On a priori knowledge of necessity

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1. A priori principles in the epistemology of modality

It is widely thought that the epistemology of (metaphysical) modality is in some sense a priori. Examples of the necessary a posteriori due to Saul Kripke (1971, 1980) and others decisively refuted the simplest idea in the vicinity: that what is necessary is always an a priori matter. To use a familiar example, it is necessary, but it is not a priori that it is necessary, that Hesperus is Phosphorus. Nevertheless, it is still widely thought that there is some fairly substantive sense in which the epistemology of modality is a priori.

Post-Kripkean advocates of a priori modal epistemology have tended to propose variants of the following principle (schema).

(*) If S knows whether it is contingent whether \( \varphi \), then S is in a position to know a priori whether \( \varphi \).

Albert Casullo considers the existence of reasons to accept a close variant of (*), (**), to be ‘the central question of modal epistemology’ (2018: 2).

(**) If it is necessary that \( \varphi \) and S knows that it is not contingent whether \( \varphi \), then S is in a position to know a priori that it is not contingent whether \( \varphi \).

According to Casullo (2010: 357–58), (**) is an ‘intuitively plausible, widely accepted principle that […] faces no clear counterexamples’.

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1 We would like to thank Beau Madison Mount, Jens Kipper, and an anonymous referee for helpful discussions and comments. This work was supported by the Alexander von Humboldt Foundation.


3 In his words: ‘Here we face the central question of modal epistemology: Is there any reason to endorse (KB), i.e., (**)? (Casullo 2018: 2).

4 We take (**) to be the most charitable reconstruction of Casullo’s (2010, 2018) principles, which differ even among themselves. According to his 2010 principle, if \( \varphi \) is necessary and S knows that \( \varphi \) is noncontingent, then S can know a priori that \( \varphi \) is contingent. This principle is plausibly too weak if ‘S can know’ is interpreted as ‘It is metaphysically possible that S knows’ (why should it not be metaphysically possible?); we thus assume Casullo has something more like ‘S in a position to know’ in mind. According to his 2018 principle, if \( \varphi \) is necessary and S knows that \( \varphi \) is noncontingent, then S knows a priori that \( \varphi \) is noncontingent. We take this principle to be too strong to be of much interest. Nothing rules out someone knowing a posteriori, e.g. by testimony, that \( \varphi \) is noncontingent.
Kripke suggests that his own examples of the necessary a posteriori conform to both (*) and (**):

All the cases of the necessary a posteriori advocated in the text have the special character attributed to mathematical statements: Philosophical analysis tells us that they cannot be contingently true, so any empirical knowledge of their truth is automatically empirical knowledge that they are necessary. This characterization applies, in particular, to the cases of identity statements and of essence. It may give a clue to a general characterization of a posteriori knowledge of necessary truths. (Kripke 1980: 159, emphasis in the original)

Thus, for example, while one cannot know a priori that it is necessary that Hesperus is Phosphorus, one can nevertheless know a priori that it is not contingent whether Hesperus is Phosphorus.

While Kripke’s famous passage has no doubt encouraged optimism about principles like (*), Kripke was right to be cautious. (**) and thereby (*) face clear counterexamples due to C. Anthony Anderson (1993: 11–13). To modify one of Anderson’s examples, consider, a necessary truth $\varphi_N$ and a contingent truth $\varphi_C$ such that a subject $S$ knows a priori that it is not contingent whether $\varphi_N$, knows a priori that it is contingent whether $\varphi_C$, but is not in a position to know a priori whether $\varphi_N$ or whether $\varphi_C$, so is not in a position to know a priori whether $\varphi_N \lor \varphi_C$. (‘$\varphi_N$’ here could be replaced by any paradigm of the necessary a posteriori, such as ‘Hesperus is Phosphorus’.) Yet—suppose further—$S$ knows, but not a priori, whether it is contingent whether $\varphi_N \lor \varphi_C$, by knowing whether $\varphi_N$. Since $S$ is in a position to know a priori whether it is contingent whether $\varphi_N \lor \varphi_C$ only if $S$ is in a position to know a priori whether $\varphi_N \lor \varphi_C$, $S$ is not in a position to know a priori whether it is contingent whether $\varphi_N \lor \varphi_C$, and the case is a counterexample to both (*) and (**).

Anderson’s counterexamples to (*) involve replacing ‘$\varphi$’ with logically complex (i.e., non-atomic) sentences. Unsurprisingly, this has encouraged the thought that true principles could be obtained by restricting (*) and other similar schematic principles to atomic sentences. Casullo (2018), for example, endorses a restriction of (**) to atomic sentences. Kipper (2017) takes on the project of trying to construct counterexamples to a similar principle restricted to atomic sentences. Rather than discuss Casullo’s and Kipper’s principles separately, we will discuss a single principle that is not importantly different from them.\(^6\)

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\(^5\) See also Kripke (1971: 153).

\(^6\) Casullo’s principle is restricted to atomic sentences that express necessary truths. (See (KB) in Casullo 2018, and see his n. 1 for the restriction to atomic sentences.) This introduces a complication that we discuss in n. 8. The principle that Kipper discusses is

\[(\text{ANC}) \quad \text{The general modal status of any minimal sentential component of any } G\text{-necessary sentence can be known priori (Kipper 2017: 3).}\]

By ‘$G$-necessary’ Kipper means noncontingent (either necessarily true or necessarily false), and the ‘general modal status’ of a sentence is either contingency or noncontingency. The principle, then, is this: If $\varphi$ is noncontingent and $\psi$ is an atomic constituent of $\varphi$, then it can be known a priori whether $\psi$ is noncontingent. Note, however, that every atomic sentence is trivially an atomic constituent of a noncontingent sentence. (For example, any sentence—and therefore any atomic sentence—$\varphi$ is a constitut-
ATOMIC: If $S$ knows whether it is contingent whether $\varphi$, then $S$ is in a position to know a priori whether is contingent whether $\varphi$,

where $\varphi$ is atomic.

Kipper (2017) purports to have counterexamples to his variant of the above, but his examples are not entirely decisive. Each involves contentious assumptions concerning what is a priori knowable about natural kinds or—even more contentiously—a combination of such assumptions with a commitment to Chalmersian epistemic two-dimensionalist ideology (Chalmers 2006, 2010). If ATOMIC is false (i.e., has a false instance), one might hope to find a more decisive refutation.

A more decisive refutation does exist, as we will show. Counterexamples to ATOMIC arise in any language that has at least two singular terms and at least one predicate for a reflexive or irreflexive relation whose reflexivity or irreflexivity (under the guise of that predicate) we are in a position to know a priori. There is no need to wade into debates about natural kind terms or to take on any epistemic two-dimensionalist commitments in order to appreciate this fact, to which §2 is devoted. In §3 we will consider some natural proposals for even more restricted forms of (*), and we will show that the same examples that refute ATOMIC arise in different forms for those further restrictions, including a draconian restriction to atomic sentences with a one-place predicate that expresses a purely qualitative property. We cannot, of course, show that there are no true and non-trivial restrictions of ATOMIC, but in light of what we do show, we take it to be clear that any such restrictions must be so draconian that the resulting principles will be fairly uninteresting. The prospects for an a priori epistemology of modality in the spirit of Kripke’s famous passage thus look very dim indeed.

2. Reflexive and irreflexive relations

There are counterexamples to ATOMIC that are as clear as Kripke’s chief examples of the necessary a posteriori—namely, identity sentences. These involve predicates for certain relations whose modal features are intimately related to identity. In fact, as far as we can tell, all non-mathematical reflexive relations expressible by relatively simple verb phrases of English give rise to counterexamples to ATOMIC. Furthermore, very many non-mathematical irreflexive (i.e., anti-reflexive) relations expressible by relatively simple verb phrases in a natural language also give rise to counterexamples to ATOMIC. (Identity and distinctness, being logical relations, are also mathematical ones.) Here is an exercise: think of an English predicate for a non-mathematical reflexive relation. It is likely that the predicate you thought of—call it ‘$R$’—satisfies both (i) and (ii).

(i) $a = b$ iff it is not contingent whether $aRb$.

ent of the noncontingent sentence $\varphi \lor (\varphi \rightarrow \varphi)$.) (ANC), then, is equivalent to the principle that the general modal status of any atomic sentence can be known priori.
(ii) For some singular terms, ‘\(a\)', ‘\(b\)', we know that it is not contingent whether \(aRb\), but we are not in a position to know a priori that it is not contingent whether \(aRb\).

But then, of course (by (ii)), we have a counterexample to ATOMIC. Here is another exercise: think of an English predicate for a non-mathematical irreflexive relation. It is quite likely that the predicate ‘\(R\)' you thought of also satisfies (i) and (ii), and thus that we have a further counterexample to ATOMIC.

Let us consider some examples. The predicates for reflexive relations you are likely to have thought of have forms like ‘is at least as \(F\) as’, ‘has the same \(F\) as’, and ‘are similarly \(F\)’. The predicates for irreflexive relations you are likely to have thought of have forms like ‘is more \(F\) than’, ‘is less \(F\) than’, and ‘have different \(Fs\)’. Now consider the following sentences.

(1) Eminem is at least as tall as Marshall Mathers.

(2) Eminem is at least as old as Marshall Mathers.

(3) Eminem is more famous than Marshall Mathers.

We are not in a position to know a priori whether it is contingent whether any of (1)–(3) are true. Yet each is atomic, and we know that each is not contingent (because we know that Eminem is Marshall Mathers), so each is a counterexample to ATOMIC. (We cannot claim much originality here: Bird (2007: 176) uses examples similar to (1)–(3) to challenge (*). What seems not to have been noticed in the literature is that such examples show that restricting principles like (*) to atomic sentences won’t do.)

This is not the end of the dialectical story, of course. An advocate of ATOMIC might try to resist these counterexamples in two ways.

First, the advocate of ATOMIC might hope to avoid the counterexamples by arguing, on Millian grounds, that all true identities are knowable a priori.\(^7\) The idea is that, since (according to Millians) a singular term such as a proper name only contributes its referent to the propositions expressed by a sentence in which it occurs, ‘\(a = b\)' and ‘\(a = a\)’ express the same proposition whenever the former is true, and since we are in a position to know the latter proposition a priori we are also in a position to know the former a priori. Arguably, then, we are in a position to know ‘\(a = b\)’ a priori when it is true, since the proposition it expresses is one that we are in a position to know a priori (under the guise of a different sentence: ‘\(a = a\)’). If so, we are in a position to know the necessity of each of (1) and (2), since it follows deductively (and therefore a priori) from the a priori knowable truths:

\[
\text{Eminem} = \text{Marshall Mathers}
\]

If Eminem = Marshall Mathers, then it is necessary that Eminem is at least as tall as Marshall Mathers.

\(^7\) Scott Soames (2002: 236–37) is a prominent defender of this view. Thanks to an anonymous referee for suggesting that we consider this objection.
If Eminem = Marshall Mathers, then it is necessary that Eminem is at least as old as Marshall Mathers.

On this view, we are also in a position to know the impossibility of (3) a priori, because it is a deductive consequence of the a priori knowable fact that Eminem = Marshall Mathers and the further a priori knowable fact:

If Eminem = Marshall Mathers, then it is not possible that Eminem is more famous than Marshall Mathers.

One could also argue from Millianism to the a priori knowability of the non-contingency of (1)–(3) by noting that, according to Millianism, we preserve the proposition expressed by each of (1)–(3) when we replace each occurrence of ‘Marshall Mathers’ in these sentences with an occurrence of ‘Eminem’. Such replacements yield:

Eminem is at least as tall as Eminem

Eminem is at least as old as Eminem

Eminem is more famous than Eminem,

each of which we are in a position to know a priori to be non-contingent.

We do not find this Millian line of thought very plausible, but no matter: even if it is correct, we will find counterexamples to ATOMIC among sentences of the same form as (1)–(3). Consider:

(4) Eminem is at least as tall as Kanye West.

(5) Eminem is at least as old as Kanye West.

(6) Eminem is more famous than Kanye West.

We know each of (4)–(6) to be contingent, but we are not in a position to know this a priori, because, for all we are in a position to know a priori, Eminem = Kanye West, in which case each of (4)–(6) is non-contingent.8

Second, someone might object to the counterexamples to ATOMIC on the grounds that (allegedly) natural language semantics teaches us that (1)–(6) are not atomic sentences, but existentially quantified sentences in which the existential quan-

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8 These examples do not refute Casullo’s version of ATOMIC, which is restricted to necessary truths (see n. 7), but others involving proper names do, even assuming Millianism. Consider, for example, a sentence of the form ‘a was born prior to b’, where a is a biological parent of b. Given the necessity of biological parenthood ‘a was born prior to b’ will be a necessary truth but not knowable a priori to be noncontingent, contrary to Casullo’s principle. (Thanks to an anonymous referee for suggesting this example.)
tifier generalizes over states (such as the state of being more famous than Kanye West). It is far from clear that natural language semantics teaches us any such thing, but let us suppose, for the sake of argument, that it does. Even at best, this objection would save the letter but not the spirit of ATOMIC and other similar principles. Presumably the philosophers who propose such principles have in mind sentences like (1)–(6) when they speak of ‘atomic sentences’, and accordingly they would think of the non-a-priori-knowability of the (non)contingency of each of (1)–(6) as a counterexample to the principles they thought they were articulating. After all, according to the kind of semantic theory we have in mind hardly any natural language expression that we ordinarily call a ‘sentence’ is an atomic sentence, and it is not plausible that schematic principles like ATOMIC are intended to have no or virtually no instances where ‘\( \varphi \)’ is replaced by a natural language sentence. But, in any case, there is no hidden structure in formal languages, and formal languages with the syntax of first-order logic provide a plethora of counterexamples to ATOMIC.

Indeed, it bears emphasis that virtually any non-trivial first-order theory that deals with contingent subject matter will provide counterexamples to ATOMIC. Virtually any such theory will have at least one two-place predicate ‘\( R \)’ such that an atomic sentence ‘\( R(a, b) \)’ is noncontingent but not a priori knowably noncontingent when ‘\( a = b \)’ is true and is contingent but not a priori knowably contingent when ‘\( a = b \)’ is false. Such a predicate might, for example, express the relation of being at least as massive, which relates or fails to relate distinct individuals contingently and relates each individual to itself necessarily.

3. Chasing woozles

At this point the a priori modal epistemologist might try to restrict the principle still further. If a restriction to atomic sentences didn’t do the trick, why not try a restriction to atomic sentences of a particular form? Consider the following.

MONADIC: If \( S \) knows whether it is contingent whether \( Pa \), then \( S \) is in a position to know a priori whether it is contingent whether \( Pa \),

where ‘\( Pa \)’ is an atomic sentence formed by applying a one-place predicate to a singular term.

Our observations about (1)–(6) do not immediately refute MONADIC. But consider the sentence

\[ \exists e (\text{ killing}(e) \land \text{ Subj}(e, \text{ Brutus}) \land (\text{Obj}(e, \text{ Caesar})) \) \]

9 The view that a sentence whose main verb is an event verb, such as ‘Brutus killed Caesar’ has an existentially quantified logical form like ‘\( \exists e (\text{ killing}(e) \land \text{ Subj}(e, \text{ Brutus}) \land (\text{Obj}(e, \text{ Caesar})) \)’, is entirely mainstream (see Kratzer 2003), but the view that state sentences like (1)–(6) have an analogous existentially quantified logical form is less so. Terence Parsons’ observation that ‘[t]here is … evidence in favour of the underlying state approach for state sentences, but there is not as much of it, and it is not as easy to evaluate, as the underlying event approach for event sentences’ (1990: 186) remains a fair assessment of the evidence 28 years later.
(3*) \( \lambda x (\text{Eminem is more famous than } x) \text{(Marshall Mathers).} \)

(3*) is a counterexample to MONADIC if and only if (3) is a counterexample to ATOMIC—after all, (3*) is logically equivalent to (3). Note that further restricting MONADIC to sentences \( 'Pa' \) where \( 'P' \) is a simple predicate won’t help: there is nothing to stop us from introducing a simple predicate \( 'F' \) that expresses the property expressed by the complex predicate \( \lambda x (\text{Eminem is more famous than } x)' \). If we do so, \( 'P' \text{(Marshall Mathers)} \) will be a counterexample to the further restricted principle. There is, of course, something ‘artificial’ about such a predicate, but no matter: an artificial counterexample is still a counterexample.

The previous sentence may encourage some readers to consider restricting the principle even further, to sentences \( 'Pa' \) where \( 'P' \) is both simple and not ‘artificial’. This is not a promising strategy, for two reasons. First, it is extremely unclear what the new restriction means. Second, insofar as we have any idea about what ‘artificial’ would have to mean for the proposal to work, it seems highly plausible that there would be some counterexamples to MONADIC that involve predicates that are both simple and not ‘artificial’. Let ‘artificial’ mean something like ‘does not occur in any possible natural language or was introduced by a stipulative definition’. While it may be difficult to imagine the property expressed by \( \lambda x (\text{Eminem is more famous than } x) \) being lexicalized in any natural language except by means of a stipulative definition, actual natural languages contain many non-stipulatively defined one-place predicates—such as ‘French’—that express properties that things have or lack in virtue of being related in some particular way to some contingently existing object—such as France. Perhaps, for all we know (assuming that France = Gaul), ‘Gaul is French’ is a counterexample to the envisioned restriction of MONADIC. And if it isn’t one, it would be astonishing if there were not a similar counterexample in some possible natural language.

But one might well think that what makes any counterexamples to MONADIC work is not the ‘artificiality’ of the predicates involved but their non-qualitativeseness. A natural further restriction to MONADIC suggests itself: restrict the principle to sentences \( 'Pa' \) where \( 'P' \) is simple and expresses a purely qualitative property. The resulting principle is so weak as to be fairly devoid of interest, but never mind that—is it true?

The answer is clearly ‘No’: Let ‘Marshall’ name the proposition that Eminem is at least as tall as Marshall Mathers. Truth is a qualitative property if anything is, and so the sentence ‘Marshall is true’ is a counterexample to the new principle if and only if (3) is a counterexample to ATOMIC.

And again, one should resist the temptation to throw in an additional anti-‘artificiality’ condition. While ‘Marshall’ may be ‘artificial’, natural languages contain many non-stipulatively introduced names for propositions: ‘Marxism’, for example, at least on one disambiguation, refers to a proposition (although it is very unclear

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10 Of course, if ‘Gaul’ is understood as referring to a certain territory, it is plausible that we are in a position to know a priori that ‘Gaul is French’ is contingent. But suppose instead that ‘Gaul’ and ‘France’ refer to the same country. On that interpretation it seems that, for all we are in a position to know a priori, ‘Gaul is French’ might be contingent in the way ‘The Congo is Belgian’ is.
which one)—a thing that we can deny, accept, and take other propositional attitudes towards.

The idea that the epistemology of modality is in some sense a priori is a popular one. We have seen that trying to precisify this idea by (*) and various natural restrictions of it hopeless. Of course we are not in a position to say that there is no way of restricting (*) so that it comes out true. Trivially, any schema with false instances can be restricted so that it has only true instances—the limiting case of a restriction that leaves it with no instances guarantees this. But we take the foregoing reflections on increasingly draconian restrictions of (*) to pretty clearly show that any true restriction of (*) will be so weak as to be devoid of epistemological interest, and that post-Kripkean advocates of a priori modal epistemology are well advised to look elsewhere for a vindication of their grand vision.

References