

An Essay on the Ancient Ideal of ‘Enraonar’

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Abstract ‘Reasoning’ can be considered a general concept that, upon speaking, is the ‘enraonar’, a Catalan word that should not be mistaken with ‘explain’ nor with ‘discuss’ which imply more detail, and cover different situations. This article is presented as an essay on the ancient ideal of ‘enraonar’. To that end, it is explained in what sense ‘enraonar’ and reason are one of the most complex phenomena thought has to deal with. Here it is argued that these natural phenomena require a systematic and ‘scientific’ study, and that without this knowledge computer science cannot simulate people’s every-day ‘enraonar’.

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1 Introduction

1.1

In Catalan, there are two words that refer to two concepts that, though they are nearly synonymous, are subtly distinguished in a way that nevertheless affects the understanding of what is said and what is understood: these are “parlar” and “enraonar”.¹ The slight difference that exists between the two is by no means insignificant: the second has a more restrictive definition than the first, not speaking incorrectly, doing so with a certain order, precision, calm and with the help of minimal but sufficient reasons to explain oneself, and at the same time, to understand and be understood as clearly as possible, to communicate with others in the best possible manner, that is, to “be reasonable”.

As is the case with other words that start with the morpheme “en”, such as “enrajolar”, “ennovular-se”, “enfilat”, “enllaçar”, “enamorar”², etc., “enraonar” indicates a positive activity in ordinary speech.

To reason is the general concept that, when done through speech, is to “enraonar”, a word not to be confused, for example, with explaining or debating, which impart more detail and refer to distinct situations. Thus, one would say, “The two friends ‘enraonan’ about the political situation,” “The professor explains the lesson,” and, “The Parliament debates the bill.” There are even books, for example [1], for which it would be appropriate to say that the author ‘enraona’ to us about the topic”, instead of “talks to us about the topic”.

Aranese makes the same distinction as Catalan between the words “parlar” and “arrasoar”, while as far as the authors of this article know, neither Spanish, nor Galician, nor Basque, nor English, nor German, nor French, nor Italian, nor Portuguese have two different words to represent the difference between speaking and “enraonar”. However, currently, it seems that Catalan, is losing the nuance between “parlar” and “enraonar”, and as the use of “enraonar” diminishes, the word “parlar” is used more and more. Taking into account the right of its speakers, the only ones who can claim ownership of a language, the authors hope that, at least in intellectual speech, the word “enraonar” may be preserved to denote a desire to speak

¹ In Spanish, both would be translated as “hablar” [Translator’s note: “to speak” in English].

² “Embaldosar” [to tile], “nublarse” [to cloud], “ensartar” (or “enhebrar”) [to string], “enlazar” [to connect], and “enamorarse” [to fall in love], respectively.

“intelligently”, a desire that we intend to manifest with the title of this article.

In order to “enraonar”, one must make use of another fundamental verb: to understand. Without understanding and being understood, one may be able to speak, but one cannot “enraonar”, to speak in a series of well-founded arguments either to examine something in depth or to try to reach some conclusion, in short, to speak, but on a basis of reason. Otherwise, knowledge will be lacking and what is said will be, in the best of cases, superficial. In order to “understand”, it is necessary to get what the speaker wishes to communicate, through voice, gesture, image, or writing. The best syntax, the best possible form of speech, these are very important, but without semantics, without the meaning of the words, without a connection to some reality, there is no way to understand or be understood.

In this article we will attempt to start down the road toward understanding linguistic utterances, “what is said”, with a certain formal modelling that, even limited to a few elemental subject/predicate statements, may potentially go further and lead to more complex statements [2]. In order to understand, it is necessary to capture the meaning, the use, the words in the language [3], and in modern speech there are a fair number of predicates the use of which allows for mathematical representation [4].

1. 2

The term “linguistic utterances” refers to real or virtual objects or elements that we will assume to be part of a subset known as the universe of discourse. Only elemental utterances of the subject/predicate type will be considered, that is, in the “x is P” form, where x is the subject in a universe X of discourse, and P is the name of a property p of the elements of X, that is, a predicate acting on X. In general, it can be said that capturing what is understood with “x is P”, for all or just some of the x’s within X, implies knowing in whole or in part the “behaviour” of P in X, a behaviour that may be static, rigid, or precise, or dynamic, flexible, or imprecise, depending on whether the elemental utterances of “x is P” are grouped, as a consequence of the behaviour of P, in a maximum of two classes or in more than two, respectively. Partial knowledge of the behaviour of P in X means knowing it in a subset of the X universe, that is, in a set we will continue to refer to as X.

How does a predicate P act on, behave in, a set X, and how can this action be “represented”? If the behaviour of P helps to group the elements of X,

can it be represented using some relational structure? Is there some binary relationship \leq_P , definable in X , that allows us to represent the behaviour of P in X , that is, that reflects the character, either static or dynamic, of P ? Elemental subject/predicate utterances are also referred to as “assertions”, and since [5], those utterance forms that are interrogatives or exclamations or curses are no longer considered assertions, as the affirmation or negation is not an elemental utterance. Note that in using only some of these linguistic expressions, such as, for example, “I beg you to come on Saturday,” “Watch your step!” “Now I would like to eat bread with tomato,” etc., it would be quite difficult to express or acquire knowledge; unlike these expressions, when uttering an assertion, the relationship between the subject and predicate enables the capturing of what is meant to be said by the simple fact that something is affirmed about something. In philosophy, perhaps for not having taken too much into account how the affirmation is affected by the fact that “ x is P ” has to do with the contextual behaviour of P in X that, often and especially when it is dynamic, is of the empirical and observational type, it does not appear that any representation has been considered using a (X, \leq_P) structure. Is this a mistake? In any case, not even a majority of assertions are based on definitions, that is, on utterances of the “if and only if” type as far as how P acts on X , but on descriptions (even, the best possible ones) of the behaviour of P in X , nor are more than a fourth of the arguments that people normally make, including philosophers and scientists, formal deductions [6]. Those who do so argue in the same way, with so-called “everyday logic” or “common sense” [7] and, if they wish to “enraonar”, they must do so in the best possible way making the best possible use of the information available in each circumstance. In addition, when using the “ x is P ” elemental utterances, it is implicitly accepted that these have some degree of “truth”, that is, that they say something about the action of P on X , and that therefore, they can be understood in terms of how the elements of X satisfy, or do not satisfy, a property p reflecting a certain “reality” that these elements present. The theoretical problem, however, rests in knowing what is understood by a degree of truth, or truthfulness, of an utterance: How can we tell if the degree of truth $t(x \text{ is } P)$ is 0 (false), is 1 (true), or some other value, without knowing intrinsically “what is” such a degree t ? Is there some such intrinsic way to define a “degree of truth” without using “external” considerations? What’s more, what sense is there in saying that $t(x \text{ is } P)$ is 1, or 0, without being able to correlate it with the truth or falsity or other utterances “ y is P ”, with y of X ?

If an affirmative answer exists, is it acceptable to simply know the truth, $t(x \text{ is } P) = 1$, or the falsity in the case that $t(x \text{ is } P) = 0$, of a single element x , that is, in a universe of discourse reduced to a single element x ? How can we describe, then, the behaviour of P ? Can P of a unitary set $\{x\}$ really be predicted? If so, the predicate would only be linked to x , and it may be necessary to have imported it previously from some other universe of discourse, perhaps wider and in which its behaviour may have been well known.

An intrinsic definition and one of the “mathematical definition” type of the (numerical) degree of truth can only come about if the “truth” is considered a magnitude, and if, therefore, its degrees are functions verifying:

‘If $x \leq_P y$, then $t_P(x) \leq t_P(y)$, for a function $t_P: X \rightarrow \{0, 1\}$ ’.

There is no other way to consistently and intrinsically introduce a definition of the degrees of truth of an utterance that is only a function of how P acts on X . Therefore, the three-item set (X, \leq_P, t_P) is a magnitude.

1. 3

A predicate P is “fertile” if, once used in a universe of discourse X according to a relationship \leq_P , it migrates or is exported under the name P^* to another universe Y , and for which there exists a bijective function $f: X \rightarrow Y$, which allows us to define

$$x \leq_{P^*} y \Leftrightarrow f^{-1}(x) \leq_P f^{-1}(y),$$

and thus the two magnitudes (X, \leq_P, t) and (Y, \leq_{P^*}, t_{P^*}) can be considered, with $t_{P^*}(x) = t_P(f^{-1}(x))$, for all of x in Y . This is, for example, the case of the predicate “old” in the universe $[0, 100]$ years, which migrates or is exported to the universe (of pure names) $[0, 1]$ with the name “big” and thanks to the bijection $f(x) = x/100$ that, at the same time and as will be shown in section 3, can serve for the degree of truth $t_{\text{old}}(x) = f(x)$.

Fertility is an important “economic” property, as it both makes it unnecessary to introduce new words and generates synonyms: for example, [in Spanish] the term “persona mayor” (“greater” person [literal]) is often used instead of “persona vieja” (old person), although one wouldn’t say “número viejo” (old number) to mean “número mayor” (greater number). Fertility, of course, contributes to the enrichment of natural language.

If P only applies to a unitary set $\{x\}$, then what fertility can P demonstrate? How can it be exported to other sets? Really, can anything else be said beyond “ x is P ”?

In addition, it mustn’t be overlooked that even if \leq_P exists but is not a pre-

order, the existence of an exact measurement that perfectly reflects the elemental meaning of P toward X [8] cannot be affirmed, as demonstrated in section 5.3.

1. 4

This is a series of questions that are worth studying as rigorously as possible to find their answers as they affect the basis of how discourses, explanations, questions and narratives are understood. Understanding is essential for the term “enraonar” to assume its full significance beyond just talking: it is, above all, to pursue the philosophical ideal of arriving at clear and distinct ideas about things, to fully know what one wants to say not just in each utterance, but in their chains. Otherwise, there can be no “knowing”. Generally, it is necessary to add that “knowing” cannot, under any circumstances, remain anchored to outdated knowledge: it must be taken into account that a lot of knowledge has an expiration date, although it is not known at any given time. For example, who today would be considered a wise person if they maintained that the Sun orbits around the Earth, had an understanding of many diseases from, not to go too far back, just 100 years ago, or hadn’t heard of integral and differential calculus introduced by Newton and Leibniz, or antibiotics and how the organism becomes accustomed to them? It is necessary to require both a regular “enraonar” and a general reasoning, to be “rational” [9], as precise as possible in the sense described by [10], that means that for its study it is best not to forego a good measure of mathematical formalism.

A combination of the rigid methodology with which science approaches reality and the way philosophy does so in a much more flexible manner, it cannot be overused: what’s more, it may even be best to look back to the spirit of precision that was typically the purpose for some thinkers who were close to trends, practically abandoned today, like the “Vienna Circle” [10] as well as the line of dialectical materialism [11], [12]; even though they were universally viewed as often at odds with each other.

NOTES

1. The importance of recognizing universe X beyond a simple list of its elements must be cited. For example, if, when one wishes to “enraonar” about the current financial situation in Europe, one does not have a certain knowledge of which are the banks and other “agents” involved, of how the worldwide financial markets act, of their relationships with governments

and how they interact with their local customers, it would be very difficult to talk about all this in a reasonable manner, to “enraonar” about this subject. Knowing the current structure of the universe of the discourse as well as possible is essential.

2. An example of certain pedagogical interest is the following: When at school the students begin to solve $a \cdot x = b$ linear equations, they have to be familiar with what they are “working with”, that is, that they are doing this with algebraic expressions of a determined type and that “they are” the universe of discourse in which it is necessary to operate according to calculus rules in relation to, at least, the operations of adding, subtracting and multiplying, which they organize and structure.

Thus, when solving a simple equation such as

$$2(x-1)+3(x+1)=0 \text{ (*)},$$

first they have to know what $2(x-1) = (2x-2)$ is, as well as $3(x+1) = (3x+3)$.

Then they will move on to the second step,

$$2(x-1) + 3(x+1) = (2x-2)+(3x+3) = (5x+1) \text{ (**)},$$

which is valid for all values of the number x . In the third step, which consists of identifying (*) as $5x+1=0$ (thanks to the equivalence of (**)), they must understand that, as it equals 0, the values of x are restricted as, for example, with $x=0$, the absurd $1=0$ is obtained, showing that the equation $5x+1=0$, equivalent to (*), is not valid for all values of x . Finally, the fourth step, consisting of isolating the thus-far “unknown” x , $x = -(1/5)$, reaching the solution of the equation (*), that is, the only value of x for which it is satisfied. Lacking this knowledge, the student will “not understand” fully what is happening, even if they get it right, let’s say, “mechanically”. The knowledge of X and the structural relationships between the elements is basic to being able to answer, step by step and in a reasoned manner, the question: For what values of x is (*) correct? The answer to which is, “Only for the value $-1/5$.”

3. An example of more general interest is that of using probabilities assigned to random events, something that, in order to be accomplished in accordance with probability theory [8], almost universally accepted today, it is necessary to know that the universe of discourse constructed by the random events must have been previously structured in the form of Boolean algebra. Otherwise, not all of the results of this theory can be used, and, for example, it may be the case that a property as important as that of that, given two events A and B , their union $A \cup B$ and intersection $A \cap B$ are also given may not be verified, which means the addition law cannot be used:

$\text{Prob}(A \cup B) + \text{Prob}(A \cap B) = \text{Prob}(A) + \text{Prob}(B)$, which is, in many cases, essential to being able to calculate probabilities and which comes from the lattice law $(A \cup B) \cap (A \cap B) = A \cap B$. Analogously, the law $\text{Prob}(A^c) = 1 - \text{Prob}(A)$ can only be used if it is certain that A^c is an event and that both Boolean laws $A \cup A^c = X$ and $A \cap A^c = \emptyset$ apply, that is, that $\{A, A^c\}$ is a perfect classification, or partition, of the universe of discourse.

4. Although they are seldom cited, it must be stated that this article is mainly inspired by the references [13], [14], [15], [16], and, above all, [17].

2 What has to be understood by 'x is P', if x belongs to X?

2.1

As stated above, knowing the universe of discourse X is essential: if it is not taken into account, it cannot be known to which objects or elements predicate P applies. For example, if $P = \text{odd}$ and $X = \text{the bell towers (of a determined place)}$, it's clear that we won't know what "x is odd" means, unless a definition, or at least, a description as specific as possible, has been introduced of what is meant by an "odd bell tower", otherwise to what happens if $X = \text{a set of natural numbers}$. It goes without saying that if an attempt is made to apply $P = \text{cacambú}$ to a well-known universe, it would be necessary to describe what is meant by "cacambú". Even with a predicate as well-known as $P = \text{Rosemary}$, and a subject as well-known as $x = \text{pencil}$, one could hardly state literally that "this pencil is rosemary", where the contrary is the case if X is a set of field herbs. The pair (X, P) is, therefore, basic: it is necessary to know what is being said of which things.

Another example comes to us from the ancient Greek world, with the set X of the "inhabitants" of Olympus and the predicate $P = \text{god}$; however, the monotheistic case in which X is a unitary set and $P = \text{God}$ is more difficult. In the first case, there is even a hierarchy, an order, among the gods; in the second there is a singular "being" which may have been reached through a second abstraction based on the previous one and different, that came about through talking about the god Zeus of Olympus. Also similar is the case of the "elements" $+\infty$ and $-\infty$ in the real number line, both introduced so that the successions of real numbers would have a limit. Now, a notable difference between "God", " $+\infty$ ", and " $-\infty$ " is that the latter concepts are added to the number line according to a definition that is in a clear relationship of consistency with the other elements of the number line, and therefore, the real number line, \mathbf{R} , can be distinguished from the extended

real number line $\mathbf{R}^* = \mathbf{R} \cup \{+\infty, -\infty\}$. It must be noted that the real number line is an abstract entity introduced, thanks to various equivalencies, based on the axioms of Peano for the natural numbers. Extending \mathbf{R} to \mathbf{R}^* is not essentially different from other extensions like that of considering the imaginary number $i = +\sqrt{-1}$, and with it, extending the real field \mathbf{R} to the complex field \mathbf{C} of elements to $+b \cdot i$, constructed theoretically based on well-understood properties such as $'(1+\sqrt{-1}) \cdot (1-\sqrt{-1})=0'$. These are extensions that present a total theoretical coherence with that which is being extended.

The example of the proposed concept of “God” is, however, very much singular, and not just because it is only applicable to the unitary set {God}, where it can only intrinsically be stated that “God is God”, and whose only element is not, in addition, well known to be “a priori”, or at least, sufficiently well described in and of itself. It is not, above all and in a plane of strict rationality, of “pure” reasoning and not experimental, for lack of a systematic, convincing, and as universally accepted as possible proof of existence and uniqueness, that is, a deductive proof that there exists a single x such that $x = \text{God}$, made using non-contradictory axioms that have previously been accepted as such, in a similar manner to how one can prove, based on rational numbers, the existence of new elements that, well defined, are not rational numbers and that in particular include numbers such as $\sqrt{2}$, which it is easy to prove is not rational, at least in introducing $+$, i $-\infty$ into \mathbf{R} . Note that, contrary to the case of extending the real number line, with “God” Olympus was diminished, or at least, it was transformed and organized again. This is also a theoretically singular concept for lack of a well-developed relational algebra between the concepts of “God”, “Energy”, “Nature”, “Living beings”, and perhaps also those that “are not alive”, which, definitely and in the 13th century, Ramon Llull [18] has tried to do in part, and in a more formal way, Kurt Gödel did the same in the 20th century [19].

2.2

Setting aside for the moment the possible existence of predicates, or names applicable to a unique element, the fact is that in order to understand well what is meant to be said with an elemental utterance “ x is P ”, how P is applied to all the elements of a determined universe of discourse must be known, that is, to those elements that are of interest and of which everyone can affirm that they “demonstrate” the property p named by P . And beyond that, an established relationship between pairs of utterances “ x is P ” and “ y

is P” that indicate the difference between both utterances as far as the verification of p on the part of x and of y must also be known; a mathematical binary relation in X, $\leq_p \subseteq X \times X$, such that $(x,y) \in \leq_p \Leftrightarrow x \leq_p y$, indicates that x does not verify p more than y verifies it, and in an understandable way, makes it possible to organize the universe of X in terms of P. An idea in line with the concept that “enraonar” inserts into X a certain “order”, “hierarchy”, or “classification”, that makes it possible to understand the discourse thanks to the use of P; thus, X goes from being “amorphous” to being organized by the structure or graph (X, \leq_p) . The predicates are concepts that are as general as could be applied to any universes, both to those with no previous known structure and others that already have one; these are the cases, respectively, of “red” applied to a sheet of paper, and of “probable” applied to events already organized in Boolean algebra.

For example, and with a predicate as common as P = full, applied to glasses that contain a liquid, little can be understood of what is meant by “this glass is full of liquid” if we do not know how to distinguish whether a glass is empty or more full than another one. In fact, we would not venture beyond considering glasses “full” in which there is no room for even one more drop of liquid and “not full” those in which a single drop can still fit, something that in common usage is not the case and that has to do with ordinary perceptive possibilities. Common language, backed by common sense, allows for a flexible use of predicates, and unless we limit ourselves to dealing with strictly mathematical concepts such as “even” or “odd”, we can say little about the term “enraonar” in the way people do [20].

It must be mentioned, however, that some mathematical concepts can be taken into account that are not precise; this is the case, for example, of the “round number” [7], an imprecise concept that applies to those natural numbers that, when decomposed into prime factors, consist of few different factors but elevated to relatively large exponents or powers with respect to their bases: thus, the number 26.38 can be said to be round, but not the number 2.5.7.13.17. Naturally, the question that is made evident immediately is, which are the two numbers that can be considered round is the rounder of them? Which of the numbers $x = 2^6 \cdot 3^8$ and $y = 2^{11} \cdot 3^9 \cdot 5^8$ is more round? This requires knowledge of whether it is $x \leq_R y$, or $y \leq_R x$, that is, it is necessary to “define”, or at least, “describe”, what is meant by “x is less round than y”, without excluding that there may be pairs of elements that are not comparable as round. On the other hand, when it comes to a precise

predicate, rigid or not graduable, the relationship \leq_P is simply converted into the identity $=_P = \leq_P \cap \leq_P^{-1}$; for example, the numbers 5 and 7 are equally odd, as well as not even and equally prime.

Naturally, a single universe X is potentially subject to the simultaneous action of diverse predicates, among them those that may be formed based on two others previously acting on X , such as, for example, the predicate **P and Q** defined as ‘ x is **P y Q**’ = $(x$ is $P)$ and $(x$ is $Q)$, an expression with two copulative conjunctions that are formally different, as the first (**y**) affects the predicates, and the second (y) the utterances, somewhat similar to how the “product” of numeric functions is defined based on the product of numbers: $(f. g) (x) = f(x).g(x)$, for all arguments x that apply; thus, if the functions f and g , is $f(x) = 2$ and $g(x) = 5$ are known, the new function $f. g$ has a value of 10 in the same point $x : (f. g) (x) = 2. 5$. Other utterances are analogous, such as “ x is P or Q ”, “ x is **NOT P**”, and “**if** x is P , **then** y is Q ”, which are defined analogously to the above for P and Q . Note that the utterances “ x is **NOT P**” and “ x is **not P**” can be considered identical if both are defined as the negation of $(x$ is $P)$; this is how “is NOT” and “is not” are identified.

2.3

It is worth noting that, at least in ordinary language, with each predicate P and in order to utilize it correctly on X , it is necessary to do so simultaneously with an antonym of the same. How could we affirm that “John is tall” without knowing that “Peter is short”, or vice versa? How do we know of the rich without knowing of the poor? How do we recognize what is cold without recognizing what is hot? This bipolarity is also present in the case of rigid predicates, although often, it is with the greatest antonym of all, the negation. For example, in mathematics, a rigid predicate in the interval $X = [0,10]$ is $P =$ “less than or equal to 4”, which applies to all numbers within $[0, 4]$ and does not apply to its complement $(4, 10]$. None of its antonyms are used to understand it because they are not needed, as with its negation $P' =$ “greater than 4” (which applies to the numbers in $(4, 10]$) is sufficient. It isn’t that there are no antonyms to this (not only that, $[2]$, they can be calculated!), but they are not used because $[0, 10]$ is already perfectly classified between $[0, 4]$ and $(4, 10]$. Instead, with the imprecise predicates of language, antonym implies the negation but not the other way around and they rarely coincide; thus, with “the glass is full”, the antonym “the glass is empty” implies the negation “the glass is not full”, but not

reciprocally. With this, anyone can make the hypothesis that, if P^a is an antonym of P , $\leq_P^a = \leq_{-P}^{-1}$, con $x \leq_{-P}^{-1} y$ and $y \leq_P x$. While the double negative $(P')'$ does not always coincide with P , the double antonym $(P^a)^a$ always coincides with P ; the opposition between linguistic terms shows the symmetrical property and the importance given to symmetry in science is well known [21], [22]. What does not immediately exist is an antonym of negation, as P' is not a linguistic term, but there is the negation of an antonym $(P^a)'$. For example, if $P = \text{tall}$, it is true that $(P^a)' = \text{not short}$, but $(P')^a = (\text{not tall})^a$ does not immediately exist in the language, and in any case, it would be necessary to introduce it. Of course, though it may exist, $(P^a)'$ is not a linguistic term. Another example, this in the case of mathematics, comes from the concept of the “transcendent” real number, difficult to capture without previously understanding that of the “algebraic” real number, which, in reality, is its negation because the two sets “transcendent numbers” and “algebraic numbers” are complementary sets on the real number line. This is an example of an irregular antonym, that is, when negation and antonym coincide.

3. True and false

3.1

The concepts of true and false, one an antonym of the other, have not only not being well defined, but they have been the cause of many discussions, [23], [24]. Setting aside a subject as unclear as what is the “Truth” with a capital T, in any case it seems that the adjectives true and false must refer to some relationship between utterances and reality. Only utterances can be described as true or false; for example, it may be that “John is not rich” can be qualified as true or false, but “rich” definitely cannot be, as it is not an utterance but rather a predicate. In the following, the concept of “true” is identified in an operational way to a magnitude that quantifies the “truth”, in lowercase, previously shown in a qualitative way by the elemental utterances “ x is P ”.

If the behaviour of predicate P is represented by a relationship \leq_P , in which

$$'x \text{ is not more } P \text{ than it is } y' \Leftrightarrow x \leq_P y,$$

then with the relational structure (X, \leq_P) , can be considered functions $t_P: X \rightarrow [0, 1]$, verifying:

- a) If $x \leq_P y$, then $t_P(x) \leq t_P(y)$
- b) If $m \in X$ is a minimal element for a \leq_P , that is, there is no element $z \in X$

such that $z \leq_p m$, then $t_p(m) = 0$.

c) If $M \in X$ is a maximal element for \leq_p , that is, there is no $z \in X$ such that $z \leq_p m$, then $t_p(M) = 1$.

These functions, which are measures [8], are called “degrees or values of truth” of the utterances “ x is P ”, and the degree of truth of “ x is P ” is usually identified with the number $t_p(x)$. If x is minimal, then it is said that “ x is P ” is totally false, and if y is maximal, then “ y is P ” is totally true. In this way, the concept “true” is represented by magnitudes (X, \leq_p, t_p) , and, of course, the “axioms” a, b, and c do not determine a unique measurement t_p ; in each case, it will be necessary to choose a one based on additional specific and reasonable conditions. For example, if X is the numeric interval $[0, 10]$ and $P = \text{large}$, in order to determine “one” measure t_p of the utterances “ x is large”, the following hypotheses can be accepted (leaving out the t_p and simplifying the notation to just t):

1. 10 is totally large, and 0 is totally not large
2. $x \leq_p y \Leftrightarrow x \leq y$, that is, “ x is less large than y ” coincides with “ x is smaller than y ” in the order of the interval $[0, 10]$.
3. If it can be said that “ x is large”, and if $y > x$, then it can also be said that y is not less large than x ,

Which means the corresponding sizes t can be defined as these t functions: $[0, 10] \rightarrow [0, 1]$ such that:

1*. $t(0)=0, t(10) = 1$

2*. If $x < y$, then $t(x) \leq t(y)$,

that is, the growing functions that go from point $(0, 0)$ to point $(10, 1)$ of the Cartesian plane. The simplest example comes from the function $t(x) = x/10$, if it can be supposed that its growth must be linear; in the case of having to suppose a quadratic growth, the function $t(x) = x^2/100$ can be taken. Additionally, and if, for example, it is $t(x)=0.8$ and $y>x$, as this implies $t(y) \geq t(x)=0.8$, then, and according to 3, y is not less large than what x is. That is, 3 is now satisfied thanks to 1* i 2*.

Note that when X is reduced to a single element x , si \leq_p exists, what remains is the unitary relationship $\{(x, x)\}$, which has neither maximals nor minimals, unless it is accepted that x is simultaneously minimal and maximal. Therefore, either $t(x)$ does not exist, or $t(x) = 0=1$, which is absurd, we are left in the “agnostic” position of not being able to say whether “ x is P ” is true or false, only able to say that it has no truth value, that there is no magnitude with $X = \{x\}$ representing the “true” predicate. This is unless,

naturally, a different definition is adopted for the truth value of x , a definition that, in a way, would have to be “artificial”, as, in any case, it would have to come from considerations outside the use of P on $X = \{x\}$. For example, if x is in some way similar to an element y of Y , a universe in which the action of predicate Q is represented by the magnitude (Y, \leq_Q, t_Q) , if $t_Q(y) = 1$, that is, y is a prototype of Q , it may be possible to transfer Q to $\{x\}$ and say that “ x is Q ” is true. With this, the “leap” to affirming that in $X = \{x\}$ everything or almost everything can be predicated, it doesn’t seem at all strange.

3.2

It’s easy to prove that the relationship $\equiv_P = \leq_P \cap \leq_P^{-1}$ is of equivalence if and only \leq_P is reflexive and transitive, that is, a pre-order. We say that a predicate is rigid when \equiv_P is an equivalence and generates a quotient set X/\equiv_P that has a maximum of two classes, and as a result, t has a maximum of two values. If these values are only 0 and 1, then we say that the predicate is classical or bivalued; of course, in this case, the quotient may consist of a single class and then t will only have a value of either 0 or 1: all utterances “ x is P ” will be either true or false. If the quotient has two classes, one will be of elements x such that “ x is P ” is true, and the other of elements y such that “ y is P ” is false.

It should be noted that, in cases where they exist, t is constant in the classes of equivalence and that, therefore, the maximum number of different values of t coincides with the number of classes of the quotient X/\equiv_P : If x , and are in the same class of equivalence, it is the case that

$$x \equiv_P y \Leftrightarrow x \leq_P y \ \& \ y \leq_P x \Rightarrow t(x) \leq t(y) \ \& \ t(y) \leq t(x) \Rightarrow t(x) = t(y).$$

NOTE. In the example above using “large”, there is a single minimal or minimum (0), a single maximal or maximum (10), and as ‘ $x \leq y \ \& \ y \leq x \Leftrightarrow x = y$ ’, the equivalence classes consist of a single element. Thus, both the quotient set has the same number of elements as $[0, 10]$, and t has this number of values as a maximum.

4. The case when speakers do not understand the predicate in the same way

In any “intelligent” conversation, the understanding of what each of the speakers says is crucial; otherwise, it would turn into a bunch of nonsense and no one could understand anything. “Enraonar” means understanding

what the other speaker wants or is trying to say, and sometimes the difficulty arises that the speakers don't reach exactly the same conclusion based on a predicate P in a universe X. A way to keep "enraonant" (talking) consists of seeking out a minimal common understanding for the concepts being discussed; at the end of the day, reaching the agreement to limit themselves to "enraonar" about the things about which they have a mutual understanding.

If there are two speakers involved and each of them understands P on X through (X, \leq_p^1) and (X, \leq_p^2) , respectively, then it may be that the intersection $\leq_p^1 \cap \leq_p^2$ is empty, or that it does not exist. In the former case, the possibility that they can understand each other is nonexistent or very small, but in the second and when they have a minimum common "meaning", given by $\leq_p = \leq_p^1 \cap \leq_p^2$, understanding is possible if both are in agreement in limiting themselves to understanding P on X for (X, \leq_p) and leaving the non-empty difference $\leq_p^i - \leq_p$ aside. Note that if the two relationships \leq_p^i ($i = 1, 2$) are pre-orders, the reflexive properties of both imply that their intersection is not empty. Of course the same can be said if instead of two speakers there are three, four or however many (Autor, 2009), and yet, the more speakers there are, the more reduced the intersection of the corresponding relationships may be.

If the two speakers previously understood the truth of the utterances "x is P" by different degrees of truth, t_p^1 y t_p^2 , then it is possible for them to understand the "common" truth using the function

$$t_p(x) = \min(t_p^1(x), t_p^2(x)), \text{ for all of } x \text{ in } X,$$

with which every utterance "x is P" is assigned a degree of truth that is the minimum of what each speaker has assigned to it on their own. The new function t_p is a degree of truth, as it verifies:

$$x \leq_p y \Leftrightarrow x \leq_p^1 y \ \& \ x \leq_p^2 y \Rightarrow t_p^1(x) \leq t_p^1(x) \ \& \ t_p^2(x) \leq t_p^2(x) \Rightarrow t_p(x) = \min(t_p^1(x), t_p^2(x)) \leq \min(t_p^1(y), t_p^2(y)) = t_p(y),$$

that is, understanding is obtained using the magnitude (X, \leq_p, t_p) .

In this way, the two speakers can arrive together at a minimum common understanding, even if each one, on their own, may come to something that goes further. In a way, it can be said that the confronted magnitudes, (X, \leq_p^1, t_p^1) and (X, \leq_p^2, t_p^2) , have been consensually compacted into (X, \leq_p, t_p) , which guarantees the speakers a minimum possibility of understanding each other. They, however, must decide previously whether, even giving up some possible conclusions each, they wish to understand each other.

5. The risk of only considering ‘true’ through grades

It must be noted that every degree t brings about, with its values in the unity interval, a linearization that cannot always be convenient: If x and y can be incomparable with respect to P , on the other hand, one of the two numbers $t(x)$, $t(y)$ will always be greater than the other. What was incomparable becomes comparable.

Once t is known in, X it can be considered a new binary relationship \leq_t , defined as

$$x \leq_t y \Leftrightarrow t(x) \leq t(y),$$

which is obviously reflexive and transitive, that is, with which the structure (X, \leq_t) is of a pre-ordered set, even if (X, \leq_p) was not. As far the linear or total character of the numeric order \leq of $[0, 1]$, the new relationship \leq_t is also linear. From $x \leq_p y \Rightarrow t(x) \leq t(y) \Leftrightarrow x \leq_t y$, it follows that

$$\leq_p \subseteq \leq_t,$$

that is, the new relationship \leq_t is wider than the old one \leq_p , and as the new one is always linear and the old one cannot be in all cases, in general the prior inclusion is strict, that is, the difference set $\leq_t - \leq_p$ will not usually be empty. When this difference is empty, that is, if $\leq_p = \leq_t$, it is said that the measure t “perfectly reflects” the use of P in X . This is the case, for example, of $P = \text{large}$ in $[0, 10]$ with $t(x) = x/10$, which is a strictly growing function with which it is immediate that as $\leq_p = \leq$ (the linear order of the interval), and also $\leq_t = \leq$, t perfectly reflects the use of “large”.

Therefore, in cases in which \leq_p is strictly contained in \leq_t , that is, that $\leq_t - \leq_p \neq \emptyset$, to consider the predicate “true” just because of its t degree “extends” what is meant to be said with it. This always happens when \leq_p is not linear, when there are elements x , y of X that are not comparable under the relationship \leq_p , that is, for which neither $x \leq_p y$, nor $y \leq_p x$.

As, given the size of X , it is often difficult to be able to completely know the relationship \leq_p , it is common to work directly with \leq_t , once a t is supposed based on the partial information gathered on \leq_p . In these cases, it must be taken into account that, when considering the linear graph (X, \leq_t) instead of (X, \leq_p) , the “meaning” of P must be expanded; the predicate will seem to say more than it really says.

The structure of X given by the use of the predicate is what determines whether its characteristic of “true” can be contemplated solely through whatever degree that, necessarily, will organize X in a linear manner. At this point it may be helpful to remember the anecdote of that famous label that appeared at M.I.T., “Lord, make the world stable, linear and Gaussian.” The

world, just as it is not always stable and Gaussian, is not always linear either, although often linear models are very useful, but not everything can be resolved with rules of threes, not by a long shot. Seeing the predicate P through a numeric degree of truth t_P can often result, at least, semantically dangerous, because it may appear that predicates more of the elements of X than it really predicates; this is why the function t_P must always be “designed” very carefully based on the maximum information that can be obtained based on the behaviour of P on X . For example, in the case of the predicate “large” in $[0, 1]$, to take

$$t_P(x) = 0, \text{ if } 0 \leq x \leq 0.7 ; t_P(x) = 1, \text{ if } 0.7 < x \leq 1,$$

means confusing the predicates “large” and “strictly larger than 0.7”, that is, just considering the numbers that are above 0.7 to be large, and, therefore, to see a predicate that is usually imprecise as a precise predicate.

6. Final Notes

6. 1

The primary objective of this article is to show that without capturing the meaning of the predicates used, that is, without knowing, even partially if not completely, the relational structure, or graph, (X, \leq_P) , when X is not reduced to a single element, it becomes difficult to be able to say that an elemental utterance “ x is P ” has been well understood. It goes without saying that it is not enough to know the “meaning” of an elemental utterance, but a few must be known, one must be able to compare them with each other in terms of how they comply with the property named by the predicate P , something that, if it is perhaps known intuitively, an attempt must be made to show here in a reasoned manner.

The second objective of this article is to shine a light, dim as it may be, on the truth or falsity of elemental utterances. This is an aspect that, once the concept of degrees of truth is introduced, it’s important to realize what it implies that there is not one single way to assign the adjectives true or false to utterances, which seems to cast doubt on the possibility that something universal called the “Truth” may exist, and that one can truly “enraonar” and not just speak. The characteristic that an utterance presents as “true” has to do with the context in which it is made: for example, in a universe with few elements, a generally imprecise predicate in a large universe can appear to be quite precise. This is the case of “old” if it predicates three people aged, respectively, 20, 30, and 90.

This is something that certainly is not essentially distinct from the assignment of a probability of an event: By conducting the “experiment” of throwing a dice and asking what is the probability of obtaining the result “5”, there is no unique answer but rather an “it depends”: if the die is perfect and thrown as carefully as possible, there is no doubt that the probability of obtaining any of the numbers 1 to 6 is $1/6$, but often dice will have imperfections, or may be weighted. For example, if the die is weighted such that out of every 10 throws the “6” comes up seven times, the probability of obtaining “6” will be $7/10$, and therefore, for the rest of the numbers from 1 to 5, they’ll be split up under $1 - 7/10 = 3/10$; therefore to each of them can be attributed a probability of $(3/10)/5 = 3/50$, and “obtaining a 5” can be assigned the probability of $3/50 = 0.06$, quite a bit smaller than the “ideal” $1/6 \sim 0.17$. Therefore, the answer to the question “what is the probability of obtaining a 5 when throwing a dice once?” depends at the very least on prior information or conditions on the dice that it is necessary to know in order to respond with reason. It also depends, obviously, on the surface it falls upon: it would be different if falling on a sandy surface, for example.

Something similar occurs with the adjective “true”; thus, in the example using “large” in the interval $[0, 10]$, the utterance “5 is large” is true with a degree $f(5)$, if f is a function between $[0, 10]$ and $[0, 1]$, which is strictly growing and verifies $f(0) = 0$ and $f(10) = 1$. If there are additional conditions allowing for f to be linear, then if $f(x) = x/10$, and $f(5) = 5/10 = 0.5$, but if these conditions demand that f be quadratic, it will be $f(x) = x^2/100$, and then the result will be $f(5) = 25/100 = 0.25$, two values between which there exists a significant difference: the second is half of the first.

To determine a degree of truth requires, beyond knowing \leq_P , also knowing additional conditions of a contextual type. To apply the adjective true to an elemental utterance requires information on the corresponding context.

6. 2

In terms of finding the degree of falsity, t^* , once the degree of truth t is known, and when dealing with concepts that are antonymous or opposite to each other, a symmetry $s: X \rightarrow X$ can be used [13], and $t^*(x) = t(s(x))$ can be taken. For example, accepting that the degree to which, in $[0, 10]$, “x is large’ is true”, is $t(x) = x^2/100$, all that needs to be done is to take $s(x) = 10 - x$ (which, obviously, is a symmetry in $[0, 10]$, because it verifies $s(0)=10$, $s(10)=0$, and $s(s(x))=x$), in order to obtain $t^*(x) = t(10-x) = (10-x)^2/100$. Thus, for example, the degree to which 5 is large is $t(5) = 0.25$, and the degree to

which it is small is also $t^*(5) = 0.25$. Note that if it were $t(x) = x/10$, it would be $t(5) = 0.5$, and with $t^*(x) = t(10-x)$, it would also be $t^*(5) = 0.5$.

In a case in which \leq_p is not linear such as \leq_{large} , which coincides with the order of the interval, then it would be necessary to use a symmetry well aligned to the increases and decreases of \leq_p in the interval. One example is the predicate “near 4”, because \leq_p is not the order \leq of $[0, 10]$, and because the previous symmetry does not yield the antonym “far from 4”, but rather the predicate “closer to 6” [8].

6. 3

The quotient set $X/=_p$, which consists of the classes $[x] = \{y \in X ; y =_p x\}$, and as it is partially ordered by the binary relationship,

$$[x] \leq_p^* [y] \Leftrightarrow x \leq_p y,$$

if and only \leq_p is a pre-order [1], it facilitates a non-numerical “measure” (and even less always linear) defined by $t(x) = [x]$, which perfectly reflects the behaviour of P and X, as it is

$$x \leq_p y \Leftrightarrow [x] \leq_p^* [y] \Leftrightarrow t(x) \leq_p^* t(y) \Leftrightarrow x \leq_t y : \leq_p = \leq_t .$$

This is an “abstract” $t: X \rightarrow X/=_p$, and therefore, and unlike what happens with the previous numerical measures, two incomparable elements under the relationship of the predicate must not need, necessarily, comparable measures (degrees of truth).

The measures defined in 3.1 can be generalized such that both them and the above can result in particular cases. It is sufficient to take, as the range of t values and instead of $[0,1]$ or $X/=_p$, a partially ordered set $(L, <)$ with a minimum 0 and a maximum 1, such that $t: X \rightarrow L$, verifies the axioms a, b, and c from 3.1. These are (abstract) measures that make it possible to consider the degrees of truth in a qualitative way and that hold certain interest in fields like Taxonomy. An example is given by taking $L = C$, the set of complex numbers $a+ib$; complex measures are often considered in Science.

6. 4

A comment is warranted with respect to some aspects related to the branch of study that the philosophers call “Pragmatics”. In order to “enraonar”, first the “speakers” must chain their elemental utterances “x is P”, “y is Q”, etc., using the connectors called “logical constants”, such as, for example, the different forms of negation, conjunction, disjunction, and conditionals,

with those that form linguistic expressions, this so that, from a pragmatic point of view, in the end the speakers themselves will be the ones who determine how each logical constant works so that the relationship of the final expression with reality is produced without betraying that of its constituent parts, that is, that only based on the elemental components can the truth of the final expression be affirmed.

The debate between the intuitionists [25] and the pragmatists [26] centred around what a logical constant is and how it works, demonstrates the difficulty of determining how to preserve, in each case, the truth, the relationship with reality. This is something difficult due, in large part, to the fact that in order to make a discourse that is not incoherent, the speakers attempt to “enraonar” using some connections between utterances and some transitions between premises or data and conclusions that preserve the truth; doing so, their objective is not very distinct from the pragmatic objective of making connections and transitions that “can be judged”, which are, finally and for inferential effects, really “what is said”.

It should be added that neither the logical constants nor the reasoning frameworks represent properties of the elements of the universe of discourse, but that only through them can what constitutes the current inferential practice be arrived at. To reason, and therefore, to “enraonar”, speakers make those chains of elemental utterances that they intuitively believe are “valid canons” of argumentation [27]; they do this in part through the logical constants and reasoning frameworks whose dynamism and flexibility allow for discourse.

Definitely, these are aspects that, moving forward, the authors will attempt to analyze with some care. In order to do so, particularly, it is necessary to see both how the relationships $\leq_{P \text{ and } Q}$, $\leq_{P \text{ or } Q}$, $\leq_{Q \text{ if } P}$, etc. can be “constructed” based on the initial \leq_P and \leq_Q as the form in which they originate, and ordinary typical reasoning frameworks can be represented, such as the “Approximate Modus Ponens”,

If P , then Q ; P^* approaches P : Q^* approaches Q , with the condition that if $P = P^*$, then $Q^* = Q$.

Only on this basis is it possible to establish coherently the possible degrees $t(x \text{ is } P \text{ and } Q)$, $t(x \text{ is } P \text{ or } Q)$, etc., as well as $t(Q^* \text{ approaches } Q)$ based on the degrees $t(\text{If } P, \text{ then } Q)$ and $t(P^* \text{ approaches } P)$, once what is meant to be said by affirming that “In universe X , a predicate approximates another predicate” is made clear.

6. 5

A predicate P is “meaningless” in X when the relationship \leq_p is empty, it has no element: $\leq_p = \emptyset$. Of course if P is meaningless in X there can be no degree of truth t_P perhaps beyond what the value 0 assigns to all elements of X .

That P is meaningless in X does not mean at all that it has to be in any other universe of discourse, for example, “yellowish” is meaningless in a universe where all elements are totally black, but it would not be in one in which there are elements of diverse colours with shades of yellow. In the first case, the only degree of truth that can be assigned is $t_P(x) = 0$ for all $x \in X$: no element is yellowish. In the second, there may be different functions t_P obtained according to, for example, the percentage of yellow it is possible to appreciate in the elements of X .

When $\leq_p \subseteq \{(x,x) ; x \in X\}$, that is, \leq_p is in part, or all, the principal diagonal of the Cartesian product $X \times X$, then it can be said that P is “almost meaningless” in X . If X is a set of pairs of socks between sizes 36 and 43, then $P = \text{size } 40$ is almost meaningless in X , because it can only be applied to those pairs of size 40 and no other pair. Although to choose a pair of socks P is important, knowing that “this pair of socks is size 40” says very little about the socks; it only describes one of their characteristics and it does not do so with others such as colour, elasticity, pressure of the elastic band, type of weave, sweat, etc., that are usually important when choosing them.

6. 6

Although in passing and only at a glance, we must also consider the case in which the universe of discourse is reduced to a single element, and an attempt is made to demonstrate that this is a singular case in which the characteristic of true or false of the only utterance therefore possible cannot be affirmed, but has to be done through non-intrinsic considerations. This is because the adjectives true and false are, essentially, of a relational character; it is difficult to apply them to isolated utterances, and what’s more, it is necessary to do so only in families (non-unitary) of isolated utterances that everyone can compare at least qualitatively. These are cases in which, if the relationship \leq_p is reflexive, the quotient set X/\equiv_p is isomorphic to the set X , as $[x] = \{x\}$. That is, these are cases that are deeply lacking conceptually and for which the assignment of a degree of truth t is quite rare as it is impossible to go further than ‘ $x \leq_p x \Rightarrow t(x) = t(x)$ ’.

Of course there are very particular situations, such as that of the name

“God” applied to a unitary set {Dios}, in which the meaning, of existence, has to necessarily come as a given due to considerations of an external order, but not considerations that are structural, intrinsic, and belonging to the use of the predicate. These are cases that could be referred to as “metaphysical”, that cannot be represented using magnitudes. It seems, then, that they are exempt from strict scientific control, and the study and justification of their “meaningful characteristic” can only correspond to philosophy or theology, which are really the fields where they are considered and which, therefore and in order to justify themselves, have to find for them the corresponding conditions of contextual validity. Otherwise, philosophy or theology may not move beyond more or less literary exercises with no direct and real relationship with “reality”; there could hardly be “philosophical or theological problems” with fertile solutions, that is, applicable outside of them as occurs with mathematical solutions. However, this does not imply that some such philosophies or theologies cannot demonstrate higher ideals of knowledge, although, when it comes to something as important today as the scientific knowledge that necessarily must form part of the “general knowledge”, they can hardly take into account the expiration date on knowledge, which is extremely important for seeing and interpreting the modern world and society.

6.7

Be that as it may, today philosophy, just like science or technology, requires control methods for its reasoning processes and for the validity of its results, and this even though such methods, rather than quantitative, may be, and may plausibly be, qualitative, some methods of control for those that, perhaps and with what has been stated here, may come across some ideas upon which to base a model. Note that even in the extreme case of a universe of discourse as a unique element, the “behavior” of any predicate would have to be represented based on contextual conditions that come from other universes, which would imply, in the end, a return to some relational structure.

It is quite possible that in the discourses upon which analysis must be realized of an essentially qualitative form, some more could be said about “x is P” based on considerations of an external order that would often be of an analogical type, that is, for similarity to other conceptual environments; with this, the controlled exportation of the knowledge involved in other universes of discourse doesn’t seem easy and it will have to be done very

carefully. This is the case, for example, of wanting to “enraonar” in legal terms, about abortion based on concepts belonging to a few determined and particular religious conceptions about the freedom of women, and in general, of how they believe the family and society should be. In a way, the different conceptions of the “truth” have been presented from philosophy, they have made the humanistic sciences separate themselves from the classical conception of the “truth” as in a well-controlled correspondence between the understanding and the thing.

Basing decisions on reason is something necessary but not sufficient; the fact is that not doing so is fairly common, either for a lack of all the information necessary, or because the information is not sufficiently trustworthy. The fact that many of the linguistic terms and frameworks of reasoning used are imprecise and/or uncertain, approximate, shows that even the safest types of reasoning cannot be fully trusted, the deductive type, as it is represented in classical logic [28], where it is modelled using precise algebraic structures such as, for example, Boolean algebra. In any case, some type of control is needed over conjectural and/or analogical reasoning, which are typical of ordinary or common-sense reasoning [29], [17] as [30] attempts to illustrate. This is not to mention the important role they play in human decision-making, emotions, feelings, prior beliefs, desires, fears, and why not spell it out: fanaticism. As [31] clearly explains, in order to improve both the lives of human beings, as well as the very stability and favourable evolution of how and where they live (human societies and the planet Earth), to accompany reason with a healthy dose of scepticism cannot be considered an indulgence; or even to accompany it with a certain scepticism of our own scepticism. It bears repeating that, in this sense, we still do not sufficiently understand either the very phenomenon known as “reason”, or the processes that intervene when making decisions.

7. Conclusion

With all that has been stated here, it remains evident that there is still much work to be done in order to study “all” the predicates, whether through magnitudes or through other algebraic and systematic methods. Yet still a door remains open just a little so that, to “enraonar”, we can also “enraonar” without too many prior concepts insufficiently understood, understanding well, and to whatever extent possible, all the linguistic terms used.

It must not be forgotten that today, both the language like “enraonar” and

reasoning are some of the most complex phenomenon thinkers face, natural phenomena that, like all such things, require systematic study of the “scientific” type, without the knowledge of which computer science, for example, could hardly well-simulate the human act of “enraonar”. To situate ourselves, how can the workings of the human brain and the thinking process, in particular, be studied without using either scientific terms or the methodologies typical of neurophysiology?

Just as the natural phenomenon of “thinking”, when it is produced in the realm of the human brain, it is not directly observable in everyday life, “reasoning”, one of its direct consequences, manifests itself in diverse ways that are directly observable, and among which to “enraonar” is the most common. Thought may only be well understood scientifically when the current neurophysiology studies manage to find out, with certainty on the brain neuron systems level, its “natural” genesis and function, and meanwhile, those who are not specialists in this branch of science must limit ourselves to developing studies of the non-experimental type, humbly remembering Wittgenstein’s assertion, “Whereof one cannot speak, thereof one must be silent.” [32]. As far as reasoning is concerned, studies conducted in formal frameworks still provide interest and certain potential fruitfulness, in which mathematical methodology can be used, like in the case of [20], and that can be fertile ground for computational sciences, and in any case, can also help improve our understanding of the phenomenon of “reason”.

Therefore, this article cannot, and will not, go further than to hint at a possible path to start down with a new theoretical and formal focus, as far as it’s possible to go, toward “enraonar”. It should not be seen, then, as any more than a first step in this direction.

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