Two Mereological Arguments against the Possibility of an Omniscient Being

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I. Introduction

Not long ago Patrick Grim presented several arguments for the conclusion that there cannot be an omniscient being.¹ These arguments seem to have important consequences for theories of quantification, properties and truth. It is not an overstatement to say that they deserve a significant amount of attention. However, I will not be considering the consequences of these arguments for theories of quantification, properties and truth in this paper. Instead, I wish to present two new arguments against the possibility of an omniscient being. My new arguments, like several of Grim’s arguments, invoke considerations of cardinality. I will argue, as Grim has, that there must be more objects in the universe than there are beliefs. Unlike Grim, though, my arguments will rely on certain mereological claims. I will refer to these two arguments, respectively, as “the Simple Argument” and as “the Complex Argument”. I use these expressions for two reasons. First, the Simple Argument requires an assumption about the mereological structure of certain beliefs that the Complex Argument doesn’t require. That assumption is that an omniscient being’s beliefs are mereological simples. That is, that an omniscient being’s beliefs have no proper parts. This assumption makes the Simple Argument easier to present and understand than the Complex Argument. The fact that the Simple Argument is easier to present and understand than the Complex Argument is the second reason for their particular names.

In this paper, I first present some preliminary assumption which will play important roles in each of arguments that follow. Then I present the Simple Argument against the possibility of an omniscient being. Next, I drop the assumption that beliefs are mereologically simple and present the Complex Argument against the possibility of an omniscient being. Finally, I note some specific consequences of these arguments for certain theistic views and make some closing remarks about my own views concerning these arguments.

II. Preliminaries

As I mentioned, the problems that I wish to present rely on certain mereological claims. The first, and least controversial of these claims is that the axioms of Ground Mereology are necessarily true of the part-whole relation. These axioms are as follows:

- **Reflexivity**: Everything is a part of itself.
- **Transitivity**: if x is a part of y and y is a part of z, then x is a part of z.
- **Anti-symmetry**: if x is a part of y and y is a part of x, then x=y.\(^2\)

Slightly more controversial, but still widely held, is that the following Strong Supplementation Principle is necessarily true of the part-whole relation:

- **Strong Supplementation Principle**: if x is not a part of y, then there is a part of x that does not overlap y.

\(^2\) These axioms are rejected by some endurantists. They will contend that the part-whole relation is a three place relation which requires a time as one of the relata. See, for example, Thomson (1983). This dispute has little bearing on the arguments that I wish to present. With appropriately time relativized versions of all of my assumptions, arguments similar to those that I am going to present may be formulated. Moreover, the conclusions of these temporally relativized arguments are as devastating to the possibility of an omniscient being as the official versions that appear below.
Finally, and perhaps most controversial is the view that Unrestricted Composition is necessarily true of the part-whole relation. With the help of plural quantifiers we may formulate Unrestricted Composition as follows:

Unrestricted Composition: For any objects, xx, there is an object y such that xx compose y.

The axioms of Ground Mereology in conjunction with the Strong Supplementation Principle and Unrestricted Composition together form Classical Exensional Mereology. My assumptions, then, amount to the claim that the axioms of Classical Extensional Mereology are necessarily true of the part-whole relation. This assumption plays a key role in the arguments that follow.

In addition to the mereological assumption above, I will also be assuming the following necessary condition on omniscience:

Omniscience Condition: Necessarily, if x is omniscient, then for every y, x believes that y exists.³

I take it that this condition is widely accepted. It follows from two claims. The first claim is that necessarily, if a being is omniscient, then everything is such that the being knows of it that it exists.⁴ The second claim is that necessarily, if someone knows something, then he believes it. It seems to me that whatever omniscience amounts to, it must be that the Omniscience Condition is true. I will make extensive use of this claim in the arguments that follow.

III. The Simple Argument

³ Or, if you prefer, necessarily, if x is omniscient, then for every y, x believes that y is self identical. My arguments will go through with either formulation of the Omniscience Condition.
⁴ This, of course, follows from the claim that necessarily, an omniscient being knows every true proposition.
The Simple Argument is an instance of a problem first noted by Gideon Rosen.\(^5\) Rosen’s considerations show that given Classical Extensional Mereology and the thesis that there are at least two things, there is no function satisfying the following conditions:

(i) For all \(x\), there is a \(y\) such that \(y = f(x)\).

(ii) For all \(x\) and for all \(y\), if \(f(x) = f(y)\), then \(x = y\).

(iii) For all \(x\) and for all \(y\), if \(y = f(x)\), then there is no \(z\) such that \(z\) is a part of \(y\).\(^6\)

What I wish to show is that given Classical Extensional Mereology, there is no omniscient being. The reason is that, roughly, if there were an omniscient being, then there would be more than two things and there would be a function satisfying the conditions given above. That function would be the function which takes as arguments any particular object whatsoever and gives as values the omniscient being’s belief that that particular object exists.

I will depart from Rosen, however, in the following respect. Instead of talking about a function satisfying the condition above, I will talk about a relation that satisfies relevantly similar conditions. This departure is beneficial because it is false that for any relation, there is a set of individuals that fall under the first relata of that relation. After all there are relations under whose scope absolutely everything must fall. For example, identity is a relation and absolutely everything is identical to itself. However, it is widely

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\(^6\) Rosen’s claim is that Unrestricted Composition is incompatible with the existence of any function satisfying the three conditions above. However, it seems that in addition to Unrestricted Composition there must be a principle which will guarantee that there are no objects \(xx\) and objects \(yy\) such that \(xx\) and \(yy\) both compose some particular object when \(xx\) and \(yy\) are wholly distinct. The Strong Supplementation Principle will guarantee that this sort of situation does not arise. Moreover, Rosen does not make explicit that there must be at least two things. However, it is clear that given a domain with a single object, a mereological simple, the identity function will satisfy each of the three conditions above and the axioms of classical extensional mereology will be satisfied as well.
accepted that for any function, there must be the set of all things which are arguments of that function. If this is correct, then there is no identity function on absolutely everything. This is because there is no set of everything. It is important to make this departure because the arguments that I present require talk about absolutely everything.

The Simple Argument proceeds in three steps. The first step is to establish that necessarily, if there is an omniscient being, then there are at least two objects. This is simple to prove. It must be the case that if there is an omniscient being, then there is at least one belief that that being has; it must be the case that an omniscient being believes that he exists. This follows from the Omniscience Condition given above. But, clearly, it must be the case that if an omniscient being believes that he exists, then there are at least two things, namely the being itself and his belief that he exists. So, necessarily, if there is an omniscient being, then there are at least two things.

The second step is to establish that necessarily, if there is an omniscient being, then there are at least as many mereological simples as there are objects in the universe. To begin, we note that, given the Omniscience Condition, it must be the case that an omniscient being believes of each thing that it exists. Moreover, it must be the case that for any two distinct things, the belief of one that it exists is distinct from the belief of the other that it exists. This follows from the fact that one belief is about one object and the other belief is about a distinct object. So, necessarily, if there is an omniscient being, then for each thing, the being has a belief that that thing exists and each of his beliefs about the existence of some particular object is distinct from any of his beliefs of a different particular object that it exists. It follows from this, and the fact established in our first step, that necessarily, if there is an omniscient being, then that being has at least
as many beliefs as there are objects in the universe. Moreover, given our assumption that an omniscient being’s beliefs are mereologically simple, it must be the case that those particular existential beliefs are mereologically simple. So, necessarily, if there is an omniscient being, then there are at least as many mereological simples as there are beliefs.

The final step of the Simple Argument is to note that if Classical Extensional Mereology is necessarily true of the part-whole relation, it cannot be the case that there are at least as many mereological simples as there are objects in the universe. This is because Classical Extensional Mereology establishes that the structure of the part-whole relation is a complete Boolean algebra without the zero element. A consequence of this fact is that the number of objects in the universe is $2^k - 1$, where $k$ is the number of mereological atoms in the universe. But, for any cardinality, $k$ (even infinite ones), $2^k - 1$ is a strictly greater cardinality than $k$. So, if Classical Extensional Mereology is necessarily true of the part-whole relation, as we have assumed, then there must be more objects in the universe than there are mereological simples. If that is the case, then it must be false that there are as many simples as there are objects in the universe. But, we just established that necessarily, if there is an omniscient being, then there are as many simples as there are objects in the universe. So, of course, it follows from these facts that necessarily, there is no omniscient being.

IV. The Complex Argument

Some people might think that beliefs have mereological structure of some kind. If this is the case, then the argument presented in the last section is no good. It is no good

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7 This is proven in Tarski (1935).
8 That is, any cardinality, $k$, greater than 1 is such that $2^k - 1$ is a strictly greater cardinality than $k$. 
because it relies on the false assumption that beliefs are mereologically simple. However, although it is true that if beliefs are complex, then the argument of the last section is unsound, it is still the case that omniscience is problematic given Classical Extensional Mereology. This is because there is an argument, similar to that of the last section, which shows that regardless of whether beliefs are complex or simple, there cannot be an omniscient being. This is the Complex Argument which I will present in this section.

We have already seen that if there is an omniscient being, then there must be at least as many beliefs as there are objects in the world. Remember that this is because according to the Omniscience Condition, it must be the case that for each object in the world, an omniscient being believes of that object that it exists. However, it can be shown that even if an omniscient being has beliefs that are mereologically complex, there must be more objects in the world than there are beliefs of that being. This argument requires an assumption about the structure of beliefs. That assumption is that any mereological sum of some of an omniscient being’s existential beliefs does not completely decompose into some distinct existential beliefs. But this assumption is plausible enough. It seems implausible to suppose that some omniscient being has some existential beliefs, say that Rochester exists and that Nicholas exists, and that the sum of those beliefs has a complete decomposition into distinct existential beliefs, say that the Eiffel Tower exists and that the Empire State Building exists. There just cannot be that degree of overlap between existential beliefs. But with this assumption, we can show that there cannot be an omniscient being.
The first step of the Complex Argument is to show that necessarily, if there is an omniscient being, then there are at least as many objects as there are pluralities of objects altogether. First, consider an arbitrarily chosen possible world. Next, consider some arbitrarily chosen objects. I will refer to these objects with the plural term ‘aa’. Now consider some arbitrarily chosen objects which are distinct from aa. I will refer to the things amongst this second plurality with the plural term ‘bb’. These objects don’t have to be wholly distinct from aa. It’s okay if, for example, some of bb are amongst aa as long as there is some object that is amongst one of them but not amongst both. Now it must be the case that if there is an omniscient being, then for each of these pluralities there corresponds a particular plurality of existential beliefs of that omniscient being. For example, there are the beliefs which are such that each of them is an existential belief about one of the objects amongst aa. I will refer to these with the phrase ‘the aa-beliefs’. Similarly, there are the beliefs which are such that each of them is an existential belief about one of the objects amongst bb. I will refer to these with the phrase ‘the bb-beliefs’.

Given Unrestricted Composition, there is an object that the aa-beliefs compose and there is an object that the bb-beliefs compose. Moreover, these objects are distinct. Remember that we chose out pluralities of objects carefully. We made sure that aa and bb were distinct pluralities. That is, we made sure that there is at least one object that is amongst one of the pluralities but not amongst both. But it follows from this that there is at least one of the omniscient being’s beliefs which is amongst either the aa-beliefs or the bb-beliefs but not amongst both. We also assumed that any mereological sum of some of an omniscient being’s existential beliefs does not completely decompose into some distinct existential beliefs. These two facts are enough to guarantee that a mereological
sum of the aa-beliefs is distinct from a mereological sum of the bb-beliefs. But since the pluralities, aa and bb, were arbitrarily chosen it follows that for any plurality of objects, there exists a corresponding mereological sum of beliefs. Moreover, none of these mereological sums is the same as any of the others. But these two things entail that there are at least as many objects as there are pluralities of objects altogether. So, if there is an omniscient being, then there are at least as many objects as there are pluralities of objects altogether. Finally, since our investigation started with an arbitrarily chosen possible world, we can conclude that necessarily, if there is an omniscient being, then there are at least as many objects as there are pluralities altogether.

Unfortunately, this conclusion is troublesome for omniscience. There is a simple plural variation on Cantor’s theorem which proves that if there are at least two things, then there are strictly more pluralities of things than there are things altogether.\(^9\) Given that this theorem is necessarily true, it follows that necessarily, if there are more than two things, then there are strictly more pluralities of things than there are things altogether. In the last section we proved that necessarily, if there is an omniscient being, then there are at least two things. But, of course, it follows from these facts and the conclusion of the last paragraph that there cannot be an omniscient being.\(^{10}\)

V. **Problems for Certain Kinds of Theism**

According to classical theism there is an omniscient being, namely God. But, it seems we have just established that there cannot be an omniscient being. So, it seems that classical theism is false. We might wonder, though, whether a particular kind of

\(^9\) See, Shapiro (1991) for a proof of a second order version of Cantor’s Theorem. The second order variables in this formulation may be read as plural quantifiers and predication may be taken as the ‘amongst’ relation which bears between the predicated object and the plurality over which the second order variables range. See Boolos (1984) for a plural interpretation of second order variables.

\(^{10}\) Thanks to Gabriel Uzquiano for extensively discussing the argument of this section with me.
theism is salvageable. For example, it might be that there is no omniscient being, but perhaps there is a nearly omniscient being.

Some kinds of theism, which have been recently defended, claim that, although God is not omniscient, God is disposed to know all truths or is infallible in his judgements.\(^\text{11}\)\(^\text{12}\) Whatever the merits of these views, it seems to me that they are subject to the same kind of arguments as those that I have presented in the text. The reason these views are subject to the same kind of arguments is that it must be the case if God is disposed to know or is infallible, then there is something that grounds these dispositions or God’s infallibility on particular matters of fact. Perhaps it is a state of God or perhaps it is a state the world is in. It could even be states of the world that are exemplified in virtue of the fact that the world stands in certain brute relations to other possible worlds. But it must be that there is something that makes it true that, for any particular proposition, God is infallible with respect to that proposition (or is disposed to know that proposition). But if there are such things, then we can reformulate the arguments of the last section to apply to these states rather than to beliefs.\(^\text{13}\)

\(^{11}\) See, Cullison (Forthcoming) for a defense of the view that God is disposed to know all truths and see Hasker (1989) for a defense of the view that God is infallible.

\(^{12}\) Depending on how dispositions and infallibility are analyzed, it may be that there are some peculiar propositions that God simply cannot come to know. For example, suppose we give a counterfactual account of being disposed to know. We say that for any proposition \(P\), if God were to consider \(P\), then God would know whether \(P\) is true. Now suppose that there is a very broad attitude that a being can bear toward an object whenever he bears an attitude relation to any proposition that is about that object. Call this attitude ‘thinking about’. Now consider the proposition that God is not thinking about Nicholas at all. This proposition may be true. Moreover, if God were to consider that proposition, then he would be thinking about Nicholas (in our very broad sense of the term). So, if God were to consider the proposition, then he would falsify it. God would of course know that it is false if he were to consider it. But what is strange is that if that particular proposition is actually true, then God could never know that it is true. We can either accept this consequence or give up on the proposed analysis of being disposed to know.

\(^{13}\) As mentioned in the last footnote, there might be some peculiarities. It might be the case that these peculiarities save these views from the arguments like those that I have presented in the text. For example, suppose that there is a true proposition that would be false if God were to consider it. Then there is the concrete state of affairs of that proposition’s being true. One might argue that if that proposition were false, then that state of affairs would not exist. If that is the case, then the peculiarities will carry over to the existence of things rather than just truths and the arguments of the last section will be spoiled.
Some people might be happy with the view that necessarily, there are some minor things that God does not know. Maybe knowing almost everything is good enough for God. Although this may be a plausible response when faced with knower paradoxes and other puzzles of omniscience, I find it implausible when it is made as a response to the arguments presented in this paper.

To see why this response seems implausible to me, consider again the arguments of the previous sections. With slight modifications, the conclusion of these arguments can be stated as follows. Necessarily, for any being there is at least one thing such that that being does not know of it that it exists. But that means that necessarily, there is at least one thing such that God does not know of it that it exists. This could be anything. Perhaps it is my shirt, or the Eiffel Tower or some widely scattered or very distant object. But the conclusion is stronger than that. Consider the property of being either blue or not blue. Like existence, this is a property that, necessarily, everything has. So, necessarily, for every object there is the proposition that that object is either blue or not blue. But, with a slightly different omniscience condition, we can reformulate the arguments from the last sections to show that necessarily, there is something such that God does not believe of it that it is either blue or not blue.\(^\text{14}\) This again could be anything, Rochester NY, the Empire State Building or even my brother, Nicholas. In fact, there are an infinite number of \textit{de re} beliefs that God does not have; at least one for every property had by every object. Moreover, there is nothing in the argument to show that, for example, the

\(^\text{14}\) \textit{Omniscience Condition*}: Necessarily, if x is omniscient, then for every object, y, x believes that y is either blue or not blue.
thing God does not believe exists is the same as the thing God does not believe is either blue or not blue.\textsuperscript{15}

Finally, unless the thing that God does not believe exists is a necessarily existing thing, it must be that possibly, there is a distinct thing such that God does not believe that it exists. But it is plausible to think that necessarily, for every necessarily existing thing, God believes that it exists. So, it is plausible to conclude that necessarily there is a thing, x, and God does not believe that x exists and possibly there is a y such that y is distinct from x and God does not believe that y exists.\textsuperscript{16}

Suppose that it is true that necessarily, there is a thing, x, and God does not believe that x exists and possibly there is a y such that y is distinct from x and God does not believe that y exists. Suppose that God does not know that some distant particle, Charlie, exists. Then if Charlie had not existed, then God would fail to know of something else that it exists. But surely if Charlie had not existed, then everything which in fact exists would have existed and nothing else would have existed. But if those counterfactuals are true, then there is something, x, such that God knows that it exists and if Charlie had not existed, then God would not know that x exists.\textsuperscript{17} But how can the existence of some distant particle play such an important role in whether or not God

\textsuperscript{15} Of course, assuming that God is perfectly rational, he should be able to derive that a thing exists if he believes that it is either blue or not blue. So, plausibly, there is a single thing that God does not have any de re beliefs about.

\textsuperscript{16} Similar principles will follow for each of the other troublesome properties such as being blue or not blue.

\textsuperscript{17} This argument is sound given a linear or strict ordering on possible worlds in the semantics for the logic of counterfactuals. However, if a Lewis is correct and there is only a weak ordering on possible worlds in the semantics for counterfactuals, then the conclusion is slightly different. Given a weak ordering, the conclusion is that if those counterfactuals are true, then there is something, x, such that God knows that it exists and if Charlie had not existed, then God might not know that x exists. This is because there might be a world where God does not know of one thing that it does not exist and an equally close world where God does not know of a distinct thing that it exists. Moreover, if everything is such that there is a world where God does not know that it exists and each of these worlds is equally close, then we may conclude the following. If those counterfactuals are true, then anything (distinct from Charlie), x, is such that God knows that it exists and if Charlie had not existed, then God might not know that x exists. But this seems like a bad conclusion as well.
knows of some other thing that it exists? It seems that if all the considerations thus far are correct, there is a strange epistemic dependence relation between some objects and God’s beliefs about other objects.

Each of the above facts seems quite unpleasant. However, these facts only scratch the surface of the problems that this argument poses for theism. Some people believe that God has made it such that every morally good person shall be rewarded. But our arguments can be reformulated to show that necessarily, there is something that is such that God does not believe of it that if it is a morally good person, then it shall be rewarded. Similarly, many Christians believe that anyone who believes in Christ will have eternal life. But our arguments can be reformulated to show that necessarily, there is something such that God does not believe of it that if it is a person who believes in Christ, then it will have eternal life. Moreover, it could be the case that God believes that everything is such that if it is a person who believes in Christ, then it will have eternal life. But he cannot make the simple valid inference for the conclusion that some particular thing is such that if it is a person who believes in Christ, then it will have eternal life.  

There is one last thing that I will note before concluding this section. All of the arguments in this paper are given in terms of beliefs. But the arguments could have been

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18 Given a latitudinarian view of de re belief, God could simply come to have a de re belief about the object that he has no beliefs about simply by giving it a name or employing Kaplan’s dthat device. So, for example, God could say “let the object which is such that I do not believe of it that it exists be called ‘Matilda”’. Once he has given it a name he can come to believe that if there is a single object which is such that he does not believe of it that it exists, then Matilda exists. Moreover, since he can run through the arguments of this paper very easily, he can come to know that there is a single object which is such that he does not believe of it that it exists. Then, by simple logic, he can conclude that Matilda exists. There are several things that we could say about this. One thing we can say is that God can perform such a trick, but if he were to do so, then he would lose one of his other beliefs. Other options are similarly strange. Perhaps the best option is to say that latitudinarianism is false. But it seems drastic to conclude that latitudinarianism is false simply to save a strange kind of theism and Classical Extensional Mereology.
given for other propositional attitude states as well. For example, one consequence for theism is that God cannot desire of everything that it is part of a morally good world. Every single propositional attitude state is similarly limited.

So it seems that God cannot have beliefs about everything, cannot have desires about everything, has attitude states that are strangely dependent upon entities that are seemingly unrelated to those states and cannot make simple valid inferences. These, though, seem like unduly strong limitations on God. These are just some of the reasons why it seems unacceptable to me for a theist to accept the conclusions of the arguments and claim that they impose minor limitations on God.

VI. Conclusion

In this paper I have presented two arguments for the conclusion that there cannot be an omniscient being. These arguments relied on some plausible assumptions and on the necessary truth of the axioms of Classical Extensional Mereology as applied to the part-whole relation. I have also noted how particularly troublesome these consequences are to certain theists. Throughout this paper my arguments have been formulated as arguments against the possibility of an omniscient being. However, these particular formulations do not accurately reflect my current view on the topics at hand. I believe that possibly there is an omniscient being. I believe that, at the very least, possibly there is a being that knows of each thing that it exists. Given these facts and the key premises in the arguments presented in this paper, I believe that the best thing to conclude is that Classical Extensional Mereology is not necessarily true of the part-whole relation. Since Ground Mereology is surely necessarily true, it seems that either Unrestricted Composition does not hold necessarily or that the Strong Supplementation Principle does
not hold necessarily. However, I leave it for further investigation which of these two principles to reject.\footnote{Thanks to Greg Fowler, Hud Hudson and Gabriel Uzquiano for reading and commenting on earlier drafts of this paper.}
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