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Efficient communication and indexicality☆

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ABSTRACT

Since sending explicit messages can be costly, people often utilize "what is not said," i.e., informative silence, to economize communication. This paper studies the efficient communication rule, which is fully informative while minimizing the use of explicit messages, in cooperative environments. It is shown that when the notion of context is defined as the finest mutually self-evident event that contains the current state, the efficient use of informative silence exhibits the defining property of indexicals in natural languages. While the efficient use of silence could be complex, it is also found that the efficient use of silence can be as "simple" as the use of indexicals in natural languages if and only if the information structure satisfies some centrality and dominance properties.

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1. Introduction

Uttering and writing words might not be as laborious as piling up bricks, yet they still consume time and cognitive energy. Consequently, people often utilize "informative silence" to economize communication. For example, suppose that a manager wants a worker to perform his routine task. Unless the worker is new to the office, the manager usually does not need to provide any explicit instruction to make the worker perform the routine; a competent worker would interpret the fact that the manager did not give any explicit instruction as the implicit instruction to perform his routine. This type of informative silence can be found in almost every practical communication; in fact, if we need to be explicit about every single detail as silence conveys no information, our verbal messages would sound like commands to a robot, and our written messages would look like a computer program. Understanding tacit communication in cooperative environments is important for economics since many economic activities rely on cooperative communication that utilizes implicit messages.

If people can communicate with silence so appropriately and broadly across various environments, the use and interpretation of informative silence need to follow "conversational logic" shared by a linguistic community, i.e., pragmatics.[1] The purpose of this paper is to investigate such a tacit communication rule based on the approach that is familiar to economic theory; the tacit communication rule is analyzed as the efficient communication rule designed by a fictitious linguistic engineer. This paper shows that the way silence conveys information in the efficient rule is analogous to indexicals in natural languages. The finding provides novel insights into how economic agents combine explicit and implicit messages in cooperative communication.

The current paper considers the following simple communication problem to analyze tacit communication in a cooperative environment. There are two agents, a speaker and a listener. There is a finite set of states, and each agent is endowed with a partitional information function where each cell in the speaker (listener)'s partition is interpreted as the speaker (listener)'s "situation" at the state. A communication rule is then defined as the sender's messaging rule that specifies whether to send an explicit message, which is costly, or remain silent, which is costless, at each state. The premise of this paper is that the tacit communication rule, i.e., the pragmatics of informative silence, is determined by a fictitious linguistic engineer who designs the efficient communication rule in the cooperative environment. The current paper then studies the efficient communication rule that fully conveys the speaker's private information while minimizing the use of explicit messages.

Due to the nature of silence, an agent cannot use informative silence to indicate two different situations that cannot be distinguished from the perspective of the listener. Thus, when the speaker uses informative silence in one situation, it could restrict the feasibility of informative silence in other situations. Since the set of indistinguishable situations for the listener can be interwoven, solving the trade-off can be complex. Moreover, because of the combinatorial nature of the problem, a small change in the prior probability could dramatically change the

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1 Pragmatics is a branch of linguistics and semiotics that studies the ways in which context contributes to meaning.

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efficient use of silence. Consequently, there is no simple regularity in the efficient use of informative silence in general.

The key to understanding the efficient use of silence is the concept of *indexicality*. In semiotics and philosophy of language, indexicals are signs or words whose reference is systematically determined by a context. For instance, the indexical "I" itself does not refer to any particular person by itself, but it refers to Mike in the context where Mike utters the word "I". That is, the word "I" operates as a function that specifies the content of "I" once the input variable "utterer" is given by a context. Similarly, the indexical "now" itself does not refer to a particular time by itself, but the content is determined once a context provides the time of utterance. In other words, an indexical indicates an object without using the name by utilizing contextual information, which is public by nature. For example, the indexical "now" exploits the fact that the date of the utterance is common knowledge between agents, whereas the indexical "I" utilizes the fact that the speaker of "I" is common knowledge between agents. In the current paper, since the subject of communication is the speaker's situation, a context is defined as the finest mutually self-evident event that contains the current state. Then, it is shown that informative silence in efficient communication rules also has an indexical property; silence itself does not refer to any particular situation by itself, but the content of silence is determined once a context is given. More specifically, given a context, call a situation or a union of situations of the speaker as an implicitly expressible event if there exists a fully informative rule that uses silence for the event. Then, informative silence in any efficient communication rule systematically refers to the most likely implicitly expressible event given the current context. It is also found that the common knowledge property in the definition of context is essential for the indexicality of informative silence; that is, if the notion of context is defined in a way that lacks common knowledge property, the efficient use of silence could fail to have indexical property.

While the indexical property can help us to find the efficient use of silence to some degree, it does not always tell us the exact use immediately. This is because informative silence could refer to a set of situations rather than a specific situation given a context, and finding out the union of situations that is most likely given a context requires combinatorial optimization. The computationally demanding nature is contrary to indexicals in natural languages, which directly refers to an entity rather than a non-trivial combination of entities in a context. This paper then characterizes the information structure in which informative silence in efficient communication rules directly refers to a specific situation rather than some combination of situations as indexicals in natural languages do. To characterize the information structure, a notion of *centrality* in the information structure is introduced. Intuitively, the speaker's situation is central in a context if her situation is relevant to every possible situation of the listener in the current context.[2] Another important property is local dominance, which is a strong version of the notion of "most common situation". It is shown that informative silence in the efficient rule directly refers to a specific situation as indexicals in natural languages do if and only if the specific situation is central and locally dominant in the current context. This result has an intriguing implication on the amount of reasoning that the use of indexicals in natural languages demands; the condition of centrality suggests that when the speaker's informative silence is as direct as indexicals in natural languages, the derivation does not require the consideration of the listener's situation that is not directly relevant to the speaker's current situation.

Directly referring to a specific situation still does not make informative silence as user-friendly as indexicals in natural languages; checking local dominance could require tedious numerical evaluations, whereas indexicals in natural languages always refer to an object intuitively given the spatial, temporal, and personal relations given by context without using any cardinal information. Thus, this paper also provides the information structure in which agents can find the efficient use of silence only from ordinal information. More specifically, I provide a condition that guarantees the local dominance condition when agents only know the probability ranking over events.

The rest of this paper is organized as follows. After the literature review, Section 2 introduces the model. In Section 3, the efficient communication rule is analyzed, and the indexical structure of informative silence is identified. Section 4 characterizes information structures in which the efficient use of silence works as intuitively as indexicals in natural languages do. Section 5 provides some discussions followed by concluding remarks in Section 6.

Related literature. The current paper contributes to the literature on "economics and language", which considers natural languages as a fundamental institution for economic activities and explains the properties of natural languages based on the method that is familiar to economics. The approach of the current paper is in line with Rubinstein (1996, 2000), which derive some properties of natural languages as if they are optimally designed by a fictitious linguistic engineer.[3] In his seminal paper, Rubinstein (1996) demonstrates that some binary relations in natural languages can be obtained from indication-friendliness, informativeness, and describability.[4] In the current paper, on the other hand, a concept in *pragmatics* is obtained from the perspective of efficiency; informative silence has indexicality when a communication rule is designed to be efficient.

While pragmatics traditionally focuses on communication in cooperative environments, the economics literature that utilizes some concepts in pragmatics has mostly focused on strategic environments, e.g., Glazer and Rubinstein (2001, 2006), and Suzuki (2017).[5] The current paper studies pragmatics in the context of cooperative communication by following the basic tenet of some major theories in pragmatics: people can communicate beyond what is explicitly stated since they share some tacit principles to infer the meaning of an implicit expression beyond what is explicitly stated.[6] Since those theories are not based on formal models, it is not easy to evaluate to what extent the current paper shares their principles. However, the current paper is at least in line with their idea that the economy of communication plays an important role in tacit communication.[7]

The current paper is also related to the literature on "communication in teams". When communication is costly, communication becomes a non-trivial economic problem even under

2 The centrality in this paper is not a metaphor; when the (interactive) information structure is represented by a graph, the central situation has the highest centrality in the graph induced by a context.

3 Another approach is based on evolutionary game theory. This approach investigates the evolutionary stability of a pre-play communication strategy in a coordination game; for example, Demichelis and Weibull (2008) and Heller (2014) show that if lying is psychologically costly, the equilibrium in which a message has a literal meaning is evolutionary stable.

4 In a similar spirit, Blume (2004) shows that a grammatical structure of natural languages, i.e., modularity, can be derived from learnability.

5 There is also the pragmatics literature that is based on game theory. For example, Benz et al. (2005) offer some game-theoretical formulations of existing concepts in pragmatics.

6 For example, Grice (1975), Horn (1984), and Sperber and Wilson (1986).

7 Grice's quantity maxim requires communication to be efficient: "Do not make your contribution more informative than is required". Horn's R Principle demands "Say no more than you must". Sperber and Wilson (1986) do not explicitly mention the economy of communication as the main communication principle. However, their notion of "optimal relevance" essentially takes into account the economy of communication.

the common interest setting. Marschak and Radner (1972) analyze the optimal information protocol in an organization based on statistical decision theory. Arrow (1974) observes that when communication is costly, people use "organizational code" to economize communication. Cremer et al. (2007) formalize the idea of organizational code and study the nature of the optimal code under different organizational structures. As in the current paper, Cremer et al. (2007) derive the organizational code as an efficient communication system in an environment where communication is costly. However, the idea of informative silence is qualitatively different from that of organizational code in Cremer et al. (2007); the premise of their paper is that agents are boundedly rational and can deal with only a fixed number of messages. Then, since their code system is coarser than their state space, the efficient code is essentially an optimal partition of the state space that balances the trade-off between interpretation costs and informativeness. On the contrary, in the current paper, the agent *can* describe her current situation as her message space is large enough. However, since sending an explicit message is costly, the main question is how she can fully convey her private information while minimizing her use of explicit messages given an information structure.

The model of this paper is built on the framework of Aumann (1976). Traditionally, the study of communication in this setting focuses on the process of two-sided communication that achieves common knowledge of posterior beliefs, e.g., Geanakoplos and Polemarchakis (1982) and Parikh et al. (1990). Since the literature of "consensus and communication" analyzes cooperative communication, their models do not adopt a game theoretical formulation as in the current paper. However, unlike in the current paper, they focus on the setting in which agents *cannot* report their private information; their main question is whether agents can reach consensus through communication *under a restricted message space* such as posterior beliefs or actions. The question of the current paper, on the other hand, is entirely different; this paper is interested in *how* an agent can efficiently convey her private information to another agent when she *can* report her private information but sending an explicit report is costly. Thus, unlike in the literature on consensus and communication, the current paper focuses on one-sided communication even though the result of the current paper can be extended to two-sided communication.

Finally, the current paper shows that when indexical silence works as intuitively as indexicals in natural languages do, the underlying information structure has to satisfy a centrality condition, which suggests that the derivation of the efficient use of silence does not require the consideration of the listener's situation that is not directly relevant to the speaker's current situation. If the property of natural languages reflects the cognitive capacity of the human mind, the centrality condition can be interpreted as "revealed simplicity". Thus, the current paper also contributes to the literature on bounded rationality that formalizes notions of simplicity and complexity relative to underlying economic problems. For example, Neyman (1985) and Rubinstein (1986) utilize finite state automata to define the complexity of strategies in repeated games. Recently, Li (2017) defines simplicity of dominant strategies, i.e., obviously-dominant strategy, based on whether the agent can identify dominance without contingent reasoning, which is similar to the finding of the current paper.

2. Model

There are two agents $i = 1, 2$. The current paper considers the partitional information model in Aumann (1976). Let Ω be a finite set of states, and let π be a prior probability. Nature draws a state $\omega \in \Omega$ according to a prior π, and an information function $\Pi_i(\omega)$ where $\Pi_i : \Omega \to 2^\Omega$ determines agent i's private information at ω; that is, agent i at ω knows that the current state is in $\Pi_i(\omega) \subset \Omega$. Let $\mathcal{P}_i$ be the range of Π_i and assume that $\mathcal{P}_i$ is a partition of Ω. In the current paper, it is convenient to call $P_i = \Pi_i(\omega)$ as agent i's **situation** at state ω. An information structure is then defined as $(\Omega, \mathcal{P}, \pi)$ where $\mathcal{P} = \{\mathcal{P}_i\}_{i=1,2}$.

Agent 1 is a speaker, and agent 2 is a listener. The speaker's set of messages is $M = \mathcal{P}_1 \cup \{\varnothing\}$ where $m = P_1$ is an explicit report that refers to the speaker's situation, whereas $m = \varnothing$ signifies "no report", i.e., silence. Since speaking and writing consume some time and energy, sending an explicit message is assumed to be costly.[8] Specifically, the cost function of messaging is $c(m)$ where $c : M \to [0, \infty)$, and assume that $c(\varnothing) = 0 < c(m) = c$ for all $m \neq \varnothing$.[9]

A **communication rule** is a mapping σ where $\sigma : \Omega \to M$ such that $\sigma(\omega) \in \{\Pi_1(\omega), \varnothing\}$ for all ω. That is, a communication rule specifies whether the speaker remains silent $\varnothing$ or report her situation at the current state $\Pi_1(\omega)$.[10] A communication rule is interpreted as pragmatics of $\varnothing$, i.e., a tacit communication rule that governs the use and the meaning of $\varnothing$.[11]

When the speaker sends a message m according to a communication rule σ, the listener updates her knowledge about the current situation from P_2 based on the message m. Specifically, let $Q_\sigma(m, P_2) \subset P_2$ be the listener's knowledge about the current situation conditional on P_2 and m given a communication rule σ; that is, $Q_\sigma(m, P_2)$ is the support of the listener's posterior belief conditional on P_2 and m given σ. It is assumed that the listener updates her knowledge about the current situation to be consistent with a communication rule σ. That is,

$$Q_\sigma(m, P_2) = P_2 \cap \{\omega : \sigma(\omega) = m\}.$$

The current paper considers a communication problem in which fully informative communication is essential; formally, a communication rule σ is **fully informative** if

$$Q_\sigma(\sigma(\omega), \Pi_2(\omega)) = \Pi_1(\omega) \cap \Pi_2(\omega)$$

for all ω. Let Σ^* be the set of fully informative communication rules.

Since sending an explicit message is costly, the current paper defines an efficient communication rule as a fully informative rule that minimizes the expected messaging cost. Formally, a communication rule σ is **efficient** if σ solves

$$\min_{\sigma \in \Sigma^*} \sum_{\omega \in \Omega} c(\sigma(\omega))\pi(\omega).$$

Having equal probabilities of events complicates the exposition of this paper without adding substance. Hence, throughout this paper, assume that

$$\sum_{\omega \in X} \pi(\omega) \neq \sum_{\omega \in X'} \pi(\omega)$$

for any $X, X' \subset \Omega$ such that $X \neq X'$.

There are two comments on the definition of efficient rules. First, since the current paper is interested in the derivation of a communication rule in cooperative environments, the designer

8 See Marschak and Radner (1972) and Arrow (1974) for fuller discussions on communication costs.

9 The result of this paper is preserved even if messaging costs are heterogeneous across messages as long as the variation is sufficiently small.

10 Note that the current paper considers a communication rule in cooperative environments. Thus, allowing reports that are coarser than P_1 or untruthful does not add any substance to the paper.

11 One could also interpret a communication rule as an explicit reporting rule that is designed by an organization. However, the analysis of this paper is primarily motivated by pragmatics.

does not need to deal with any incentive issue; the focus of this paper is on how cooperative communication can be *economized* by the use of informative silence. Second, one might consider that if communication is costly, focusing on fully informative communication rules is not always natural since the quantity of communication could be suboptimal. One interpretation of the current setting is that this paper considers the situation where the cost of uttering words is not significant enough to sacrifice informativeness of communication. Another justification would be that even though the current paper has no explicit cost–benefit analysis for the optimal quantity of communication, the current setting is compatible with the consideration of the optimal quantity. To see this, suppose that we compute the optimal quantity of communication under an "original" information structure $(\mathcal{P}^0, \pi)$. Then, we can interpret agent 1's partitions in the current setting $\mathcal{P}_1$ as the coarsening of the original partitions that reflect the optimal quantity of communication.

The current paper does not adopt the game theoretical formulation even though the most natural purpose of cooperative communication would be achieving successful coordination. The rationale of the current approach is that even if communication is explicitly modeled as a strategy of the speaker in a common interest game, the extra game theoretical structure does not help us to provide an additional insight into the problem; any fully informative communication rule can be supported as an equilibrium communication whereas there is no clear-cut selection criterion for an equilibrium communication strategy except for "optimal equilibrium". Then, since the current paper is interested in how exactly people would use informative silence, it is reasonable to analyze the efficient communication rule directly as in other papers that study "optimal languages", e.g., Rubinstein (1996, 2000), and Cremer et al. (2007). A non-game theoretical treatment of communication can also be found in the literature on consensus and communication such as Geanakoplos and Polemarchakis (1982), which also focus on cooperative communication.

3. Efficient communication rule and indexical silence

This section analyzes efficient communication rules. To start with, I provide the properties of fully informative and admissible rules. I then show that the efficient use of silence exhibits indexicality, a special type of context dependence property that is common in natural languages.

3.1. Preliminary analysis

Whenever the speaker's message updates the listener's knowledge about the current state, the speaker's message refines the listener's information partition. A **cover** of P_2, denoted by S, is a collection of P_1 such that (i) $P_1 \cap P_2 \neq \emptyset$ for all $P_1 \in S$, and (ii) $P_2 \subseteq \{\omega : \Pi_1(\omega) \in S\}$.[12] In short, the cover of P_2 is the set of all possible situations of the speaker from the perspective of the listener in the situation P_2. Let $S(P_2)$ denote the cover of P_2. Clearly, since $\mathcal{P}_i$ is a partition of Ω, each P_2 has a unique cover.

Example 1. Suppose $\Omega = \{1, 2, 3, 4, 5, 6, 7\}$, and $\mathcal{P}$ is such that

$$\mathcal{P}_1 = \{\{1, 7\}, \{2, 3\}, \{4, 5\}, \{6\}\},$$
$$\mathcal{P}_2 = \{\{1, 2\}, \{3, 4, 5, 6\}, \{7\}\}$$

Then, the cover of each P_2 is $S(\{1, 2\}) = \{\{1, 7\}, \{2, 3\}\}$, $S(\{3, 4, 5, 6\}) = \{\{2, 3\}, \{4, 5\}, \{6\}\}$, and $S(\{7\}) = \{1, 7\}$.

By using the concept of cover, fully informative communication rules can be characterized concisely.

Lemma 1. *A communication rule is fully informative if and only if every cover S has at most one $P_1 \in S$ such that $\sigma(\omega) = \varnothing$ for all $\omega \in P_1$.*

Proof. If part: First, if $\sigma(\omega) = \Pi_1(\omega)$, then $Q_\sigma(\sigma(\omega), \Pi_2(\omega)) = \Pi_2(\omega) \cap \Pi_1(\omega)$. Second, suppose that, for some P_2', there exists only one $P_1' \in S(P_2')$ such that $\sigma(\omega) = \varnothing$ for all $\omega \in P_1'$. Then,

$$Q_\sigma(\varnothing, P_2') = P_2' \cap \{\omega : \sigma(\omega) = \varnothing\} = P_2' \cap P_1'$$

for all $\omega \in P_2' \cap P_1'$.

Only if part: Suppose that $\sigma(\omega) = \varnothing$ for all $\omega \in P_1' \cup P_1''$ where $P_1', P_1'' \in S$ in a fully informative rule. Then, since $P_1', P_1'' \in S$, there exists $\omega' \in P_1'$ such that $P_1' \cap \Pi_2(\omega') \neq \emptyset$ and $P_1'' \cap \Pi_2(\omega') \neq \emptyset$. Hence, if the listener receives $\varnothing$ at ω', her updated information partition is

$$Q_\sigma(\varnothing, \Pi_2(\omega')) \supseteq \Pi_2(\omega') \cap (P_1' \cup P_1'') \supsetneq \Pi_2(\omega') \cap P_1'$$

That is, σ is not fully informative, a contradiction. □

The idea of Lemma 1 is straightforward. Note that the listener cannot distinguish one silence from another silence. Thus, if the listener cannot distinguish two situations, and the speaker expresses one of them with silence, the speaker never uses silence to express another situation in any fully informative communication rule.

Before moving to the analysis of efficient communication rules, it is useful to study admissible communication rules, which are reasonable but not necessarily efficient. Formally, given $(\Omega, \mathcal{P})$, a fully informative communication rule σ is **admissible** if there is no fully informative rule σ' such that

$$\sum_{\omega \in \Omega} c(\sigma'(\omega))\pi(\omega) < \sum_{\omega \in \Omega} c(\sigma(\omega))\pi(\omega).$$

under any π such that $supp(\pi) = \Omega$. In other words, if a communication rule σ is admissible, there is no other rule σ' that is superior to σ regardless of the prior probability. Clearly, any efficient rule is admissible.

To provide a property of admissible rules, let $\mathcal{S}(P_1)$ be the set of all covers that contain P_1.

Proposition 1. *If a communication rule σ is admissible, then, given any $P_1 \in \mathcal{P}_1$, there exists $S \in \mathcal{S}(P_1)$ such that, for some $P_1' \in S$, $\sigma(\omega) = \varnothing$ for all $\omega \in P_1'$.*

Proof. See Appendix. □

There are two types of silence in admissible rules. The first type can be found in the situation where no communication is required. Suppose that P_1' is mutually self-evident, i.e., $\Pi_2(\omega) \subset P_1'$ for all $\omega \in P_1'$. Then, $S(P_2) = \{P_1'\}$ for all $P_2 \subset P_1'$, and thus $S = \{P_1'\}$ for all $S \in \mathcal{S}(P_1')$. Proposition 1 then suggests that $\sigma(\omega) = \varnothing$ for all $\omega \in P_1'$. In short, no admissible rule uses an explicit message when it is common knowledge that the listener has nothing to learn from the speaker at the current state.

The second type of silence, which is the primary interest of this paper, is *informative silence*. Proposition 1 suggests that the use of informative silence cannot be too conservative in admissible rules; given any situation P_1, the speaker uses informative silence for at least one situation that is "relevant" to P_1 from the perspective of the listener.

[12] In topology, an element of a cover is often restricted to a *subset* of a covered set. In the current paper, however, even if $P_1 \supsetneq P_2$, P_1 is called the cover of P_2.

3.2. Efficiency and indexicality

Optimization problems with a real variable can often be parameterized so that a small change in a parameter-value generates a small change in the solution. However, such a parametric regularity is harder to obtain in combinatorial optimization; in fact, in the efficient communication problem, a very small change of π can dramatically alter the efficient use of silence. To see this, consider the following example.

Example 2. Suppose $\Omega = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{P}$ is such that

$$\mathcal{P}_1 = \{\{1, 2\}, \{3, 4\}, \{5, 6\}\},$$
$$\mathcal{P}_2 = \{\{1, 2, 3\}, \{4, 5, 6\}\}.$$

If $(\pi(1), \pi(2), \pi(3), \pi(4), \pi(5), \pi(6)) = (0.1, 0.04, 0.25, 0.26, 0.2, 0.15)$, the efficient communication rule is

$$\sigma(\omega) = \begin{cases} \Pi_1(\omega) & \text{if } \omega \in \{1, 2, 5, 6\} \\ \varnothing & \text{if } \omega \in \{3, 4\}. \end{cases}$$

On the other hand, if $(\pi(1), \pi(2), \pi(3), \pi(4), \pi(5), \pi(6)) = (0.1, 0.06, 0.25, 0.24, 0.2, 0.15)$, then the efficient communication rule is

$$\sigma(\omega) = \begin{cases} \varnothing & \text{if } \omega \in \{1, 2, 5, 6\} \\ \Pi_1(\omega) & \text{if } \omega \in \{3, 4\}. \end{cases}$$

Note that under both cases,

$$\sum_{\omega\in\{3,4\}} \pi(\omega) > \sum_{\omega\in\{5,6\}} \pi(\omega) > \sum_{\omega\in\{1,2\}} \pi(\omega).$$

Thus, even if we restrict our attention to the very simple partitional structure with the certain probability ranking over P_1, there is no simple regularity in the efficient use of silence.

Even if there is no simple regularity in the efficient use of silence, there can be some general property in the structure of the efficient rule. The key observation to obtaining a structural property is the following: if two informative silences, which are indistinguishable, can refer to two separate situations, the *context* of use has to determine the meaning of each silence. In semiotics and philosophy of language, signs and words whose references are *systematically* determined by the context of use are called *indexicals*.[13] For instance, the word "I" itself does not refer to any particular person by itself, but it refers to Mike in the context where Mike utters the word "I". That is, the word "I" operates as a function that specifies the content of "I" once the input variable "utterer" is given by a context.[14] Put differently, an indexical refers to an object without relying on the name of the object by *exploiting contextual information, which is public by nature*; the indexical "I" exploits the fact the utterer of "I" is common knowledge in the communication, whereas indexical "she" utilizes the fact that a woman who is, say, standing in front of agents, is common knowledge between them.

To identify the indexical structure of informative silence in the efficient rule, we need to define the notion of context for the current communication problem. In the current paper, since the subject of communication is the speaker's situation P_1, it is natural to define the context of communication at ω as the set of states that is common knowledge at the current state ω between agents.[15] Hence, define the **context** of communication at ω as the *finest mutually self-evident event* Z that contains ω, denoted by $Z(\omega)$. Formally, let $\mathcal{E}(\omega)$ be the set of mutually self-evident events that contain ω. That is,

$$\mathcal{E}(\omega) = \{X \subset \Omega : \omega \in X, \Pi_1(\omega') \subset X, \Pi_2(\omega') \subset X \text{ for all } \omega' \in X\}.$$

Then, the context of communication at ω is the smallest set in $\mathcal{E}(\omega)$.[16] Hence, if the current context is Z, it is common knowledge between the speaker and the listener that the current state is somewhere in Z.

Example 3. Suppose $\Omega = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathcal{P}$ is such that

$$\mathcal{P}_1 = \{\{1, 2\}, \{3, 8\}, \{5, 6\}, \{7, 4\}\},$$
$$\mathcal{P}_2 = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7\}, \{8\}\}.$$

Then, the context at each state is

$$Z(\omega) = \begin{cases} \{1, 2, 3, 8\} & \text{if } \omega \in \{1, 2, 3, 8\} \\ \{4, 5, 6, 7\} & \text{if } \omega \in \{4, 5, 6, 7\}. \end{cases}$$

To formalize the indexicality of $\varnothing$ in the current communication problem, let $X \subset \Omega$ and define

$$\Gamma_\sigma(X) = X \cap \{\omega \in \Omega : \sigma(\omega) = \varnothing\}.$$

That is, $\Gamma_\sigma(X)$ is the set of states in X at which a communication rule σ uses silence.

Now, let Σ^* be the set of fully informative communication rules and define

$$\mathcal{X}_Z = \{X \in 2^{\mathcal{P}_1} : X = \Gamma_\sigma(Z) \text{ for some } \sigma \in \Sigma^*\}.$$

Note that $X \in 2^{\mathcal{P}_1}$ is a situation or a union of situations of the speaker. Thus, $\mathcal{X}_Z$ is a collection of situations and unions of situations of the speaker in the context Z where some fully informative rule uses silence. That is, the communication remains fully informative as long as the speaker uses silence for $X \in \mathcal{X}_Z$. Then, let us call $X \in \mathcal{X}_Z$ an **implicitly expressible event** in a context Z.

Let X^*_Z be the most likely implicitly expressible event in the context Z. Formally, X^*_Z solves

$$\max_{X\in\mathcal{X}_Z} \sum_{\omega\in X} \pi(\omega).$$

Note that X^*_Z is unique since, by assumption, there is no tie in the probabilities of events.

Definition 1. The message $\varnothing$ at ω in σ is **indexical silence** if

$$\Gamma_\sigma(Z(\omega)) = X^*_{Z(\omega)}.$$

That is, if $\varnothing$ at ω is indexical silence, it refers to the most likely implicitly expressible event at $Z(\omega)$.

As the indexical meaning of "I" is "the utterer of I", the indexical meaning of $\varnothing$ is "the most likely implicitly expressible event". Moreover, as the specific person referred by "I" is systematically determined according to the indexical meaning of "I" once the context is given, the specific event referred by $\varnothing$ is also systematically determined according to the indexical meaning of $\varnothing$ once the context Z is known.

13 "I", "tomorrow", and "here" are typical examples. Linguistic indexicals are often called deixis. When the meaning of deixis is determined by antecedent, it is called anaphora.

14 One of the most influential formulations of indexicals is provided by Kaplan (1989). The central concepts in his formulation are "context", "character", and "content". Roughly speaking, "content" is the reference of an indexical; "context" is a variable that fixes the reference of an indexical; "character" is a function that specifies the content of an indexical given a context. The current paper follows the basic idea of Kaplan (1989).

15 Context is a term that is used very widely. The current paper adopts the knowledge-based context that considers context as a set of background assumptions shared by the speaker and the listener. See Stalnaker (2014) for a fuller discussion on this approach. Note that "physical context" such as spatio-temporal location and "linguistic context" that consists of past utterances can also be accommodated to knowledge-based approach when the state space is properly constructed.

16 Note that $\mathcal{E}(\omega)$ always contains Ω.

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Proposition 2. *A communication rule σ is efficient if and only if the message $\varnothing$ at any $\omega \in \Gamma_\sigma(\Omega)$ is indexical silence.*

Proof. See Appendix. □

The proof exploits the property of context; since the context is common knowledge at any state, the set of all contexts forms a partition of Ω, and the messaging cost minimization problem is context-separable. Thus, one can solve the efficient communication problem by the divide-and-conquer procedure; solve the minimization problem for each context separately and merge the solutions to obtain the efficient communication rule. Then, by duality, the solution of the messaging cost minimizing problem can be obtained as the maximization of the unambiguous use of silence given a context.

In the proof of Proposition 2, we can find that the common knowledge property of context Z plays a crucial role. In fact, the common knowledge property of context is essential for the indexicality of informative silence.

Observation *Suppose that, contrary to the current setting, the notion of context Z is defined in a way that lacks common knowledge property.*[17] *Then, the efficient use of silence does not always exhibit indexicality.*

Without the common knowledge property of context Z, the cost minimization problem is not always context-separable. Hence, there is no guarantee that the globally efficient use of silence coincides with the efficient use of silence within the context. Consequently, there is no indexical meaning of silence given a context in the efficient rule; silence could refer to, say, the second most likely implicitly expressive event in one context, while it refers to the most likely implicitly expressive event in another context lacking indexical meaning. If people can appropriately and flexibly use informative silence across various contexts in reality, it is natural to expect that informative silence has indexicality. In this sense, one could argue that the current definition of context, which makes informative silence indexical, is the right one.

Under some information structure, agents can find the efficient use of silence almost immediately from the indexical property.

Example 4. Suppose that an information structure is such that $\Pi_1(\omega) \subsetneq \Pi_2(\omega)$ for all ω. Then, $X^*_{Z(\omega)} = \phi(\omega)$ where $\phi(\omega)$ solves

$$\max_{P_1 \subset Z(\omega)} \sum_{\omega' \in P_1} \pi(\omega').$$

A notable special case of Example 4 is the informed-uninformed agent setting. In this case, any fully informative rule uses silence at most one situation. Thus, the speaker uses silence to indicate the speaker's ex-ante most common situation in the efficient rule.

In the following example, finding the efficient use of silence based on the indexical property demands rather careful inspection.

Example 5. Suppose $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, and $\mathcal{P}$ is such that

$$\mathcal{P}_1 = \{\{1, 2\}, \{3\}, \{4, 5\}, \{6, 7\}, \{8\}, \{9, 10\}, \{11\}\},$$
$$\mathcal{P}_2 = \{\{1, 8\}, \{2, 3\}, \{4\}, \{5, 6\}, \{7, 9\}, \{10, 11\}\}.$$

Suppose $\sum_{\omega\in\Pi_1(\omega')} \pi(\omega) \le \sum_{\omega\in\Pi_1(\omega'')} \pi(\omega)$ if $\omega' < \omega''$, and the inequality is strict if $\Pi_1(\omega') \ne \Pi_1(\omega'')$.

The information structure has two possible contexts $Z' = \{1, 2, 3, 8\}$ and $Z'' = \{4, 5, 6, 7, 9, 10, 11\}$. Then, the set of implicit expressible events given Z' and that given Z'' are

$$\mathcal{X}_{Z'} = \{\{1, 2\}, \{3\}, \{8\}, \{3, 8\}\},$$
$$\mathcal{X}_{Z''} = \{\{4, 5\}, \{6, 7\}, \{9, 10\}, \{11\}, \{4, 5, 9, 10\}, \{6, 7, 11\}, \{4, 5, 11\}\}$$

From the indexical property, if σ^* is efficient, then $\Gamma_{\sigma^*}(Z') = \{3, 8\}$ and $\Gamma_{\sigma^*}(Z'') = \{6, 7, 11\}$. Thus,

$$\sigma^*(\omega) = \begin{cases} \varnothing & \text{if } \omega \in \{3, 6, 7, 8, 11\} \\ \Pi_1(\omega) & \text{if } \omega \notin \{3, 6, 7, 8, 11\}. \end{cases}$$

3.3. Complexity of indexical silence

As we saw in Example 5, when silence refers to an event that consists of more than one situations given a context, the user of indexical silence needs to find out the exact combination of situations that maximizes the use of silence. Since this is a combinatorial optimization problem, one needs to check all "reasonable combinations" of the use of silence to find out the solution.

To provide some idea about how demanding the computation can be, suppose an information structure has a context Z with $P'_1 \subset Z$ such that

$$Z = \{\omega' : \Pi_1(\omega') \in S \text{ where } S \in \mathcal{S}(P'_1)\}.$$

Suppose $|S| = k$ for all $S \in \mathcal{S}(P'_1)$, $S \cap S' = P'_1$ for all $S, S' \in \mathcal{S}(P'_1)$, and let $|\mathcal{S}(P'_1)| = l$. Then, the number of implicitly expressible events in Z is

$$|\mathcal{X}_Z| \ge (k-1)^l + 1.$$

Thus, the number of implicitly expressible events in one context grows exponentially in the number of relevant covers l. Note that the burden of a large $|\mathcal{X}_Z|$ is not only the number of comparisons but also the computation of $\sum_{\omega\in X_Z} \pi(\omega)$ for every $X_Z \in \mathcal{X}_Z$.

If indexical silence is interpreted as the efficient use of silence designed by an organization, the computational burden might not be so problematic. However, if indexical silence is considered as a tacit communication rule for ordinary conversation, i.e., pragmatics, the complexity of computing the use of indexical silence would be troublesome. This calls for the analysis of the next section.

4. Direct and ordinal indexicals

This section analyzes the information structure in which indexical silence works as intuitively as indexicals in natural languages do. There are two major differences between indexical silence and indexicals in natural languages. First, indexicals in natural languages "directly" refer to an entity given a context, whereas indexical silence could refer to a non-trivial combination of situations given a context.[18] Second, indexicals in natural languages do not rely on the cardinal information in the communication environment such as probability.

[17] For example, one might define $Z(\omega)$ as the smallest event that is mutual knowledge at ω.

[18] For instance, in Example 5, the informative silence refers to the combination of situations $\{6, 7\}$ and $\{11\}$ in the context Z'' rather than directly referring to a single situation in the context.

4.1. Direct indexicals

To capture the directness of indexicals in natural languages, let $\phi(\omega)$ be the speaker's situation that solves

$$\max_{P_1 \subset Z(\omega)} \sum_{\omega' \in P_1} \pi(\omega').$$

That is, $\phi(\omega)$ is the *speaker's most common situation* in a context $Z(\omega)$.

Definition 2. The message $\varnothing$ at ω in σ is **direct indexical** if

$$\Gamma_\sigma(Z(\omega)) = \phi(\omega).$$

That is, if $\varnothing$ is direct indexical, informative silence refers to a *single* situation of the speaker that is most likely in the current context. Note that the efficient use of silence is not always direct indexical. To see this, consider Example 3. The silence at $\omega \in \{3, 4\}$ in the efficient rule under the first π is direct indexical since $\varnothing$ directly refers to the single situation given the context; in contrast, the silence at $\omega \in \{1, 2, 5, 6\}$ in the efficient rule under the second π is not direct indexical since $\varnothing$ refers to the union of situations given the context.

To provide the properties of information structures in which the efficient use of silence is direct indexical, let $\mathcal{S}(P_1)$ be the set of all covers that contain P_1.

Definition 3. The speaker's situation P_1 is **central** in a context Z if $S(\Pi_2(\omega)) \in \mathcal{S}(P_1)$ for all $\omega \in Z$.

That is, if a situation is central in a context, it is contained by all covers in the context.[19] Intuitively, if a situation is central in a context, the situation is "relevant" to the listener at any state in the context. Note that some context could fail to possess a central situation. Thus, one could interpret that a context is "simple" in terms of the information structure when the context contains a central situation.

Example 6. Consider the information structure in Example 5. Recall that this information structure has two contexts $Z' = \{1, 2, 3, 8\}$ and $Z'' = \{4, 5, 6, 7, 9, 10, 11\}$. The situation $\{1, 2\}$ is central in Z' since $S(\Pi_2(\omega)) \in \mathcal{S}(\{1, 2\}) = \{S(\{1, 8\}), S(\{2, 3\})\}$ for all $\omega \in Z'$. On the other hand, no situation is central in Z'' since there is no $P_1 \subset Z''$ that is contained by all covers in Z''.

Lemma 2. *If the message $\varnothing$ at ω' in an efficient communication rule σ is direct indexical, then $P(\omega')$ is central in $Z(\omega')$.*

Proof. See Appendix. □

The idea behind Lemma 2 is as follows. Suppose that the speaker uses silence for a non-central situation in a context, Then, we can always find another situation at which if the speaker uses silence, the listener can distinguish this silence from another silence. Consequently, the speaker needs to use silence for more than one situation in the context, precluding direct indexical silence.

To provide another necessary condition for direct indexicals, let

$$H(\omega) = \{\omega' : \Pi_1(\omega') \in S \text{ where } S \in \mathcal{S}(\Pi_1(\omega))\}.$$

[19] The notation of centrality here is not a metaphor. To see this, consider an undirected graph $G = (V, E)$ where $V = \mathcal{P}_1$, and $P_1P_1' \in E$ iff $P_1 \neq P_1'$ and $P_1, P_1' \in S(P_2)$ for some $P_2 \in \mathcal{P}_2$. If P_1 is central in Z, any vertex in Z is directly connected to P_1.

Definition 4. The speaker's situation $\Pi_1(\omega')$ is **locally dominant** if

$$\sum_{\omega \in \Pi_1(\omega')} \pi(\omega) \geq \sum_{\omega \in \Gamma_\sigma(H(\omega'))} \pi(\omega)$$

for all σ that is fully informative and $\sigma(\omega') = \Pi_1(\omega')$.

When the speaker's current situation is locally dominant, the probability of having the current situation is higher than that of any implicitly expressible event that does not use silence for the current situation in the context.

Lemma 3. *If the message $\varnothing$ at ω' in an efficient communication rule σ is direct indexical, then $\Pi_1(\omega')$ is locally dominant.*

Proof. See Appendix. □

The next proposition states that centrality and local dominance are not only necessary but also sufficient conditions for direct indexicals.

Proposition 3. *The message $\varnothing$ at ω' in the efficient communication rule σ is direct indexical if and only if $\Pi_1(\omega')$ is central in $Z(\omega')$ and locally dominant.*

Proof. See Appendix. □

The next result is immediate from Proposition 3.

Corollary 1. *Suppose σ is an efficient communication rule. The message $\varnothing$ at any $\omega \in \Gamma_\sigma(\Omega)$ is direct indexical if and only if every context Z has a central and locally dominant situation.*

The simplest information structure that satisfies the condition in Corollary 1 is that in Example 4.

4.2. Ordinal indexicals

As I mentioned earlier, the second gap between indexicals in natural languages and indexical silence is that the former can operate solely with ordinal information such as spatial, temporal, and personal relations in context, whereas indexical silence could require rather demanding quantitative evaluations. That is, even though the agent can find whether a situation is central or not from the partitional structure, checking whether a central situation is locally dominant or not could demand the exact computation. The rest of this section investigates a condition under which the agent can verify the local dominance condition solely from the ordinal information in the information structure.

To state the condition, let $\psi(P_2)$ be the solution of

$$\max_{P_1 \in S(P_2)} \sum_{\omega \in P_1 \cap P_2} \pi(\omega).$$

That is, $\psi(P_2)$ is the speaker's most common situation given P_2, which is unique by assumption.

Definition 5. The speaker's situation P_1 is **consistently most common** if $\psi(P_2) = P_1$ for all P_2 such that $S(P_2) \in \mathcal{S}(P_1)$.

Note that agents do not need to know the exact value of $\pi(\omega)$ to find the consistently most common situation; they just need to know the probability ranking over the set of $P_1 \cap P_2$.

Proposition 4. *The message $\varnothing$ at ω' in the efficient communication rule is direct indexical if $\Pi_1(\omega')$ is central in $Z(\omega')$ and consistently most common.*

Proof. See Appendix. □

One might wonder whether there is a simpler way to guarantee the local dominance condition than checking the condition of consistently most common. For instance, does having a situation that has a higher probability than any other situation in the context guarantee the local dominance? The next example shows that the answer is negative.

Example 7. Suppose $\Omega = \{1, 2, 3, 4, 5, 6\}$, and $\mathcal{P}$ is such that

$\mathcal{P}_1 = \{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$,

$\mathcal{P}_2 = \{\{1, 2, 3\}, \{4, 5, 6\}\}$.

Moreover, suppose

$\pi = (0.15, 0.24, 0.2, 0.21, 0.1, 0.1)$.

The situation $\{3, 4\}$ has a higher probability than $\{1, 2\}$ and $\{5, 6\}$. However, the combination of two situations $\{1, 2, 5, 6\}$ has a higher probability than $\{3, 4\}$, and the communication rule that uses $\varnothing$ for $\{1, 2, 5, 6\}$ is fully informative. Thus, $\{3, 4\}$ is not locally dominant.

5. Discussions

5.1. Efficient communication rules with cognitive cost

Unless agents recurrently face the same communication problem, it might be too demanding to expect them to use informative silence efficiently beyond direct indexical in practice. In fact, if we take into account the cognitive cost of finding the use and meaning of non-direct indexical silence, the cognitive cost could be even higher than the cost of using explicit messages.

When the cognitive cost is incorporated, the definition of an efficient communication rule needs to be revised. Suppose that it is costly to use non-direct indexical because of the cognitive cost. Specifically, assume that

$$C_\sigma(m, \omega) = \begin{cases} c' & \text{if } m = P_1 \\ c'' & \text{if } m = \varnothing \text{ and } \Pi_1(\omega) \subsetneq \Gamma_\sigma(Z(\omega)) \\ 0 & \text{if } m = \varnothing \text{ and } \Pi_1(\omega) = \Gamma_\sigma(Z(\omega)) \end{cases}$$

That is, if $\varnothing$ refers to a set of situations rather than a particular situation given a context, the use of informative silence is costly. Then, the efficient communication rule under $C_\sigma(m, \omega)$ solves

$$\min_{\sigma \in \Sigma^*} \sum_{\omega \in \Omega} C_\sigma(m, \omega)\pi(\omega).$$

Observation Suppose $c' < c''$. In the efficient communication rule under $C_\sigma(m, \omega)$, the speaker uses $\varnothing$ at ω if and only if $\omega \in \phi(\omega)$.

Under $C_\sigma(m, \omega)$, an explicit report is less costly than $\varnothing$ whenever $\varnothing$ refers to a set of situations in the current context. Thus, in the efficient communication rule under the cost function, $\varnothing$ indexically refers to the most common speaker's situation given a context. Note that $\phi(\omega)$ is not necessarily central and locally dominant even though the efficient use of silence under $C_\sigma(m, \omega)$ is always direct indexical.

In one shot problem, the cognitive cost might often be higher than the messaging cost unless the message requires some lengthy report. However, when the communication problem is recurrent, the cognitive cost might not be so expensive once an agent learns the efficient use of silence. Note that it is also costly to remember the rule, especially when the use of informative silence in rare contexts. Thus, whether the cognitive cost is lower than the messaging cost depends on the nature of a communication problem.

5.2. Efficient communication rules in games

As explained in Section 2, the current paper directly analyzes the efficient communication rule since our primary interest is in cooperative environments where the game theoretical formulation does not provide additional insights. Needless to say, when there is some conflict of interest between the speaker and the listener, the game theoretical approach becomes essential since the speaker might not wish to reveal her information anymore. However, the analysis of the efficient communication rule can be extended to the strategic environment.

To see the claim, suppose that the speaker and the listener have different payoff functions that depend on the listener's action a and the state ω. The speaker sends a message given her situation P_1, and the listener chooses an action a. Assume that the speaker's message space is $M = \mathcal{P}_1 \cup \{\varnothing\}$, and that the cost of an explicit message is c. Note that P_1 is just a label, and so the speaker can send a message $P_1 \neq \Pi_1(\omega)$ at ω. First, we analyze the equilibrium communication under $c = 0$. Suppose that we are interested in the equilibrium strategy $\hat{\sigma}$, and let $\hat{\Sigma}$ be the set of communication rules that have the same informational content as $\hat{\sigma}$, i.e., if $\sigma \in \hat{\Sigma}$, σ is relabeling of $\hat{\sigma}$. Then, if the equilibrium that supports $\hat{\sigma}$ is strict and c is sufficiently small, the efficient communication rule of the equilibrium strategy $\hat{\sigma}$ can be obtained by solving our familiar problem

$$\min_{\sigma \in \hat{\Sigma}} \sum_{\omega \in \Omega} c(\sigma(\omega))\pi(\omega)$$

That is, given an equilibrium communication strategy, we can analyze the efficient communication problem as in the current paper if the cost of messaging is not too large.

In a communication game with some conflict of interest, some equilibria might not be strict. In that case, the exact efficient equilibrium communication cannot be obtained by the above approach since the equilibrium strategy can be distorted by the communication cost. However, if the cost is small, and the equilibrium communication strategy is continuous in the change of the messaging cost, the above method still provides a good approximation for the efficient communication rule.

Note that when there is some conflict of interest between the speaker and the listener, the meaning of informative silence cannot be determined by $(\Omega, \mathcal{P}, \pi)$ since the equilibrium communication depends on the payoff function. Then, since a context $Z(\omega)$ alone cannot determine the meaning of informative silence, the informative silence loses the indexical property. This is analogous to the fact that "yesterday" does not indicate one day before the date of the utterance when the speaker has an incentive to tell a lie about the date. In fact, people often utilize the original meaning of an indexical to confuse the listener in a strategic environment in practice.

6. Conclusion

Many cooperative economic activities rely on a conversation, which often utilizes informative silence to convey information. The current paper analyzed the pragmatics of informative silence as the efficient communication rule designed by a fictitious linguistic engineer. It is shown that when the notion of context is defined as the finest mutually self-evident event that contains the current state, the efficient use of informative silence exhibits the defining property of indexicals in natural languages. That is, as indexical "I" systematically refers to the person who utters the word given a context, informative silence in the efficient rule systematically refers to the most likely implicitly expressible event given a context. Furthermore, while the efficient use of silence could be complex, it is also found that the efficient use of silence

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is as intuitive as the use of indexicals in natural languages if and only if the information structure satisfies some centrality and dominance properties. Thus, if the nature of indexicals in natural languages reflects the cognitive capacity of the human mind, it could be the case that ordinary people can utilize indexical silence only if the information structure is sufficiently simple.

The idea of this paper has a wide range of economic applications. In fact, whenever some messaging cost is introduced to an economic model with communication, the efficient use of silence becomes a relevant problem. One direct application can be found in team decision problems under uncertainty where team members communicate before coordinating their actions. In this application, an agent explicitly reports his/her information in the efficient rule only if his/her current information is not the most common given the current context. Another application could be found in a principal–agent model where the principal suggests a project based on her private information, and the agent chooses a project and his effort level. If there is no conflict of interest in the dimension of the project choice, the principal uses informative silence as a message of "business as usual" in the efficient rule. As it is discussed in Section 5.2, the application of informative silence is not limited to the common interest setting; the current approach can be extended to study the use of informative silence in cheap talk games with a conflict of interest.

Appendix

A.1. Proof of Proposition 1

Suppose σ is admissible, but some P_1'' has no $S \in \mathcal{S}(P_1'')$ such that, for some $P_1' \in S$, $\sigma(\omega) = \varnothing$ for all $\omega \in P_1'$. Then, $\sigma(\omega) = \Pi_1(\omega)$ for any ω such that $\Pi_1(\omega) \in S$ where $S \in \mathcal{S}(P_1'')$. Now, consider the alternative rule σ' such that

$$\sigma'(\omega) = \begin{cases} \varnothing & \text{if } \omega \in P_1'' \\ \sigma(\omega) & \text{if } \omega \in \Omega \backslash P_1'' \end{cases}$$

Note that since $\sigma'(\omega) = \Pi_1(\omega)$ for any ω such that $\Pi_1(\omega) \in S \backslash \{P_1''\}$ where $S \in \mathcal{S}(P_1'')$,

$$Q_{\sigma'}(\sigma(\omega), \Pi_2(\omega)) = P_1'' \cap \Pi_2(\omega)$$

for all $\omega \in P_1''$. Then, since $\sigma(\omega) = \Pi_1(\omega)$ for any ω such that $\Pi_1(\omega) \in S \backslash \{P_1''\}$ where $S \in \mathcal{S}(P_1'')$, and σ is fully informative, σ' is also fully informative. Moreover,

$$\sum_{\omega \in \Omega} c(\sigma(\omega))\pi(\omega) - \sum_{\omega \in \Omega} c(\sigma'(\omega))\pi(\omega) = c \sum_{\omega \in P_1''} \pi(\omega) > 0$$

under any π such that $supp(\pi) = \Omega$. But then, σ is not admissible, a contradiction.

A.2. Proof of Proposition 2

If part: If the message $\varnothing$ at any $\omega \in \Gamma_\sigma(\Omega)$ is indexical silence, then, for any fully informative communication rule σ',

$$\sum_{\omega' \in \Gamma_{\sigma'}(Z(\omega))} \pi(\omega') \leq \sum_{\omega' \in \Gamma_\sigma(Z(\omega))} \pi(\omega')$$

for all ω. Since $Z(\omega)$ is mutually self-evident at every ω, the set of contexts is a partition of Ω. Hence,

$$\bigcup_{\omega \in \Omega} \Gamma_\sigma(Z(\omega)) = \Gamma_\sigma(\Omega).$$

Therefore, for any fully informative communication rule σ',

$$\sum_{\omega' \in \Gamma_{\sigma'}(\Omega)} \pi(\omega') \leq \sum_{\omega' \in \Gamma_\sigma(\Omega)} \pi(\omega').$$

Then, since

$$\sum_{\omega' \in \Omega} c(\sigma(\omega'))\pi(\omega') = c - c \sum_{\omega' \in \Gamma_\sigma(\Omega)} \pi(\omega'),$$

it follows that, for any fully informative communication rule σ',

$$\sum_{\omega' \in \Omega} c(\sigma'(\omega'))\pi(\omega') \geq \sum_{\omega' \in \Omega} c(\sigma(\omega'))\pi(\omega').$$

Since any communication rule with indexical silence is fully informative, σ is an efficient rule.

Only if part: Suppose that σ is an efficient rule, but $\varnothing$ in σ is not indexical silence at some $\tilde{\omega} \in \Gamma_\sigma(\Omega)$. Then, $\Gamma_\sigma(Z(\tilde{\omega})) \neq X^*_{Z(\tilde{\omega})}$. Now consider an alternative communication rule σ'' such that

$$\sigma''(\omega) = \begin{cases} \sigma(\omega) & \text{if } \omega \notin Z(\tilde{\omega}) \\ \tilde{\sigma}(\omega) & \text{if } \omega \in Z(\tilde{\omega}) \end{cases}$$

where $\tilde{\sigma}$ is some fully informative communication rule such that $\Gamma_{\tilde{\sigma}}(Z(\tilde{\omega})) = X^*_{Z(\tilde{\omega})}$. Note that σ'' is a fully informative communication rule by construction. Then, since $\Gamma_\sigma(Z(\tilde{\omega})) \neq X^*_{Z(\tilde{\omega})}$ and $\Gamma_{\sigma''}(Z(\tilde{\omega})) = X^*_{Z(\tilde{\omega})}$,

$$\sum_{\omega \in \Gamma_\sigma(Z(\tilde{\omega}))} \pi(\omega) < \sum_{\omega \in \Gamma_{\sigma''}(Z(\tilde{\omega}))} \pi(\omega).$$

Moreover, by construction,

$$\sum_{\omega \in \Omega \backslash \Gamma_\sigma(Z(\tilde{\omega}))} \pi(\omega) = \sum_{\omega \in \Omega \backslash \Gamma_{\sigma''}(Z(\tilde{\omega}))} \pi(\omega).$$

Then, since $\cup_{\omega \in \Omega} \Gamma_\sigma(Z(\omega)) = \Gamma_\sigma(\Omega)$,

$$\sum_{\omega \in \Gamma_\sigma(\Omega)} \pi(\omega) < \sum_{\omega \in \Gamma_{\sigma''}(\Omega)} \pi(\omega).$$

But then,

$$\sum_{\omega' \in \Omega} c(\sigma(\omega'))\pi(\omega') > \sum_{\omega' \in \Omega} c(\sigma''(\omega'))\pi(\omega').$$

That is, σ is not efficient, contradicting the hypothesis.

A.3. Proof of Lemma 2

Suppose the message $\varnothing$ at ω' in the efficient communication rule σ is direct indexical. Then, $\Gamma_\sigma(Z(\omega')) = \phi(\omega')$. Thus, the expected cost of σ is

$$c - c \sum_{\omega \in \Gamma_\sigma(\Omega \backslash Z(\omega'))} \pi(\omega) - c \sum_{\omega \in \phi(\omega')} \pi(\omega).$$

Now suppose $\phi(\omega')$ is not central in $Z(\omega')$. Then, there exists $S' \notin \mathcal{S}(\Pi_1(\omega'))$ such that $Z(\omega') \supset \{\omega : \Pi_1(\omega) \in S'\}$. Let $P_1' \in S'$ be a situation that is not an element of any cover in $\mathcal{S}(\Pi_1(\omega'))$. Then, consider the alternative communication rule σ'.

$$\sigma'(\omega) = \begin{cases} \varnothing & \text{if } \omega \in \phi(\omega') \cup P_1' \\ \Pi_1(\omega) & \text{if } \omega \in Z(\omega') \backslash (\phi(\omega') \cup P_1') \\ \sigma(\omega) & \text{if } \omega \in \Omega \backslash Z(\omega') \end{cases}$$

Note that since $P_1' \in S'$ is not an element of any cover in $\mathcal{S}(\phi(\omega'))$, P_1' is the only one situation with $\varnothing$ in $S' \notin \mathcal{S}(\phi(\omega'))$. Then, from Lemma 1 together with the fact that σ is efficient, σ' is fully informative rule.

The expected cost of σ' is

$$c - c \sum_{\omega \in \Gamma_{\sigma'}(\Omega \backslash Z(\omega'))} \pi(\omega) - c \sum_{\omega \in \phi(\omega') \cup P_1'} \pi(\omega).$$

But then, since $\sum_{\omega \in \phi(\omega')} \pi(\omega) < \sum_{\omega \in \phi(\omega') \cup P_1'} \pi(\omega)$, the expected cost of σ' is strictly lower than that of σ. That is, σ is not efficient, a contradiction.

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A.4. Proof of Lemma 3

Suppose the message $\varnothing$ at ω' in the efficient communication rule σ is direct indexical but $P(\omega')$ is not locally dominant. Then, there exists a fully informative rule σ' such that

$$\sum_{\omega\in\Pi_1(\omega')}\pi(\omega) < \sum_{\omega\in\Gamma_{\sigma'}(H(\omega'))}\pi(\omega)$$

and $\sigma'(\omega') = \Pi_1(\omega')$. Now consider the following alternative communication rule

$$\sigma''(\omega) = \begin{cases}\varnothing & \text{if } \omega\in\Gamma_{\sigma'}(Z(\omega'))\\ \Pi_1(\omega) & \text{if } \omega\in Z(\omega')\backslash\Gamma_{\sigma'}(Z(\omega'))\\ \sigma(\omega) & \text{if } \omega\in\Omega\backslash Z(\omega')\end{cases}$$

Since σ and σ' are fully informative, and $Z(\omega')$ is mutually self-evident, σ'' is fully informative. The expected cost of σ'' is

$$c - c\sum_{\omega\in\Gamma_\sigma(\Omega\backslash Z(\omega'))}\pi(\omega) - c\sum_{\omega\in\Gamma_{\sigma'}(Z(\omega'))}\pi(\omega).$$

On the other hand, since the message $\varnothing$ at ω' is direct indexical in σ, $\Gamma_\sigma(Z(\omega')) = \phi(\omega')$. The expected cost of σ is

$$c - c\sum_{\omega\in\Gamma_\sigma(\Omega\backslash Z(\omega'))}\pi(\omega) - c\sum_{\omega\in\phi(\omega')}\pi(\omega).$$

Note that, from Lemma 2, if $\varnothing$ is direct indexical in an efficient rule σ, then $\Pi_1(\omega')$ is central in $Z(\omega')$. I claim that if $\Pi_1(\omega')$ is central in $Z(\omega')$, then $Z(\omega') = H(\omega')$. To see this, note that whenever $\omega\in P_1$ and $\omega\in H(\omega')$, then $\omega''\in H(\omega')$ for any $\omega''\in P_1$. Then, since $Z(\omega')$ is mutually self-evident, $Z(\omega')\supset P_1$ for all $P_1\in S$ where $S\in\mathcal{S}(\Pi_1(\omega'))$. Hence, $Z(\omega')\supset H(\omega')$. Now suppose that $Z(\omega')\supsetneq H(\omega')$. Then, since $Z(\omega')$ is mutually self-evident, there exists $P_1\subset Z(\omega')$ such that $P_1\notin S$ for any $S\in\mathcal{S}(\Pi_1(\omega'))$. Then, $\Pi_1(\omega')$ is not central in $Z(\omega')$, contradicting the hypothesis that $Z(\omega')\supsetneq H(\omega')$. Hence, $Z(\omega') = H(\omega')$.

From the first inequality of this proof and $Z(\omega') = H(\omega')$,

$$\sum_{\omega\in\phi(\omega')}\pi(\omega) < \sum_{\omega\in\Gamma_{\sigma'}(Z(\omega'))}\pi(\omega).$$

But then, σ is not efficient, a contradiction.

A.5. Proof of Proposition 3

Since "only if" part has already been established by Lemma 2 and 3, we need to show "if" part. Suppose that the message $\varnothing$ at ω' in the efficient communication rule σ is not direct indexical. Then, $\Gamma_\sigma(Z(\omega'))\neq\phi(\omega')$. The expected cost of σ is then

$$c - c\sum_{\omega\in\Gamma_\sigma(\Omega\backslash Z(\omega'))}\pi(\omega) - c\sum_{\omega\in\Gamma_\sigma(Z(\omega'))}\pi(\omega).$$

Now consider the following alternative rule,

$$\sigma'(\omega) = \begin{cases}\varnothing & \text{if } \omega\in\phi(\omega')\\ \Pi_1(\omega) & \text{if } \omega\in Z(\omega')\backslash\phi(\omega')\\ \sigma(\omega) & \text{if } \omega\in\Omega\backslash Z(\omega')\end{cases}$$

Since σ is fully informative and only $\phi(\omega')$ in $Z(\omega')$ has $\varnothing$, σ' is fully informative from Lemma 1. The expected cost of σ' is

$$c - c\sum_{\omega\in\Gamma_\sigma(\Omega\backslash Z(\omega'))}\pi(\omega) - c\sum_{\omega\in\phi(\omega')}\pi(\omega).$$

Now suppose $\phi(\omega')$ is central in $Z(\omega')$ and locally dominant. Then, $Z(\omega') = H(\omega')$. Thus,

$$\sum_{\omega\in\phi(\omega')}\pi(\omega) \geq \sum_{\omega\in\Gamma_\sigma(Z(\omega'))}\pi(\omega).$$

From the assumption, there is no tie in the probabilities of events. Thus, whenever the above inequality holds, it must be strict. But then, the expected cost of σ is strictly higher than that of σ', contradicting the assumption that σ is efficient.

A.6. Proof of Proposition 4

Suppose $\Pi_1(\omega')$ is consistently most common. Then,

$$\sum_{\omega\in\Pi_1(\omega')\cap P_2}\pi(\omega) > \max_{P_1\in S(P_2)\backslash\{\Pi_1(\omega')\}}\sum_{\omega\in P_1\cap P_2}\pi(\omega)$$

for any P_2 such that $S(P_2)\in\mathcal{S}(\Pi_1(\omega'))$.

Note that, since $\Pi_1(\omega')$ is central in $Z(\omega')$, we have

$$\sum_{P_2\subset Z(\omega')}\max_{P_1\in S(P_2)\backslash\{\Pi_1(\omega')\}}\sum_{\omega\in P_1\cap P_2}\pi(\omega) \geq \sum_{\omega\in\Gamma_\sigma(Z(\omega'))}\pi(\omega)$$

for any fully informative rule σ such that $\sigma(\omega') = \Pi_1(\omega')$. Moreover,

$$\sum_{P_2\subset Z(\omega')}\sum_{\omega\in\Pi_1(\omega')\cap P_2}\pi(\omega) = \sum_{\omega\in\Pi_1(\omega')}\pi(\omega),$$

Then, we have

$$\sum_{\omega\in\Pi_1(\omega')}\pi(\omega) > \sum_{\omega\in\Gamma_\sigma(Z(\omega'))}\pi(\omega)$$

for any σ that is fully informative and $\sigma(\omega') = \Pi_1(\omega')$. That is, $\Pi_1(\omega')$ is locally dominant. Then, since $\Pi_1(\omega')$ is central in $Z(\omega')$, $\varnothing$ at ω' is direct indexical from Proposition 3.

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