MATHEMATICS AS AN EXACT AND PRECISE LANGUAGE OF NATURE

Afsar Abbas

Institute of Physics, Bhubaneswar - 751005, India afsar@iopb.res.in

ABSTRACT

One of the outstanding problems of philosophy of science and mathematics today is whether there is just "one" unique mathematics or the same can be bifurcated into "pure" and "applied" categories. A novel solution for this problem is offered here. This will allow us to appreciate the manner in which mathematics acts as an exact and precise language of nature. This has significant implications for Artificial Intelligence.

Human vocal cords are capable of producing a very large (but not unlimited) number of sounds of varying range in wavelength and frequency. As these sounds are to be distinguishable to the human ear, the number of acceptable sounds in a particular language get still more limited. When combined together into bunches, these provide a very large number of sound options available for humans to communicate with. Every language however chooses from amongst these large number of options to decide upon a few acceptable ones. Thus a limited number of sound combinations are chosen in a language to provide them with meaning and then the social group enforces their usage.

Clearly to start with a child has several options. It starts to make various sounds, experiments with them and relishes in them. But to survive and to be able to communicate with others it learns that only a few sounds are acceptable to the social group that it belongs to. So, if a child insists on calling a cat a "dog" and a dog a "cat" it soon learns to use the proper acceptable sound when it has to inform its parents as to whether it was a dog or a cat that bit him. A "rose" in English and "gulab" in Hindi or Urdu are different words for the same object. But the associations they are meant to convey are significant in those languages. Sound and its association is an arbitrary property of a particular language. Clearly cultural and sociological factors determine as to how a particular language develops.

Note that within the periphery of a particular language a sound which does not fall within the acceptable category is considered gibberish. For each acceptable sound in a language there are clearly many more gibberish sounds. Thus the range of gibberish sounds outside any language is much larger than that of acceptable sounds in the language.

The above statements are supported by the following definitions of language. Noam Chomsky defines it thus, "A language is a set (finite or infinite) of sentences each finite in length and constructed out of a finite set of elements" (Chomsky(1957)). Also as per Trager, "A language is a system of arbitrary vocal symbols by means of which the members of a society interact in terms of their" total culture" (Trager(1949)).

Just as English is the language of residents of London and Hindi or Urdu is of those residing in Delhi, mathematics is the language of scientists. And as the scientists when using mathematics are communicating about nature, mathematics turns out to be the language of nature.

The fact that mathematics is the language of nature has been known to

scientists for a long time. For example Galileo Galileo Had said (Galileo (1616)), "Philosophy (ie physics) is written in this grand book - I mean the universe - which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth".

Bertrand Russell (Russell (1931)) said, "Ordinary language is totally unsuited for expressing what physics really asserts, since the words of everyday life are not sufficiently abstract. Only mathematics and mathematical logic can say as little as the physicist means to say". James Jeans enthusiastically stated (Jeans (1930)), "God is a mathematician". And work to show that indeed mathematics is the language of nature has been actively pursued (Redhead (1975), Alan and Peat(1988), French (1999), Omnes (2005)).

It is clear that just about all the scientists and most of the philosophers would feel that mathematics is indeed the language of nature. However the mathematics that is usable as a language of nature is called "applied mathematics". Inherent in this word "applied" is the fact that there is a lot of mathematics which is not applied. This is the so called "pure" mathematics. That is, mathematics which has found no application in a description of nature. It acts outside any physical framework - a pure construct of human intellect as many a mathematician would have us believe. In fact, it is a dream of every mathematician to discover/invent a mathematics which can be labelled as "pure" - that is uncorrupted by any "lowly" application to the real world. There must be a thrill in creating something that is absolutely independent of any existing thing/concept/idea. Hardy boasted (Hardy (1940)), "I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

So clearly - though mathematics may be the language of nature (ie. the "applied" part), most of it is not (ie the "pure" part). What is that mathematics? It seems to have a Platonic world of its own. The logical positivists (Carnap (1995 Edition)) tried to understand this dichotomy by arguing that knowledge has two sources - the logical reasoning and the empirical experience. According to them logical reasoning shall lead to the analytical a priori knowledge. That would embrace the field of pure mathematics. While the

empirical experience would lead to the synthetic a posteriori knowledge and this would correspond to the applied mathematics. Explicitly or implicitly such a "dichotomous" point of view of the intrinsic structure of mathematics is held by most scientists, mathematicians and philosophers today. However, if indeed mathematics is the language of nature, then how come there is a smaller reservoir of language which nature communicates with (ie applied mathematics) but there is a larger reservoir of unused language (ie pure mathematics)? How and why does this unused language (" pure mathematics ") come into existence? No one has any reasonable understanding of it at present. This is an extremely unsatisfactory state of affairs and demands further inquiry. Though not often explicitly stated, this problem remains today as one of the most outstanding open issues in science/mathematics and its philosophy.

If anyone has doubts as to the seriousness of the issue, one need only read Reuben Hersh (Hersh (2005)). Therein he was reviewing Ronald Omnes' new book on philosophy of physics and mathematics (Omnes (2005)). He quotes Omnes, "The consistency of mathematics is therefore tantamount to the existence of mathematically expressible laws of nature." Thereafter Hersh goes on to say bitterly, "Minor glitches can be shrugged off. (A few oddball branches of math like higher set theory and nonstandard logics may not be physical, but who cares?) ". If this is how leading authorities feel about the issue, then what could be more urgent? Hersh further quotes Omnes," What exactly is the extent of the present mathematical corpus that is in relation to the mathematics of physics? I cannot say that I have analyzed this question carefully, but I considered it from time to time when reading papers in theoretical physics and mathematics". Hence clearly up to now, no philosopher or mathematician or scientist has understood what mathematics" really "is. Here I offer a novel solution to the conundrum.

Just as a child discovers the "correct" words to use for specific objects or ideas through social interactions, similarly a scientist learns the appropriate word for a particular physical reality by interacting with nature. However, while the word "rose" for a particular flower is culture determined and varies from language to language, the mathematical word for a particular physical object is exact and specific. It turns out that nature is very demanding and requires strict adherence to clear-cut mathematical rules to reveal its reality to scientists. Only through a tortuous and painstaking process of basically hit and trial method along with some judicious guesswork is it that a scientist

discovers the "correct" mathematical word or expression for a particular physical object or phenomena.

So basically, a priori, to a scientist there would exist a large number of mathematical options/words to accurately describe a particular physical phenomenon. One tries all. There is bound to be a stage where confusion reigns. That would be the initial stage wherein more than one mathematical model or terminology may appear to be applicable. History of science tells us that slowly with time and much effort, the physical reality will manifest itself by demanding and forcing upon scientists only one particular and unique mathematical structure. That will be the stage that the scientist would have discovered the "exact" word/phrase for that particular physical object/phenomena. No ambiguity about that (more on it below). Thus nature has allowed the scientist to read that particular "word".

The whole purpose of science is to continue to read nature through this "exact" mathematical language. Acquiring a larger vocabulary of this mathematical language leads to a greater fluency with nature.

Sometimes scientists would have to develop ab initio the necessary mathematics to understand physical reality. For example, to understand the empirically determined Kepler's laws of planetary motion, Newton had to develop the requisite calculus to do so. The very fact that Newton was actually able to acquire the necessary mathematical vocabulary made it possible for him to appreciate the effects of gravity. The physical 'book' of gravity was 'read' only because the necessary mathematics could be simultaneously developed. It was to 'read' the other physical effects as well, that Newton's contemporary, Leibniz was independently developing the required mathematics of calculus. Hence the requisite mathematical language of calculus was basic and essential to an understanding of gravity and dynamics in physical nature. Simply put, had it not been possible to develop the language of calculus, one would not have been able to read nature any better.

The basic mathematics of calculus could be developed by scientists themselves (ie. Newton and Leibniz) as fortunately it did not necessarily require a too sophisticated pre-existing mathematical framework. Their work was simplified by the fact that the foundations of calculus had already been laid by earlier mathematicians. It was not just for nothing that Newton had stated that he had risen on the shoulders of giants.

As there appears to be some confusion in the minds of many as to the issue of priority here, may I quote Richard Courant and Herbert Robbins

on this matter (Courant and Robbins (1996) p. 398), "With an absurd oversimplification, the "invention" of calculus is sometimes ascribed to two men, Newton and Leibniz. In reality, calculus is the product of a long evolution that was neither initiated nor terminated by Newton and Leibniz, but in which they played a decisive part."

Very often development in science is hampered if the adequate and appropriate mathematical framework does not exist. Therefore, had the algebra of tensors not been developed by Einstein's contemporary mathematicians, he would never have been able to give his equation of motion in General Theory of Relativity in 1915. This equation gives the force of gravity as an entirely pure geometry on the left hand side of the equation while all the other forcesstrong, weak and electromagnetic which describe all the matter particles and radiation, sit on the right hand side of the equation. This may be called the Ultimate Equation relating space, time and matter. This extremely beautiful and revealing equation describing nature could not have been 'read' but for the fortuitous contemporary development of tensor algebra.

However, note that if the ideas presented here are correct in mathematics is indeed the language of nature, then this would allow "anyone" (who is sufficiently prepared) to read it. And indeed in the case of the General Theory of Relativity the German mathematician A. Hilbert simultaneous to Einstein, had "read" the same equation. This is being revealed as further facts about the General Theory of Relativity are coming to light in recent years. This is in contrast to the earlier popular opinion that the General Theory of Relativity was the mysterious creation of Einstein and none other. That is, as per this opinion, had Einstein not been born there would have been no General Theory of Relativity today. Misreading of history can indeed make one appear like a blind person groping in a maze. (This groping would be in addition to what all scientists/mathematicians/philosophers have to do anyway as part of their work to understand and read nature - quite a demanding and challenging job in itself!). That view was at complete variance with the fact that correct mathematics is indeed the language of nature and practically anyone (proviso sufficient mathematical and scientific background is there) can "read" it!

As an attestation to the fact that mathematics is the language of nature, history of science is replete with examples where more than one scientist, 'read' the same language independently. The above example of Newton vs Leibniz and Einstein vs Hilbert are but two such cases.

When the quark model of particle/high energy physics was being developed to understand the structure of the umpteen number of particles being discovered experimentally in the 1950's and 1960's, a priori there were several group theoretical mathematical candidates for a scientific description of reality: the groups G_2 , F_2 , SU(2)xU(1) and SU(3) were good candidates for it. It was basically through the method of hit and trial that it was found that SU(3) group was the correct candidate for such a description. It was found that to understand the structure of particles like proton, neutron etc it was necessary to assume that they were made up of three kind of quarks which were named as up, down and strange (in the accepted nomenclature at present). As such nature 'forced' the scientists to read the word "SU(3)" in the quark model. Plainly stated they found that no other mathematical 'word' can do the job appropriately!

It maybe noted here that two scientists, Murray Gell-Mann and G. Zweig, independently and practically simultaneously, had come to the same conclusion that indeed SU(3) was the relevant group as discussed above. So to say, they both had been able to "read" nature correctly. Hence this is another example, in addition to the cases of Newton/Leibniz and Einstein/Hilbert as pointed out above, where proper mathematics, being the language of nature, allows itself to be "read" by more than one person at the same time.

Another example from particle/high energy physics is that of Quantum Chromodynamics, the theory of the strong interaction. In the 1970's and 1980's a priori several groups SU(2), U(2), SO(3) and SU(3) were reasonable candidates as the group theoretical/mathematical 'words' to describe strong interaction consistently. But experimental information and mathematical consistency forced the group SU(3) as the relevant group for the theory of the strong interaction to be built upon. It has been meticulously checked and found that SU(3) and none other is the right "word" for Quantum Chromodynamics or the theory of strong interaction. No scientist could have even in his wildest dreams ever thought of such a scenario right up to the 1960's.

So also is the example of the so called the Standard Model of particle physics which is built around the group SU(3)xSU(2)xU(1). This is the most successful model in particle physics as of now. This mathematical structure or word is the result of judicious speculation, meticulous experimentation and sheer hard work on the part of scientists all over the world during the last 100 years or so. It is important to realize that no other mathematical description can do what the Standard Model can do. Not that scientists did

not try other "words". They did - in fact they tried very hard indeed. But they always failed and were forced to accept the above group.

Another proof that indeed mathematics is the correct language of nature is the following. It also turns out that often a particular 'word' in mathematics is used for more than one physical situations. For example the group SU(2) as a mathematical language can describe the so called 'spin' degree of freedom and other independent 'isospin' or 'nusospin' degrees of freedom in physics (Abbas(2005)). Words/sounds do have independent existence. We give them a particular meaning by association. We match/map them to whatever physical object/concept we wish. For example I may call a nectar 'honey' and use the same word for my wife. As far as I am concerned the same word 'honey' is accurately describing/mapping the reality of nectar as well as my wife. Thus in the mathematical description of nature the group SU(2) can stand as the "word' for different physical entities like spin, isospin and nusospin. Also as we described a little earlier the group SU(3) stood for quarks in the quark model as well as for another independent framework of describing the strong interaction as the gauge force built up around this group - the so called Quantum Chromodynamics. This is a further proof that mathematics correctly read (the SU(2) or SU(3) groups here) is indeed the language of nature.

Well, good enough. However, this must be true for all that part of mathematics which can be labelled as "applied". But this constitutes only a small part of the whole mathematical edifice that exists today. What about the huge amount of "pure" mathematics. On the basis of what has been stated so far, "pure" mathematics should therefore be understood so as to belong to the honourable category of "gibberish". No offense meant, but as far as the language of nature is concerned, if the relevant applied mathematics is the exact and accurate vocabulary of nature, then necessarily the 'pure' mathematics must be treated as 'gibberish' in the framework of what one understands as a "language".

Just as a child can produce a large number of gibberish sounds in ordinary language so can a mathematician produce a huge amount of 'gibberish' mathematics. Just as the structure of the physical reality allows us to produce a large amount of gibberish sounds so also the mathematical reality of nature seems to be structured in a manner that it allows us to conceive of a huge amount of mathematical "gibberish". But the history of science is full of instances of mathematics which was considered as 'pure', as what the

discoverers of the same would have us believe, and which later turned out to be extremely valuable in physical applications. For example as to Hardy (who as we stated earlier was proud of the purity of his mathematics) it turned out that some of his work on infinite series in number theory today finds deep applications in cryptography/communication theory etc. When discovered, the mathematical matrices were thought to be beyond any applications in nature. Today, these form the bread and butter of quantum physicists. Hence what may be a mathematical "gibberish" today may turn out to attain the status of a proper and correct word in the language to describe nature as relevant "applied" mathematics.

If "all" of mathematics can be used to explain one or the other aspect of nature, then there would be no "gibberish" mathematics left. That would mean that whatever human mind is capable of producing in mathematics just can not go beyond some application or the other to nature. It is hard to say at this stage whether this is correct. Further work has to be done to see if this be indeed so. At this stage though, it appears that there is indeed a huge amount of mathematics which finds no application in the description or understanding of nature.

It should also be obvious that the way scientists learn to read the book of nature, this should be independent of any cultural, sociological, historical and personal bias. The mathematical - physical reality lies beyond our physical and existential limitations. The example of S. Ramanujan, S N Bose et al would clearly show that this is indeed so. Coming from completely different social and cultural background, these scientists/mathematicians were still able to read the book of nature in its exact mathematical formulation. In addition what they read could also be read by all the other scientists correctly. And as such, it must be that the mathematical mapping of physical reality, if done in the proper manner, is accurate and exact.

Amongst those who believe that mathematics is the language of nature there is still another issue of misunderstanding - and that is the issue of embedding. It is commonly believed that "as mathematics is more fundamental and larger in content, physics/science should be embedded in mathematics". For example (French (1999))," The relationship between mathematics and science is clearly of fundamental concern in both the philosophy of mathematics and the philosophy of science One possibility is to employ a model - theoretic framework in which "physical structures" are regarded as embedded in "mathematical ones" ". Continuing in the same strain, (Redlich

(1975)), "it is an "empirical - historical fact" that theories in physics can be represented as mathematical structures". Similar view is also expressed by Omnes (Omnes (2005)), "When a physical theory ... requires mathematics in the formalized corpus .. one can make the axioms necessary for the theory explicit, at least in principle, and follow the unfolding of ideas from these axioms into the mathematical corpus."

As per this view, mathematics forms a basic and fundamental structure and physics arises as a later structure which tries to gain legitimacy by embedding itself in this already preexisting structure. But this model leads to several interpretational problems. This is necessarily artificial in content as clearly this model is completely at variance with what has been presented by us above. As per what we are saying here, in fact it is actually physics which maps the primitive and basic reality of nature and in as much as mathematics is the language of nature, it continues to 'read' this book of nature. There is no question of embedding here.

Many persons (Russell (1931), Jeans (1930), Alan and Peat (1988), Redhead (1972), Shapiro (1977), French (1999), Omnes (2005)), including this author, have talked of mathematics as being the language of science or nature. If this is so, then the ability to handle mathematics should be linked to the ability to handle ordinary language grammar. However in a recent study Rosemary Varley et al (2005) studied three men with brain damage which affected their ability to handle grammar. However Varley et al found that these men retained full ability to do computations including recursion. They could even deal with structure - dependent concepts such as mathematical expressions with brackets etc. This clearly shows that the ability to handle the language of mathematics is entirely independent of one's ability to handle ordinary language. Thus though both (human languages say English/Hindi/Urdu and mathematics) are languages, these are essentially different in as much as they register differently in human brain. This difference should be basic rather than accidental.

Hence scientists, when making up theories have to find judicious combinations of these disparate aspects of the two languages to communicate with each other. So no wonder experts in one or the other of these two 'languages' miss appreciation of the total reality. Clearly this supports our contention here, as that of mathematics being the 'exact' language of nature.

As we have shown above, there are actually two independent "languages". Firstly the everyday language (like English, Hindi and Urdu) and secondly

the language of mathematics. The first one is imprecise and fuzzy while the other one is precise and exact. It would have been rather puzzling if these two were to register and be controlled by the same area of the brain and in the same manner. Because then it would have been hard to understand as to how the same area of the brain could produce imprecise and fuzzy language at the same time that it was producing another independent language which was precise and exact. It shows consistency of the ideas presented here that Varley et al found that indeed the two languages arise intrinsically in a different manner within the human brain.

If these two modes of languages register differently in the human brain, then it is possible that they arose at different periods of time during human evolution, due to different requirements of adaptation needed for them to become essential for survival. The first, the spoken language, arose as a result of social interaction and as a result of demands of survival for food etc. While the second, the language of mathematics, arose as a result of man's interaction with nature. As man spends more time with nature than with other human beings, the second language must be more naturally acquired than the first one. This point is also supported by the fact that other creatures have been interacting with nature for a longer period of time than what we humans have been doing. Do they have a language of mathematics? Indeed they do. When a bird needing to feed two chicks in its nest, actually brings back two insects to feed them, then indeed it has acquired the rudiments of mathematics. Hence it is clear that humans in the course of evolution must have learnt elements of the language of nature well before they learnt to speak. Therefore, it may come as a surprise to some, but mathematics as a language of nature, albeit in a more elementary form, must have been available to species other than homo sapiens. Indeed current research shows that acquisition of spoken language may be a much later development in human evolution. In fact, the growth of the human brain and the faculty of (spoken) language acquisition may have been simultaneous (Deacon (1992)).

"Mistakes" are an inherent part of mathematics. Do these mistakes occur accidentally or are they basic to mathematics itself? Rene Descartes thought mistakes by mathematics were inadvertent. Charles Peirce thought that these were due to lapse of memory. Ludwig Wittgenstein stated that actually mathematics was a subject in which it was possible to make mistakes. In fact in mathematics it is impossible not to make mistakes. Riemann used

what he called "Dirichlet's Principle" incorrectly. Hilbert incorrectly thought that he had proven the Continuum Hypothesis. There are umpteen examples. These are only for such recorded and well known written cases. But when an individual scientist/mathematician, during his private moments, in trying to go beyond known mathematics, keeps on making mistakes. He struggles through a maze of mistakes and then arrives at whatever he thinks is consistent mathematics and is "publishable". Only these are what we hear of and what one talks of. These private mistakes should also be considered "mistakes" in mathematics. These are too innumerable and are often well kept secrets (as never uttered) to be recorded here!

Just as a child when speaking a language tries to experiment with sounds and names, it discovers new sounds (gibberish) and in fact enjoys doing it. That gibberish would be proven to be a "mistake" due to social pressures and ultimately abandoned by the child. But clearly such mistakes are part of the very process of speech learning. So also are mistakes in mathematics. As nature is very demanding and requires strict adherence to its "intrinsic" mathematics so even gibberish mathematics would have rules of consistency. Hence though mistakes would be made these would be subsequently corrected. In fact mistakes would be an inevitable part of discovering new mathematics. This analogy too shows that indeed mathematics is a "language" (of nature).

A thought, on as to how two different mathematics - pure and applied arise. In everyday language I am free to visualize objects which are half man - half woman or part horse - part dog - part snake or part cow - part human - part bird - part elephant etc. I can visualize groups of these as coexisting with humans. In such cases my imagination allows me to cook up all kind of "realities" and thus think that I am able to visualize that these may have some kind of existence in a Platonic world of its own. However quite clearly we should call all these objects and their interactions etc as nothing but gibberish, in as much as we know that these do not correspond to any realistic objects. However it may be that in spite of being unaware of geology and palaentology, that on observing a lizard at close quarters. I may be able to imagine of a time when earth may have had extremely large and dangerous lizards living on it. That hypothesis may later on be actually proven to be correct by palaentologists as those creatures having been dinosaurs. So there is a small, though non-zero chance that some of my gibberish imaginings may turn out to reflect some aspect of reality in future.

In the same manner, mathematics being a precise and exact language of nature - may allow us to cook up new "realities". Imagine new mathematics - which is part this and part that, leave an axiom here, add a lemma there, bring in some geometry and add some algebra etc. Do I have a consistent mathematical framework? I may make mistakes as discussed above, but then these would be corrected in due course of time. However, the fact that the known mathematics on the basis of which I am trying to go beyond, does have consistency, hence my new mathematics should have consistency too. If not then we abandon it. Since different known mathematical subdisciplines are related to each other, hence this new mathematics may have similar consistency and inter-relationships. I may call my new structure as "pure" mathematics. However, this would be "gibberish" mathematics as we discussed earlier. It is nevertheless possible that in future the same structure may find applications in the description of nature. Hence the Platonic world of mathematics would be no more real than the Platonic world we discussed in the social context above.

Phonologists have shown that phonemes in individual language families are quite different. This is so because as we grow up we acquire certain pronunciation habits that are determined by the sound patterns permitted in a particular language (Hjelmslev (1970)).

An anecdote would not be out of order. A British Scientist was in Japan to attend an International Conference. During a session, a young Japanese student gave a presentation of his work. The transparencies were written in English. After the oral presentation, a senior Japanese scientist, sitting next to him, turned to him and said, "He is working under me. What do you think of the work?". The British replied. "I don't really know. I would have understood it better had he spoken in English." To which the Senior Japanese replied, "But he was speaking in English!".

So though a written language may be "read" by anyone in principle, the spoken language demands proper pronunciation which too identifies a language. As Roman Jakobson has said (Jakobson and Halle (1956)),

"As regards the combination of linguistic elements there exists an increasing degree of freedom. But when dealing with the combination of distinctive features to phonemes. freedom does not exist for the individual speaker - the code has already established all the possibilities that can be realized in the given language."

As we have stated here, in terms of the appropriate applied mathematics,

we are learning the proper words in the language of nature, But as per above, how do we "pronounce" it? As we saw from the anecdote, proper pronunciation or lack thereof can provide or destroy universality in a language. If indeed proper mathematics is the language of nature then its "pronunciation" should be universal and exact too.

But what would one identify as "pronunciation" in mathematics? Here I would like to present an hypothesis as to what should be taken as "pronunciation" when communicating in the language of nature.

Group theory of mathematics has been extensively applied in physics. It it used to classify particles and in fact by a mathematical process called " gauging" these actually even define forces between particles. A priori there are several groups as candidates to be used to describe a particular physical phenomenon. These are for example: infinite series of groups SO(n), SU(n) and Sp(n) (for any $n = 1, 2, 3, 4, 5, \dots$ to infinity) and G2, F4. E6, E8 etc. So why was it that in the 1960's scientists discovered that to give proper description of reality of particles only the group SU(3) with three quarks labelled up, down and strange was the "correct" procedure? Why was it that it was the group combination SU(3) x SU(2) x U(1) that was found to be necessary in the successful Standard Model of particle physics. I propose here that a priori all these groups were options for "sound production" to provide different "pronunciations" for that particular "word" in the language of nature. Nature being precise and exact chose the "pronunciation" as "SU(3)" for the quark model discussed above. And similarly it is the sound pronunciation which is fixed in SU(3)xSU(2)xU(1). Hence as per the suggestion here the exactness in the group representation is precise and exact fixing of the sound pronunciation by nature so that unlike the spoken language, there is no ambiguity in mathematics.

Particles have fixed quantum numbers which are used to identify them. For example lepton numbers for electron and neutrino, baryon number for quarks, protons and charge number for electron, protons etc. What is the nature of these quantum numbers? In terms of what has been stated above this is just to fix the pronunciation to describe and identify the different "races"/classes in the "genealogy"/classification of matter in nature.

Scientists have also discovered that it is an empirical fact that the law of gravity and that of electromagnetic forces is inverse square of distance and not any other, say a cubic or a fraction or any other power of distance. This too, in view of what has been stated above, should be understood as a precise

reading of the language of nature. What the corresponding "words" imply and how these should be interpreted in the context of a language is an open issue and calls for work in future.

Within the field of Artificial Intelligence, it is important to know as to how humans actually acquire knowledge, which is so intrinsically related to language. Clearly the fact that the spoken human language is basically different from the language of nature (mathematics), should be a significant fact for AI scientists. Language is more complex than what was thought of so far. Hence we have to redefine what we mean by intelligence in the first place. This prompts for further work.

In summing up, mathematics is a precise and exact language. As such most of the sounds/words in it are gibberish (pure mathematics) and the rest are relevant and useful sounds (applied mathematics) which maps the physical reality in an accurate manner. In the explanation presented here there is no dichotomy of mathematics as in the view of say the logical positivists (and may I say that of most of the philosophers of science as well). In addition nature also allows us to be able to "pronounce" the words correctly.

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