

# Peirce, Pedigree, Probability

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## Abstract

An aspect of Peirce's thought that may still be underappreciated is his resistance to what Levi calls *pedigree epistemology*, to the idea that a central focus in epistemology should be the justification of current beliefs. Somewhat more widely appreciated is his rejection of the subjective view of probability. We argue that Peirce's criticisms of subjectivism, to the extent they grant such a conception of probability is viable at all, revert back to pedigree epistemology. A thoroughgoing rejection of pedigree in the context of probabilistic epistemology, however, *does* challenge prominent subjectivist responses to the problem of the priors.

**Keywords.** Bayesian; common prior; merging of opinions; Peirce; problem of the priors; subjective probability

## 1 Preamble

Significant effort in the epistemological enterprise has been directed at developing accounts of how our beliefs are justified. According to some, beliefs depend for their justification on their relationships to indubitable foundational propositions or sense data. According to others, while justificatory chains are required, these chains never bottom out in foundational justifications but go on forever. According to still others, a belief is justified if it was acquired by a reliable process. A presupposition motivating these efforts is that the justification of currently held beliefs should be a central concern of epistemology. Levi calls epistemology starting from this presupposition “pedigree epistemology” for its preoccupation with the backwards-looking task of scrutinizing the origins of currently held beliefs. As Levi sees it, the rejection of this presupposition is the greatest insight in the pragmatist tradition as initiated by Peirce.

We can also detect at least an implicit challenge to the preoccupation with pedigree in the thinking of some probabilists of the subjectivist school. Perhaps this is a small wonder. Some of the central figures in the development of the subjective view of probability were influenced by pragmatism, and subjectivism and pragmatism may seem like natural bedfellows. According to Galavotti, “Of the main interpretations of probability, namely frequentism, logicism, subjectivism and propensionism, subjectivism is the closest to the pragmatist perspective first and foremost for the centrality ascribed to man as an agent acting in the world, and

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the idea that human action is guided by belief” (2019, p. 1). Yet, Peirce explicitly rejects the subjectivism of his day, which he refers to as the “conceptualistic” view. We argue that Peirce’s case against conceptualism gives rise to a genuine tension in his epistemology. Despite explicit disavowals of a significant justificatory role for pedigree in epistemology in both his early and later writings, some of Peirce’s attacks on the subjective view of probability depend crucially on pedigree requirements.

We have two main points in this essay. The first, just mentioned, is that Peirce himself seems to have failed to fully appreciate the consequences of rejecting pedigree epistemology. We make this case in Section 3 after rehearsing Peirce’s views on pedigree and their extensive development by Isaac Levi in Section 2. Second, the importance of the rejection of pedigree is not restricted to Peirce scholarship, but extends to contemporary debates in probability. For instance, attempts to recover forms of objectivity for subjective Bayesianism make crucial appeals to the pedigree of probabilistic judgments. If we are to dispense with pedigree requirements, prominent approaches to the problem of the priors fail, as we illustrate in Section 4. Our concern is not to advocate the rejection of pedigree, but to investigate some significant consequences of such a rejection in the context of probability.

## 2 Peirce on Pedigree

In certain striking passages, Peirce seems to strenuously reject what Levi calls “pedigree epistemology,” the idea that the focus in epistemology should be on justifying currently held beliefs by scrutinizing the source and formation of them. One of the most interesting passages in this connection is in “The Fixation of Belief,” when Peirce introduces his “belief-doubt model” of inquiry.

The irritation of doubt causes a struggle to attain a state of belief. I shall term this struggle inquiry [...]. With the doubt, therefore, the struggle begins, and with the cessation of doubt it ends. Hence, the sole object of inquiry is the settlement of opinion. We may fancy that this is not enough for us, and that we seek, not merely an opinion, but a true opinion. But put this fancy to the test, and it proves groundless; for as soon as a firm belief is reached we are entirely satisfied, whether the belief be true or false. [...] The most that can be maintained is, that we seek for a belief that we shall think to be true. But we think each one of our beliefs to be true, and, indeed, it is mere tautology to say so. (1877, W 3:247–248)

Identifying the fixation of belief as the purpose of inquiry specifies initiation and termination conditions for inquiry. Doubt initiates inquiry; belief terminates it.

According to Peirce, the belief-doubt model has dramatic consequences for epistemology. “That the settlement of opinion is the sole end of inquiry,” he writes, “[...] sweeps away, at once, various vague and erroneous conceptions of proof” (1877, W 3:248). Peirce gives three examples. First, despite what “[s]ome philosophers have imagined,” putting “a proposition into the interrogative form” is not sufficient to initiate genuine inquiry. Instead, “[t]here must be a real and living doubt.” Second, a “common idea” is that “demonstrations must rest on some ultimate and absolutely indubitable propositions.” These might be first principles or propositions representing sense data. According to Peirce’s belief-doubt model, this is

misguided. We are “entirely satisfied” once a firm opinion is established regardless of the opinion’s connection to some putative foundational propositions or its origins more generally: “in point of fact, an inquiry, to have that completely satisfactory result called demonstration, has only to start with propositions perfectly free from all actual doubt. If the premises are not in fact doubted at all, they cannot be more satisfactory than they are.” Third, continuing to “argue a point” among parties already convinced of it is “without a purpose.” “When doubt ceases,” Peirce claims, “mental action on the subject comes to an end.” For an individual agent, the backward-looking task of justifying some presently held belief is similarly without a purpose; with the cessation of doubt, genuine inquiry into the matter—i.e., the struggle to attain a state of belief—comes to an end.

It may be tempting to interpret Peirce’s belief-doubt model as a bit of speculative psychology. In our view, this interpretation is mistaken (see also (Kasser, 1999)). The model can serve as the basis for a serious and substantial epistemological outlook. The person to do the most to develop this line of Peirce’s thought is Isaac Levi. As Levi himself remarks, “In my judgment, the belief-doubt model represents the greatest insight in the pragmatist tradition, and I have sought to preserve it in my own thinking” (1991, p. 163). Rehearsing some general aspects of Levi’s development of the model may help to make some of the key ideas more precise and the model on the whole more plausible. It is Levi’s elaboration of the belief-doubt model and the concomitant rejection of pedigree epistemology that we will rely on.

According to Levi, an agent’s full beliefs serve as his or her *standard for serious possibility*. Only hypotheses consistent with an agent’s full beliefs are serious possibilities. Not only does this identify a function for full belief—something that many probabilistic theories fail to do for qualitative belief—but, in so doing, it also specifies the source of judgments of possibility, a crucial component of any feasible epistemology since judgments of possibility are prerequisites for both theoretical inquiry and practical deliberation.

An immediate corollary of the thesis that an agent’s full beliefs serve as her standard for serious possibility is that it is not a serious possibility that any of her full beliefs is false. The negation of a hypothesis that an agent believes is trivially inconsistent with her beliefs and, therefore, not a serious possibility. Other pragmatism-influenced writers sometimes express sympathy with this view. Quine, for example, writes, “Within our own total evolving doctrine we can judge truth *as earnestly and absolutely* as can be” (p. 25 1960, emphasis added). Levi takes the rejection of pedigree epistemology, then, to be the rejection of the idea that an agent’s *beliefs* or *point of view* stands in need of justification. On this view, to repeat, it is not a serious possibility from the agent’s point of view that her full beliefs—her *very standard* for judging error—contain error. Once a belief has been fixed, it

has become a premise, evidence, settled assumption, or part of the ‘background knowledge’ to be used in subsequent inquiries into the credential of other statements as well as in practical deliberations aimed at moral, political, economic, or other practical objectives. Whether [...] a theory, law, statistical claim, or observation report, and regardless of the grounds on which it has been added, its status as [a full belief] has been settled and the grounds on which it has been added no longer matter. (Levi, 1980, p. 2)

Pedigree epistemology, by contrast, attempts to inquire into the grounds for current beliefs. But in order for this undertaking to be non-trivial, it would have to be conducted from *some*

*other* point of view or standard which the agent, by assumption, does not endorse.

It is worth emphasizing the Peircean origins of this view since it seems to contrast with Peirce's fallibilism which has been the subject of widespread discussion. *Fallibilism* is the view that we can never be certain of any of our (non-logical) beliefs. Levi argues that the thesis that an agent's full beliefs serve as her standard for serious possibility is in fact inconsistent with fallibilism given corrigibilism (1980, pp. 14–15). *Corrigibilism* is the view that full beliefs are revisable. As Peirce puts it, "the scientific spirit requires a man to be at all times ready to dump his whole cart-load of beliefs, the moment experience is against them" (1896, CP 1.55). Fallibilism restricts full belief in the certainty sense—the sense implied by identifying the function of full belief with the standard for serious possibility—to the logical truths. But since Levi assumes that the logical truths are included in any minimal set of rational beliefs (i.e., logical inconsistencies are never serious possibilities), there can be no revision of full beliefs, contradicting corrigibilism. While Peirce views corrigibilism as an important aspect of fallibilism, by distinguishing the two claims, it is possible to preserve corrigibilism in the face of the inconsistency. Despite this conflict with fallibilism, Peirce nevertheless seems to have the *standard for serious possibility* function of belief in mind when he identifies the cessation of doubt as the end of inquiry, and asserts that inquiries beginning from premises free from all actual doubt are "demonstrations" and as satisfactory as they can possibly be. As late as 1905 (nearly three decades after "The Fixation of Belief"), Peirce writes,

But in truth, there is but one state of mind from which you can "set out," namely, the very state of mind in which you actually find yourself at the time you do "set out"—a state in which you are laden with an immense mass of cognition already formed, of which you cannot divest yourself if you would; and who knows whether, if you could, you would not have made all knowledge impossible to yourself? Do you call it *doubting* to write down on a piece of paper that you doubt? If so, doubt has nothing to do with any serious business. But do not make believe; if pedantry has not eaten all the reality out of you, recognize, as you must, that there is much that you do not doubt, in the least. Now that which you do not at all doubt, you must and do regard as *infallible, absolute truth*. (1905, CP 5.416, emphasis added)

In such passages, Peirce seems to agree with Levi about the role of fixed beliefs.<sup>1</sup>

As Levi immediately stresses after introducing this conception of full belief, retaining corrigibilism means that the status of a belief as fixed is not secure for all time. An agent can still have grounds to revise her beliefs. This is central to the reorientation in epistemology that the belief-doubt model occasions. The focus in epistemology, according to Levi, should not be on justifying our current point of view—which is pointless because it is the standard by which justification is assessed—but on improving it. Put another way, *revision* of belief is the proper locus of concern about justification. Improvements are measured against some end or set of ends. A theme common to many pragmatist thinkers is that there are two primary ends in inquiry: as James puts it, "Believe truth! Shun error!" (1896, p. 209). Levi interprets the two goals as obtaining valuable information (whether true or false) and avoiding error (1991, p. 81). This shift in focus places special emphasis on a theory of belief revision for making improvements, and recognizes a tension between achieving two different goals. Accordingly, Levi develops an involved epistemic decision theory that governs belief revision, one suitable

to evaluating trade offs between the demands of acquiring valuable information and avoiding error.<sup>2</sup> Justified revisions are those that are optimal given the goals of inquiry.

An agent's point of view, for Levi, consists not only of her full beliefs, but also of subjective judgments of both probability and value (including judgments of informational value relevant to epistemic decision problems).<sup>3</sup> Judgments of possibility can be thought of as the basis on which (subjective) probability spaces are constructed. A standard for serious possibility partitions the set of hypotheses, marking hypotheses consistent with it as serious possibilities and hypotheses inconsistent with it as not. According to Levi and others, subjective probability judgments make finer distinctions among the set of possibilities, assigning *degrees* of belief (cf. de Finetti, 1975, p. 23). The various elements of an agent's point of view, as Levi conceives of them, are *commitments* to standards of evaluations of options in both epistemic and practical decision problems. An agent may fail to live up to her commitments; but she ought to take steps to better approximate them when she recognizes deviation and is able. It is in this commitment sense that deductive closure (and the inclusion of logical truths in every belief state that follows from it), for example, is rationally mandatory for full belief according to Levi. Perhaps it makes sense to think of pedigree requirements as a type of rationality requirement. Not all rationality requirements, however, concern pedigree. Deductive closure might be considered a rationality requirement, though not one appealing to the pedigree of an agent's beliefs. As Levi sees it, deductive closure of a set of beliefs is simply a matter of explicitly representing the standard for serious possibility to which an agent is committed: the same set of hypotheses is consistent with a set of full beliefs as with its deductive closure.

Commitments in general, like the special case of full beliefs, are revisable. The appropriate focus in revising commitments is again improvement rather than origins. But what revisions are justified depend on the agent's point of view. One point worth noting is that having doubt about a particular hypothesis is distinct from having doubt about one's probabilistic judgment about that hypothesis.<sup>4</sup> For Levi, pedigree is no more relevant for an agent's (subjective) judgments of probability than it is for an agent's (subjective) judgments of serious possibility. A commitment to assessing possibilities in terms of certain personal probabilities, like a commitment to a standard for serious possibility, is a resource in subsequent inquiries. Even though it is subject to revision, and such revisions stand in need of justification, once the commitment has been made, the grounds on which it was adopted are no longer relevant. Surprisingly, Peirce seems to disagree.

### 3 Peirce's Panning of Personal Probability

Peirce rejects a subjectivist or conceptualistic view of probability. It is in his reasons for this rejection, we will argue, that we can discern a pedigree demand. His own conception of probability is objectivist. For him, probability attaches to *modes of inference* or *arguments*. In "The Fixation of Belief," Peirce already observes that "the question of validity [of an argument] is purely one of fact and not of thinking" (1877, W 3:244), and in that sense objective. In "The Doctrine of Chances," Peirce explicitly turns to probability. In line with the pragmatic principle he outlined in "How to Make Our Ideas Clear," Peirce declares that in order "to get a clear idea of what we mean by probability, we have to consider what real and sensible difference there is between one degree of probability and another" (1878, W 3:279).

An argument is more precisely “always conceived as a member of a genus of arguments all constructed in the same way, and such that, when their premisses are real facts, their conclusions are so also,” and “[i]f the argument is demonstrative, then this is always so; if it is only probable, then it is for the most part so.” Now the “real and sensible difference between one degree of probability and another, in which the meaning of the distinction lies,” is that

in the frequent employment of two different modes of inference, one will carry truth with it oftener than the other. [...] [I]n the long run, there is a real fact which corresponds to the idea of probability, and it is that a given mode of inference sometimes proves successful and sometimes not, and that in a ratio ultimately fixed. [...] We may, therefore, define the probability of a mode of argument as the proportion of cases in which it carries truth with it. (1878, W 3:280)

Peirce thus advances here, more specifically, a frequentist conception of probability.<sup>5</sup> He would later tweak this into a conception of probability as a *disposition* to produce long-run frequencies.<sup>6</sup> But Peirce always held an objectivist conception of probability, and always rejected a subjectivist or *conceptualistic*<sup>7</sup> view of probability.<sup>8</sup>

Peirce’s most extensive and best known criticism of conceptualism occurs in the “The Probability of Induction.” We will here examine Peirce’s objections against conceptualism on the basis of the two parts of his critique in this paper: one aimed at his own reconstruction of a best case for conceptualism, the other aimed at the principle of insufficient reason. We argue that in both cases his criticisms derive from his view that *probability must be based in fact*, and that this assertion, insofar as it allows for a conceptualist notion of probability at all, raises a pedigree demand.

### 3.1 Probability Must Express a Fact

Peirce starts his critique in “The Probability of Induction” by building a case *for* conceptualism. This part of Peirce’s critique is also discussed in detail by [Kasser \(2016\)](#).

Peirce first allows that, even if probability pertains to arguments, “we may, nevertheless, speak of the chance of an event absolutely, meaning by that the chance of the combination of all arguments in reference to it which exist for us in the given state of our knowledge” (1878, W 3:293).<sup>9</sup> Furthermore, “it is incontestable that the chance of an event has an intimate connection with the degree of our belief in it [...] there is a feeling of believing, and this feeling does and ought to vary with the chance of the thing believed, as deduced from all the arguments.” He observes that there is one quantity that is “peculiarly appropriate” to “serve as a thermometer for the proper intensity of belief.” This is, in modern terms, the logarithm of the odds of the thing believed,<sup>10</sup> which results in an account of “a proceeding which men often resort to, under the name of balancing reasons” that foreshadows, if not coincides with, a notion of “weight of evidence” in modern Bayesianism.<sup>11</sup> Peirce concludes,

These considerations constitute an argument in favor of the conceptualistic view. The kernel of it is that the conjoint probability of all the arguments in our possession, with reference to any fact, must be intimately connected with the just degree

of our belief in that fact; and this point is supplemented by various others showing the consistency of the theory with itself and with the rest of our knowledge. (1878, W 3:294–295)

Having allowed this much, Peirce starts his criticism. He declares,

But probability, to have any value at all, must express a fact. It is, therefore, a thing to be inferred upon evidence. (1878, W 3:295)

Peirce invites us to consider the formation of such a degree of belief. Specifically, we seek to “form a probable judgment of the color” of a bean (hidden from us) that has been randomly taken out of a bag filled with beans of two different colors, by drawing with replacement beans from this bag. But “the whole utility of probability is to insure us in the long run,” and “that assurance depends, not merely on the value of the chance, but also on the accuracy of the evaluation”:

In short, to express the proper state of our belief, not one number but two are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. [...] [W]hen our knowledge is very slight, [the second] may be even more important than the probability itself; and when we have no knowledge at all this completely overwhelms the other, so that there is no sense in saying that the chance of the totally unknown event is even (for what expresses absolutely no fact has absolutely no meaning), and what ought to be said is that the chance is entirely indefinite. We thus perceive that the conceptualistic view, though answering well enough in some cases, is quite inadequate. (1878, W 3:295–296)

The inadequacy that Peirce perceives rests on the idea that one’s estimate of an objective probability should be accompanied by some indication of one’s confidence in (or the robustness of, or the amount of evidence supporting) this estimate.<sup>12</sup> Developing an account of the second factor, or more generally what Keynes would later call the “weight of an argument” or “evidential weight” behind a credence (1921, ch. 6; sect. 26.7),<sup>13</sup> is an interesting challenge for subjectivism. But as Kasser (2016, p. 646) also notes, this challenge has been taken up in various ways in the modern Bayesian literature, and Peirce’s perceived inadequacy is not generally taken to refute the idea of subjective probability.<sup>14</sup>

More fundamental than this particular point of critique is a stipulation that Peirce makes earlier. This is the stipulation that there must be a direct connection between degree of belief and objective probability. In Peirce’s reconstruction of a viable conceptualism, the thermometer for the proper intensity of belief must track the objective probability (more precisely, the objective odds) of the thing believed. In this sense, Peirce stipulates, in his reconstruction, that degrees of belief must “express” objective statistical facts. But there is no reason a conceptualist need go along with Peirce’s stipulation that probability must always “express” a statistical fact. In this stipulation, which already takes the objective notion of probability to be the fundamental one, there appears an element of question-begging against the conceptualist.<sup>15</sup>

This element is more pronounced in some of Peirce’s other writings. In his review of Venn’s *The Logic of Chance*, expanding on Venn’s dismissal of conceptualism, Peirce evokes

probability's mandatory basis in fact to dismiss a subjective notion of probability altogether. He notes that "conceptualists have not undertaken to say what is meant by 'degree of credence.' They would probably pronounce it indefinable and indescribable" (1867, W 2:99), and offers an appraisal of the use of the notion that is less charitable than the above:

That is, indeed, the precise and only use or significance of these fractions termed probabilities: they give security in the long run. Now, in order that the degree of credence should correspond to any truth in the long run, it must be the representation of a general statistical fact, – a real, objective fact. And then, as it is the fact which is said to be probable, and not the belief, the introduction of "degree of credence" at all into the definition of probability is as superfluous as the introduction of a reflection upon a mental process into any other definition would be, – as though we were to define man as "that which [...] ought to be conceived as a rational animal." (1867, W 2:100)

In effect, Peirce's stipulation of probability's factual basis here becomes indistinguishable from a claim that the conceptualist notion of probability is superfluous because it is not the notion of probability as correctly defined, by Venn and Peirce, in terms of objective fact. Of course, as an argument against conceptualism, such a claim simply begs the question.

Nevertheless, when further on Peirce discusses how Venn has "fallen into some conceptualistic errors of his own," he again talks about credences that express objective probabilities. What he says there, like what he says in "The Probability of Induction," points at a further diagnosis, aside from the flat-out rejection of the conceptualist notion, of the upshot of his stipulation that probability must be rooted in fact.

Peirce considers the case where we have statistical data about how many Englishmen who go to Madeira live through the first year, and about how many consumptives who go to Madeira live through the first year, but "[t]here are no certain data for the least approximation to the proportion of consumptive Englishmen who die in Madeira during the first year" (1867, W 2:101). Peirce now states that

[...] since there is nothing in the data to show what this ratio is, the probability in question is unknown. But a "degree of credence" or "credibility," to be logically determined, must, as we have seen, be an expression of probability in the [objective] sense; and therefore this "degree of credence" (supposing it to exist) is unknown. "We know not what to believe," is the ordinary and logically correct expression in such cases of perplexity. (1867, W 2:102)

This point is echoed, recall, in the above quote from "The Probability of Induction," where Peirce states that "there is no sense in saying that the chance of the totally unknown event is even [...] what ought to be said is that the chance is entirely indefinite." Now what Peirce demands here is that our credences about objective probabilities are actually based on information about the latter. If "there is nothing in the data" to base our credences on, the latter must remain "unknown," too. We thus need to be able to offer a good reason—in particular, statistical data—for holding particular credences: probability is "a thing to be inferred upon evidence." In other words, the particular credences we hold stand in need of justification. This is a pedigree demand.

We return to the diagnosis of Peirce’s stipulation below, but first resume our discussion of his critique of conceptualism in “The Probability of Induction.” The remainder of Peirce’s critique in this paper is concerned with the principle of insufficient reason, and culminates in his well-known “if universes were as plenty as blackberries” charge. We briefly rehearse his argument, to conclude that his main objection against insufficient reason stems from the same stipulation: it is the wider complaint that subjective probabilities cannot generally be rooted in fact.

### 3.2 Antecedent Probabilities Do Not (Generally) Express Facts

The principle of insufficient reason, mainly associated with Laplace, states that in the absence of reason to think one possibility more likely than the other, we should distribute our credences over possibilities equally.<sup>16</sup> Peirce takes this to be a core principle of the conceptualism he attacks. Thus, considering the case of a single hypothesis, he writes that “[i]n the conceptualistic view of probability, complete ignorance, where the judgment ought not to swerve either toward or away from the hypothesis, is represented by the probability 1/2” (1878, W 3:296).

Peirce first produces an example to show that the principle of insufficient reason is inconsistent.<sup>17</sup> The general point here, that (a naive application of) the principle allows for multiple conflicting uniform probability distributions, is subsumed in Peirce’s subsequent and more elaborate discussion in the context of inductive inference, to which we turn now. This part of Peirce’s argument has also been discussed (and defended) in detail by Burch (2010).<sup>18</sup>

Thus Peirce turns his attention to what he elsewhere calls the doctrine of “inverse probabilities,” which refers to the subjectivist derivation, using Bayes’s rule, of a posterior probability distribution on the basis of evidence and, unavoidably, some prior distribution. Peirce first advances his central distinction between “[e]xplicative, analytic, or deductive” and “[a]mpliative, synthetic, or (loosely speaking) inductive” reasoning (1878, W 3:297), and notes that while probabilistic calculations can only be of the first kind, conceptualists maintain that their method of inverse probabilities actually constitutes ampliative reasoning.<sup>19</sup> The suggestion is that this cannot be right; and Peirce proceeds to identify the catch. Unsurprisingly, this is the prior distribution.

Peirce starts with an example he attributes to Quetelet, where, on having seen the tide rise  $m$  times, we assign probability  $\frac{m+1}{m+2}$  to the prediction that it rises again. This is an instance of Laplace’s infamous rule of succession, and Peirce notes, by taking  $m = 0$ , that this rule relies on the principle of insufficient reason for a prior distribution.<sup>20</sup> He continues,

The manner in which it has been reached has been by considering a number of urns all containing the same number of balls, part white and part black. [...] But the only possible reason for drawing any analogy between such an arrangement and that of Nature is the principle that alternatives of which we know nothing must be considered as equally probable. But this principle is absurd. (1878, W 3:298)

In particular, “[t]here is an indefinite variety of ways of enumerating the different possibilities, which, on the application of this principle, would give different results.” Peirce then proceeds to work out in detail an alternative and “far better” (1883, W 4:442) way of carving

out the elementary possibilities, which, however, leads to a distribution such that “the colors of the balls already drawn have no influence on the probability of any other being white or black” (1878, W 3:300). That is, we would be led to the conclusion that “Nature is a pure chaos, or chance combination of independent elements, in which reasoning from one fact to another would be impossible.”

This is an expression of the observation, made before by Boole,<sup>21</sup> that an otherwise natural-looking choice of uniform prior distribution can encode an assumption that induction is impossible.<sup>22</sup> But Peirce draws a more sweeping conclusion. This is that *no* prior over “possible universes” makes sense:

The relative probability of this or that arrangement of Nature is something which we should have a right to talk about if universes were as plenty as blackberries, if we could put a quantity of them in a bag, shake them well up, draw out a sample, and examine them to see what proportion of them had one arrangement and what proportion another. (1878, W 3:300–301)

We see that Peirce’s critique of the principle of insufficient reason is a special instance of the general complaint that the “doctrine of inverse probability” requires prior or “antecedent probabilities” which must generally lack a factual basis. Thus in a later entry for the *Dictionary of Philosophy and Psychology*, Peirce writes,

If these antecedent probabilities were solid statistical facts, like those upon which the insurance business rests, the ordinary precepts and practice would be sound. But they are not and cannot, in the nature of things, be statistical facts. What is the antecedent probability that matter should be composed of atoms? Can we take statistics of a multitude of different universes? [...] All that is attainable are subjective probabilities, or likelihoods, which express nothing but the conformity of a new suggestion to our prepossessions; and these are the source of most of the errors into which man falls, and of all the worst of them. (1902, CP 2.777)

In short, this is a reiteration of the demand that probability must be rooted in fact.

### 3.3 Plain Dismissal or Pedigree Demand?

We now return to the question of how to diagnose Peirce’s stipulation that probability rest in fact. What exactly is the charge against the conceptualist position that Peirce derives from it?

We noted above that one possible interpretation of the upshot of Peirce’s stipulation is that there is simply no place for conceptualist probability, because only the objective, factual, notion of probability makes sense. Probabilities must *be* statistical facts. This would make Peirce’s charge, as an argument against conceptualism, a clearly question-begging one.<sup>23</sup>

There is a milder interpretation. Burch (2010, p. 432) writes that

[...] one should not leap to the erroneous view that Peirce would have rejected as false or confused all subjectivist interpretations of probability. Rather, Peirce’s objectivism had two noteworthy features. First, it was an insistence that in the overwhelming majority of cases in which the probability of something is of

concern, what is immediately and directly at issue is a relative frequency of real events. [...] Second, Peirce's objectivism was the claim that if and when the issue does happen to be the degree of belief that should rationally be accorded to some proposition  $B$  given knowledge of some proposition  $A$ , then the objectively understood probability  $P(B|A)$  is the most reasonable measure of this degree. So, rather than rejecting all subjectivist uses of  $P(B|A)$ , Peirce subordinated them to objectivist uses by making them dependent on objectivist uses.

Similarly, [Levi \(2004a, p. 262\)](#) writes,<sup>24</sup>

I am inclined to reconstrue Peirce's view as one that admits that inquirers may assign subjective degrees of probabilistic belief to hypotheses provided those degrees of belief can be grounded or justified by knowledge of objective, statistical, or frequency probability. His objection is to taking numerically determinate judgments of subjective probability seriously in scientific inquiry when they lack such grounding.

More precisely, in Levi's understanding of Peirce, "judgements of numerically determinate belief probability are derived in accordance with principles of 'direct inference' from information about objective statistical probabilities of outcomes of an experiment of some kind" ([2011, p. 41](#)).<sup>25</sup>

On this milder interpretation, Peirce is not completely begging the question against the conceptualist, in that he simply only allows for an objectivist notion of probability. He allows for conceptualist probability qua intensity of belief; however, he demands that a probabilistic judgment has a basis in statistical fact. On this interpretation of Peirce, credences must be justified by such a basis in fact: a pedigree demand.

Note that a precondition of this pedigree demand is that conceptualist probability is subordinated to objective probability. That is, one can only meaningfully hold credences for tracking or estimating objective probabilities. A probability that "matter should be composed of atoms" is automatically disqualified, because such probabilities cannot correspond to any objective probabilities to track, unless one (absurdly, according to Peirce) imagines "a multitude of universes." But one can meaningfully hold credences to track statistics like mortality rates or "those upon which the insurance business rests." The further pedigree demand is that these credences must actually be grounded in information or evidence about these statistical facts. Thus one's credences about mortality rates, or the chance of a consumptive Englishman surviving his trip to Madeira, would still be unjustified if not based on solid statistical information. The reading of Burch and Levi even seems to be that one must have full knowledge of the relevant statistical facts.<sup>26</sup>

We do not think Peirce's writings clearly single out either the one or the other interpretation. But we think we can say the following. Insofar as Peirce allows for a conceptualist notion of probability at all (insofar as, if only for the sake of argument, he does not let his stipulation slide into a claim that conceptualist probability is simply meaningless or superfluous), it is a demand that one must be able to offer a good reason for the subjective probabilities one holds. One must possess statistical evidence or indeed full knowledge in support of one's estimates. The demand is for a justification for the credences one has, and this is a pedigree demand.<sup>27</sup> We can therefore conclude that, insofar as Peirce's demand that probability is based in fact allows for subjective probability at all, it is a pedigree demand.

#### 4 Pedigree in Present-Day Personalism: The Problem of the Priors

Jettisoning pedigree requirements in the context of contemporary theorizing about probability has substantial and interesting consequences. Certain contemporary views in the philosophy of probability can be understood as making strong appeals to the pedigree of epistemic states, even if they are not typically thought of in precisely this way. Here, we consider approaches to the problem of the priors for subjective probability. A common thought is that rational, scientific inquiry should produce objective knowledge. Certainly, much scientific theory is often regarded as objective. Yet, according to “mainstream” Bayesianism, opinions are subjective. Bayesian agents express opinions over some space of possibilities in the form of prior probability distributions. These priors reflect subjective degrees of belief and are constrained only by the axioms of probability theory. These axioms leave ample room for widely divergent opinions. But Bayesians need some way of achieving intersubjective agreement, the worry goes, since “the requirement of intersubjectivity is what makes science objective” (Quine, 1990, p. 5). The *problem of the priors* is a worry about the prospect of conjuring sufficient objectivity from these priors to sustain the picture of science and rational inquiry as objective.

Many subjectivists seem to regard the problem as less pressing than it is sometimes presented. This includes some of the foundational thinkers in the subjectivist tradition. Suppes complains about the lack of attention some of these thinkers have paid to the problem: “The absence of [attention to the origins of priors] can be seen in the rather laconic views about prior probabilities themselves, as expressed by the three most important foundational thinkers on the Bayesian viewpoint in the 20th century, namely, Ramsey, de Finetti and Savage” (2007, p. 441). A few pages later, he notes, “What is remarkable about the views of these three foundational thinkers is that none of them ventures very deeply into the psychological or common sense side of how, in fact, subjective probabilities are formed” (2007, p. 445). What Suppes sees as remarkable can also be seen as a natural consequence of an anti-pedigree epistemological stance. If we ought to focus on improving our current point of view rather than justifying it, justifying priors or discerning their origins may fail to be a high priority—or even sensible.

In an English summary of a joint paper written in Italian with de Finetti, Savage remarks, “The subjective point of view sees all probabilities as the opinions of some person and initial probabilities as his initial opinions, that is, at the beginning of some context under discussion” (1972, p. 143). Continuing, Savage, like Peirce, recognizes that agents must “set out” from their present points of view, with the information they have:

According to the subjective view, probabilities must often be based on vague, uncertain, and fragmentary information. We think it unrealistic and impractical to confine the definition of probability to those limiting situations where impressions of symmetry or frequent experience tend to relieve the formation of opinion from embarrassments of vagueness and interpersonal disagreement. (1972, pp. 143–144)

Rather than focusing on the backward-looking task of justifying initial opinions, Savage suggests looking forward to inquiry: “The effects of disparity between the initial judgments of people or of vagueness in the initial judgments of one person are often largely eliminated by a sufficiently revealing experiment” (1972, p. 145).<sup>28</sup>

Yet the general impulse to justify priors persists in some quarters.<sup>29</sup> Earman points out that, for those concerned to address the problem of the priors, there are two main ways for Bayesians to respond. The first is to constrain priors. The second is to wash them out (1992, pp. 138–39).<sup>30</sup> The second strategy is particularly popular: “The most common and in many ways the most effective Bayesian response to the subjectivity objection is the convergence response [...] [t]he differences in scientists’ priors will be washed out over time, that is, they will become less and less relevant to scientists’ opinions as more and more evidence comes in” (Strevens, 2017, p. 84). In what follows, we argue that both of these strategies for responding to the problem of the priors make appeals to pedigree. One point we want to particularly emphasize is that the second strategy cannot operate independently of the first. Results establishing that priors wash out have preconditions that effectively constrain justified probability judgments to those obtainable from appropriate origins.

#### 4.1 Common Priors

Perhaps the most straightforward reply to the problem of the priors is to deny that initial interpersonal variation in probability judgment is legitimate. In economics, for instance, this view is sometimes known as the *Harsanyi doctrine*. As Aumann observes, “John Harsanyi (1968) has argued eloquently that differences in subjective probabilities should be traced exclusively to differences in information—that there is no rational basis for people who have always been fed precisely the same information to maintain different subjective probabilities. This, of course, is equivalent to the assumption of equal priors” (1976, pp. 1237–1238).<sup>31</sup> Morris claims that the common prior assumption “has become an article of faith among economists” (1995, p. 227). One important result, Aumann’s Agreement Theorem, establishes the surprising fact that two agents cannot “agree to disagree” about the probability of an event provided they satisfy the common prior assumption and their posteriors are common knowledge. Through Aumann’s result and others (e.g., Harsanyi, 1968; Milgrom and Stokey, 1982), the common prior assumption has assumed a significant role in game theory and microeconomics.

The Harsanyi doctrine emerged in somewhat different guises in other areas of research on probability. Some *objective* Bayesians champion formalisms that allegedly deliver a *uniquely* rational prior. Carnap (1950) famously suggested an objective-logical prior distribution for an explicatum of the notion of degree of confirmation, though he soon weakened this to a continuum of permissible functions. Jaynes claims objective priors must be sought if Bayesian methods are to play a role in science: “if the methods are to have any relevance to science, the prior distribution must be completely ‘objective’, in the sense that it is independent of the personality of the user [...] The measure of success [...] is just the extent to which we are able to eliminate all personalistic elements and create a completely ‘impersonalistic’ theory” (1968, p. 117). Such objectivist approaches, like the common prior assumption in economics, are prime examples of Earman’s first strategy for responding to the problem of the prior. Differences in probability judgments emerge only if there is a failure of rationality or agents have yet to fully share their information. A number of criticisms have been leveled against objectivist methods (Seidenfeld, 1986; Earman, 1992; Howson and Urbach, 2006). While we are sympathetic to these objections, we refrain from entering into that discussion. Our point here is that variants of the Harsanyi doctrine place pedigree requirements on rational probability judgments: agents’ current views should be obtainable from conditioning

a common prior or else are unjustified.

The Harsanyi doctrine hardly commands universal assent among probabilists. “[N]one of the arguments in support of the common prior assumption is compelling,” writes Morris in his survey (1995, p. 243). Edwards, Lindman, and Savage also demur: “Your opinions about a coin can of course differ from your neighbor’s. For one thing, you and he may have different bodies of relevant information. We doubt that this is the only legitimate source of difference of opinion. Hence the personal in personal probability” (1963, p. 197). Savage is not “distressed” by this lack of agreement and seems to sense no great urgency to offer a sure-fire reply to the problem of the priors:

The criteria incorporated in the personalistic view do not guarantee agreement on all questions among all honest and freely communicating people, even in principle. That incompleteness, if one will call it such, does not distress me, for I think that at least some of the disagreement we see around us is due neither to dishonesty, to errors in reasoning, nor to friction in communication [...] (1954, pp. 67–68)

While intersubjective agreement is not guaranteed for Bayesians on Savage’s account, there are certain mechanisms that generate agreement in certain circumstances. For example, different agents may, with some frequency, make similar prior probability judgments based on similar assessments of symmetry, even if, *pace* Harsanyi, such agreement is not rationally mandated: “there are partitions that diverse people all consider nearly uniform, though not compelled to that agreement by any postulate of the theory of personal probability” (1954, p. 68). In other cases, evidence and inquiry may bring about agreement among different priors. The viability of this approach, rather than requiring that agents set out with exactly the same opinion, requires merely that their priors stand in a certain relationship to one another that is less demanding than identity. To elevate this possibility to an inevitability, however, imposes a certain pedigree requirement on probability judgments which we explain next.

## 4.2 Priors Wash Out

If the problem of the priors lacks a compelling solution, then, some say, subjective probability is unable to account for the rationality and objectivity of science. But, as Huttegger observes, “a Peircean response to this charge [...] has been noted by many authors [...] The basic thought is that disagreement often turns out to be transient and, as more evidence becomes available, a consensus may emerge. The underlying empiricist credo is that experience trumps any initial belief state; diverging opinions are just a sign that not enough evidence has accumulated yet” (2015, p. 613). (There is some irony in subjective Bayesians turning to some of Peirce’s ideas to vindicate their viewpoint given Peirce’s attacks on subjectivism.) In 1878, Peirce articulates the outlines of a conception of scientific method leading to consensus through the acquisition of evidence:

They may at first obtain different results, but, as each perfects his method and his processes, the results will move steadily together toward a destined centre. So with all scientific research. Different minds may set out with the most antagonistic views, but the progress of investigation carries them by a force outside of themselves to one and the same conclusion. This activity of thought by which we

are carried, not where we wish, but to a foreordained goal, is like the operation of destiny. No modification of the point of view taken, no selection of other facts for study, no natural bent of mind even, can enable a man to escape the predestinate opinion. This great law is embodied in the conception of truth and reality. The opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real. That is the way I would explain reality. (1878, W 3:273)

The “Peircean” response to the problem of the priors is an important and widely-discussed aspect of Bayesian theory. Writing more than half a century before Huttegger, Edwards, Lindman, and Savage remark, “Although your initial opinion about future behavior of a coin may differ radically from your neighbor’s, your opinion and his will ordinarily be so transformed by application of Bayes’ theorem to the results of a long sequence of experimental flips as to become nearly indistinguishable. This approximate merging of initially divergent opinions is, we think, one reason why empirical research is called ‘objective’” (1963, p. 197).<sup>32</sup> Suppes shares a similar perspective:

It is of fundamental importance to any deep appreciation of the Bayesian viewpoint to realize the particular form of the prior distribution expressing beliefs held before the experiment is conducted is not a crucial matter [...] For the Bayesian, [...] the existence of a systematic method for reaching agreement is important [...] The well-designed experiment is one that will swamp divergent prior distributions with the clarity and sharpness of its results, and thereby render insignificant the diversity of prior opinion. (1966, p. 204)

More recently, Cisewski et al. write in connection with merging results, “We follow, for example, Peirce in *requiring* that sound scientific methodology provides investigators with the resources to resolve interpersonal disagreements through shared evidence” (2017, p. 73, emphasis added). The idea, in short, is that, under certain conditions, when Bayesians update their prior opinions on shared evidence *via* conditionalization, initial disagreements give way to intersubjective agreement. Supporting this sort of response to the problem of the priors are deep theorems establishing conditions under which priors wash out or opinions *merge* (Blackwell and Dubins, 1962; Gaifman and Snir, 1982). As Huttegger observes, “The Blackwell-Dubins theorem specifies under what conditions two conditional probability measures will become and remain close. If the two probability measures are viewed as prior degrees of belief, this means that agents will expect their conditional beliefs to be approximately the same in the long run” (2015, p. 613).

There are a number of reservations that one might voice in response to using the merging theorem to respond to the problem of the priors.<sup>33</sup> The fundamental limitation of this response for our purposes, however, is that merging of opinions does not hold generally. Instead, opinions merge if, and essentially only if, priors stand in a certain relation called *absolute continuity*. One probability function  $P$  is absolutely continuous with respect to another probability function  $Q$  if, to any event that  $Q$  assigns probability 0,  $P$  assigns probability 0, too. Informally and roughly, the Blackwell-Dubins theorem says that if  $P$  is absolutely continuous with respect to  $Q$ , then  $P$  assigns probability 1 to agreeing with  $Q$  in the limit of inquiry. A form of the converse implication has also been established (Kalai and Lehrer, 1994, Theorem 2). When absolute continuity is relaxed in a very mild, conceptually-motivated way,

not only do we lose merging of opinions, we open the door to opinion polarization—the case in which opinions grow increasingly “further apart”—even when agents update on a shared, infinite stream of evidence (Nielsen and Stewart, 2021, Theorem 3).

Requiring that all agents’ priors be absolutely continuous with respect to each other, however, is a substantial constraint on the form that priors are permitted to take. If we are to accept absolute continuity with respect to some privileged prior or set of priors as a requirement on probability judgments, current probability judgments are subject to a pedigree requirement. Any agent’s current probability judgments must be obtainable by conditioning a prior chosen from a privileged class of priors, otherwise, it seems we must conclude, they are unjustified. If we do not think of satisfying the preconditions of the theorem as mandatory, then it is difficult to see the merging theorems as offering a satisfactory solution to the problem of the priors. We have, instead, that some probabilities happen to achieve intersubjective agreement; others do not. Critics of the Bayesian paradigm may fail to be impressed with this much “objectivity.” Rationality does not necessarily—or even typically, perhaps—secure intersubjective agreement after all. Huttegger is admirably less coy than many Bayesians in discussing this limitation of the merging theorem: “It has to be emphasized, though, that the conditions that are required for [merging of opinions] to hold are far from being trivially satisfied. Our priors may fail to be absolutely continuous, and there may be no way you could persuade me to change my priors by appealing to my rationality” (2015, p. 636).

It may be tempting to respond by arguing that absolute continuity—to be more precise, the requirement that all rational agents take priors from the *same* class of mutually absolutely continuous probabilities—is a legitimate rationality requirement. At least in small probability spaces, many may find it eminently reasonable. Regularity, the requirement that every non-empty event be assigned positive probability, enjoys fairly widespread support (e.g., Shimony, 1955; Lewis, 1980; Skyrms, 1995), and every prior is trivially absolutely continuous with respect to a regular prior. If regularity is required, then, mutual absolute continuity follows for all legitimate priors. But we want to make three points about this sort of response. First, absolute continuity is non-trivial in general probability spaces, and regularity generally mathematically unachievable (in standard probability theory). In general spaces, restricting priors to a class of mutually absolutely continuous probabilities in order to appeal to the merging theorem is tantamount to requiring a uniform and permanent dogmatism about contingent matters: rationality, on this view, requires all agents to essentially dismiss a certain set of contingent hypotheses out of the gate. Since there are various classes of mutually absolutely continuous priors, which contingent hypotheses get dismissed is arbitrary without further motivation for a particular set of mutually absolutely continuous priors.<sup>34</sup> Assuming mutual absolute continuity in general spaces, according to Morris, “seems scarcely weaker than assuming the common prior assumption” (1995, p. 237). Second, even in discrete probability spaces, the regularity route to absolute continuity is at odds with the account of full belief sketched above. The set of serious possibilities that are candidates to bear positive probability is subject to revision according to changes in full beliefs, and different agents have different sets of full beliefs. Third, and most importantly for our purposes, a requirement that current probability judgments result from priors selected from a certain class is, in any event, a form of pedigree requirement for probabilistic beliefs. An agent’s current probability judgments stand in need of justification in the sense that they are subject to criticism for lack of appropriate origin in much the same way that an agent’s current full beliefs are subject to

criticism for lack of appropriate origins according to, say, foundationalists.

## 5 Peroration

A striking, persistent, and perhaps under-emphasized feature of Peirce’s thinking about epistemology is his resistance to the importance of pedigree. We find expressions of such resistance early on in “The Fixation of Belief,” for example, and much later on within four years of his final publication. His resistance to pedigree, however, does not seem to transfer to contexts in which he discusses subjective interpretations of probability. When Peirce offers arguments against subjectivism, allowing that the interpretation makes at least minimal sense, he objects to it for pedigree reasons: subjective priors do not admit the appropriate sort of justification *via* grounding in statistical fact.

What would resistance to pedigree look like in probabilistic contexts? The sort of view that is most consonant with a rejection of pedigree, we submit, is not one that attempts to address the problem of the priors either by rejecting any interpersonal variation as illegitimate or by constraining variation to a class for which merging of opinions obtains. In either case, an agent’s current probability judgments would be unjustified if they did not result from an “appropriate” prior, either from a prior in common with other inquirers or from a prior in a class of mutually absolutely continuous priors. To say more than Savage, to say that rationality invariably leads to intersubjective agreement, for instance, requires imposing pedigree requirements on subjective probability judgments. While many epistemologists may be sympathetic to pedigree requirements, on Peirce’s belief-doubt model, such requirements are illegitimate. Jettisoning pedigree requirements for probability judgments requires rethinking the nature and extent of objectivity available to subjectivists.

## Notes

1. It is impossible to deny that Peirce ridicules this very view at points. He writes, for instance, that “infallibility in scientific matters seems to me irresistibly comical” (1897, CP 1.9). Elsewhere, he writes, “We cannot be absolutely certain that our conclusions are even approximately true” (1897, CP 1.141). Levi’s proposal to resolve the tension between remarks like these and those cited above is to distinguish fallibility, which he rejects and thinks Peirce should too, and corrigibility, which he and Peirce accept and which he thinks is what Peirce is really after in certain passages discussing fallibilism (1980, pp. 14–19). For a different resolution of this apparent tension in Peirce’s thinking, see (Boyd, 2012).
2. One immediate question about this project is how such a decision theory could justify “contracting” or giving up a full belief, a belief about which an agent recognizes no serious possibility of error. Levi provides two decision-theoretic justifications in terms of the aims of inquiry (1991, p. 117).
3. More precisely, adapting an idea from Carnap, Levi takes *confirmational commitments* rather than probability functions as the relevant probabilistic component of an agent’s point of view. A confirmational commitment is a function that associates any state of full belief with a probability function (or, more generally, a set of probability functions, allowing for imprecise probabilities). So a confirmational commitment is like a plan for probabilistic assessments whatever the evidence or background knowledge happens to be. Levi, at least at certain points, interprets a confirmational commitment as specifying the “seriously permissible” probability judgments relative to a given state of full belief (1980, p. 89).
4. For a Savage-inspired argument that Levi frequently gives that agents cannot coherently suspend judgment between or be in doubt about different subjective probability assessments—in the context of precise probabilities at least—see, e.g., (2004b, p. 463).

5. As Peirce himself notes, “[t]he conception of probability here set forth is substantially that first developed by Mr. Venn, in his *Logic of Chance*” (1878, W 3:281). See Verburgt (2021) on the elements of pragmatism in Venn’s thinking, including a comparison of his and Peirce’s ideas about probability and belief.
6. In a endnote added to “The Doctrine of Chances” (1910, CP 2.664), Peirce writes about the “would-be” of a die to turn up in a certain way, thus foreshadowing the modern propensity interpretation of probability (see, e.g., Suárez, 2013).
7. This term Peirce also took from Venn. We will also use the modern terms “subjective,” “personalist,” and “Bayesian” here. The main issue concerns the legitimacy of a *credal* or *degree of belief* account of probability.
8. “Nobody can go further than I in condemnation of this way of using probability, which completely vitiates the theory and practice of Inductive and Abductive reasoning, has set back civilization, and has corrupted ideals” (1902, CP 2.101).
9. By the *chance* Peirce here actually refers to “the ratio of favorable to unfavorable cases,” as opposed to the probability, the ratio of favorable to all cases (i.e., all applications of the relevant argument that yield a true conclusion to all applications of the argument). In modern terms, if we cast the relevant event as an *hypothesis*  $H$  and the relevant premises as *evidence*  $E$ , Peirce’s chance of the event corresponds to the *odds* for  $H$  given  $E$ , that is, the ratio  $P(H|E)/P(-H|E)$ .
10. That is, in modern notation,  $\log P(H|E) - \log P(-H|E)$ . Peirce here also refers to “Fechner’s psycho-physical law” of the logarithmic relationship between the physical and psychological intensity of a stimulus (1878, W 3:294). Interestingly, according to Stigler (1978, p. 248), experimental work that Peirce did (1884, CP 7.21–35) shows that despite being an objectivist he “was also one of the first individuals (perhaps the very first) to experimentally elicit subjective or personal probabilities, determining that these probabilities varied approximately linearly with the log odds.”
11. In particular, the notion of Good (1985). See Levi (2011) and Kasser (2016) for details and discussion.
12. Peirce’s own approach, that he presents immediately after (1878, W 3:301–303), is essentially the modern Neyman-Pearson approach to confidence interval estimation—which arguably does involve both factors (Levi, 2011, p. 47; Kasser, 2016, p. 646).
13. Note that this is different from Good’s notion of weight of evidence mentioned in note 5. See again Levi (2011) and Kasser (2016).
14. Popper formulated a similar challenge as “the paradox of ideal evidence” (1959, pp. 454ff); for a reply see Jeffrey (1983, pp. 196f). Bayesian accounts of “evidential weight” include the introduction of notions of resiliency (Skyrms, 1977; Joyce, 2005) and the shift to imprecise credences (Hill, 2019).
15. Kasser, referring in an endnote to Levi, writes that “the construal of probabilities and chances in terms of frequencies doesn’t (yet) beg any issue between conceptualists and materialists. If conceptualists can account for the legitimacy of probabilistic inferences in terms of degrees of belief, then the view will have been vindicated, for Peirce’s purposes” (2016, p. 635). As we will discuss next, however, this same stipulation does play a more clearly question-begging role in other passages. At the same time, our main point will be that there is an alternative diagnosis of Peirce’s general stipulation that probability must be based in fact, on which his charge against conceptualism is not so much question-begging as a pedigree demand. In relation to this we will below also engage with Levi’s interpretation.
16. As Laplace (1814, p. 4) puts it, we assign equal probability to “equally possible cases, that is to say, to cases whose existence we are equally uncertain of.” See Zabell (1988) for a history of the principle. Versions of the principle are nowadays usually referred to as “the principle of indifference,” after Keynes (1921, ch. 4).
17. This example, concerning the color of the hair of inhabitants of Saturn, is a slightly more elaborate version of Keynes’s book example (1921, ch. 4). Suppose we do not know the color of a particular book in our library. In particular, we do not know whether it is red; hence, by insufficient reason, we equally assign credence  $1/2$  to it being red and to it being non-red. But by analogous reasoning, we do the same for it being blue and non-blue, and it being green and non-green. This leads to an incoherent set of probability assignments.
18. Burch simply accepts Peirce’s stipulation that “subjectivism cannot be basic and autonomous. It must be

posterior to and derivative from objectivism” (2010, p. 432), and approvingly summarizes Peirce’s analysis of the flaw in the use of inverse probability as follows: “In effect, then, a question about objective probabilities has been ‘answered’ by illicitly smuggling into the picture a subjectivist view and by substituting a subjective probability for the objective probability that is plainly called for. The Method [of inverse probability] breaks down [...] because we cannot rationally make any of the crucial objective probability assignments required by the Method. It is exactly at this point that the user of the Method of Inverse Probabilities silently purloins the objectivist account of probability from the scene and slyly substitutes for it the subjectivist account” (pp. 438-439). Dismissing non-objective prior probabilities out of hand in this way, Burch here actually provides a nice illustration of how to understand Peirce’s critique in a fully question-begging way. Yet see section 3.3.

19. See Merrill (1975); Levi (2004a) for more on Peirce’s classification of different types of (probabilistic) inference.
20. Burch (2010, p. 437) assigns this “single burst of brilliant illumination” a central place in Peirce’s argument. It should be noted that there is nothing particularly insightful about the observation that the rule of succession is rooted in the principle of insufficient reason; this is how Laplace derived it, as Peirce also notes in the following. See Zabell (1989) for more details on the rule of succession and its history.
21. If we “let the assumed principle be, that *all possible constitutions of the system of balls are equally probable*” (Boole, 1854, p. 370) then “past experience does not in this case affect future experience” (1854, p. 371) . Peirce mentions Boole when he repeats the point in (1883, W 4:408–450).
22. Later instances of this observation include Carnap (1950, p. 565), who discusses the confirmation function  $c_f$  that relies on a “principle of equiprobability of individual distributions” that he locates in Peirce, Keynes, and Wittgenstein, and that would be “tantamount to the principle never to let our past experiences influence our expectations for the future.” A more recent example are the so-called no-free-lunch theorems in machine learning, that purport to show that no possible learning algorithm is better than random guessing, but that crucially rely on such a uniformity assumption about chances (see Sterkenburg and Grünwald, 2021).
23. One might insist that it is unfair to call Peirce’s charge question-begging, as his own objectivist conception of probability is rooted in his independently-motivated pragmatic principle. However, the relevant stipulation that probability must be rooted in fact, as Peirce uses it in his charge against conceptualism, is not a necessary consequence of his principle. As a case in point, the same principle can be used to motivate a behavioral conception of subjective probability along the lines of (Ramsey, 1990), (de Finetti, 1975), and (Savage, 1954).
24. Also see his (1995, pp. 65-67; 2004a, p. 269; 2011, pp. 41-42). Notably, however, Levi appears to take a different view in (2004b, pp. 472-475).
25. That is, via “probable deductions of the form ‘The proportion  $\rho$  of the  $M$ ’s are  $P$ ’s;  $S$  is an  $M$ ; therefore it follows, with probability  $\rho$ , that  $S$  is  $P$ ’” (Levi, 2004a, p. 260).
26. On the discussion of the consumptive Englishmen, Levi writes that “Peirce chastised Venn on the grounds that the statistical probability of a consumptive Englishman’s surviving is unknown. [...] There is, in effect, no way to ground judgments of credence in knowledge of statistical probability” (1995, p. 66).
27. We can distinguish between justifying current probability judgments—pedigree epistemology—and justifying *revisions* of probability judgments in light of knowledge of statistical data when we have it. The former seems to be Peirce’s concern; the latter does not challenge the notion of prior probabilities.
28. According to Levi, Savage and de Finetti endorse a form of Peircean “tenacity” in thinking that priors are only updated by conditionalization. Since Levi takes the fundamental probabilistic component of an agent’s point of view to be a confirmational commitment, i.e., a map associating states of full belief with probability functions, conditionalization itself is not *really* a revision in probabilistic point of view since it does not involve revising an agent’s confirmational commitment. Bayesians who restrict changes in judgments of probability to conditionalization of a fixed prior require that agents pick a confirmational commitment and stick to it come what may. It is in this sense that Levi—who allows for revision of confirmational commitments—accuses them of endorsing a form of tenacity. The result, he says, is dogmatism and arbitrariness (1974, p. 406).
29. It is worth remarking that the particular sorts of justifications that Peirce is concerned to refute no longer find wide support among personalists. Like Peirce, many subjectivists are wary of the principle of insufficient reason. Savage, for instance, observes, “Most holders of personalistic views do not find the principle of

insufficient reason compelling” (1954, p. 64). Others continue to reinforce this observation (Seidenfeld, 1979; Levi, 1982). For this reason, Peirce’s attacks on personalism by way of attacks on the principle of insufficient reason cannot rightly be regarded as persuasive when it comes to many contemporary subjectivist views. Similarly, Peirce’s charge that subjectivists “have not undertaken to say what is meant by ‘degree of credence’” seems unsustainable following the work of de Finetti, Ramsey, and Savage.

30. A third is probabilistic opinion pooling (Genest and Zidek, 1986; Dietrich and List, 2014). Huttegger points out that pooling, unlike the washing out approach, relies on more than coherence and conditionalization (2015, p. 613). That’s true. But, in some forms, it also dispenses with the constraints on priors required for washing out.
31. About opinion pooling, Aumann remarks, “It seems to me that the Harsanyi doctrine is implicit in much of this literature; reconciling subjective probabilities makes sense if it is a question of implicitly exchanging information, but not if we are talking about ‘innate’ differences in priors” (1976, p. 1238). If Aumann is right, opinion pooling also relies on constraining priors in responding to the problem of the priors. When the common prior assumption holds for some group of inquirers, a case can be made that, under certain conditions, geometric pooling enjoys special normative status due to its equivalence with supra-Bayesianism, the Bayesian response to learning the opinion of others (Bacelli and Stewart, 2021).
32. In other passages, Savage clearly recognizes the limitations of this sort of reply, that it only applies to initial opinions that happen to be sufficiently similar, for example. Furthermore, he writes, “it is typically true of any observational program, however extensive but prescribed in advance, that there exist pairs of opinions, neither of which can be called extreme in any precisely defined sense, but which cannot be expected, either by their holders or any other person, to be brought into close agreement after the observational program” (1954, p. 68).
33. For example, it may be possible, for all the theorem says, for  $P$  to fail to *actually* achieve consensus with  $Q$  in the limit. It is just that  $P$  assigns probability 0 to such an outcome. The Blackwell-Dubins theorem, in other words, only makes a claim about what happens “almost-surely” from the perspective of  $P$ , i.e., with  $P$ -probability 1. It is not clear to us that the merging theorem, given this almost sure hedge, delivers on Peirce’s program. For an investigation of the even more restrictive assumptions required to remove the almost sure hedge, see Nielsen (2018, Sec. 4). A second reservation about interpreting the merging theorem as a Peircean response to the problem of the priors concerns corrigibility. Yet another assumption of the theorem is that evidence forms a *filtration*. An intuitive way to think about that technical condition is that evidence only increases, or, put another way, that agents, at some point, only *add* to their stock of full beliefs. If we allow that agents might also have reasons to contract beliefs (let alone change their “initial” priors), the applicability of the theorem becomes even more fraught.
34. See Earman (1992) and Huttegger (2015) for further reflections on the status of absolute continuity. Nielsen and Stewart argue that absolute continuity is not a rationality requirement, nor is it empirically motivated, nor does it mark an interesting conceptual distinction between disagreement and radical disagreement for an independently-motivated account of disagreement (2021, pp. 65–66).

## References

- Aumann, R. J. (1976). Agreeing to disagree. *The Annals of Statistics* 4(6), 1236–1239.
- Bacelli, J. and R. T. Stewart (2021). Support for geometric pooling. *The Review of Symbolic Logic*, doi.org/10.1017/S1755020320000416.
- Blackwell, D. and L. E. Dubins (1962). Merging of opinions with increasing information. *The Annals of Mathematical Statistics* 33(3), 882–886.
- Boole, G. (1854). *An Investigation of the Laws of Thought*. London: Macmillan.
- Boyd, K. (2012). Levi’s challenge and Peirce’s theory/practice distinction. *Transactions of the Charles S. Peirce Society* 48(1), 51–70.

- Burch, R. (2010). If universes were as plenty as blackberries: Peirce on induction and verisimilitude. *Transactions of the Charles S. Peirce Society* 46(3), 423–452.
- Carnap, R. (1950). *Logical Foundations of Probability*. Chicago, IL: The University of Chicago Press.
- Cisewski, J., J. Kadane, M. Schervish, T. Seidenfeld, and R. Stern (2017). Standards for modest Bayesian credences. *Philosophy of Science* 85(1), 2018.
- de Finetti, B. (1972). *Probability, Induction, and Statistics*. New York: John Wiley & Sons.
- de Finetti, B. (2017, originally published in 1975). *Theory of Probability: A Critical Introductory Treatment*. John Wiley & Sons.
- Dietrich, F. and C. List (2014). Probabilistic opinion pooling. In A. Hájek and C. Hitchcock (Eds.), *Oxford Handbook of Probability and Philosophy*. Oxford University Press.
- Earman, J. (1992). *Bayes or Bust?* Cambridge, MA: MIT Press.
- Edwards, W., H. Lindman, and L. J. Savage (1963). Bayesian statistical inference for psychological research. *Psychological Review* 70(3), 193–242.
- Gaifman, H. and M. Snir (1982). Probabilities over rich languages, testing and randomness. *The Journal of Symbolic Logic* 47(03), 495–548.
- Galavotti, M. C. (2019). Pragmatism and the birth of subjective probability. *European Journal of Pragmatism and American Philosophy* 11(XI-1), 1–13.
- Genest, C. and J. V. Zidek (1986). Combining probability distributions: A critique and an annotated bibliography. *Statistical Science* 1(1), 114–135.
- Good, I. J. (1985). Weight of evidence: A brief survey. In J. Bernardo, M. DeGroot, D. Lindley, and A. Smith (Eds.), *Bayesian Statistics 2: Proceedings of the Second Valencia International Meeting*, pp. 249–270. North-Holland.
- Harsanyi, J. C. (1968). Games with incomplete information played by “Bayesian” players part III. The basic probability distribution of the game. *Management Science* 14(7), 486–502.
- Hill, B. (2019). Confidence in belief, weight of evidence and uncertainty reporting. *Proceedings of Machine Learning Research* 103, 235–245.
- Howson, C. and P. Urbach (2006). *Scientific Reasoning: The Bayesian Approach* (3<sup>rd</sup> ed.). Chicago, IL: Open Court Publishing.
- Huttegger, S. M. (2015). Merging of opinions and probability kinematics. *The Review of Symbolic Logic* 8(04), 611–648.
- James, W. (2000, originally published in 1896). The will to believe. In *Pragmatism and Other Writings*. New York, NY: Penguin.
- Jaynes, E. T. (1968). Prior probabilities. *IEEE Transactions on Systems Science and Cybernetics* 4(3), 227–241.
- Jeffrey, R. C. (1983). *The Logic of Decision* (2nd ed.). Chicago: University of Chicago Press.
- Joyce, J. M. (2005). How probabilities reflect evidence. *Philosophical Perspectives* 19, 153–178.

- Kalai, E. and E. Lehrer (1994). Weak and strong merging of opinions. *Journal of Mathematical Economics* 23(1), 73–86.
- Kasser, J. (1999). Peirce’s supposed psychologism. *Transactions of the Charles S. Peirce Society* 35(3), 501–526.
- Kasser, J. (2016). Two conceptions of weight of evidence in Peirce’s *Illustrations of the Logic of Science*. *Erkenntnis* 81, 629–648.
- Keynes, J. M. (1921). *A Treatise on Probability*. London: Macmillan.
- Laplace, P.-S. (1814). *Essai Philosophique sur les Probabilités*. Paris: Courcier. Page numbers refer to English translation of the 5th French edition (1825) by I.D. Dale, volume 13 of *Sources in the History of Mathematical and Physical Sciences*, New York: Springer, 1994.
- Levi, I. (1974). On indeterminate probabilities. *The Journal of Philosophy* 71(13), 391–418.
- Levi, I. (1980). *The Enterprise of Knowledge*. MIT Press, Cambridge, MA.
- Levi, I. (1982). Ignorance, probability and rational choice. *Synthese* 53(3), 387–417.
- Levi, I. (1991). *The Fixation of Belief and Its Undoing: Changing Beliefs through Inquiry*. Cambridge University Press.
- Levi, I. (1995). Induction according to Peirce. In K. L. Ketner (Ed.), *Peirce and Contemporary Thought: Philosophical Inquiries*, pp. 59–93. New York: Fordham University Press.
- Levi, I. (2004a). Beware of syllogism: Statistical reasoning and conjecturing according to Peirce. In C. Misak (Ed.), *The Cambridge Companion to Peirce*, Chapter 11, pp. 257–286. Cambridge: Cambridge University Press.
- Levi, I. (2004b). The logic of consistency and the logic of truth. *Dialectica* 58(4), 461–482.
- Levi, I. (2011). The weight of argument. In S. M. D. Brandolini and R. Scazzieri (Eds.), *Severe Uncertainty: Rationality and Plausible Reasoning*, pp. 39–58. Palgrave.
- Lewis, D. (1980). A subjectivist’s guide to objective chance. In W. L. Harper, R. Stalnaker, and G. Pearce (Eds.), *IFS*, pp. 267–297. Springer.
- Merrill, G. (1975). Peirce on probability and induction. *Transactions of the Charles S. Peirce Society* 11(2), 90–109.
- Milgrom, P. and N. Stokey (1982). Information, trade and common knowledge. *Journal of Economic Theory* 26(1), 17–27.
- Morris, S. (1995). The common prior assumption in economic theory. *Economics & Philosophy* 11(2), 227–253.
- Nielsen, M. (2018). Deterministic convergence and strong regularity. *The British Journal for the Philosophy of Science* 71(2020), 1461–1491.
- Nielsen, M. and R. T. Stewart (2021). Persistent disagreement and polarization in a Bayesian setting. *The British Journal for the Philosophy of Science* 72(1), 51–78.
- Peirce, C. S. (CP). *The Collected Papers of Charles Sanders Peirce*, Volume I–VIII (1931–1958). Cambridge, MA: Harvard University Press.

- Peirce, C. S. (W). *Writings of Charles Sanders Peirce: A Chronological Edition*, Volume 1–7 (1982–2010). Bloomington: Indiana University Press.
- Popper, K. (2002, originally published in 1959). *The Logic of Scientific Discovery*. London: Routledge Classics.
- Quine, W. V. O. (1990). *Pursuit of Truth*. Cambridge, MA: Harvard University Press.
- Quine, W. V. O. (2013, originally published in 1960). *Word and Object*. MIT Press.
- Ramsey, F. P. (1990). Truth and probability. In D. H. Mellor (Ed.), *Philosophical Papers*, pp. 52–109. Cambridge University Press.
- Savage, L. (1972, originally published in 1954). *The Foundations of Statistics*. New York: John Wiley and Sons.
- Seidenfeld, T. (1979). Why I am not an objective Bayesian: Some reflections prompted by Rosenkrantz. *Theory and Decision* 11(4), 413–440.
- Seidenfeld, T. (1986). Entropy and uncertainty. *Philosophy of Science* 53(4), 467–491.
- Shimony, A. (1955). Coherence and the axioms of confirmation. *The Journal of Symbolic Logic* 20(01), 1–28.
- Skyrms, B. (1977). Resiliency, propensities and causal necessity. *The Journal of Philosophy* 74(11), 704–713.
- Skyrms, B. (1995). Strict coherence, sigma coherence and the metaphysics of quantity. *Philosophical Studies* 77(1), 39–55.
- Sterkenburg, T. F. and P. D. Grünwald (2021). The no-free-lunch theorems of supervised learning. *Synthese* 199, 9979–10015.
- Stigler, S. M. (1978). Mathematical statistics in the early States. *The Annals of Statistics* 6(2), 239–265.
- Strevens, M. (2017). Notes on Bayesian confirmation theory. <http://www.strevens.org/bct/BCT.pdf>. Unpublished Notes.
- Suárez, M. (2013). Propensities and pragmatism. *The Journal of Philosophy* 60(2), 61–92.
- Suppes, P. (1966). A Bayesian approach to the paradoxes of confirmation. *Studies in Logic and the Foundations of Mathematics* 43, 198–207.
- Suppes, P. (2007). Where do Bayesian priors come from? *Synthese* 156(3), 441–471.
- Verburgt, L. M. (2021). Pragmatism at Cambridge, England before 1900. *British Journal for the History of Philosophy* 29(1), 84–105.
- Zabell, S. L. (1988). Symmetry and its discontents. In B. Skyrms and W. L. Harper (Eds.), *Causation, Chance, and Credence*, pp. 155–190. Dordrecht: Kluwer.
- Zabell, S. L. (1989). The rule of succession. *Erkenntnis* 31, 283–321.