Substantia
An International Journal of the History of Chemistry

Firenze University Press
www.fupress.com/substantia

Citation: P Thyssen, A. Ceulemans (2020) Particular Symmetries: Group Theory of the Periodic System. *Substantia* 4(1): 7-22. doi: 10.13128/Substantia-671

Received: Oct 15, 2019

Revised: Dec 19, 2019

Just Accepted Online: Dec 21, 2019

Published: Mar 11, 2020

Data Availability Statement: All relevant data are within the paper and its Supporting Information files.

Competing Interests: The Author(s) declare(s) no conflict of interest.

Research Article

Particular Symmetries: Group Theory of the Periodic System

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Abstract. To this day, a hundred and fifty years after Mendeleev's discovery, the overal structure of the periodic system remains unaccounted for in quantum-mechanical terms. Given this dire situation, a handful of scientists in the 1970s embarked on a quest for the symmetries that lie hidden in the periodic table. Their goal was to explain the table's structure in group-theoretical terms. We argue that this symmetry program required an important paradigm shift in the understanding of the nature of chemical elements. The idea, in essence, consisted of treating the chemical elements, not as particles, but as states of a superparticle. We show that the inspiration for this came from elementary particle physics, and in particular from Heisenberg's suggestion to treat the proton and neutron as different states of the nucleon.We provide a careful study of Heisenberg's last paper on the nature of elementary particles, and explain why the Democritean picture of matter no longer applied in modern physics and a Platonic symmetry-based picture was called for instead. We show how Heisenberg's Platonic philosophy came to dominate the field of elementary particle physics, and how it found its culmination point in Gell-Mann's classification of the hadrons in the eightfold way. We argue that it was the success of Heisenberg's approach in elementary particle physics that sparked the group-theoretical approach to the periodic table. We explain how it was applied to the set of chemical elements via a critical examination of the work of the Russian mathematician Abram Ilyich Fet the Turkish-American physicist Asim Orhan Barut, before giving some final reflections.

Keywords. Periodic system, group theory, symmetry, elementary particle approach, period doubling, Madelung rule.

At the heart of chemistry lies the Periodic System of Chemical Elements. Since Mendeleev's discovery in 1869 — 150 years ago — the Periodic System has figured as the undisputed cornerstone of modern chemistry. No lecture theatre or scientific laboratory is complete without a copy of the periodic table adorning its walls. From time to time, a new chemical element is added to the taxonomic chart. But its overall structure has remained the same ever since it was developed in the 1860s. "Such has been the scientific and cultural impact of Dmitri Mendeleev's periodic table of the elements that many people assume it is essentially complete", writes Eric Scerri in a recent *Nature* special on the Periodic System.[1]

In reality, however, Mendeleev's iconic chart has remained something of a mystery till the present day. When examining the overall structure of the standard periodic table, two defining features stand out: (1) the organisation of the elements in *s*-, *p*-, *d*- and *f*-blocks which reflects the particular filling order of the orbitals for many-electron systems, and (2) the so-called period doubling — the fact that all periods occur in pairs of equal length, except for the first period. Despite the quantum revolution in the 1920s, both of these characteristic features remain in need of explanation. Quantum chemistry can predict the states of every individual element, but it has great difficulties in treating the Periodic System as a whole.

As a result, chemists commonly use the so-called Madelung rule to rationalize the orbital filling order and to predict the onset of the *s*-, *p*-, *d*- and *f*-blocks in the periodic table. As a welcome extra, the period doubling emerges as a natural consequence of the Madelung rule. But the Madelung rule has never been derived from first principles and remains a purely empirical (or *lexicographic*) rule — a useful mnemonic without quantum mechanical underpinning.

In 1969, a century after Mendeleev's discovery, the Swedish physicist Per-Olov Löwdin (1916–2000) noted how remarkable it was that "the simple [Madelung] rule has not yet been derived from first principles".[2] The quest for an *ab initio* derivation of the Madelung rule came to be known as the *Löwdin challenge*. Allen and Knight called it the "oldest and largest standing problem in quantum chemistry".[3] Many claims to a successful derivation have been published, but all have been dismissed.

As a result, the Madelung rule has witnessed several critical attempts to bury it once and for all.[4] But each time, it has found proponents who have called it back from the grave, and for good reason. The Madelung rule, after all, successfully describes the overall architecture of the Periodic System. It is this aspect of the Madelung rule, in particular, that endows it with explanatory power. It is this aspect also that drew the attention of a handful of group theoreticians in the 1970s, whose work will be the focus of this essay.

As so often happens in the history of science, the insight to study the Periodic System from a group-theoretical perspective cropped up almost simultaneously at several places in Europe and North-America around 1970. The pioneers included the Turkish-American physicist Asim Orhan Barut (1926–1994) in Boulder (Colorado), Octavio Novaro (1939–2018) in Mexico City (Mexico), Valentin N. Ostrovsky (1945–2006) in Saint-Petersburg (USSR), and Abram Ilyich Fet (1924–2007) in Novosibirsk (USSR), each with their respective co-workers.[5]

In their quest for the symmetries that lie hidden in the Periodic System, each of these teams worked independently. Their hope was that symmetry might provide a key to the System's secrets. Since no quantum mechanical derivation of the Madelung rule was known, an important target of their research became the group-theoretical derivation of the Madelung rule. If successful, this project also held the promise of explaining the period doubling in a group-theoretical, rather than quantum mechanical, way.

In this essay, we explore some of the attempts to explain the Periodic System in group-theoretical terms.[6] Our focus will be on the contributions by Abram Ilyich Fet and Asim Orhan Barut. We will not discuss the work of Octavio Novaro and Valentin Ostrovsky. The reason for this is quite simple. Although each team had the same goal in mind — *viz.* the derivation of the Madelung rule and the period doubling — their approaches differed significantly. Novaro and Ostrovsky took a traditional *atomic physics approach*, whereas Fet and Barut adopted an *elementary particle approach*. Let us briefly explain both approaches.

Historically, when simple quantum systems were studied, such as the hydrogen atom or the harmonic oscillator, the Hamiltonians of those systems were exactly known, and their symmetries under various transformations could be directly studied.[7] Since most of these systems belong to the domain of atomic physics, this was called the *atomic physics approach*.[8] Both Ostrovsky

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[1] Scerri (2019).

[2] Löwdin (1969, 332).

[3] Allen & Knight (2002, 83).

[4] For some recent criticisms, see Wang & Schwarz (2009), Schwarz & Wang (2010), Schwarz & Rich (2010) and Schwarz (2010). However, as described in Thyssen & Ceulemans (2017), one really should consider the orbital correlation diagram between two lexicographic orderings: the hydrogenic order and the Madelung order. Both are limiting cases, with the actual systems lying in between. Be that as it may, there is no doubt that the actual ground state configurations of the elements are much closer to the Madelung rule than to the hydrogenic rule.

[5] Some key publications are Barut (1972a), Barut (1972b), Novaro & Wolf (1971), Novaro & Berrondo (1972), Novaro (1973), Novaro (1989), Novaro (2006), Ostrovsky (2004), Ostrovsky (2006), Byakov et al. (1976), Fet (2010), and Fet (2016). For more recent additions to this literature, see Kibler (1989) and Thyssen & Ceulemans (2017).

[6] A detailed account of the symmetry groups involved and the current status of the group-theoretical approach is presented in the recent book by Thyssen & Ceulemans (2017).

[7] The Hamiltonian of a system corresponds to the sum of the kinetic and potential energies for all the particles in the system, and thereby provides a detailed description of that system.

[8] The distinction between the atomic physics approach and the elemen-

(together with Demkov) and Novaro (together with Berrondo) followed this approach when they attempted to construct a Hamiltonian for the Periodic System.[9]

Coming up with such a Hamiltonian, however, proved extremely difficult. This was due, in part, to the fact that no *ab initio* derivation of the Madelung rule existed. Both Fet and Barut therefore felt the need for another approach. They found their inspiration in the recent developments in elementary particle physics, and in particular in the work of the German physicist Werner Karl Heisenberg (1901–1976) and American physicist Murray Gell-Mann (1929–2019). Their approach was therefore called the *elementary particle approach*.

The aim of our essay is threefold. First and foremost, to show that the elementary particle approach required an important paradigm shift in the understanding of the nature of chemical elements. As we will demonstrate, the idea, in essence, consisted of treating the chemical elements, not as particles (as in the atomic physics approach), but as states of a superparticle. Second, our essay retraces the origin of this paradigm shift via the developments in elementary particle physics in the 1960s and the work of Heisenberg, all the way back to Plato (428–348 BC). It was Heisenberg's deep respect for Plato, after all, that led him to propose treating the proton and neutron, not as elementary particles, but as different states of the nucleon. Third, our essay aims to highlight the inevitable tension that the symmetry program created between the formal mathematical treatment and the under-lying physical reality. This becomes particularly clear when comparing the work of Fet and Barut.

OUTLINE

Our essay is structured as follows. In section 1, we briefly introduce the two main characters of our paper: Abram Ilyich Fet and Asim Orhan Barut. In section 2, we provide a careful study of Heisenberg's last paper on the nature of elementary particles. What might feel like a long detour, will turn out crucial to understand the approaches by Fet and Barut. We explain why according to Heisenberg the traditional Democritean picture of matter no longer applied to modern physics, and why a Platonic symmetry-based picture of matter was called for instead. According to this picture, the elementary particles are only material realizations of certain 'particular' symmetries. Indeed, according to Heisenberg, it was not the particles, but their 'particular' symmetries that were truly fundamental.

In section 3, we explain what Heisenberg precisely meant by this philosophical claim via a brief study of isospin. We also show how Heisenberg's Platonic philosophy came to dominate the field of elementary particle physics, and how it found its culmination point in Gell-Mann's classification of the hadrons in the *eightfold way*. In section 4, we return to the Periodic System. We demonstrate that it was the success of Heisenberg's approach in elementary particle physics that sparked the group-theoretical approach to the Periodic System.

In section 5, we show that the history of this approach was marked by the continuous tension between the attraction to beautiful mathematical structures, and the need to keep contact with physical reality. We illustrate this via a critical examination of the work of Fet, in comparison to the work of Barut.

1. BIOGRAPHICAL PRELUDE

Abram Ilyich Fet

Abram Ilyich Fet was a Russian mathematician and philosopher. According to his wife, Ludmila P. Petrova-Fet, and his colleague Rem G. Khlebopros, Fet "belonged to a particular 'species of human' that is becoming extinct today".[10] While he mainly worked in mathematics and physics, he also explored biology, chemistry, economics, history, sociology, psychology, literature, music and the arts. As a dissident of the Soviet regime, he got dismissed twice from research institutes. In the years of unemployment, he nevertheless continued to do science on his own, living from casual translations.

His interest in the periodic table came through his collaboration in the early 1970s with the acclaimed Soviet physicist Yuri Borisovich Rumer (1901–1985). Rumer was convinced of the importance of symmetry groups for the natural sciences in general. He studied the symmetries of the genetic code with the help of B. G. Konopel'chenko, and the symmetries of elementary particles with Fet. The latter work culminated in the publication of a monograph on *The Theory of Unitary Symmetry Groups*.[11]

Having studied the symmetries of biology and physics, Rumer and Fet decided to embark on a "non-traditional" project, as Rumer later phrased it in a letter to the academician M. A. Leontovich (1903–1981) in 1973. They would study the symmetries of the Periodic Sys-

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tary particle approach was first made by Ostrovsky (2006).

[9] A typical example is the attempt by Ostrovsky and Demkov to develop a Hamiltonian based on Maxwell's fish eye potential. See Demkov & Ostrovsky (1972) and also Ceulemans & Thyssen (2018).

[10] Gladky et al. (2015, 283).

[11] See Rumer & Fet (1970).

tem of chemical elements. Inspired by Gell-Mann's work in elementary particle physics, they decided to apply the same elementary particle approach to the periodic table.

Their first paper appeared in 1971 in the journal *Teoreticheskaya i Matematicheskaya Fizika*.[12] Numerous papers and conference proceedings followed in the ensuing decade as Fet continued to develop their initial ideas. In 1984, Fet wrote a monograph, entitled *Symmetry of the Chemical Elements*, which presented a summary of his work on the Periodic System. However, his book was only published by the *Novosibirsk Academy* in 2010, more than a quarter of a century later, and three years after Fet's passing.

In the foreword to Fet's book, Khlebopros explains that Fet's work was edited in 1984 by the Siberian publishing department *Nauka*. Everything was ready for publishing. Even the cover had been approved by the *Arts Council*. But all of a sudden the book was withdrawn from publication, and the type matter was decomposed. The reason for this became clear a little later: on 8 October 1986, Fet was dismissed from work "due to noncompliance with the position held based on the performance evaluation." Fet, in other words, lacked publications; he did not live to see his reputation vindicated.[13]

According to the author's widow, though, Fet was fired for reasons which were entirely political and had no relation to science.[14] Khlebopros suggested that it had to do with Fet's personality: "A talented mathematician and physicist, a very well-educated and intelligent person with a sense of dignity and independence, he was, of course, envied and hated by ungifted science bureaucrats".[15] Recently, an English translation of Fet's monograph was published by *De Gruyter*.[16]

Asim Orhan Barut

Born in Malatya (Turkey) in 1926, Asim Orhan Barut studied at the Eidgenössische Technische Hochschule (ETH) in Zurich (Switzerland), where he obtained his under-graduate diploma in 1949 and his PhD in 1952.[17] After postdoctoral work in theoretical physics at the University of Chicago from 1953 to 1954, Barut served as an assistant professor at Reed College from 1954 to 1955 and at Syracuse University from 1956 to 1961. In 1962, Barut became a faculty member at the University of Colorado (Boulder), where he served for 32 years until his untimely death in 1994 at the age of 68.

Like Fet, Barut had broad interests which ranged from physics to politics, philosophy and religion.[18] But his true love was mathematical physics, and in particular group theory. Barut published more than 500 scientific papers, and authored 6 books.[19] He was also a devoted teacher and sought-after speaker — "his teaching style was blackboard and chalk" — and he travelled the globe to teach and speak at numerous summer schools and workshops.[20]

In 1971, Barut was the visiting Erskine Fellow at the University of Canterbury in Christchurch (New Zealand), where he also attended the Rutherford centennial symposium on the structure of matter. His stay in New Zealand gave rise to two important publications in connection with our topic — the symmetry of the Periodic System. The first one was a small booklet which contained the notes of his lectures as Erskine Fellow on "Dynamical Groups and Generalized Symmetries in Quantum Theory". The second one was his contribution to the proceedings of the Rutherford centennial symposium on the "Group Structure of the Periodic System".[21]

There are important similarities but also crucial differences in the works of Fet and Barut. As we already mentioned in the introduction, both Fet and Barut were greatly inspired by Heisenberg's and Gell-Mann's achievements in elementary particle physics, and both wondered to what extent the elementary particle approach could be applied to the Periodic System. The key to this approach, as we will argue, was a radical revision of the nature of the chemical elements. Fet and Barut were forced to treat the element, not as composite particles, but as states of a superparticle. In order to fully grasp the need for this paradigm shift, we will have to consider the works of Heisenberg and Gell-Mann. This will be done in sections 2 and 3. We will turn to the contributions of Fet and Barut in sections 4 and 5. It is also here that the differences between both will begin to shine through. Fet occupied a position at the mathematical end of the spectrum, whereas Barut's position was more balanced between mathematics and physics.

2. HEISENBERG'S PLATONIC PHILOSOPHY

Heisenberg's last paper was published posthumously.[22] It was devoted to the nature of elementary parti-

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[12] Rumer & Fet (1971).

[13] See Fet (2010).

[14] Private communication with Ludmila P. Petrova, January 4, 2011.

[15] See Fet (2010).

[16] Fet (2016).

[17] Scully (1998).

[18] Dowling (1998).

[19] On top of that, he also co-edited another 25 books.

[20] Scully (1998).

[21] See Barut (1972a) and Barut (1972b).

[22] Heisenberg passed away on 1 February 1976; his paper appeared in

cles. The question "What is an elementary particle?" had haunted Heisenberg for most of his scientific career. According to Heisenberg, "certain erroneous developments in particle theory [...] are caused by a misconception by some physicists that it is possible to avoid philosophical arguments altogether." "Starting with poor philosophy", Heisenberg continued, "they pose the wrong questions." As we intend to show in this section, Heisenberg had come to the conviction that the traditional Democritean picture of matter no longer applied, and that it had to be replaced by a Platonic one.[23] The idea that "in the beginning was the particle", in other words, had to be replaced by "in the beginning was symmetry".[24]

In the beginning was the particle

For over 2500 years, scientists and philosophers have pondered what would happen if one continued to divide matter into ever smaller constituents. Would this process go on *ad infinitum* or would one reach a point where no further division was possible? Is matter continuous or discrete?

Different (reductionist) answers were offered by different pre-Socratic philosophers. The material monists (Thales, Anaximander and Anaximenes) thought that matter was composed of a single material substance. The material pluralist Empedocles, on the other hand, claimed that all matter was composed of four *roots*: fire, air, earth and water.[25] It was Plato who first referred to these roots as στοιχειον (*stoicheion* or *elements*) in his major cosmological dialogue *Timaeus*, and who associated the four classical elements with the Platonic solids.

However, according to Heisenberg, the best-known answer to the above questions was given by the pre-Socratic philosopher Democritus.[26] Democritus (like his teacher Leucippus) was a materialist who postulated that all matter was ultimately composed of *atoms* — small, (physically) indivisible, immutable and indestructible units of matter. Indeed, the Greek word ατομον (*atomon*) literally means "indivisible" or "uncuttable". The philosophical atoms of Democritus were too small for us to see, and came in a variety of shapes and sizes. They were infinite in number and in constant motion, colliding with each other in an otherwise empty vacuum (or *void*).[27]

Plato's pluralistic doctrine was very different from Democritus' atomistic doctrine, and despite Plato's influence at the time, it was Democritus who emerged victoriously in the long run. In Heisenberg's opinion, "the strongest influence on the physics and chemistry of the last century undoubtedly came from the atomism of Democritos".[28] Bertrand Russell, in his *History of Western Philosophy*, concurred that the atomistic doctrine of Leucippus and Democritus "was remarkably like that of modern science".[29] Indeed, in the 18th-century, John Dalton (1766–1844) proposed that each chemical element is composed of a unique type of atom with characteristic atomic weight.[30] Like the philosophical atoms of Democritus, Dalton's chemical atoms could not be created, nor divided into smaller constituents or destroyed during chemical processes.[31]

The growing particle zoo

For nearly one century, the chemical atoms were thought to be the smallest possible units of matter. However, with the discovery of the electron by Sir Joseph John Thomson (1856–1940) in 1897, it became apparent that Dalton's atoms were not elementary after all. After the discoveries of the proton in 1917 and the neutron in 1932, the Rutherford–Bohr model of the atom was proposed with a central atomic nucleus of positively charged protons (p^+) and neutral neutrons (n^0), surrounded by a cloud of negatively charged electrons (e^-).

Despite their revolutionary character, these discoveries did not put into question the atomism of Democritus. On the contrary, "the electron, the proton and pos-

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the March edition of the journal *Physics Today*. See Heisenberg (1976a). It was based on a translation of his opening lecture to the *German Physical Society*'s spring meeting, given on 5 March 1975. The original version of his talk was published in the February 1976 issue of *Naturwissenschaften*. See Heisenberg (1976b).

[23] The materialistic interpretation of Democritus' atomic theory is due to Aristotle. Democritus himself thought of the atoms as immaterial entities, in full agreement with Plato's ideas. In that sense, Heisenberg's conviction to replace particles with symmetry principles was not in reaction to a Democritean picture of matter, but rather to the Aristotelian view of atomic theory. However, since our aim is historical (rather than philosophical) accuracy, we will keep with Heisenberg's terminology when representing his ideas on the nature of elementary particles.

[24] Heisenberg (1976a), quotations on p. 32.

[25] Aristotle later added a fifth element to this list of earthly and corruptible elements. The aether or quintessence (*quinta essentia*) was a heavenly substance and formed the constituent of all the stars and planets in the Universe.

[26] Heisenberg (1976a).

[27] The atomistic doctrine of Democritus was further refined by Epicurus and popularised by the Roman poet Lucretius in the first-century BC in his poem *De Rerum Natura* (*The Nature of Things*). See Lucretius (2007).

[28] Heisenberg (1976a, 37).

[29] Russell (1946, 84).

[30] Dalton (1808).

[31] Chalmers (2009).

sibly the neutron could, it seemed, be considered as the genuine atoms, the indivisible building blocks, of matter", dixit Heisenberg.[32] The idea thus originated that all matter is ultimately composed of three fundamental particles: protons, neutrons and electrons. Since they seemed immutable, and their number was therefore fixed, physicists called them *elementary particles*. The elementary particles of modern physics became the modern analogue of the philosophical atoms of Democritus.

This sparse ontology came to an abrupt end in 1947 with the discovery of *pions* by Cecil Powell (1903–1969) in cosmic ray experiments. The pions (π^+, π^0 and π^-), moreover, were observed to disintegrate into yet another class of particles, *muons* (μ^+, μ^0 and μ^-). For example:

$$\pi^+ \rightarrow \mu^+ + \nu$$

The situation only worsened with the construction of particle accelerators. By accelerating particles to tremendous velocities, and forcing them into head-on-collisions, a plethora of new particles were discovered in the 1950s. Among these were the *kaons* (K^+, K^-, K^0 and $\bar{K}^0$), the *lambda* particle (Λ^0), the *sigma* particles (Σ^+, Σ^0 and Σ^-, as well as Σ^{*+}, Σ^{*0} and Σ^{*-}), the *xi* particles (Ξ^0, Ξ^-, Ξ^{*0} and Ξ^{*-}) and the *delta particles* (Δ^{++}, Δ^+, Δ^0 and Δ^-).

In the early 1940s, the Universe was a simple place, composed of only three fundamental particles. By the early 1960s, the Universe had turned unfathomably complicated, with over 30 "fundamental" particles. The parsimonious ontology of the 1940s, in other words, had given way to a baroque ontology in the 1960s, in seeming contradiction with Occam's well-known razor. As we shall see, an entirely new way of looking at the elementary particles was needed before order could be restored in the growing particle zoo.

The loss of elementarity

For Heisenberg, the discovery of the particle zoo was ample evidence that the materialistic picture no longer applied in modern physics. "In the physics of elementary particles of our time," wrote Heisenberg, "good physics has sometimes been unconsciously spoiled by poor philosophy" — referring to the atomistic doctrine of Democritus.[33]

The problem according to Heisenberg was not that physicists were now forced to take these 30 odd particles as elementary. On the contrary, the problem was that their elementary nature was called into question by recent experimental findings.

For example, when an electron (e^-) and a positron (e^+) collide at low energy, they annihilate, producing two gamma-ray photons (γ):

$$e^- + e^+ \rightarrow \gamma + \gamma$$

The reverse reaction, electron-positron creation, also occurs. Here, a high energy photon is converted into an electron-positron pair:

$$\gamma \rightarrow e^- + e^+$$

Clearly then, electrons and positrons are not immutable. They can be created and annihilated. "They are not "elementary" in the original meaning of the word", wrote Heisenberg.[34]

Another example of the breakdown of the materialistic picture is provided by radioactive β^- decay, such as the decay of carbon-14 into nitrogen-14. In order to change the parent nuclide ${}^{14}_{6}\mathrm{C}$ into the daughter nuclide ${}^{14}_{7}\mathrm{N}$ (a process known as *nuclear transmutation*), a neutron must be converted into a proton. Due to the conservation of electric charge and lepton number, this must be accompanied by the emission of an electron and an electron antineutrino ($\bar{\nu}_e$):

$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}_e$$

Generalising, β^- decay always involves the transmutation of a neutron into a proton:

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

The reverse process is observed in β^+ decay (or positron emission), with a proton turning into a neutron:

$$p^+ \rightarrow n^0 + e^+ + \nu_e$$

Clearly then, protons and neutrons are not immutable. They can be transmuted into one another. No particle is more elementary than the other one.

What these, and other empirical findings, showed according to Heisenberg, was that the question "What do these particles consist of?" had become meaningless. After all, from the point of view of β^- decay, one might (naively) consider the neutron to be a compound particle, consisting of a proton, an electron and an electron antineutrino. But from the point of view of β^+ decay, it is

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[32] Heisenberg (1976a, 37).

[33] Heisenberg (1976a, 37).

[34] Heisenberg (1976a, 32).

not the neutron, but the proton that is compound, consisting of a neutron, a positron and an electron neutrino. "Experimentally, the concept of "dividing" had lost its meaning", blurted Heisenberg. In Heisenberg's opinion, this fading of the distinction between elementary particles and compound particles was probably "the most important experimental result of the last fifty years".[35]

Plato and that sort of thing

Since the materialistic picture of matter no longer applied in modern physics, a paradigm shift was called for. According to Heisenberg, "If we wish to compare the results of present-day particle physics with any of the old philosophies, the philosophy of Plato appears to be the most adequate".[36]

Heisenberg had a deep love and appreciation for Plato. According to David Peat, "his scientific attitudes reflect a debt to philosophy and in particular his respect for Plato." Heisenberg concurred that "My mind was formed by studying philosophy, Plato and that sort of thing".[37]

Heisenberg's father, August Heisenberg (1869–1930), was a scholar of ancient Greek philology and modern Greek literature; he became a professor of philology at the University of Munich in 1910 when Heisenberg was nine years old. In 1911, the young Heisenberg entered the *Maximilians-Gymnasium*. At that time, it was still common practice to place more emphasis upon classical Greek and Latin than on the sciences and mathematics. All of this contributed to Heisenberg's classical-humanistic education.

In his teenage years, as a result of the political turmoil in Munich after the First World War,[38] the young Heisenberg became part of the Cavalry Rifle Command No. 11. Their headquarters were located in the *Theological Training College*, opposite the University. Heisenberg often retired to the roof of the college with a Greek school edition of Plato's Dialogues. "There, lying in the wide gutter, and warmed by the rays of the early morning sun," Heisenberg later recalled, "I could pursue my studies in peace." It was there, in the spring of 1919, that Heisenberg first read Plato's cosmological treatise, the *Timaeus*.[39]

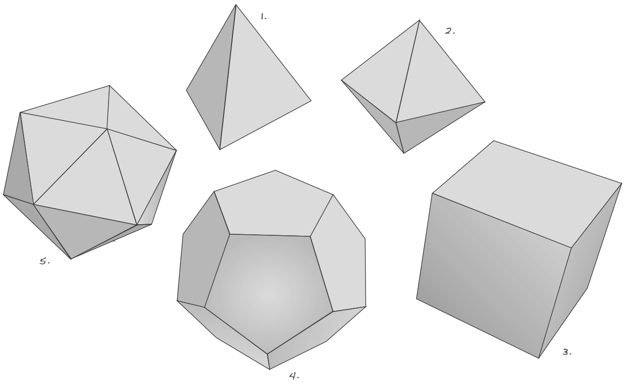

Figure 1. The Platonic solids: 1. tetrahedron; 2. octahedron; 3. cube (or hexahedron); 4. dodecahedron; and 5. icosahedron.

Platonic solids in the Timaeus

Plato believed the Universe had been created out of chaos by a Demiurge using the four elements — fire, air, earth, and water — as basic building blocks.[40] Plato associated each of these elements with one of the five Platonic solids. The element fire was thus identified with the pointy tetrahedron; air with the smooth octahedron; earth with the bulky and weighty cube; and water with the fluid and nearly spherical icosahedron (Figure 1).[41]

Empedocles, who first introduced the four elements, believed the elements could be mixed in various proportions but were themselves immutable and indestructible. What makes Plato's "theory of everything" so exciting is that the elements are no longer elementary. Each regular polyhedron, after all, is constructed from regular polygons. The tetrahedron, octahedron and icosahedron are built from (respectively 4, 8 and 20) equilateral triangles; the cube (or hexahedron) is built from 6 squares. The elements can therefore be broken down into triangles and squares and recombined to create new elements.[42]

For example, two particles of fire can be broken down into 8 equilateral triangles and recombined to form one particle of air:

$$\underset{4}{\text{fire}} + \underset{4}{\text{fire}} \rightarrow \underset{8}{\text{air}}$$

Likewise, a particle of water, consisting of 20 triangles, can transmute into five particles of fire, or two particles of air and one of fire:

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[35] Heisenberg (1976a), quotations on p. 33.

[36] Heisenberg (1976a, 38).

[37] See Peat (1996, 3) and Heisenberg (1996, 6).

[38] Specifically, the rise and fall of the Bavarian Soviet Republic in Munich during the German Revolution of 1918–1919.

[39] Heisenberg (1971, 8).

[40] See Plato (1976).

[41] The fifth Platonic solid, the dodecahedron, was used for the Universe as a whole. Aristotle later conjectured that it represented the aether which made up the celestial heavens.

[42] Plato (1976, 1259).

$$\underset{20}{\text{water}} \rightarrow \underset{(5\times4)}{5\times\text{fire}}$$

Notice though that since earth is made up from squares, it cannot be transmuted into any of the other elements. These elemental transmutations resemble the ones described above for elementary particles.

In the beginning was symmetry

To Heisenberg, "the whole thing seemed to be wild speculation []. It saddened me to find a philosopher of Plato's critical acumen succumbing to such fancies." Yet one aspect of Plato's account captured his imagination. "I was enthralled by the idea that the smallest particles of matter must reduce to some mathematical form," wrote Heisenberg. In his opinion, "the elementary particles in Plato's *Timaeus* are finally not substance but mathematical forms".[43]

What is more, these mathematical forms — triangles and squares, and the Platonic solids they make up — are highly symmetrical. What is fundamental, in other words, are not the material particles themselves, but the mathematical symmetries underlying them. This Platonic way of thinking moreover seemed applicable to modern physics. According to Heisenberg, "our elementary particles are comparable to the regular bodies of Plato's *Timaeus*".[44] As Heisenberg explained:

> *So far we had always believed in the doctrine of Democritus, which can be summarised by: "In the beginning was the particle." We had assumed that visible matter was composed of smaller units, and that, if only we divided these long enough, we should arrive at the smallest units, which Democritus had called "atoms" and which modern physicists called "elementary particles." But perhaps this entire approach has been mistaken. Perhaps there was no such thing as an indivisible particle. [] In the beginning was symmetry!*[45]

According to Heisenberg, it was not the elementary particles, but the symmetries that lie beyond them, that are truly fundamental. The elementary particles are but material realizations of these underlying symmetries.[46] One eloquent model of such 'particular symmetries' will be presented in the next section.

3. THE SYMMETRY OF ELEMENTARY PARTICLES

In order to make Heisenberg's position more concrete, we will briefly look at the example of *isospin*. After all, the concept of isospin was introduced in 1932 by Heisenberg himself, soon after the discovery of the neutron by Sir James Chadwick (1891–1974) that same year.

Protons and neutrons are sometimes called *nucleonic particles* because they are the components of atomic nuclei. Despite their difference in electric charge, the proton and neutron are nearly identical in all other respects. Both are fermions, and both have almost the same mass.[47] Heisenberg was baffled by this consanguinity, and intent on uncovering the reason for it.

When two or more particles have the same mass (or energy), they are said to be *degenerate*. Degeneracies are a tell-tale sign that there is a symmetry lurking in the background. Symmetry is all about the interplay between change and permanence; it is about the quest for permanence in a world of constant flux. More precisely, an object is said to be *symmetric* when there is a transformation (*change*) that leaves certain aspects of the object fixed (*permanence*). Rotating a ball around its centre, for example, leaves its overall appearance unchanged. Hence, the ball is said to be spherically symmetric.

The same applies to the nucleonic particles. If someone were to exchange a proton for a neutron — as we saw happens during β decay — it would be practically impossible to tell, given their similarity in mass. Indeed, the strong interaction force cannot, as a matter of fact, distinguish protons from neutrons.[48]

In view of all this, Heisenberg suggested treating the proton and neutron, not as two distinct elementary *particles*, but as two possible *states* of one and the same particle, which he called the *nucleon*. Heisenberg did not have to look far to find an equivalent quantum system that also appears in two possible states. Since the so-called Stern–Gerlach experiment, it was known that the electron has a *spin*, which can adopt two states, commonly denoted as spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$.[49] In the same way, Heisenberg proposed the nucleon has an *isospin*, which can adopt two states, denoted as $|p^+\rangle$ and $|n^0\rangle$.

Both spin and isospin are characterised by the same symmetry group: the Special Unitary group of degree 2, or SU(2) group. The SU(2) group is an example of a

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[43] Heisenberg (1971), quotations on p. 8.

[44] Heisenberg (1971, 241).

[45] Heisenberg (1971, 133).

[46] See also Peat (1987).

[47] To be specific, $m_{p+} = 938.272046$ MeV/C^2, and $m_{n0} = 939.565378$ MeV/C^2. Fermions are particles that obey Fermi–Dirac statistics, as opposed to bosons which obey Bose–Einstein statistics.

[48] It is only the (weaker) electromagnetic force that makes the distinction on the basis of their difference in charge.

[49] Gerlach & Stern (1922).

Lie group, named after the Norwegian mathematician Sophus Lie (1842–1899).[50] Let us only note here that the fundamental representation of SU(2) is a *doublet*. The spin up and spin down states of the electron form an SU(2) spin doublet; the proton and neutron form an SU(2) isospin doublet.

It is here that Heisenberg crossed the conceptual line between *particles* and *states*. On the one side are two nucleonic particles that are clearly related to each other as they have nearly the same mass. On the other side are the two degenerate states of a spin system that is described by the SU(2) group. The connection consists in associating the two nucleons (*particles*) with the two components (*states*) of the SU(2) doublet.

This was perhaps the first time that such a connection was made. It predates the discovery by Gell-Mann (and others) of the SU(3) symmetry of hadronic matter — to be discussed in the next section — by more than three decades. Above all, it offers support to Heisenberg's conviction that symmetries are more fundamental than particles.

With the help of Heisenberg's isospin, all elementary particles can be assigned into isospin *multiplets*. The pions π^+, π^0 and π^-, for instance, are assigned to an isospin triplet, as are the sigma particles Σ^+, Σ^0 and Σ^-. The delta particles Δ^{++}, Δ^+, Δ^0 and Δ^- form an isospin quartet; the xi particles Ξ^- and Ξ^0 constitute an isospin doublet, and the lambda particle Λ^0 an isospin singlet.

From the SU(2) symmetry point of view, particles within a multiplet are identical. Just as the spherical symmetry of a ball allows one to rotate one orientation into another, the SU(2) symmetry allows one to transform the particles of an isospin multiplet into one another.

The eightfold way

The American physicist Murray Gell-Mann (1929–2019) took Heisenberg's idea a step further in the 1950s and 1960s. For reasons which are beyond the scope of this article, Gell-Mann introduced a new quantum number, which went by the name of *strangeness*, and was denoted *S*. The proton and neutron, for example, were assigned strangeness $S = 0$; the sigma and lambda particles $S = -1$, and the xi particles $S = -2$.

Gell-Mann subsequently ordered the particles on the basis of their isospin component T_3 and strangeness *S*. This process is illustrated in Figure 2 for the baryons n^0, p^+, Σ^-, Σ^0, Σ^+, Λ^0, Ξ^- and Ξ^0.[51] The result is an *octet* of particles, with six particles at the corners of a regular hexagon, and two more particles at the centre. Inspired by the *Eightfold Path* of Buddhism, Gell-Mann named his classification scheme the *eightfold way*.[52]

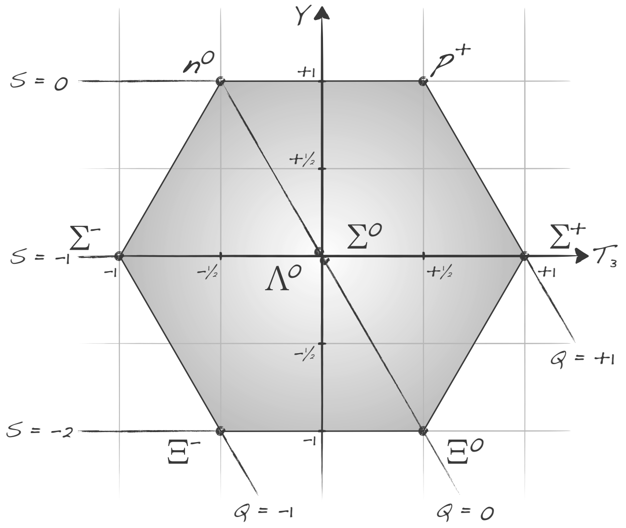

Figure 2. The baryon octet.

Particles along the same horizontal line in Figure 2 form the familiar isospin multiplets. On the upper line, we have the proton-neutron doublet; on the lower line the xi doublet, and on the middle line the sigma triplet superposed with the lambda singlet.

Gell-Mann realised that the eightfold way pointed at a hidden symmetry. Just as the isospin multiplets are representations of the SU(2) group, the baryon octet is a representation of the larger SU(3) group. Indeed, from the SU(3) symmetry point of view, the baryons are no longer treated as distinct particles. Instead, they are taken to be the states of a *superparticle*. This implies that the members of the baryon octet can be transformed into one another. As a result, the strong force cannot distinguish them. The interchange of one baryon for another goes unnoticed in strong interactions.

Broken symmetry

From the SU(2) point of view, the proton and neutron are identical; they appear as two faces of the same nucleonic coin. Hence, in order to tell them apart, the SU(2) symmetry has to be broken. This is done by imposing the quantisation of the charge operator, which

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[50] A full account of the SU(2) group (and the others to follow) is given in Thyssen & Ceulemans (2017).

[51] The name "baryon" refers to the Greek word βαρύς for "heavy".

[52] See Gell-Mann & Ne'eman (2000).

breaks the SU(2) symmetry to the U(1) symmetry. It is only at that point that the proton and neutron regain their identities, and that one can meaningfully distinguish them.

The same principle applies to Gell-Mann's eightfold way. From the SU(3) point of view, all baryons are identical. Hence, in order to tell them apart, the SU(3) symmetry has to be broken. As a first step, the SU(3) symmetry can be broken to the SU(2) symmetry. The SU(3) octet is then split into the familiar SU(2) submultiplets: the nucleon doublet, sigma triplet, lambda singlet and xi doublet (indicated by the horizontal lines in Figure 2). From that point onwards, particles from different isospin multiplets can no longer be transmuted into one another; they are no longer identical. This series of symmetry breakings is typically represented by a chain of subgroups:

$$SU(3) \supset SU(2) \supset U(1)$$

The importance of symmetry breaking cannot be overstated. As the world unfolds, and the phenomena take place, the initial ideal symmetries break down, and only remnants remain, as with Plato's ideals. According to most physicists today, it is the breaking of symmetry that makes the world an interesting and variegated place to live in. As the French physicist Pierre Curie appropriately said: "*C'est la dissymétrie qui crée le phénomène*".[53]

With the eightfold way, the zoo of particles was finally classified, and order was restored to the world of elementary particles. Most importantly, when Gell-Mann drew up his classification schemes, some seats remained unoccupied, hinting at the existence of as yet undiscovered particles. Like Mendeleev a century earlier, Gell-Mann predicted the existence of the eta meson (η^0) and the omega baryon (Ω^-), which were discovered soon afterwards. Such was the predictive force of symmetry. "I was playing around with the particles. [Mendeleev] was playing around with the elements," said Gell-Mann in 1997.[54] "It was natural to make a comparison between them, although I think Mendeleev's work was much more important." Gell-Mann was ultimately awarded the Nobel Prize in 1969, a century after Mendeleev's development of the periodic table.

The key to these revolutionary developments in elementary particle physics was the move from materialism to idealism. As Heisenberg explained, symmetries are ontologically prior to particles. Symmetries represent the fundamental level of reality, whereas particles only constitute a secondary level of reality. The elementary particles, after all, only emerge from these symmetries by a series of symmetry breaking steps, and therefore have a derivative status.

4. THE SYMMETRY OF CHEMICAL ELEMENTS

It did not take long before the same group-theoretical approach was applied to the zoo of chemical elements. The situation with the Periodic System in the 1970s, after all, resembled the one in elementary particle physics in the 1950s. As we explained in sections 2 and 3, when the zoo of elementary particles was discovered, their internal dynamics were still shrouded in mist.[55] The exact Hamiltonian for these hadronic systems was not known, and another approach was called for. Instead of inferring the symmetry group of the system from the behaviour of the Hamiltonian under various operations, the symmetry group was simply postulated on the basis of the known empirical data and phenomenology of particle reactions. That is, instead of adopting an atomic physics approach, Heisenberg and Fet opted for a phenomenological elementary particle approach.

The goal of Fet and Barut was to apply the same phenomenological approach to the Periodic System. After all, despite the developments in quantum mechanics and computational chemistry, the internal dynamics of many-electron systems was also still shrouded in mist. Both Fet and Barut therefore took the structure of the Periodic System as empirical input and looked for a particular symmetry group that could explain this data.[56] Not surprisingly, the key to their approach was once again the move from Democritus to Plato, which required a radical revision of the nature of chemical elements, as we now intend to explain.

The nature of chemical elements

Heisenberg did not treat the proton and neutron as distinct particles, but as distinct states of one superparticle: the nucleon.[57] In a similar vein, Fet and Barut did not treat the chemical elements as distinct elements, but as distinct states of a superparticle, which was later named the *baruton* in honour of Barut for his contri-

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[53] Curie (1894, 127).

[54] Quoted from an interview between Gell-Mann and the former editor-in-chief of *Science News*, Tom Siegfried on September 16, 1997 in Santa Fe, New Mexico. See also Siegfried (2002).

[55] Quantum chromodynamics was only developed in the 1970s.

[56] To be specific, the empirical data consisted of the various period lengths which were assumed to be the dimensions of the various multiplets of the symmetry group.

[57] Gell-Mann similarly treated the baryons, not as distinct particles, but as distinct states of some baryonic matter.

butions to the symmetries of the Periodic System.[58] The chemical elements, in other words, were no longer treated as concrete, physical particles with an internal substructure. The structural conception of the atom was thus excluded from the consideration of these group theoreticians.

This had at least two crucial advantages. First, by treating the chemical elements as states of a *single* quantum system, the Periodic System was being studied as a whole. Contrast this with the atomic physics approach, where each element was treated as a *separate* quantum system. Second, by stripping the atoms from their physical content, the link with quantum mechanics was entirely lost. What remained, was an abstract 'group-theoretical' atom, a structureless non-composite entity, without internal dynamics. Fet and colleagues, for example, emphasised that their approach was "*not* a theory of electronic shells".[59] As a result, there was no mention of electronic configurations, orbitals or quantum numbers. By ignoring the internal substructure of the elements, Fet and Barut could thus circumvent the traditional quantum mechanical challenges, such as the Löwdin challenge referred to in the introduction.

Yet another advantage of the elementary particle approach can be mentioned. Heisenberg and Gell-Mann did not know of the possible substructure of the elementary particles when they studied their symmetries. Yet, the eightfold way did pave the way towards the discovery of *quarks*, the constituents of all elementary particles.[60] Both Fet and Barut wondered whether a group-theoretical study of the Periodic System might similarly pave the way to a deeper understanding of the substructure of the elements and new insights in the internal (quantum) dynamics of many-electron systems.

Fet was well aware of these advantages, and mentioned them on more than one occasion. Interestingly, he also referred to the work of Barut and Novaro and made an important remark about the difference with his own work: these authors, in his opinion, considered "the symmetry developed as a symmetry of the electron shells only, not distinguishing it from the Bohr model".[61] In contrast, in his own perspective the atom system was considered as a whole.[62] Later on, he repeated this claim by stressing the novelty of his approach in the most explicit terms:

> *We'd like to point out again the most important distinct feature of the theory suggested: while the Bohr model considers one element as a separate quantum system (and the atomic number is included in the theory as a parameter, so the number of quantum systems is the same as the number of elements), our model considers the atoms of all possible elements as the states of a unified quantum system, linkable to each other by symmetry group action.*[63]

Despite these claims, it is difficult to maintain that there is a fundamental difference with the perspective in Barut's work, who explicitly asked in his Rutherford lecture: "Are there (global) quantum numbers which would characterize the elements as different 'states' of a single system? All elements would then constitute a single 'multiplet'." Barut then expressed the atomic numbers, not as parameters, but as functions of these quantum numbers.[64]

The symmetry group of the Periodic System

Having thus introduced the baruton, whose states are the chemical elements, the primary challenge for Fet and Barut was to find the symmetry of the baruton (just like Heisenberg had identified the SU(2) group as the symmetry of the nucleon, and Gell-Mann the SU(3) group as the symmetry of the eightfold way). The principal key turned out to be the hydrogen atom.

The symmetries of the hydrogen atom were well-known. Fock had shown that the hydrogen atom possesses rotational symmetry not only in three dimensions but also in four. This rotational symmetry was described by the Special Orthogonal group in 4 dimensions, also known as the Fock group or SO(4) group. As a result, all the hydrogen orbitals of fixed were grouped in SO(4) multiplets of dimension n^2.[65]

The ultimate goal, however, was to treat the *entire* set of hydrogen orbitals, regardless of their principal quantum number n, as a single symmetric object. This called for a so-called *covering group* which would contain the SO(4) group as a subgroup. The orbitals would then form a single infinite-dimensional multiplet of this covering group.

It was only in the sixties of the previous century that this goal was achieved. One of the first proposals came

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[58] Wulfman (1978).

[59] Byakov et al. (1976, 3).

[60] It is telling that Heisenberg, as a true Platonist, remained extremely skeptical about the possible existence of quarks, as this seemed to herald back the Democritean materialism. For him, the quark hypothesis was perhaps useful as a mathematical tool, but it certainly did not provide a picture of reality. "Even if quarks should be found (and I do not believe that they will be)," said Heisenberg, "they will not be more elementary than other particles, since a quark could be considered as consisting of two quarks and one anti-quark, and so on." Quoted from Peat (1987).

[61] Fet (2010, 154).

[62] See also Kibler (2018).

[63] Fet (2010, 155).

[64] See Barut (1972a), quotation on p. 84.

[65] See Fock (1935).

from Barut in 1964. He found an extension of the Fock group, known as SO(4,1), which was able to pack all the discrete states of hydrogen into one infinite-dimensional multiplet.[66] Within a year, two young doctoral students (classmates and childhood friends) in Moscow, Ilya A. Malkin and Vladimir Ivanovich Man'ko (°1940), took this idea a bit further and extended the group to SO(4,2).[67]

The SO(4,2) group describes the conformal or scaling transformations of spacetime. In a later development Barut and Haugen considered a further extension to scale trans-formations of mass and charge.[68] This yields a theoretical framework that incorporates the Maxwell equations, and ultimately the photon. The SO(4,2) group is thereby enlarged to the inhomogeneous conformal group IO(4,2) with 21 parameters. However the physical significance of these conformal generators remains a recurrent matter of debate.[69]

All of these groups are called *conformal symmetries*. From the SO(4,2) symmetry point of view, any hydrogen orbital can be transformed into any other orbital. But the SO(4,2) group also provided an excellent starting point for the group-theoretical study of the Periodic System. Since the chemical elements could be labelled by the same set of four quantum numbers as were used to describe the hydrogen orbitals, the SO(4,2) group served as an ideal candidate to describe the symmetry of the baruton. Both Fet and Barut recognised the conformal symmetry of hydrogen as the master equation from which to start.

From the SO(4,2) symmetry point of view, all chemical elements are identical. The SO(4,2) group, in other words, can *transmute* any chemical element into any other. It can be compared with the *philosophers' stone*, although the transformations induced by the conformal group are of course not physical but merely mathematical. In order to distinguish the chemical elements, the SO(4,2) symmetry has to be broken. It is only by shattering the SO(4,2) group that the elements regain their identities.

The next challenge therefore was to find a proper symmetry breaking that would explain the ordering of the elements in the Periodic System. It is here that the real differences between the treatments of Fet and Barut became clear as both proposed a different symmetry breaking chain. As we will explain in the next section, Fet's approach occupied a position at the mathematical end of the spectrum, whereas Barut's approach retained the link with physics and chemistry to a larger extent.

5. THE MADELUNG RULE AND PERIOD DOUBLING

We evaluate Fet's proposal, as it was described in his monograph on the *Symmetry of the Chemical Elements*.[70] Several introductory chapters of Fet's book are devoted to the construction of the conformal SO(4,2) group for the hydrogen system. In Chapter 4, Fet devoted an extensive discussion to the concept of isospin. Fet had a special interest in representing this example, since later on the SU(2) group would have to come to his rescue, when he was struggling with the period doubling in the periodic table. Of importance at present are chapter 5 and 6. In chapter 5, Fet exposed his views on the symmetry of the periodic table. In chapter 6, he confronted his views with chemical evidence.

In chapter 5, Fet first explained the conformal symmetry and then also introduced the Madelung rule as an observation of the basic regularity in the periodic table. Both Fet and Barut agreed that the Madelung rule offered the most concise explanation of the periodicity. Following this rule, one could regroup the elements of the periodic table in subsets, with the same n and l, and insert these in an (n,l) matrix. The Madelung rule traces a zigzag path through this matrix, which guided both Barut and Fet. In doing so, they observed a distinctive feature of the periodic table, namely that it seems to consist of two separate twin tables. This is the well-known period doubling. The difference between both is the parity of $n + l$. But here, the treatments of Fet and Barut diverged.

Barut solved the riddle of the period doubling by considering a symmetry breaking from SO(4,2) to SO(3,2). He had studied this group chain earlier with Bohm in a study on hadronic matter and found that the mother representation of SO(4,2) splits into two identical representations of SO(3,2).[71] Note that there are no quantum characteristics that discriminate these two subgroup representations. As far as SO(3,2) is concerned, they have the same symmetry. They are distinguished in odd and even according to the parity of $n + l$, but we do not have a symmetry operator in the model to determine this parity.

Here appears a critical turning point in Fet's work, which characterises the author as a mathematician of one piece, not willing to compromise on a matter of principle. Fet reminded the reader that the l quantum number is not really a quantum number, in the sense that it does not correspond to an eigenvalue of

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[66] Barut et al. (1965).

[67] Malkin & Man'ko (1966).

[68] Barut & Haugen (1972).

[69] Jaekel & Reynaud (1998).

[70] Our present analysis of Fet's book is based on a personal copy, which was given to us by Fet's widow. The manuscript was translated for us by Jewgienij Liszczuk., see Fet (2016).

[71] Barut & Bohm (1970).

an operator of the enveloping algebra. It only serves for the development of the square of the total angular momentum which is given as $l(l + 1)$, and so this value is always even.

In the eyes of Fet, the unavoidable consequence was that the $n + l$ sum of the Madelung rule *had no group sense*. No compromise was possible: "That is why from the view of the elements group description, the $n + l$ number should not be included in the 'lexicographic rule' formulation. Therefore we should replace the Madelung indexing by another one, which would also logically describe the properties of the elements, but which would be free from this disadvantageous feature".[72]

Fet concluded that there are thus two separate periodic tables, each of which follows a hydrogen sequence and must thus be described as SO(4,2), but with n and l redefined. For the odd sequence, instead of the quantum number v, he defined a pseudo principal quantum number as:

$$v = ½\ (n + l + 1)$$

The odd sequences are then mapped onto the new as follows:

$$\begin{gathered} 1s \rightarrow 1s \\ 2p\ 3s \rightarrow 2p\ 2s \\ 3d\ 4p\ 5s \rightarrow 3d\ 3p\ 3s \\ 4f\ 5d\ 6p\ 7s \rightarrow 4f\ 4d\ 4p\ 4s \end{gathered}$$

Hence this sequence has become a perfect SO(4,2) representation again. Likewise, for the even sequence, one has to apply

$$v = ½\ (n + l)$$

which will turn the even sequence in an equivalent system:

$$\begin{gathered} 2s \rightarrow 1s \\ 3p\ 4s \rightarrow 2p\ 2s \\ 4d\ 5p\ 6s \rightarrow 3d\ 3p\ 3s \\ 5f\ 6d\ 7p\ 8s \rightarrow 4f\ 4d\ 4p\ 4s \end{gathered}$$

These two tables are like Heisenberg's nucleonic matter, forming the states of a spin-like doublet. The resulting symmetry group is the combination of both symmetries. In mathematical terms, this corresponds to the product of an SO(4,2) like group and an isospin-like group: SU(2) ⊗ SO(4,2).

In light of Barut's alternative, Fet's proposal appears artificial. It is true that there is no proper operator for l in the SO(4,2) group, but the symmetry breaking from SO(4,2) to SO(3,2) generates exactly the doubling that is observed. Indeed, in this process only operators which either preserve $n + l$ or change $n + l$ by two units are possible. So this group preserves the parity of the sum and is thus the perfect rationale for the existence of an odd and an even half of the periodic table.

This is a valuable insight which we owe to Barut. Fet was aware of Barut's Rutherford lecture, but he missed the point of the argument.[73] The crucial point of the doubling is not the individual value of l, nor n, but only the parity of their sum. And clearly, this is the property that is conserved in SO(3,2).

Later in the chapter, Fet also took into account the spin quantum number of the electron, which allowed all orbitals to be occupied by two electrons. So this was a further doubling, requiring an extra SU(2) group. However, this group was not an artificial construct but simply the true spinor characteristic. T treatment which then followed, however, was quite remarkable again, since Fet combined the spin quantum number ½ with the angular momentum l, thus dividing the $4l + 2$ elements of every manifold into two submanifolds with respectively $2l$ and $2l + 2$ elements. In physical terms, this means that every manifold (except for $l = 0$) is divided into two spin-orbit levels: a lower one with $j = l - ½$, and an upper one with $j = l + ½$. This is at odds with the quantum mechanical description of the elements, which certainly indicates that for the lighter elements spin-orbit coupling is not ruling the ground state terms.

The chemical data

Chapter 6 displayed chemical data to strengthen Fet's case. He took as an example the ionisation potential of the main group elements from boron to neon. According to Fet's claim, this graph should consist of two different trends: one corresponding to the spin-orbit doublet {B, C}, and another one for the spin-orbit quartet {N, O, F, Ne}. The data were plotted in a way to emphasise the existence of two separate parts, with a dashed border line in between.[74]

Nonetheless, this way of drawing the graph was unable to hide that no distinction of the kind is at stake. Indeed, there is a linear increase from boron to nitrogen; the break does not occur between carbon and nitrogen, but between nitrogen and oxygen. The reason for this is

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[72] Fet (2010, 177).

[73] See Barut (1972a).

[74] See Fet (2010, 194).

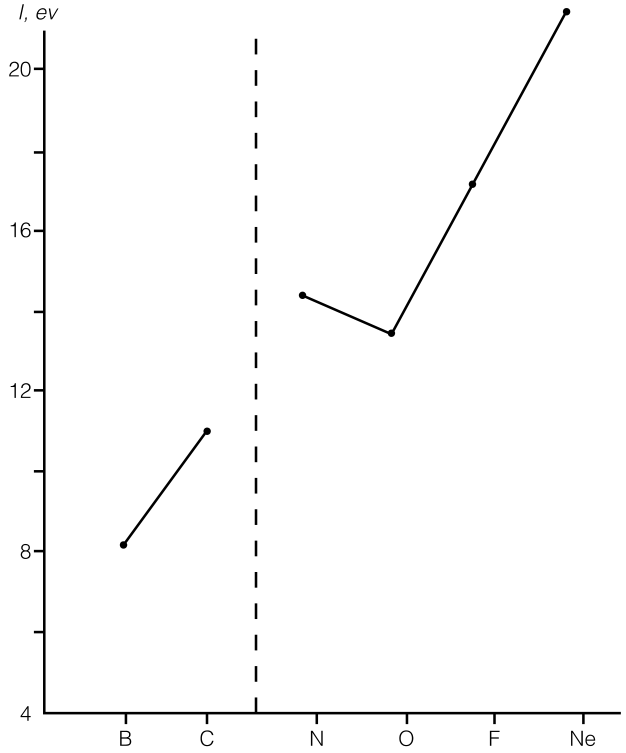

Figure 3. First ionization potentials. [Figure adapted from Fet, 2010, 194]

perfectly clear. It is due to electronic repulsion: in nitrogen, the 2*p* shell is half-occupied, with three electrons nicely distributed in space, at a maximal distance of each other. In oxygen, the nuclear charge increases so all 2*p* valence electrons are expected to feel an increased charge, and it would be more difficult to ionise them. On the other hand, one cannot avoid occupying one of the 2*p* orbitals twice. These two electrons are doomed to occupy the same region in space and to repel each other more strongly. This effect more than offsets the increase in the attraction of the nucleus, and thus the ionisation potential drops.

Similar discrepancies between Fet's claims and the actual data can be found in other properties, such as the dissociation energies of the diatomics.[75] As the number of electrons increases, multiple bonding becomes possible, and the strength of the diatomic bond increases accordingly in an uninterrupted linear correlation from boron to nitrogen. The highest stability is reached for dinitrogen N_2 since it realises a triple bond, based on the sp-hybridization. The bonding in O_2 and F_2 is smaller due to the occupation of antibonding orbitals and finally vanishes for neon.

Perhaps Fet as a mathematician was less susceptible to such chemical explanations. Nevertheless, the graphical representations of his claims were highly misleading.

6. THE LIMITS OF SYMMETRY

When Heisenberg proposed to consider the proton and neutron as the two sides of the same isospin coin, a paradigm shift was set into motion. The materialistic interpretation of the world consisting of particles gave way to a new understanding which views the particles as representations of symmetry groups. Heisenberg depicted this as the confrontation between the atomism of Democritus versus the idealism of Plato. Symmetries, not particles, were taken to be fundamental. They represented the deepest ontological level, whereas the particles only had a derivative status. "In the beginning was symmetry", exclaimed Heisenberg on more than one occasion.[76] The culmination of Heisenberg's symmetry program was attained when Gell-Mann introduced the eightfold way, which provided a classification of all hadronic matter, and which led to the successful prediction of two new elementary particles. To some extent, the ability of a system to make successful predictions echoes Mendeleev's belief in the periodic law that enabled him to make detailed predictions for certain unknown elements (such as gallium, germanium and scandium). It is thus no surprise that the symmetry program was also applied to the periodic system, even though such attempts were relatively scarce.

The success of the symmetry program did not stay confined to the hermetic circles of elementary particle physics, but as this contribution has illuminated, it inspired new perspectives on the periodicity of Mendeleev's table as a hallmark of an as yet unidentified underlying symmetry group. Here as well, the key to the symmetry program was the move from materialism to idealism. The chemical elements were no longer treated as particles, but as states of a superparticle, the baruton, whose symmetry was described by the conformal group SO(4,2). From the perspective of this group, the chemical elements had lost their identities, and merely functioned as different states of a single quantum system. It was only by a controlled breaking of the SO(4,2) symmetry that the elements regained their chemical and physical identities.

As we noted, several groups started the group-theoretical study of the Periodic System almost simultaneously in the early seventies of the previous century. In this account we devoted particular attention to the

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[75] Fet (2010, 199).

[76] Heisenberg (1976a), quotations on p. 32.

contributions by Fet and Barut, both of whom adopted the elementary particle approach of Heisenberg and Gell-Mann. In comparing the work of Fet and Barut, we also illustrated the tension between a formal mathematical treatment and an underlying physical. Fet approached the problem from a rigorous mathematical point of view. The result was a formal scheme that accommodates the chemical elements, but on the other hand did not advance our knowledge of the structure of the periodic table, nor reflected the actual chemical and physical properties of the elements. In that sense, Barut approached the problem from a much more physical and chemical point of view.

As Heisenberg already warned in his last paper, for a theory to be not only successful but also useful, it should not restrict itself to a description of phenomena but also offer an understanding. There is the danger to get lost in the mathematical details of a theory by focusing too much on its structural aspects, and to 'loose touch' this way with physical reality. It is not always easy to find the right balance between mathematics and physics. While the formal system, set up by Fet, perhaps fell short of achieving this balance, other contributions opened a much more promising perspective. Here we mention especially the legacy of Asim Barut who explained the group-theoretical origin of the period doubling from a much more physical and chemical point of view. The original line of thinking in the work of the late Ostrovsky is also worth mentioning, although Ostrovsky adopted an atomic physics approach.[77] Recently, the introduction of non-linear Lie algebras has provided a synthesis of the key elements of both (atomic physics and elementary particle) approaches. This has expanded the study of the Periodic System into a different realm, where its intriguing structure might finally reveal its secrets.[78]

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[77] See Barut (1972a) and Ostrovsky (2006).

[78] See Thyssen & Ceulemans (2017).

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