prior probability $P_i(\alpha_i)$ to the observed combination of cosmological parameters, with their delicate balance between overall flatness and small perturbations. Yet, by parity of reasoning, a defender of T_2 can apply anthropic reasoning following the lead of the proponent of T_1. Restricting consideration to the anthropic subset of Σ_2 should raise the probability of the observed cosmological parameters substantially, following Weinberg in treating large gravitationally bound systems as a necessary condition for observers. As before, the details will depend on delicate questions regarding the extent of the anthropic subset. But it is surely the case that many quite distinctive ensembles look the same when restricted to their anthropic subsets. This point can be illustrated using Eddington's example of a selection effect. Eddington noted that fishermen should not infer the absence of fish smaller than the gaps in their nets from their inability to catch them; similarly, they should not conclude that two lakes have similar overall populations of fish based on similar distributions of large fish.

These three aspects of anthropic predictions are implicit in discussions of EI, although ironically they are often regarded as virtues. It is supposedly a virtue of an anthropic prediction to be "robust" under variations in the ensemble Σ and choice of measure. In other words, some features of the ensemble Σ are expected to be quite generic, independent of the finer details of the dynamics. In addition, even if one cannot isolate a unique measure, there may be a class of reasonable measures that yield the same anthropic predictions. Similarly, the effect of choosing a particular anthropic subset is also argued to have a minimal impact. These arguments provide some assurance that anthropic predictions can be made in spite of our ignorance about the relevant fundamental physics and lack of a convincing resolution of the measure problem. Yet these same features insure that anthropic predictions cannot reveal anything in quantitative detail regarding the underlying dynamics producing the multiverse ensemble.

Wilczek (2007) comments that the intrusion of selection arguments in fundamental physics represents a "lowering of expectations": in a multiverse, we will not be able to compare theory and experiment to one part in a billion, as in other historical cases in the development of physics. To be more specific, and perhaps departing from Wilczek's view, anthropic predictions provide little basis for establishing a specific dynamical theory governing the creation of the multiverse. This clear from Weinberg's discussion of Λ (discussed in §3), in which the prediction does not depend on exactly which multiverse hypothesis one accepts. Using the word "prediction" begins to seem almost purely rhetorical. One might hit a less ambitious target, such as arguing that anthropic selection in a multiverse offers better explanations than alternatives such as intelligent design (as in Susskind 2006). Yet such a defense of the multiverse falls far short of the high standard of evidential constraints on theory achieved in other areas of physics. The line of argument above aims to establish that it falls short not only due to the inaccessibility of physics at the appropriate energy scales, which is admittedly an important challenge, but also because anthropic predictions cannot constrain the physics generating the multiverse ensemble.

7 Alternatives

My negative assessment of the probative value of anthropic predictions, given in §5 - §6, does not entirely undermine EI, as there are other arguments in its favor of a quite different character. They are often presented alongside anthropic predictions by proponents of EI, but it seems important to clarify the nature of these other arguments. Here I will briefly discuss four other lines of argument in favor of EI, based on: (i) philosophical considerations, (ii) direct evidence, (iii) elimination of competing ideas, and (iv) treating EI as a consequence of fundamental physics.

Some advocates of the multiverse make their case, partially or completely, based on broadly philosophical or metaphysical considerations, without reference to empirical results. Tegmark's position seems to be primarily based on considerations regarding the applicability of mathematics (see, e.g., his 2007 review). While this line of argument certainly deserves further discussion, I do not have space to assess it here. There are also consequences of EI that apparently do not take the form of anthropic predictions as I have characterized them. The discovery of the imprint of a bubble collision in the CBR, with appropriate characteristics, would provide direct evidence of interactions with another "pocket universe," potentially vindicating aspects of EI (see, e.g., Aguirre and Johnson 2011). Neither of these lines of argument are threatened by my challenges to the evidential value of anthropic predictions.

The third line of argument in favor of the multiverse treats it as a "last resort" (see, e.g., Guth 2007), to be accepted because of the failure of other proposals. Grant for the sake of argument that a fundamental theory with finely-tuned fundamental constants is not acceptable. Thus, as in the case of Λ, we demand a further account for the observed value of the constant, and survey our options. The case in favor of the multiverse based on Λ is perhaps best regarded as an eliminative argument: no existing theory offers a successful dynamical account for the small value of Λ, and there are good reasons to doubt that any will be forthcoming. Once all the alternatives have been eliminated as impossible, the only choice remaining is to accept an anthropic explanation, however improbable, as the truth — namely, the value of Λ is a parochial feature of habitable pocket universes in a multiverse. Multiverse advocates need to make a case that an anthropic explanation should not be eliminated out of hand, as they once were. Whether this style of argument leads to the correct conclusion depends upon whether the best theory was included in the initial survey, or else the eliminative step merely leads to the best of a bad lot.

Two questions arise in assessing such an eliminative argument. First, are criteria for eliminating theories appropriate? In this discussion it is not straightforward incompatibility with observations, but rather the need for unacceptably finely-tuned fundamental constants that is used as the criteria for eliminating theories. This leads to the second question. Have we cast the net broadly enough in drawing up a list of alternatives? The strength of the conclusion depends on whether we have done so.

These are both important questions, but in response to either question anthropic predictions have a limited role. The predictions merely establish that a proposed multiverse is in fact compatible with observed values of some fundamental constant(s). Anthropic predictions function as a compatibility check rather than a source of positive evidence. So, in conclusion, an eliminative argument sidesteps the worries about anthropic predictions raised above, and shifts the burden of proof to motivating the criteria for elimination and providing assurance that the true theory is among those under consideration.

EI can also be defended indirectly as the consequence of an independently well-tested theory, as in the standard account of it as a consequence of inflation reviewed briefly in §2 above. This is a promising line of defense for multiverse theories, and is immune to the argument of the previous two sections insofar as: (1) the case for the multiverse-generating theory does not itself rest on anthropic predictions, and (2) the prediction of the multiverse follows directly from well-tested aspects of the theory. Pursuing both points in relation to inflation and EI leads into a thicket of questions which I will take up in the next section.

8 Inflation and EI Reconsidered

Conventional wisdom holds that EI is a natural consequence of inflation. The empirical success of inflation, briefly reviewed in §2 above, then offers indirect support for EI. In addition, conventional wisdom also maintains that EI answers the initial state problem of inflation, as I will explain below. Against this conventional picture, I will argue that accepting EI undermines the original empirical case in favor of inflation: the consequences of inflation are demoted to anthropic predictions, subject to the critical arguments in §5 - §6 above. One way to avoid this unhappy result is to sever the link between inflation and EI. The section ends by briefly summarizing doubts about the case for EI, based on concerns about extending theories beyond their domain of applicability.

EI is sometimes presented as the answer to inflation's own fine-tuning problem.[30] Inflation's original motivation was to eliminate fine-tuning in the standard big bang model, such as the horizon and flatness problems. But rather than eliminating fine-tuning, inflation apparently pushes it to a different aspect of the theory — namely, the choice of the effective potential $V(\phi)$ and the initial state of the inflaton field ϕ. The effective potential must satisfy various conditions to produce an appropriate spectrum of density perturbations (in standard slow-roll inflation). In order to trigger inflation, the field must be in a uniform state over scales larger than the Hubble radius.[31] Penrose (1986) argued on general thermodynamic grounds that the initial conditions for inflation (granting an appropriate effective potential) must in fact be less probable than the required initial state of the standard big bang model (motivating the work mentioned in §4.1; cf. Penrose 2004, Chapter 28). In response to such criticisms, advocates of EI invoke anthropic selection effects: the form of the potential and the initial condition of the inflaton field at early times must have been appropriate to trigger inflation in our region of the multiverse, to set the conditions necessary for the existence of observers.

Yet this reasoning, once it is accepted, allows for a more direct response to the original fine-tuning problems motivating inflation. As noted above in §6, in a standard FLRW universe with infinite spatial sections, there presumably are regions on the scale of the observed universe with the appropriate initial conditions for the gravitational degrees of freedom. These regions may be deemed "improbable" in some sense; but by parity of reasoning, this is as irrelevant as the "improbability" of the initial state required for ϕ. Taking anthropic selection into account, it is plausible that these

[30] See, e.g., Linde's early discussions of chaotic inflation.

[31] Vachaspati and Trodden (2000) show that inflation requires uniformity over a super-Hubble-radius patch, granted several plausible assumptions; see Goldwirth and Piran (1992) for a systematic review of the initial conditions problem for inflation.

conditions are probable once we condition on an appropriate anthropic subset $\mathscr{A}$.[32] Thus anthropic reasoning can be used to pre-empt inflation, and it is unclear how an advocate of inflation can draw the line between its legitimate use (in the defense against inflation's initial conditions problem) vs. an illegitimate pre-emptive strike.

More importantly, on this approach the "predictability crisis" applies to the familiar "predictions" of inflation. The usual features associated with the output of slow-roll inflation will only be realized in some parts of the multiverse, and the objections above to using anthropic predictions to establish theories apply. To be clear, I take this to be an objection to a particular way of making an empirical case for inflation, based on the solution of fine-tuning problems in big bang cosmology. There is another position put succinctly by Liddle and Lyth (2000, p. 5):

> By contrast to inflation as a theory of initial conditions, the model of inflation as a possible origin of structure in the Universe is a powerfully predictive one. Different inflation models typically lead to different predictions for observed structures, and observations can discriminate strongly between them. ... Inflation as the origin of structure is therefore very much a proper science of prediction and observation.

Here I do not have the space to address whether inflation "as a theory of structure formation" can be subjected to empirical test, while avoiding the concerns raised above regarding anthropic predictions.

These conclusions can be avoided if inflation does not in fact lead to EI. Whether inflation leads to EI is a point of contention in the physics literature. Generally the discussions of how EI arises require extending calculations into regimes in which they may not be reliable. Turok (2002), for example, offers a scathing criticism of the stochastic approach to EI (p. 3459): "The calculations so far presented to justify eternal inflation in fact break every known principle in theoretical physics ..." The stochastic calculations treat the evolution of the inflaton field in a classical background spacetime and ignore the back-reaction of the fields on the spacetime. This may be a useful approximation in the slow-roll regime of inflation, but it is not clear that it leads to reliable results in the fluctuation dominated regime studied in EI. Although I do not have space to pursue the point here, the other ways in which inflation can lead to EI similarly depend on extending existing theories into new domains, where their reliability is doubtful.

9 Conclusions

Papers on EI routinely invoke successful anthropic predictions as powerful evidence in its favor. But the rhetoric of predictive success barely hides a number of pressing questions in foundations of physics and methodology. Cosmologists acknowledge that the measure problem is a substantial obstacle to testing theories via anthropic predictions (§4). I have argued that there are two further obstacles, in §5 and §6, respectively. The first is providing a justification for introducing probabilities in this context, either in the form of objective chances or as epistemic probabilities based on an assumption of "typicality." Second, the evidential value of anthropic predictions is minimal: they may serve as a compatibility check for a particular multiverse theory, but offer no hope of more detailed constraints on the theory. An advocate of EI may take anthropic predictions as a useful side

[32]Note that this response will apply only to properties that vary across different regions of the cosmological model; including, presumably, uniformity and amplitude of perturbations but not the value of the cosmological constant Λ.

benefit of EI rather than the main reason for accepting it. There are four other arguments for EI, described briefly in §7. Finally, the reasoning employed by advocates of EI threatens to undermine the original case in favor of inflation, as discussed in §8.

None of this goes to show that EI is incorrect; we may well live in the EI multiverse. But anthropic predictions alone provide little or no evidence that this is the case.

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