Language and the Self-Reference Paradox

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Heinz Von Forester characterizes the objects "known" by an autopoietic system as eigen-solutions, that is, as discrete, separable, stable and composable states of the interaction of the system with its environment. Previous articles have presented the FBST, Full Bayesian Significance Test, as a mathematical formalism specifically designed to access the support for sharp statistical hypotheses, and have shown that these hypotheses correspond, from a constructivist perspective, to systemic eigen-solutions in the practice of science. In this article several issues related to the role played by language in the emergence of eigen-solutions are analyzed. The last sections also explore possible connections with the semiotic theory of Charles Sanders Peirce.

> *"If the string is too tight it will snap, but if it is too loose it will not play."*
> Siddhartha Gautama

> *"The most beautiful thing we can experience is the mysterious. It is the source of all true art and all science. He to whom this emotion is a stranger, who can no longer pause to wonder and stand rapt in awe, is as good as dead: His eyes are closed."*
> Albert Einstein

1 Introduction

In Stern (2007a) it is shown how the eigen-solutions found in the practice of science are naturally represented by statistical sharp hypotheses. Statistical sharp hypotheses are routinely stated as natural *laws*, conservation *principles* or invariant *transforms*, and most often take the form of functional equations, like $h(x)=c$. The article also discusses why the eigen-solutions' essential properties of discreteness (sharpness), stability, and composability, indicate that considering such hypotheses in the practice of science is natural and reasonable. Surprisingly, the two standard statistical theories for testing hypotheses, classical (frequentist p-values) and orthodox Bayesian (Bayes factors), have well known and documented problems for handling or interpreting sharp hypotheses. These problems are thoroughly reviewed, from statistical, methodological, systemic and epistemological perspectives.

The same article presents the FBST, or Full Bayesian Significance Test, an unorthodox Bayesian significance test specifically designed for this task. The mathematical and statistical properties of the FBST are carefully analyzed. In particular, it is shown how the FBST fully supports the test and identification of eigen-solutions in the practice of science, using procedures that take into account all the essential characteristics pointed by Von Forester. In contrast to some alternative belief

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calculi or logical formalisms based on discrete algebraic structures, the FBST is based on continuous statistical models. This makes it easy to support concepts like sharp hypotheses, asymptotic convergence and stability, and these are essential concepts in the representation of eigen-solutions. The same article presents cognitive constructivism as a coherent epistemological framework that is compatible with the FBST formalism, and vice-versa. We will refer to this setting as the Cognitive Constructivism plus FBST formalism, or CogCon+FBST framework for short.

The discussion in Stern (2007a) raised some interesting questions, some of which we will try to answer in the present article. The first question relates to the role and the importance of language in the emergence of eigen-solutions and is discussed in section 2. In answering it, we make extensive use of the William Rasch "two-front war" metaphor of cognitive constructivism, as exposed in Rasch (2000). As explained in section 4, this is the war against dogmatic realism at one front, and against skepticism or solipsism, at the second. The results of the first part of the paper are summarized in section 5. To illustrate his arguments, Rasch uses some ideas of Niels Bohr concerning quantum mechanics. In section 3, we use some of the same ideas to give concrete examples of the topics under discussion. The importance (and also the mystery) related to the role of language in the practice of science was one of the major concerns of Bohr's philosophical writings, see Bohr (1987, I-IV), as exemplified by his famous "dirty dishes" metaphor:

> Washing dishes and language can in some respects be compared. We have dirty dishwater and dirty towels and nevertheless finally succeed in getting the plates and glasses clean. Likewise, we have unclear terms and a logic limited in an unknown way in its field of application – but nevertheless we succeed in using it to bring clearness to our understanding of nature. Bohr (2007)

The second question, posed by Søren Brier, which asks whether the CogCon+FBST framework is compatible with and can benefit from the concepts of Semiotics and Peircean philosophy, is addressed in section 6. In section 7, I present my final remarks.

Before ending this section a few key definitions related to the concept of eigensolution are reviewed. As stated in Maturana and Varela (1980), the concept of recurrent state is the key to understand the concept of cognitive domain in an autopoietic system.

> Living systems as units of interaction specified by their conditions of being living systems cannot enter into interactions that are not specified by their organization. The circularity of their organization continuously brings them back to the same internal state (same with respect to the cyclic process). Each internal state requires that certain conditions (interactions with the environment) be satisfied in order to proceed to the next state. Thus the circular organization implies the prediction that an interaction that took place once will take place again. If this does not happen the system maintains its integrity (identity with respect to the observer) and enters into a new prediction. In a continuously changing environment these predictions can only be successful if the environment does no change in that which is predicted. Accordingly, the predictions implied in the organization of the living system are not predictions of particular events, but of classes of inter-actions. Every interaction is a particular interaction, but every prediction is a prediction of a class of

interactions that is defined by those features of its elements that will allow the living system to retain its circular organization after the interaction, and thus, to interact again. This makes living systems inferential systems, and their domain of interactions a cognitive domain. (Maturana & Varela, 1980, p.10)

The epistemological importance of this circular (cyclic or recursive) regenerative processes and their eigen (auto, equilibrium, fixed, homeostatic, invariant, recurrent, recursive) states, both in concrete and abstract autopoietic systems, are further investigated in Von Forester and Segal:

The meaning of recursion is to run through one's own path again. One of its results is that under certain conditions there exist indeed solutions which, when reentered into the formalism, produce again the same solution. These are called "eigen-values," "eigen-functions," "eigen-behaviors," etc., depending on which domain this formation is applied -in the domain of numbers, in functions, in behaviors, etc. (Von Forester & Segal, 2001, p. 145)

Objects are tokens for eigen-behaviors. Tokens stand for something else. In exchange for money (a token itself for gold held by one's government, but unfortunately no longer redeemable), tokens are used to gain admittance to the subway or to play pinball machines. In the cognitive realm, objects are the token names we give to our eigen-behavior. When you speak about a ball, you are talking about the experience arising from your recursive sensorimotor behavior when interacting with that something you call a ball. The "ball" as object becomes a token in our experience and language for that behavior which you know how to do when you handle a ball. This is the constructivist's insight into what takes place when we talk about our experience with objects. (Von Forester & Segal, 2001, pp. 127-128)

Von Forester also establishes several essential characteristics of these eigen-solutions, as quoted in the following paragraph from Von Forester. These essential characteristics can be translated into very specific mathematical properties, that are of prime importance when investigating several aspects of the CogCon+FBST framework.

Eigenvalues have been found ontologically to be discrete, stable, separable and composable, while ontogenetically to arise as equilibria that determine themselves through circular processes. Ontologically, Eigenvalues and objects, and likewise, ontogenetically, stable behavior and the manifestation of a subject's "grasp" of an object cannot be distinguished. (Von Forester, 2003c, p. 266)

2 Eigen-solutions and Language

Goudsmit (1998, Objects as warrants for eigenvalues), finds an apparent disagreement between the form in which eigen-solutions emerge, according to Von Forester and Maturana:

Generally, Von Forester's concept of eigenvalue concerns the value of a function after a repeated (iterative) application of a particular operation. ... This may eventually result in a stable performance, which is an eigenvalue of the observer's behavior. The emerging objects are warrants of the existence of these eigenvalues. ... contrary to Von Forester, Maturana considers the

consensuality of distinctions as necessary for the bringing forth of objects. It is through the attainment of consensual distinctions that individuals are able to create objects in language. (Goudsmit, 1998, sec. 2.3.3)

Confirmation for the position attributed by Goudsmit to Von Forester can be found in several of his articles. In Von Forester, for example, one finds:

I propose to continue the use of the term 'self-organizing system,' whilst being aware of the fact that this term becomes meaningless, unless the system is in close contact with an environment, which possesses available energy and order, and with which our system is in a state of perpetual interaction, such that it somehow manages to 'live' on the expenses of this environment. ... So both the self-organizing system plus the energy and order of the environment have to be given some kind of pre-given objective reality for this view points to function.(Von Foerster, 2003a, p. 3)

Confirmation for the position attributed by Goudsmit to Maturana can also be found in several of his articles. In Maturana, for example, one finds:

Objectivity. Objects arise in language as consensual coordinations of actions that in a domain of consensual distinctions are tokens for more basic coordinations of actions, which they obscure. Without language and outside language there are no objects because objects only arise as consensual coordinations of actions in the recursion of consensual coordinations of actions that languaging is. For living systems that do not operate in language there are no objects; or in other words, objects are not part of their cognitive domains. ... Objects are operational relations in languaging. (Maturana (1988, sec. 9, p. iv)

The standpoint of Maturana is further characterized in the following paragraphs from Brier:

The process of human knowing, is the process in which we, through languaging, create the difference between the world and ourselves; between the self and the non-self, and thereby, to some extent, create the world by creating ourselves. But we do it by relating to a common reality which is in some way before we made the difference between 'the world' and 'ourselves' make difference, and we do it on some kind of implicit belief in a basic kind of order 'beneath it all'. I do agree that it does not make sense to claim that the world exists completely independently of us. But on the other hand it does not make sense to claim that it is a pure product of our explanations or conscious imagination." "...it is clear that we do not create the trees and the mountains through our experiencing or conversation alone. But Maturana is close to claim that this is what we do.(Brier, 2005, p. 374)

In order to understand the above comments, one must realize that Maturana's viewpoints, or at least his rhetoric, changed greatly over time, ranging from the ponderate and precise statements in Maturana and Varela (1980), to some extreme positions assumed in Maturana (1991, p. 30-52, see next paragraph). Maturana must have had in mind the celebrated quote by Albert Einstein at the beginning of this article.

Einstein said, and many other scientists have agreed with him, that scientific theories are free creations of the human mind, and he marveled that through them one could understand the universe.

The criterion of validation of scientific explanation as operations in the praxis of living of the observer, however, permit us to see how it is that the first reflection of Einstein is valid, and how it is that there is nothing marvelous in that it is so. (Maturana, 1991, p. 36)

Scientific explanations arise operationally as generative mechanisms accepted by us as scientists through operations that do not entail or imply any supposition about an independent reality, so that in fact there is no confrontation with one, nor is it necessary to have one even if we believe that we can have one. (Maturana, 1991, p. 38)

Quantification (or measurements) and predictions can be used in the generation of a scientific explanation but do not constitute the source of its validity. The notions of falsifiability (Popper), verificability, or Confirmation would apply to the validation of scientific knowledge only if this were a cognitive domain that revealed, directly or indirectly, by denotation or connotation, a transcendental reality independent of what the observer does. (Maturana, 1991, p. 38)

Nature is an explanatory proposition of our experience with elements of our experience. Indeed, we human beings constitute nature with our explaining, and with our scientific explaining we constitute nature as the domain in which we exist as human beings (or languaging living systems). (Maturana, 1991, p. 44)

Brier further contrasts the standpoint of Maturana with that of Von Forester:

Von Forester is more aware of the philosophical demand that to put up a new epistemological position one has to deal with the problem of solipsism and of pure social constructivism." "The Eigenfunctions do not just come out of the blue. In some, yet only dimly viewed, way the existence of nature and its 'things' and our existence are intertwined in such a way that makes it very difficult to talk about. Von Forester realizes that to accept the reality of the biological systems of the observer leads into further acceptance about the structure of the environment. (Brier, 2005, p. 375)

While the position adopted by Von Forester appears to be more realistic or objective, the one adopted by Maturana seems more idealistic or (inter) subjective. Can these two different positions, which may seem so discrepant, be reconciled? Do we have to choose between an idealistic or a realistic position, or can we rather have both? This is one of the questions we address in the next sections.

In Stern (2007a) we used an example of physical eigen-solution (physical invariant) to illustrate the ideas in discussion, namely, the speed of light constant, c. Historically, this example is tied to the birth of Special Relativity theory, and the debacle of classical physics. In this article we will illustrate them with another important historical example, namely, the Einstein-Podolsky-Rosen paradox. Historically, this example is tied to questions concerning the interpretation of quantum mechanics. This is one of the main topics of the next section.

3 The Languages of Science

At the end of the 19th century, classical physics was the serene sovereign of science. Its glory was consensual and uncontroversial. However, at the beginning of the 20th century, a few experimental results challenged the explanatory power of classical physics. The problems appeared in two major fronts that, from a historical

perspective, can be linked to the theories (at that time still non existent) of special relativity and quantum mechanics.

At that time, the general perception of the scientific community was that these few open problems could, should and would be accommodated in the framework of classical physics. Crafting sophisticated structural models such as those for the structure of ether (the medium in which light was supposed to propagate), and those for atomic structure, was typical of the effort to circumvent these open problems by artfully maneuvering classical physics. But physics and engineering laboratories insisted, building up a barrage of new and challenging experimental results.

The difficulties with the explanations offered by classical physics not only persisted, but also grew in number and strength. In 1940 the consensus was that classical physics had been brutally defeated, and relativity and quantum mechanics were acclaimed as the new sovereigns. Let us closely examine some facts concerning the development of quantum mechanics (QM).

One of the first steps in the direction of a comprehensive QM theory was given in 1924 by Louis de Broglie, who postulated the particle-wave duality principle, which states that every moving particle has an associated *pilot wave* of wavelength $\lambda = h/mv$, where h is Planck's constant and mv is the particle's momentum, that is, the product of its mass and velocity. In 1926 Erwin Schrödinger stated his wave equation, capable of explaining all known quantic phenomena, and predicting several new ones that where latter confirmed by new experiments. Schrödinger theory is known as orthodox QM, see Tomonaga (1962) and Pais (1988) for detailed historical accounts. Orthodox QM uses a mathematical formalism based on a complex wave equation, and shares much of the descriptive language of de Broglie's particle-wave duality principle.

There is, however, something odd in the wave-particle descriptions of orthodox QM. When describing a model we speak of each side of a double faced wave-particle entity, as if each side existed by itself, and then inextricably fuse them together in the mathematical formalism. Quoting Cohen:

> Notice how our language shapes our imagination. To say that a particle is moving in a straight line really means that we can set up particle detectors along the straight line and observe the signals they send. These signals would be consistent with a model of the particle as a single chunk of mass moving (back and forth) in accordance with Newtonian particle physics. It is important to emphasize that we are not claiming that we know what the particle is, but only what we would observe if we set up those particle detectors.(Cohen, 1989, p. 87)

From Schrödinger's equation we can derive Heisenberg's uncertainty principle, which states that we can not go around measuring everything we want until we pin down every single detail about (the classical entities in our wave-particle model of) reality. One instance of the Heisenberg uncertainty principle states that we can not simultaneously measure a particle position and momentum beyond a certain accuracy. One way of interpreting this instance of the Heisenberg uncertainty principle goes as follows: In classical Newtonian physics our particles are "big enough" so that our measurement devices can obtain the information we need about the particle without

disturbing it. In QM, on the other hand, the particles are so small that the measurement operation will always disturb the particle. For example, the light we have to use in order to illuminate the scene, so we can see where the particle is, has to be so strong, relative to the particle size, that it "blows" the particle away changing its velocity. The consequence is that we cannot (neither in practice, nor even in principle) simultaneously measure with arbitrary precision, both the particle's position and momentum. Hence, we have to learn how to tame our imagination and constrain our language.

The need to exercise a strict discipline over what kinds of statements to use was a lesson learned by 20th century physics – a lesson that mathematics had to learn a bit earlier. A classical example from set theory of a statement that cannot be allowed is the Russell's catalog (class, set), defined in Robert (1988, p. x) as: "The 'catalogue of all catalogues not mentioning themselves.' Should one include this catalogue in itself? ... Both decisions lead to a contradiction!"

Robert (1988) indicates several ways to avoiding this paradox (or antinomy). All of them imply imposing a (very reasonable) set of rules on how to form valid statements. Under any of these rules, Russell's definition becomes an invalid or ill-posed statement and, as such, should be disregarded, see also Halmos (1998, chapters 1 & 2) and Dugundji (1966, chapter 1). Measure theory (Borel, Lebesgue, Haar, etc.) was a fundamental achievement of 20th century mathematics. It defines measures (notions such as mass, volume and probability) for parts of R^n. However not all parts of R^n are included, and we must refrain of speaking about the measure of inadmissible (non-measurable) sets, see Ulam (1943) for a short article, Kolmogorov and Fomin (1960/1999) for a standard text, and Nachbin (1965) for extensions pertinent to the FBST formalism. The main subject in Robert (1988) is Non Standard Analysis, a form of extending the languages of both Set Theory and Real Analysis, see the observations in section 6.6 and also Davis (1977, sec. 3.4), Goldblatt (1998) and Nelson (1987).

All the preceding examples of mathematical languages have one thing in common: When crafting a specific language, one has to carefully define what kinds of statements are accepted as valid ones. Proper use of the language must be constrained to valid statements. Such constraints are necessary in order to preserve language coherence.

The issue of what kinds of statements should be accepted as valid in QM is an interesting and still subsisting issue, epitomized by the famous debate at the Brussels Solvay conference of 1930 between Niels Bohr and his friend and opponent Albert Einstein. Ruhla (1992, chapters 7 & 8) and Baggott (1992, under the topic *hidden variables*) give very intuitive reviews of the subject, requiring minimal mathematical expertise. Without the details concerning the physics involved, one can describe the debate as: While Bohr suggested very strict rules for admissible statements in QM, Einstein advocated for more amiable ones. In 1935 Einstein, Podolsky and Rosen suggested a *gedankenexperiment*, known as the EPR paradox, as a compelling argument supporting Einstein's point of view. D. Bohm, in 1952 and J. Bell, in 1964, contributed to the debate by showing that the EPR paradox could lead to concrete

experiments providing a way to settle the debate on empirical grounds. It was only in 1972 that the first EPR experiment could be performed in practice. The observational evidence from these experiments seems to favor Bohr's point of view!

One of today's standard formalisms for QM is Abstract QM, see Hughes (1992) for a very readable text and Cohen (1989) for a concise and formal treatment. For an alternative formalism based on Niels Bohr's concept of complementarity, see Bohr (1987, I-IV) and Costa and Krause (2004). Other formalisms may also become useful, see for example Kolmanovskii and Nosov (1986, sec.2.3) and Zubov (1983). Abstract QM, which is very clean and efficient, can be stratified in two layers. In the first layer, all basic calculations are carried out using an algebra of operators in (Rigged) Hilbert spaces. In a second layer, the results of these calculations are interpreted as probabilities of obtaining specific results in physical measurements, see also Rijsbergen (2004). One advantage of using the stratified structure of abstract QM is that it naturally avoids (most of) the danger of forming invalid statements in QM language. Cohen provides the following historical summary:

> Historically, ... quantum mechanics developed in three stages. First came a collection of ad hoc assumptions and then a cookbook of equations known as (orthodox) quantum mechanics. The equations and their philosophical underpinning were then collected into a model based on mathematics of Hilbert space. From the Hilbert space model came the abstraction of quantum logics. (Cohen, 1989, p. vii)

From the above historical comments we draw the following conclusions:

3.1. Each of the QM formalisms discussed in this section, namely, de Broglie wave-particle duality principle, Schrödinger orthodox QM and Hilbert space abstract QM, operates like a language. Maturana stated that objects arise in language. He seems to be right.

3.2. It seems also that new languages must be created (or discovered) to provide us the objects corresponding to the structure of the environment, as stated by Von Forester.

3.3. Exercising a strict discipline concerning what kinds of statements can be used in a given language and context, seems to be vital in many areas.

3.4. It is far from trivial to create, craft, discover, find and/or use a language so that it works, providing us the "right" objects (eigen-solutions).

3.5. Even when everything looks (for the entire community) fine and well, new empirical evidence can bring our theories down as a castle of cards.

As indicated by an anonymous referee, abstract formalisms or languages do not exist in a vacuum, but sit on top of (or are embedded in) natural (or less abstract) languages. This bring us to the interesting and highly relevant issues of hierarchical language structures and constructive *ladders* of objects, including interdependence analyses between objects at different levels of such complex structures, see Piaget (1975) for an early reference. For a recent concrete example of the scientific relevance

of such interdependences in the field of Psychology, using a factor analysis statistical model, see Shedler and Westen (2004, 2005); These issues are among of the main topics addressed in Stern (2006b, 2007b) and other forthcoming articles.

4 The Self-Reference Paradox

The conclusions established in the previous section may look reasonable. In 3.4, however, what exactly are the "right" objects? Clearly, the "right" objects are those objects we more or less clearly see and can point at, using as reference language the language we currently use.

There! I have just fallen, head-on, into the quicksands of the self-reference paradox. Don't worry (or do worry), but note this: The self-reference paradox is unavoidable, especially as long as we use English or any other natural human language.

Rasch (2000, p. 73, 85) has produced a very good description of the self-reference paradox and some of its consequences:

> having it both ways seems a necessary consequence. ... One cannot just have it dogmatically one way, nor skeptically the other. ... One oscillates, therefore, between the two positions, neither denying reality nor denying reality's essentially constructed nature. One calls this not idealism or realism, but (cognitive) constructivism. (Rasch, 2000, p. 73)

> What do we call this oscillation? We call it paradox. Self-reference and paradox -- sort of like love and marriage, horse and carriage. (Rasch, 2000, p. 85)

Cognitive constructivism implies a double rejection: that of a solipsist denial of reality, and that of any dogmatic knowledge of the same reality. Rasch uses the "two front war" metaphor to describe this double rejection. Carrying the metaphor a bit further, the enemies of cognitive constructivism could be portrayed, or caricatured, by saying that Dogmatism despotically requires us to believe in its (latest) theory. Its statements and reasons should be passively accepted with fanatic resignation as infallible truth; Solipsism's anarchic distrust wishes to preclude any established order in the world. Solipsism wishes to transform us into autistic skeptics, incapable of establishing any stable knowledge about the environment in which we live. We refer to Caygill (1995, entry: dogmatism) for a historical perspective on the Kantian use of some of the above terms.

Any military strategist will be aware of the danger in the oscillation described by Rasch, which alternately exposes a weak front. The enemy at our strong front will be subjugated, but the enemy at our weak front will hit us hard. Rasch sees a solution to this conundrum, even recognizing that this solution may be difficult to achieve:

> There is a third choice: to locate oneself directly on the invisible line that must be drawn for there to be a distinction mind / body (system / environment) in the first place. Yet when one attempts to land on that perfect center, one finds oneself oscillating wildly from side to side, perhaps preferring the mind (system) side, but over compensating to the body (environment) side --or vice versa. The

history of post-Kantian German idealism is a history of the failed search for this perfect middle, this origin or neutral ground outside both mind and body that would nevertheless actualize itself as a perfect transparent mind/body within history. Thus, much of contemporary philosophy that both follows and rejects that tradition has become fascinated by, even if trapped in, the mind/body oscillation. (Rasch, 2000, p. 85)

5 Objective Idealism and Pragmatism

We are now ready for a few definitions of basic epistemological terms. These definitions should help us build epistemic statements in a clear and coherent form according to the CogCon+FBST perspective.

> 5.1. Known (knowable) Object: An actual (potential) eigen-solution of a given system's interaction with its environment. In the sequel, we may use a somewhat more friendly terminology by simply using the term Object.
> 5.2. Objective (how, less, more): Degree of conformance of an object to the essential properties of an eigen-solution.
> 5.3. Reality: A (maximal) set of objects, as recognized by a given system, when interacting with single objects or with compositions of objects in that set.
> 5.4. Idealism: Belief that a system's knowledge of an object is always dependent on the systems' autopoietic relations.
> 5.5. Realism: Belief that a system's knowledge of an object is always dependent on the environment's constraints.
> 5.6. Solipsism, Skepticism: Idealism without Realism.
> 5.7. Dogmatic or Metaphysical Realism: Realism without Idealism.
> 5.8. Realistic or Objective Idealism: Idealism and Realism.
> 5.9. "Something in itself": This expression, used in reference to a specific object, is a marker or label for ill posed statements.

Cog-Con+FBST assumes an objective and idealistic epistemology. definition 5.9 labels some ill posed dogmatic statements. Often, the description of the method used to access something in itself looks like:

- Something that an observer would observe if the (same) observer did not exist, or
- Something that an observer could observe if he made no observations, or
- Something that an observer should observe in the environment without interacting with it (or disturbing it in any way), and many other equally nonsensical variations.

Some readers may not like this form of labeling this kind of invalid statement, preferring to use, instead, a more elaborate terminology, such as "object in parenthesis" (approximately) as object, "object without parenthesis" (approximately) as something in itself, and so forth. There may be good reasons for doing so; for example, this elaborate language has the advantage of automatically stressing the differences between constructivist and dogmatic epistemologies (see Maturana, 1988;

Maturana & Poerksen, 2004); Steier, 1991). Nevertheless, we have chosen our definitions in agreement with some very pragmatic advice given in Bopry:

> Objectivity as defined by a (dogmatic) realist epistemology may not exist within a constructivist epistemology; but, part of making that alternative epistemology acceptable is gaining general acceptance of its terminology. As long as the common use of the terms is at odds with the concepts of an epistemological position, that position is at a disadvantage. Alternative forms of inquiry need to co-opt terminology in a way that is consistent with its own epistemology. I suggest that this is not so difficult. The term objective can be taken back. (Bopry, 2002, p. 13)

Among the definitions 5.1 to 5.9, definition 5.2 plays a key role. It allows us to say how well an eigen-solution manifests Von Forester's essential attributes, and consequently, how good (objective) is our knowledge of it. However, the degree of objectivity can not be assessed in the abstract, it must be assessed by the means and methods of a given empirical science, namely the one within which the eigen solution is presented. Hence, definition 5.2 relies on an "operational approach," and not on metaphysical arguments. Such an operational approach may be viewed with disdain by some philosophical schools. Nevertheless, for C.S.Peirce it is "*The Kernel of Pragmatism*," CP 5.464-465:

> "Suffice it to say once more that pragmatism is, in itself, no doctrine of metaphysics, no attempt to determine any truth of things. It is merely a method of ascertaining the meanings of hard words and of abstract concepts. ... All pragmatists will further agree that their method of ascertaining the meanings of words and concepts is no other than that experimental method by which all the successful sciences (in which number nobody in his senses would include metaphysics) have reached the degrees of certainty that are severally proper to them today; this experimental method being itself nothing but a particular application of an older logical rule, 'By their fruits ye shall know them'. (CP 5.464-465)

Definition 5.2 also requires a belief calculus specifically designed to measure the statistical significance, that is, the degree of support of empirical data to the existence of an eigen-solution. In Stern (2007a) we showed why confirming the existence of an eigen-solution naturally corresponds to testing a sharp statistical hypothesis, and why the mathematical properties of FBST e-values correspond to the essential properties of an eigen-solution as stated by Von Forester. In this sense, the FBST calculus is perfectly adequate to support the use of the term Objective and correlated terms in scientific language. Among the most important properties of the e-value mentioned in Stern (2007a), we find:

Continuity: Give a measure of significance that is smooth, i.e. *continuous* and *differentiable*, on the hypothesis parameters and the sample statistics, under appropriate regularity conditions of the statistical model.
Consistency: Provide a *consistent*, that is, asymptotically convergent significance measure for a given sharp hypothesis.

Therefore, the FBST calculus is a formalism that allow us to assess, continuously and consistently, the objectivity of an eigen-solution, by means of a convergent significance measure, see Stern (2007a). We should stress, once more, that achieving

comparable goals using alternative formalisms based on discrete algebraic structures may be, in general, rather difficult. Hence, our answer to the question of how to land on Rasch's perfect center is: Replace unstable oscillation by stable convergence!

Any dispute about objectivity (epistemic quality or value of an object of knowledge), should be critically examined and evaluated within this pragmatic program. This program (in Luhmann's sense) includes the means and methods of the empirical science in which the object of knowledge is presented, and the FBST belief calculus, used to evaluate the empirical support of an object, given the available experimental data.

Even if over optimistic (actually hopelessly utopian), it is worth restating Leibniz' flag of Calculemus, as found in Gerhardt:

> Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Suffciet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: Calculemus. (Gerhardt, 1890, v. 7, pp. 64-65)

A contemporary translation could read:

> Actually, if controversies were to arise, there would be no more need for dispute between two philosophers, rather than between two statisticians. For them it would suffice to reach their computers and, in friendly understanding, say to each other: Let us calculate!

6 The Philosophy of C. S. Peirce

In the previous sections we presented an epistemological perspective based on a pragmatic objective idealism. Objective idealism and pragmatism are also distinctive characteristics of the philosophy of C. S. Peirce. Hence the following question, posed by Søren Brier, that we examine in this section: Is the CogCon+FBST framework compatible with, and can it benefit from, the concepts of semiotics and Peircean philosophy?

In Stern (2007a) we had already explored the idea that eigen-solutions, as discrete entities, can be named, that is become signs in a language system, as pointed by Von Forester in Segal:

> There is an additional point I want to make, an important point. Out of an infinite continuum of possibilities, recursive operations carve out a precise set of discrete solutions. Eigen-behavior generates discrete, identifiable entities. Producing discreteness out of infinite variety has incredibly important consequences. It permits us to begin naming things. Language is the possibility of carving out of an infinite number of possible experiences those experiences which allow stable interactions of your-self with yourself. (Segal, 2001, p. 128)

We believe that the process of recursively "discovering" objects of knowledge, identifying them by signs in language systems, and using these languages to "think" and structure our lives as self-conscious beings, is the key for understanding concepts such as signification and meaning. These ideas are explored, in a great variety of

contexts, in Bakken and Hernes (2002), Brier (1995), Ceruti (1989), Efran, Lukens, and Lukens (1990), Eibel-Eibesfeldt (1970), Ibri (1992), Piaget (1975), Wenger, Pea, Brown, and Heath (1999), Winograd and Flores (1987) and many others. Conceivably, the key underlying common principle is stated by Brier :

> The key to the understanding of understanding, consciousness, and communication is that both the animals and we humans live in a self-organized signification sphere which we not only project around us but also project deep inside our systems. Von Uexküll calls it "Innenwelt" (Brier 2001). The organization of signs and the meaning they get through the habits of mind and body follow very much the principles of second order cybernetics in that they produce their own Eigenvalues of sign and meaning and thereby create their own internal mental organization. I call this realm of possible sign processes for the signification sphere. In humans these signs are organized into language through social self-conscious communication, and accordingly our universe is organized also as and through texts. But of course that is not an explanation of meaning. (Brier, 2005, p. 395)

When studying the organization of self-conscious beings and trying to understand semantic concepts such as signification and meaning, or teleological concepts such as finality, intent and purpose, we move towards domains concerning systems of increasing complexity that are organized as higher hierarchical structures, like the domains of phenomenological, psychological or sociological sciences. In so doing, we leave the domains of natural and technical sciences behind, at least for a moment, see Brent and Bruck (2006) and Muggleton (2006), in last month's issue of *Nature* (March 2006, when this article was written), for two perspectives on future developments.

As observed in Brier (2001), the perception of the objects of knowledge, changes from more objective or realistic to more idealistic or (inter) subjective as we progress to higher hierarchical levels. Nevertheless, we believe that the fundamental nature of objects of knowledge as eigen-solutions, with all the essential characteristics pointed out by Von Forester, remains just the same. Therefore, a sign, as understood in the CogCon+FBST framework, always stands for the following triad:

S-1. Some perceived aspects, characteristics, and so forth, concerning the organization of the autopoietic system.
S-2. Some perceived aspects, characteristics, and so forth, concerning the structure of the system's environment.
S-3. Some object (discrete, separable, stable and composable eigen-solution based on the particular aspects stated in S-1 and S-2) concerning the interaction of the autopoietic system with its environment.

This triadic character of signs bring us, once again, close to the semiotic theory of C. S. Peirce, offering many opportunities for further theoretical and applied research. For example, we are currently using statistical psychometric analyses in an applied semiotic project for the development of software user interfaces (for related examples see Ferreira, 2006). We defer, however, the exploration of these opportunities to forthcoming articles.

In the remainder of this section we focus on a more basic investigation that, we believe, is a necessary preliminary step that must be undertaken in order to acquire a clear conceptual horizon that will assist a sound and steady progress in our future research. The purpose of this investigation is to find out whether the CogCon+FBST framework can find a truly compatible ground in the basic concepts of Peircean philosophy. We proceed establishing a conceptual mapping of the fundamental concepts used to define the Cog-Con+FBST epistemological framework into analogous concepts in Peircean philosophy. Before we start, however, a word of caution: The work of C.S.Peirce is extremely rich, and open to many alternative interpretations. Our goal is to establish the compatibility of CogCon+FBST with one possible interpretation, and not to ascertain reductionist deductions, in any direction.

The FBST is a continuous statistical formalism. Our first step in constructing this conceptual mapping addresses the following questions: Is such a formalism amenable to a Peircean perspective? If so, which concepts in Peircean philosophy can support the use of such a formalism?

6.1 Probability and Statistics: The FBST is a probability theory based statistical formalism. Can the probabilistic concepts of the FBST find the necessary support in concepts of Peircean philosophy? We believe that Tychism is such a concept in Peircean philosophy, providing the first element in our conceptual mapping. In CP 6.201 Tychism is defined as: "... the doctrine that absolute chance is a factor of the universe."

6.2 Continuity: As stated in the previous section, the CogCon+FBST program pursues the stable convergence of the epistemic e-values given by the FBST formalism. The fact that FBST is a belief calculus based on continuous mathematics is essential for its consistency and convergence properties. Again we have to ask: Does the continuity concept used in the FBST formalism have an analogous concept in Peircean philosophy? We believe that the analogy can be established with the concept of Synechism, thus providing the second element in our conceptual mapping. Synechism is defined as: "that tendency of philosophical thought which insists upon the idea of continuity as of prime importance in philosophy and, in particular, upon the necessity of hypotheses involving true continuity" (CP 6.169).

6.3 Eigen-Solutions: A key epistemological concept in the CogCon +FBST perspective is the notion of eigen-solution. Although the system theoretic concept of Eigen-solution cannot possibly have an exact correspondent in Peirce's philosophy, we believe that Peirce's fundamental concept of "Habit" or "Insistency" offers an adequate analog. Habit, and reality, are defined as: "The existence of things consists in their regular behavior" (CP 1.411). "Reality is insistency. That is what we mean by 'reality'. It is the brute irrational insistency that forces us to acknowledge the reality of what we experience, that gives us our conviction of any singular. (CP 6.340).

However, the CogCon+FBST concept of eigen-solution is characterized by Von Forester by several essential properties. Consequently, in order for the conceptual mapping under construction to be coherent, these characteristics have to be mapped accordingly. In the following paragraphs we show that the essential properties of sharpness (discreteness), stability and *compositionality* can indeed be adequately represented.

> 6.3a—Sharpness: The first essential property of eigen-solutions stated by Von Forester is discreteness or sharpness. As stated in Stern (2007a), it is important to realize that, in the sequel, the term 'discrete', used by Von Forester to qualify eigen-solutions in general, should be replaced, depending on the specific context, by terms such as lower-dimensional, precise, sharp, singular, etc. As physical laws or physical invariants, sharp hypotheses are formulated as mathematical equations.

Can Peircean philosophy offer a good support for sharp hypotheses? Again we believe that the answer is in the affirmative. The following quotations should make that clear (see also Eisele, 1976, p.136-137 & CP 6.203).

> an object (a thing) IS only in comparison with a continuum of possibilities from which it was selected. (Ibri, 1992, p. 84)

> Existence involves choice; the dice of infinite faces, from potential to actual, will have the concreteness of one of them. (Ibri, 1992, p. 85)

> .as a plane is a bi-dimensional singularity, relative to a tri-dimensional space, a line in a plane is a topic discontinuity, but each of this elements is continuous in its proper dimension. (Ibri, 1992, p. 85)

> Whatever is real is the law of something less real. Stuart Mill defined matter as a permanent possibility of sensation. What is a permanent possibility but a law? (CP 1.487)

> In fact, habits, from the mode of their formation, necessarily consist in the permanence of some relation, and therefore, on this theory, each law of nature would consist in some permanence, such as the permanence of mass, momentum, and energy. In this respect, the theory suits the facts admirably. (CP 1. 415)

> 6.3b—Stability: The second essential property of eigen-solutions stated by Von Forester is stability. As stated in Stern (2005), a stable eigen-solution of an operator, defined by a fixed-point or invariance equation, can be found (built or computed) as the limit of a sequence of recursive applications of the operator. Under appropriate conditions (such as within a domain of attraction, for instance) the process convergence and its limiting eigen-solution will not depend on the starting point.

A similar notion of stability for an object-sign complex is given by Peirce:

That for which it (a sign) stands is called its object; that which it conveys, its meaning; and the idea to which it gives rise, its interpretant. The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit. (CP 1.339)

6.3c — Compositionality: The third essential property of eigen-solutions stated by Von Forester is compositionality. As stated in Stern (2007a) and Borges and Stern (2006), compositionality properties concern the relationship between the credibility, or truth value, of a complex hypothesis, H, and those of its elementary constituents, Hj, j =1 ...k. Compositionality is at the very heart of any theory of language, see Noeth (1995). As an example of compositionality, see Peirce quotations below. Peirce discusses the composition of forces, that is, how the components are combined using the parallelogram law.

If two forces are combined according to the parallelogram of forces, their resultant is a real third... Thus, intelligibility, or reason objectified, is what makes Thirdness genuine. (CP 1.366)

A physical law is absolute. What it requires is an exact relation. Thus, a physical force introduces into a motion a component motion to be combined with the rest by the parallelogram of forces.(CP 6.23)

In order to establish a minimal mapping, there are two more concepts in CogCon+FBST to which we must assign adequate analogs in Peircean philosophy.

6.4 Extra variability: In Stern (2007a) the importance of incorporating all sources of noise and fluctuation, that is, all the extra variability statistically significant to the problem under study, into the statistical model is analyzed. The following excerpt from Peirce indicates that his notion of falibilism may be used to express the need for allowing and embracing all relevant (and in practice inevitable) sources of extra variability. According to Peirce, falibilism is "the doctrine that there is no absolute certainty in knowledge"

There is no difficulty in conceiving existence as a matter of degree. The reality of things consists in their persistent forcing themselves upon our recognition. If a thing has no such persistence, it is a mere dream. Reality, then, is persistence, is regularity. ... as things (are) more regular, more persistent, they (are) less dreamy and more real. Fallibilism will at least provide a big pigeon-hole for facts bearing on that theory. (CP 1.175)

6.5 Bayesian statistics: FBST is an Unorthodox Bayesian statistical formalism. Peirce has a strong and unfavorable opinion about Laplace's theory of inverse probabilities.

The majority of mathematical treatises on probability follow Laplace in results to which a very unclear conception of probability led him. ... This is an error often appearing in the books under the head of 'inverse probabilities.' (CP 2.785)

Due to his theory of inverse probabilities, Laplace is considered one of the earliest precursors of modern Bayesian statistics. Is there a conflict between CogCon+FBST and Peirce's philosophy? We believe that a careful analysis of Peirce's arguments not only dissipates potential conflicts, but also reinforces some of the arguments used in Stern (2007a).

Two main arguments are presented by Peirce against Laplace's inverse probabilities. In the following paragraphs we will identify these arguments and present an up-to-date analysis based on the FBST (unorthodox) Bayesian view:

6.5a—Dogmatic priors vs. Symmetry and Maximum Entropy arguments:

> Laplace maintains that it is possible to draw a necessary conclusion regarding the probability of a particular determination of an event based on not knowing anything at all about [it]; that is, based on nothing. ... Laplace holds that for every man there is one law (and necessarily but one) of dissection of each continuum of alternatives so that all the parts shall seem to that man to be *"egalement possibles"* in a quantitative sense, antecedently to all information. (CP 2.764)

The dogmatic rhetoric used at the time of Laplace to justify ad hoc prior distributions can easily backfire, as it apparently did for Peirce. Contemporary arguments for the choice of prior distributions are based on MaxEnt formalism or symmetry relations (see Dugdale, 1996; Eaton, 1989; Kapur, 1989; Nachbin, 1965). Contemporary arguments also examine the initial choice of priors by sensitivity analysis, for finite samples, and give asymptotic dissipation theorems for large samples (see DeGroot, 1970; Gelman et al., (2003); Stern, 2004). We can only hope that Peirce would be pleased with the contemporary state of the art. These powerful theories have rendered ad hoc priors unnecessary, and shed early dogmatic arguments into oblivion.

6.5b—Assignment of probabilities to (sharp) hypotheses vs. FBST possibilistic support structures:

> Laplace was of the opinion that the affirmative experiments impart a definite probability to the theory; and that doctrine is taught in most books on probability to this day, although it leads to the most ridiculous results, and is inherently self-contradictory. It rests on a very confused notion of what probability is. Probability applies to the question whether a specified kind of event will occur when certain predetermined conditions are fulfilled; and it is the ratio of the number of times in the long run in which that specified result would follow upon the fulfillment of those conditions to the total number of times in which those conditions were fulfilled in the course of experience. (CP 5.169)

In the second part of the above excerpt Peirce expresses a classical (frequentist) understanding of having probability in the sample space, and not in the parameter space, that is, he admits predictive probability statements but does not admit epistemic probability statements. The FBST is a Bayesian formalism that uses both predictive and epistemic probability statements, as explained in Stern (2007a). However, when we examine the reason presented by Peirce for adopting this position, in the first part

of the excerpt, we find a remarkable coincidence with the arguments presented in Stern (2003, 2004, 2006, 2007a) against the orthodox Bayesian methodology for testing sharp hypotheses: The FBST *does not* attribute a *probability* to the theory (sharp hypothesis) being tested, as do orthodox Bayesian tests, but rather a degree of *possibility*. In Stern (2003, 2004, 2006, 2007a) we analyze procedures that attribute a probability to a given theory, and came to the exact same conclusion as Pierce did, namely, those procedures are absurd.

> 6.6 Measure Theory: Let us now return to the Peircean concept of Synechism, to discuss a technical point of contention between orthodox Bayesian statistics and the FBST unorthodox Bayesian approach. The FBST formalism relies on some form of Measure theory, see comments in section 3. De Finetti, the founding father of the orthodox school of Bayesian statistics, feels very uncomfortable having to admit the existence of non-measurable sets when using measure theory in dealing with probabilities, in which valid statements are called events, see de Finetti (1975, 3.11, 4.18, 6.3, & appendix). Dubins and Savage (1976, p. 8) present similar objections, using the colorful gambling metaphors that are so characteristic of orthodox (decision theoretic) Bayesian statistics. In order to escape the constraint of having non-measurable sets, de Finetti readily proposes a deal: to trade off other standard properties of a measure, like countable (σ) additivity:

> Events are restricted to be merely a subclass (technically a σ-ring with some further conditions) of the class of all subsets of the base space. In order to make σ-additivity possible, but without any real reason that could justify saying to one set 'you are an event,' and to another you are not.' (di Finetti, 1975, v. 2, p. 259)

In order to proceed with our analysis, we have to search for the roots of de Finetti's argument, roots that, we believe, lay outside de Finetti's own theory, for they hinge on the perceived structure of the continuum. Bell states:

> the generally accepted set-theoretical formulation of mathematics (is one) in which all mathematical entities, being synthesized from collections of individuals, are ultimately of a discrete or punctate nature. This punctate character is possessed in particular by the set supporting the 'continuum' of real numbers --the 'arithmetical continuum'. (Bell, 1998, p. 2)

Among the alternatives to arithmetical punctiform perspectives of the continuum, there are more geometrical perspectives. Such geometrical perspectives allow us to use an arithmetical set as a coordinate (localization) system in the continuum, but the "ultimate parts" of the continuum, called infinitesimals, are essentially nonpunctiform, that is, non point like. Among the proponents of infinitesimal perspectives for the continuum one should mention G. W. Leibniz, I. Kant, C. S. Peirce, H. Poincaré, L. E. J. Brouwer, H. Weyl, R. Thom, F. W. Lawvere, A. Robinson, E. Nelson, and many others. I refer you to Bell (2005) for an excellent general

historical review, and to Robertson (2001) for the ideas of C. S. Peirce. In the infinitesimal perspective,

> any of its (the continuum) connected parts is also a continuum and, accordingly, divisible. A point, on the other hand, is by its nature not divisible, and so (as stated by Leibniz) cannot be part of the continuum. (Bell, 1998, p.3)

According to Bell, in Peirce's doctrine of synechism, he championed the retention of the infinitesimal concept in the foundation of the calculus, both because of what he saw as the efficiency of infinitesimal methods, and because he regarded infinitesimals as constituting the "glue" causing points on a continuous line to lose their individual identity (Bell, 2005, p. 211). Bell further characterizes Perice's views with the following quotations: "The very word continuity implies that the instants of time or the points of a line are everywhere welded together," and "[the continuum] does not consist of indivisibles, or points, or instants, and does not contain any except insofar as its continuity is ruptured" (Peirce quoted in Bell, 1998, p. 208).

De Finetti's argument on non-measurable sets implicitly assumes that all point subsets of R^n have equal standing, that is, that the continuum has no structure. Under the arithmetical punctiform perspective of the continuum, de Finetti's objection makes perfect sense, and we should abstain from measure theory or alternative formalisms, as does orthodox Bayesian statistics. This is how Peirce's concept of synechism helps us to overcome a major obstacle (for the FBST) presented by orthodox Bayesian philosophy, namely, the objections against the use of measure theory.

At this point it should be clear that my answer to Søren Brier's question is emphatically affirmative. From Søren Brier's comments and suggestions it is also clear how well he knew the answer when he asked me the question. As a maieutic teacher however, he let me look for the answers my own way. I can only thank him for the invitation that brought me for the first time into contact with the beautiful world of semiotics and Peircean philosophy.

7 Final Remarks

The physician Rambam, Moshe ben Maimon (1135-1204), then caliphate of Cordoba, wrote *Shmona Perakim*, a book on psychology (medical procedures for healing the human soul) based on fundamental principles exposed by Aristotle in *Nicomachean Ethics* (see Olitzky, 2000; Rackham, 1926). Rambam explains how the health of the human soul depends on always finding the *straight path* (derech y'shara) or *golden way* (shvil ha-zahav), at the perfect center between the two opposite extremes of excess (odef) and scarcity (choser), (see Maimonides, 2001, v.1: "Knowledge," ch. 2 & "Temperaments," §1, §2):

> The straight path is the middle one, that is equidistant from both extremes.... Neither should a man be a clown or jokester, nor sad or mourning, but he should be happy all his days in serenity and pleasantness. And so with all the other qualities a man possesses. This is the way of the scholars.

Every man whose virtues reflect the middle, is called a chacham... a wise man. (Maimonides, 2001, v. 1, ch. 2, § 1 & 2)

Rambam explains that a (always imperfect) human soul, at a given time and situation, may be more prone to fall victim of one extreme than to its opposite, and should try to protect itself accordingly. One way of achieving this protection is to offset its position in order to (slightly over-) compensate for an existing or anticipated bias.

At the dawn of the 20th century, humanity had in classical physics a paradigm of science handing out unquestionable truth, and faced the brutality of many totalitarian states. Dogmatism had the upper hand, and we had to protect ourselves accordingly.

At the beginning of the 21st century we are enjoying the comforts of a hyperactive economy that seems to be blind to the constraints imposed by our ecological environment, and our children are being threatened by autistic alienation through the virtual reality of their video games. It may be the turn of (an apathetic form of) solipsism.

Finally, Rambam warns us about a common mistake: Protective offsets may be a useful precautionary tactic, or even a good therapeutic strategy, but should never be considered as a virtue per se. The virtuous path is the straight path, neither left of it nor right of it, but at the perfect center.

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