

Randomization and Fair Judgment in Law and Science

Julio Michael Stern*,
Marcos Antonio Simplicio†,
Marcos Vinicius M. Silva‡,
Roberto A. Castellanos Pfeiffer§

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Abstract

Randomization procedures are used in legal and statistical applications, aiming to shield important decisions from spurious influences. This article gives an intuitive introduction to randomization and examines some intended consequences of its use related to truthful statistical inference and fair legal judgment. This article also presents an open-code Java implementation for a cryptographically secure, statistically reliable, transparent, traceable, and fully auditable randomization tool.

Keywords: Randomization; Truthful inference; Fair judgment; Judicial autonomy and independence.

*Institute of Mathematics and Statistics of the University of Sao Paulo
jstern@ime.usp.br

†Polytechnic School of the University of Sao Paulo mjunior@larc.usp.br

‡Polytechnic School of the University of Sao Paulo mvsilva@larc.usp.br

§Law School of the University of Sao Paulo, roberto.pfeiffer@usp.br

κληρω νυν πεπαλασθε διαμπερες ος κε λαχησιυ :

*Let the lot be shaken for all of you,
and see who is chosen.* Iliad, VII, 171.

מְדֻבְּרִים יִשְׁבֵּית הַגּוֹרֵל וּבֵין עֲצוּמִים יִפְרָד

*Casting the dice puts judgment quarrels to rest and keeps
distinct essential powers duly separated.* Proverbs 18:18.

1 Introduction

Francisco Antonio Dória has had a consistent interest in randomness and chaos and, together with his collaborators, has investigated fundamental aspects of such phenomena. This article is our contribution to the Festschrift celebrating Doria's 75th birthday.

This article analyses some pragmatical aspects of applying randomization in empirical science and law, considers some philosophical implications or premises justifying or motivating these applications, and offers some tools that promote good randomization practices. *The Cardsharps* (1594) marks the beginning of the independent career of the great Italian master Michelangelo Merisi da Caravaggio (1571-1610). This painting displays a wealthy but innocent looking boy playing cards with his opponent, a cardsharp, that cheats in two ways: On the one hand, the cardsharp hides in his belt spurious cards that he intends to use in illegitimate ways; on the other hand, a sinister looking and strategically positioned accomplice gives him access to privileged and undue information. Finally, the cardsharp carries a dagger, hinting at the dangers lurking in this environment of misrepresentation and deception.

Caravaggio gives a beautiful depiction of some themes discussed in this article. First, the social importance of activities involving randomization, that is, the random setting of some variable of interest, like the drawing of dice or, in this painting, the distribution of playing cards. Second, it suggests the question – Why to randomize?



Figure 1: *The Cardsharps* (1594), by Michelangelo Caravaggio

that is – Why should a rational agent abdicate the opportunity of making a deterministic choice introducing, on purpose, a random component in making a decision? If so – What is the role played by randomization? Finally – How to randomize? that is – What dangers could jeopardize a randomization process? and, if necessary – How to shield or immunize the process against these dangers?

In order to answer these questions, we have to pay attention to some topics in Statistics, Computer Science and Cryptography; in addition, we have to examine some details concerning the design of empirical trials or the operation of legal systems. In this article, we investigate each one of the questions just raised, looking for an intuitive understanding of the role(s) played by randomization.

In the final sections of the paper, we present an easy to use, open-code, traceable, auditable, secure, and statistically sound randomization tool that is ready for use in empirical trials and legal applications. This kind of secure randomization tool can prevent the possibility of misrepresentation and deception, as depicted in

the painting by Caravaggio. Moreover, even in situations where no misdeeds actually occur, the use of such a tool can be beneficial by fostering public confidence in the soundness of important decisions, by strengthening the resilience of public institutions, and by favoring the peaceful resolution of conflicts.

2 Social Importance of Randomization

Gambling and lotteries exchange billions of Dollars every day worldwide. Hence, ensuring honesty and transparency in these activities should already be considered a meritorious task. However, since ancient times, sortition (i.e., selection by lottery) is used for many other purposes. In the Iliad, one of the oldest texts of western culture (approx. 1200BC), the Argonauts (crew of the ship Argo) selected a man to execute a dangerous task by sortition – see this paper’s first opening quotation. In the same manner, modern societies often resort to sortition for drafting. Figure 2 displays some photographs related to compulsory enlistment for service in the USA, namely, military drafting during the Civil (left) and the Vietnam (center) wars, and selection for jury duty (right).

In order to gain public trust, the sortitions for the Vietnam war were conducted in public view: Balls with calendar dates were placed in a transparent urn and some anniversary dates were then picked, giving the (un)lucky winners the opportunity to serve their country in the battlefield. A post hoc statistical analysis of these drawings revealed a significant bias favoring latter days of the calendar, corresponding to the last balls placed inside the urn, an unexpected effect of an ill-conceived randomization process that generated misunderstanding, frustration and conflict.

Figure 2 (right) shows a letter calling a citizen for jury duty in the USA. In this process, an eligible citizen was chosen at random by running a computer program. Post hoc analyses of this randomization process revealed no significant bias or any other statistical anomaly. Nevertheless, the code of these computer programs were never made public, making the randomization process opaque and



Figure 2: Draft lotteries in war and peace

non-verifiable, thus generating mistrust and resentment.

Finally, in the world of science, good clinical trials are conducted by (double) blind and random attribution of patients to two or more distinct treatments. The objective of such a trial is to find out if a new or alternative treatment is significantly better than the old or standard one, according to well-established statistical criteria. In this situation, some frequently asked questions are: Why should a patient's treatment be selected at random? Why not give him or her the freedom to chose his or her proffered treatment? Why hide from a patient information about his or her own treatment?

3 Why to Randomize?

Imagine a clinical trial where patients are free to choose a treatment according to their own will. Among patients, there will be rich and poor, people with different degrees of instruction, people with better or worst networks for support, etc. Obviously, rich, well educated, and well connected patients will have better access to good information and advice and, therefore, will be prone to make better decisions. Moreover, these same patients likely have better overall living conditions and, therefore, even with the same treatment, might have a better chance of recovery. Hence, this freedom of choice would automatically introduce *confounding effects*: After the trial is over, we would not be able to (completely) discern the beneficial and adverse consequences of distinct treatments from

consequences of preexisting conditions.

Similar unwanted interference is generated by the *placebo* and *nocebo* confounding effects. If a patient knows to be receiving either a new, experimental and possibly wonderful drug, or else an old and possibly not very effective drug, his or her moral may be, respectively, lifted or depressed. That, in turn, may affect his or her overall health and chance of recovery. This is why, in a good clinical trial, treatment information is denied (blinding or censorship) to patients, and commonly also to their direct caretakers (double-blinding).

There are in the medical literature plenty of examples of clinical trials that came to wrong conclusions in consequence of such confounding effects. The best known antidotes against these confounding effects rely in some form of randomization. The idea is to chose a patient's treatment based on a random variable that is independent of any potentially confounding variable. In so doing, the random element in the choice of treatment has the effect of breaking causal links that should not interfere with the experiment, allowing the trial to adequately focus on the causal links of interest – see Stern (2008) and Pearl (2009) for further details.

Finally, let us consider the use of randomization in the legal system, like the selection of jurors or judge(s) for a given case. Figure 3 displays two pictures from ancient Egypt. On the left, a stone carving of approx. 2400BC shows two merchants using a two-pan balance to correctly measure amounts of goods for a fair commercial transaction. On the right, the Hunefer papyrus, of approx. 1275BC, shows the scale used by Maat, the goddess of justice, where the heart and the (de)merits of a man are measured. It should be clear that these two scales are essentially distinct – they belong to distinct contexts. The figure at the center suggests the possibility of “mixing” these two essences: Perhaps Maat could make a more benevolent assessment in the scale of justice if she, or her priests, received goods of commercial valuable... There we have, once more, a confounding effect, characterized by spurious influences between powers belonging to essentially distinct systems:



Figure 3: Essentially different powers: Economy and Justice

in this case, the economic system and the justice system.

How to avoid such confounding effects caused by spurious influences, fostering autonomous decisions in a strong and independent justice system? Surprisingly, the Hebrew bible already offers very good advice at this respect, as stated in the second opening quotation of this paper. Interestingly, the Hebrew root **עצם**, *etzem*, whose literal meaning is bone, also generates words meaning essence (the etymological origin of the English word), strength, power and the modern Hebrew word for independence.

Judges, even if perfectly honest, do not come to court as a blank slate, nor should they. Every judge has his or her own history of decisions and opinions. Hence, if the selection of judges could be influenced by the litigants or other interested parties, the richer, better informed, well connected, or otherwise more powerful parties would likely have an advantage in directing the case to a judge sympathetic to their arguments. For this reason, in many modern democracies, the *distribution* of a new judicial case must take the form of a random choice among the available judges or courts qualified to judge it.

4 How to Randomize?

Previous sections discussed several applications of randomization and explanations of why to use it. This section describes some desirable characteristics of such a randomization process, including:

Statistical honesty: In a set of sortitions (random selections) in the system, the probability of any group of outcomes should be exactly as prescribed by the established rules.

Cryptographic security: The outcome of the sortition should be unpredictable; moreover, no external agent should be able to influence the randomization process, even if the agent knows in detail the randomization mechanism being used and has state of the art knowledge of all relevant technologies involved.

Transparency: All relevant information about the sortition process must be of public knowledge, including any pertinent detail about the randomization mechanisms being used.

Auditability: All relevant occurrences of an actual randomization process must be traceable and auditable. Furthermore, it should not be possible to conceal any improper use of the randomization system.

The first two requirements are of technical nature, stipulating that, in the randomization process, we should use “honest dice that cannot be tampered with” – or else a more convenient device, like a computational algorithm that adequately mimics all the relevant characteristics of “honest dice”. For more technical discussions on these characteristics, see Marcondes et al (2019), Saa and Stern (2019) and Silva et al. (2020).

In order to emphasize the importance of the last two requirements, let us discuss a form of cheating known as *rerandomization*. In this kind of cheating, the agent responsible for a sortition has the privilege of using the randomization mechanism out of public scrutiny, examine the outcome, and chose at will either to make this process and its outcome public, or to hide this first try and randomize a second time, as if the first try never happened. Imagine for example the classic process of picking a ball from a transparent urn. However, instead of making a live presentation, the sortition ceremony is recorded for broadcasting at a later time. A dishonest agent could repeat and record the process twice, and only release

the recording that best fits his or her goals, as if it were the only recording ever made. It should be clear that the repeated use of this subterfuge gives the agent in charge some latitude to pick and choose, biasing the final outcome according to his or her convenience.

Authority, Transparency and Understanding

Why is transparency even required in a randomization process? Would it not be possible, or even easier, to anchor the credibility of the process on a *principle of authority*? If a given authority is responsible for a randomization process, doesn't the requirement of transparency imply an implicit doubt? If so, doesn't the requirement of transparency imply disrespect for the same authority?

These are basic questions in philosophy of law, that can only be answered in a context that specifies the fundamental values and goals chosen by a given society. Niklas Luhmann (1985, 1989), a celebrated scholar in philosophy of law, postulates that the fundamental goal of the justice system is – “the congruent generalization of normative expectations”. That is, the final objective of the legal system is the construction of a harmonious society, where citizens have a coherent view of what constitutes a good set of rules for social behavior (normative expectations). Moreover, a legal system should provide mechanisms that stimulate citizens to conform to normative expectations and inhibit their transgression.

This conception of law requires from every citizen a well founded trust that the justice system is efficient and fair, preferably obtained by conscious understanding of laws and regulations and their forms of implementation. Moreover, a justice system conceived according to such principles is weak or fragile if sustained on blind faith on ad hoc authority, but strong and resilient if sustained by a conscious, engaged, and participative community. The articles of Silva et al. (2020) and Stern (2018) expand these ideas.

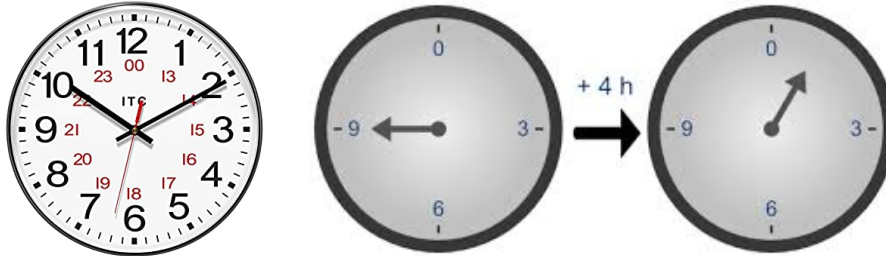


Figure 4: Modulo 12 or Clock Arithmetic

5 Modular Arithmetic and Trusted Roulottes

In this section we discuss intuitive ideas for how to implement an honest randomization device that all interested parties can trust. In following sections of this article we offer a viable technological solution to the problem of randomization that satisfies all requirements stipulated in previous sections, following the general ideas hereby discussed.

Modular arithmetic is an integer arithmetic system in which numbers “wrap around” after reaching a maximum value, m , called the modulus. A familiar example is the standard reading of a clock. After noon (12 o’clock), we restart counting from 1, 2, ... (p.m.). Notice that, in Figure 5, the position corresponding to noon (or midnight) is marked either by the modulus value, $m = 12$, or by the value *zero* – that is mathematically more convenient. In general, for positive integers, n and $m > 1$, we define $n \bmod m$ (read n modulo m) as the remainder of the division of n by m . For example, see Figure 5 (left): $13 \bmod 12 = 1$, $14 \bmod 12 = 2$, ... $23 \bmod 12 = 11$, and $24 \bmod 12 = 12 \bmod 12 = 0$. Figure 5 (right) illustrates the modular arithmetic operation $(9 + 4) \bmod 12 = 1$.

Now imagine we have a game using a roulette or wheel of fortune, see Figure 5, with k participants, also known as the *stakeholders*, all of them wanting the privilege of spinning the roulette, and not trusting anyone else to do the job. How can we break this deadlock?

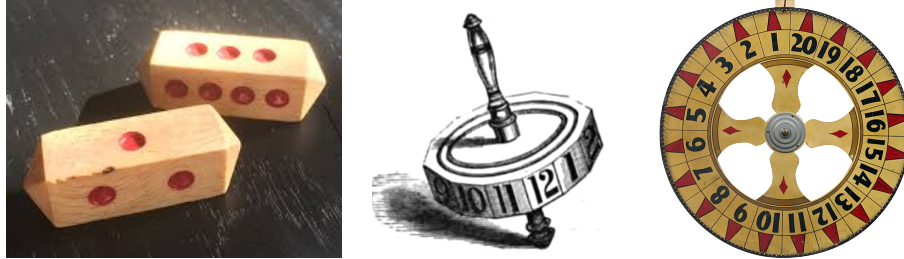


Figure 5: Rolling Dice and Spinning Roulettes

We can solve the aforementioned impasse using the following protocol:

1. Provide each stakeholder with a well-balanced roulette, marked according to the numbers set $\{0, 1, 2 \dots (m - 1)\}$;
2. Ask each stakeholder to spin his roulette *honestly*, that is, with a not fully controlled and strong enough initial impulse so to produce any of the possible outcomes, $\{0, 1, 2 \dots (m - 1)\}$, with the same probability, $(1/m)$. Moreover ask each stakeholder to use his roulette *independently*, that is, to do so without sharing any information with other stakeholders or interested parties;
3. Collect and add, using modulo m arithmetic, the results produced by each one of the k stakeholders in order to produce the final result: $n_f = (n_1 + n_2, \dots + n_k) \bmod m$.

We can guarantee that the final result, n_f , produced by this protocol is equivalent to an “honest roulette”, as long as at least one (any one) of the k stakeholders does his job as required. This guarantee is a corollary of the following theorem: Let x and y be independent random variables in $\{0, 1, 2 \dots (m - 1)\}$. Then, if any one of these random variables, x or y , is uniformly distributed, so is $z = (x + y) \bmod m$. Imagine, for example, that variable x is not random at all, but rather a known constant, c , namely, the initial state of the roulette. Furthermore, imagine that y is independent

of x and uniformly distributed in $\{0, 1, 2 \dots (m - 1)\}$. Under these conditions, the theorem states that the final state of the roulette, $z = c + y$ is uniformly distributed, corresponding to the intuitive idea that, when using a well-constructed roulette, a strong enough impulse will produce a final outcome that “forgets” the initial state of the roulette. For further details and formal mathematical analyses, see Scozzafava (1993).

In many applications in statistics, clinical trials, and complex sortitions, we need a random variable x uniformly distributed in the interval $[0, 1[$ of the real line. In computational procedures, this continuous variable can be approximated by a fraction n/m , where m is a large integer, and $n \in \{0, 1, 2, \dots (m - 1)\}$. This fraction can be translated to standard floating point notation, and then be further transformed into random variables with several probability distributions of interest in statistical modeling, see Hamersley (1964), Ripley (1987). Such uniform or non-uniform random variables can, in turn, be used in dynamic clinical trials, haphazard intentional sampling, adaptive sampling procedures, and other complex applications of interest in statistical modeling and decision science, see for example Fossaluza (2015) and Lauretto et al. (2012, 2017) and the bibliography therein.

In the sequel, we describe a software implementing the protocol outlined in this section, including all necessary precautions in order to guarantee cryptographical security. In this software, every stakeholder is required to input a random number n between 0 and $m = 9,999,999$. Each stakeholder has the responsibility of producing his or hers random 7-digit decimal number in a way he or she finds appropriate.

Random decimal digits can be easily produced, for example, by rolling 10-faced dice, available in several shapes with the required symmetry conditions, see Stern (2011). Figure 6 exhibits 10-faced dice shaped as a pentagonal trapezohedron (left) and a pentagonal antiprism (center), and also 20-faced dice shaped as an icosahedron (right); in order to produce a random decimal digit using icosahedral dice, the user should read only the last digit for outcomes ranging



Figure 6: Polyhedral dice for decimal digits

from 10 to 20.

6 Intuitive Cryptology

One crucial requirement of the random drawing approach described in Section 5 is that the the roulettes are run independently. Otherwise, a dishonest stakeholder S_i could wait until the results of all roulettes are revealed, and then run his/her own roulette for manipulating the final outcome of the drawing: for example, suppose that the sum of the contributions from all stakeholders except S_i is $n = 3$ for, say, $m = 12$; after learning this value of n , S_i could force his/her own roulette to give $n_i = 2$, thus obtaining $n_f = n + n_i \bmod m = 5$ as the final (manipulated) outcome of the drawing.

To ensure this independence property, Silva et al. (2020) builds upon the properties of hash-based bit-commitment mechanisms. Intuitively, a hash function H is a cryptography construct analogous to fingerprinting for humans, as illustrated in Figure 7 – we refer the reader to Beutelspacher (1994), Bultel et al. (2017) and Fellows and Koblitz (1994) for intuitive introductions on key ideas of cryptology, and to Rogaway and Shrimpton (2004) for a concise but formal explanation of properties of cryptographic hash functions. Specifically, given the fingerprint for an unknown human being, it takes a lot of computational power to look all around the world for the owner of that fingerprint; on the other hand, given a fingerprint and the corresponding human, it is quite easy to check whether

or not they match, and it is hard (almost impossible) to find two different people with the same fingerprint (even considering identical twins).

Similarly, suppose that someone computes the hash of a number n , i.e., a value $h = H(n)$, which acts as a “fingerprint” for n ; then, if only h is revealed, but n is kept secret, there is no simple mechanism for finding the value of n . Of course, one could test every possible value of n , checking if a guess n_i is such that $H(n_i) = h$, just like finding the owner of a fingerprint given only the fingerprint itself. However, the computational effort for performing such brute-force attack would be very large. Actually, in practice the cost for hash functions would be even larger than searching for a human who owns a fingerprint: while there are a few billions of humans in the world, a hash function can be used in such a manner that the number of tests would be as large as the number of atoms in the whole planet Earth! For this purpose, it suffices to combine the value of n with a large and unpredictable (e.g., random) mask r when computing the hash, i.e., to make $h = H(r, n)$. As a result, even if there are only a few possible values of n to be tested, determining whether or not a guess n_i is correct would require testing all possible values of r too. Therefore, it suffices to use a large-enough mask r (e.g., 256-bits) to ensure that any attempt of determining n via brute force would be computationally infeasible.

When both n and h are revealed, on the other hand, it is easy to verify whether or not they match: it suffices to compute $H(n)$ directly, and check if the output of this computation is identical to the provided value of h . However, like different humans should not have the same fingerprint, it is computationally hard to find two distinct values of n (say, n_1 and n_2) that have the same hash h . Hence, once $h_1 = H(n_1)$ is revealed, one can say that the person who revealed it is “committed” to revealing n_1 , i.e., it would be hard to trick someone into believing that h_1 was computed from any other input $n_2 \neq n_1$.

Such properties are used by Silva et al. (2020) to build a two-phase procedure for ensuring the fairness of random draws:

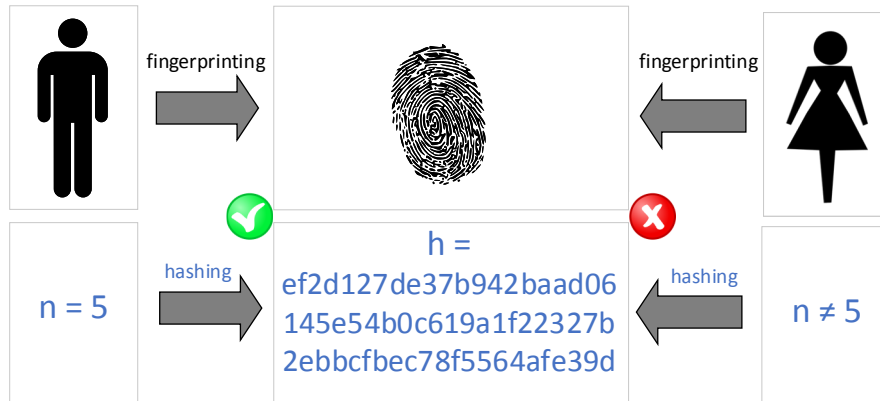


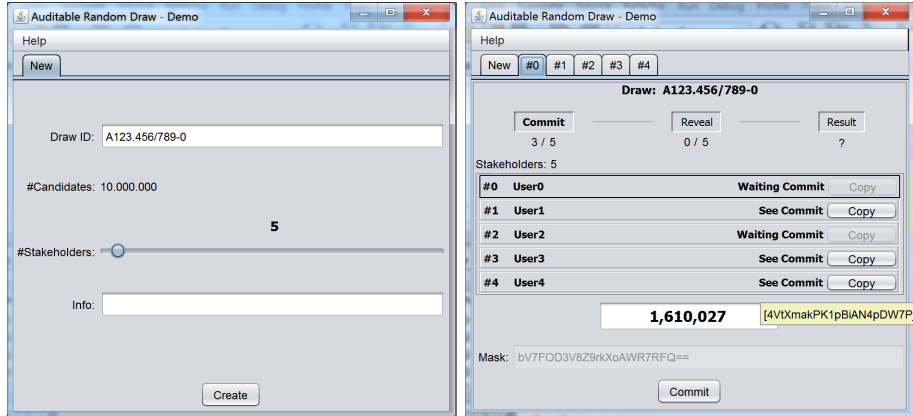
Figure 7: Hash functions and their similarity with human fingerprinting.

1. Commitment phase: first, each stakeholder S_i runs a roulette (honestly or not), getting a value n_i as result. Then, S_i computes the hash of n_i , denoted h_i , and reveals only h_i to the other stakeholders, keeping n_i itself secret. This prevents S_i from learning the roulette results from his/her peers, and vice-versa.
2. Reveal phase: only after all hashes are received, every stakeholder S_i reveals its own n_i . The outcome of the drawing is then computed locally by S_i by adding every n_i together using modular arithmetic as explained in Section 5. In this case, even if S_i is malicious and tries to delay the revelation of n_i until it learns the partial outcome of the drawing from the values revealed by his/her peers, it would be already too late: after revealing h_i in the commitment phase, S_i has no choice but to reveal the already chosen n_i , rather than some other value that might lead to a more desirable (but unfair) drawing outcome.

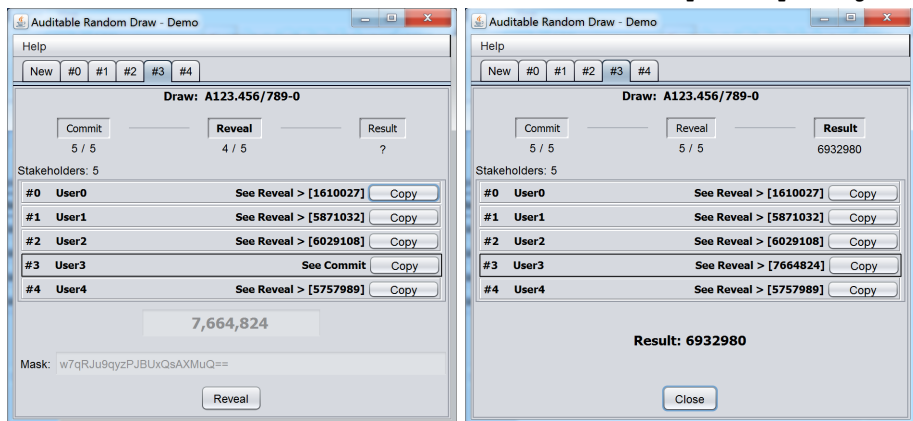
7 *Java* Implementation

We developed a simple Java library for implementing the protocol described in Silva et. al (2020), and made it available under the MIT License at <https://doi.org/10.24433/CO.6108166.v1> This library can, thus, be freely adapted for the needs of specific application scenario. To help in this task, we also provide a simple proof-of-concept graphical interface for testing purposes, which is depicted in Figure 8. More precisely, this figure shows:

- (a) A simple configuration interface for drawing a number among $m = 10,000,000$ candidates, i.e., from 0 to 9,999,999. The number of stakeholders participating in the drawing and additional metadata related to it can also be defined.
- (b) A snapshot of the Commitment phase, as seen by Stakeholder S_0 in a drawing involving 5 stakeholders. In this snapshot, S_0 is then free to choose a number n_0 to commit, which is combined with a random mask for better security against brute force attacks. Meanwhile, S_1 , S_3 and S_4 have already sent the hashes h_1 , h_3 and h_4 of their own commitments, n_1 , n_3 and n_4 , respectively; as a result, these stakeholders cannot modify the chosen values n_1 , n_3 and n_4 anymore.
- (c) A snapshot of the Reveal phase, as seen by Stakeholder S_3 . The figure shows that S_3 is the only one who has not yet revealed the chosen value for n_3 , while all of his/her peers have already revealed n_0 , n_1 , n_2 and n_4 . Nevertheless, S_3 can only reveal the correct n_3 (and corresponding mask), since the revealed value must match the committed value h_3 .
- (d) The completion of the protocol, when one of the eligible numbers (namely, 6,932,980) is picked with uniform probability based on all stakeholders' contributions n_0 , n_1 , n_2 , n_3 and n_4 .



(a) Random drawing with 5 stakeholders (S_0, S_1, S_2, S_3 and S_4) and modulus value $m = 10,000,000$. (b) S_1, S_3 and S_4 after commitment, as seen by S_0 . Value committed by S_4 is $h_4 = 4VtCmakPK1pBiAN4pDW7Pj\dots$



(c) S_0, S_1, S_2 and S_4 in reveal phase. (d) Drawing result: $1,610,027 + 5,871,032 + 6,029,108 + 7,664,824 + 5,757,989 \pmod{10,000,000} = 6,932,980$.

Figure 8: Proof-of-concept Java implementation: screenshots

8 Final Remarks

Previous articles of this research group have explored the need of randomization procedures in legal systems, like the random assignment (distribution) of legal cases to individual judges or courts, the sortition of jurors for a given case, etc., see Marcondes et al. (2019), Saa and Stern (2019), Silva et al. (2020). Moreover, these papers provide extensive discussions on how to build honest (statistically non-biased) and cryptographically secure procedures and protocols, on the sociological and political importance of using fully transparent and auditable procedures, and on the positive effects of using procedures fully compliant with the aforementioned desiderata in the constitution of strong and autonomous legal institutions.

Finally, breaking away from vicious old habits can always be stimulated by respectful criticism, by firm encouragement, and by making available user friendly tools that facilitate the adoption of virtuous new habits without the imposition of additional difficulties beyond the already heavy load of overcoming corporate inertia. This paper provides such a tool, fully compliant with all technical desiderata, user friendly, written in freely available and open source code. The authors hope it will be soon put to use by Brazilian legal institutions and, if necessary, stand ready to help in this endeavor.

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A True Polymath

A tribute to Francisco Antonio Doria



Editors
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Décio Krause

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