CHAPTER 7

Mathematical Cooperation

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1. SOCIAL MODELS

Complex dynamical systems theory provides a modeling strategy for social systems, which are usually too complicated to model without a theory which allows chaos and bifurcation. These new models contribute to the hermeneutical circle for evolving social structures, in which mathematical help in understanding may be very welcome. Even the simplest social systems, such as two persons or two nations, tax our intuitive cognitive strategies. Dynamical models may be used as navigational aids for cooperation, or conflict resolution, in many situations in which good will prevails, yet does not suffice. Here we give a few examples of this art of building social models, *erodynamics*.

2. FIRST STEPS

Newton, soon after his development of the calculus in 1666, became interested in world history and prehistory. He pursued applications of astronomy to the *chronology of ancient kingdoms*, and probably envisioned dynamical models for cultural evolution. Our first recorded models for social systems are the Verhulst model for population growth, of 1837, and Richardson's model for the arms race, of 1919. This sequence accelerated after World War II with the syntheses of general systems theory and cybernetics. In the mathematical branch of these movements, *systems dynamics*, we have the extensive development of models for factories, cities, nations, the world monetary system, and many other systems. The work of Jay Forrester was central to this growth. The independent development of dynamical systems theory after Poincaré remained aloof from social applications until recently, and now a reunion of these two branches of mathematics is underway. In the Poincaré lineage, came the development of catastrophe theory by René Thom, its extensive application to social systems by Christopher Zeeman, and new dynamical models for economic systems by Steve Smale, in the 1970s. Since then, chaos theory has discovered systems with complex structure, and systems dynamics has discovered chaos.

3. ARMS RACES

Lewis Frye Richardson was an English physicist, meteorologist, and Quaker. A conscientious objector in the first World War, he served as an ambulance driver on the front lines in France, and saw a great deal of death and suffering. He decided to devote his life to the elimination of war. He developed a *linear model* for the arms race between two nations, in which a spiral of increasing armaments in each nation resulted from mathematical laws, as shown in Figure 1. He felt that the individual nations caught in this kind of dynamic were innocent victims of an out-of-control global system. He submitted a paper on this model to a journal, fully confident that another war could be averted. However, as the paper was rejected, the second World War began. After this rejection, Richardson continued his work, trying to justify the model on the basis of actual armament statistics. In these efforts, he founded the field of *politicometrics*. In 1935, Gregory Bateson adapted the Richardson model to the process of the division of a culture into subcultures, analogous to differentiation in biological systems. He called this universal dynamical process for the development of a schism a *Richardsonian process of schismogenesis*.[1] Richardson's life work was published posthumously in 1968. In the 1970s, Isnard and Zeeman replaced this with a *nonlinear model*, the cusp catastrophe of Thom's theory, shown in Figure 2. They applied their model to the original arms race context of Richardson's work, showing how the model fit a situation of schismogenesis, in which the voting population of a democratic nation split into *hawks* and *doves*. Zeeman also adapted the cusp to model *anorexia nervosa*, an emotional disease in which gluttony and fasting alternate.[2] In 1985, Kadyrov, a systems scientist in Moscow, put together two of these cusp models into a *double-cusp model* for two nations engaged in an arms

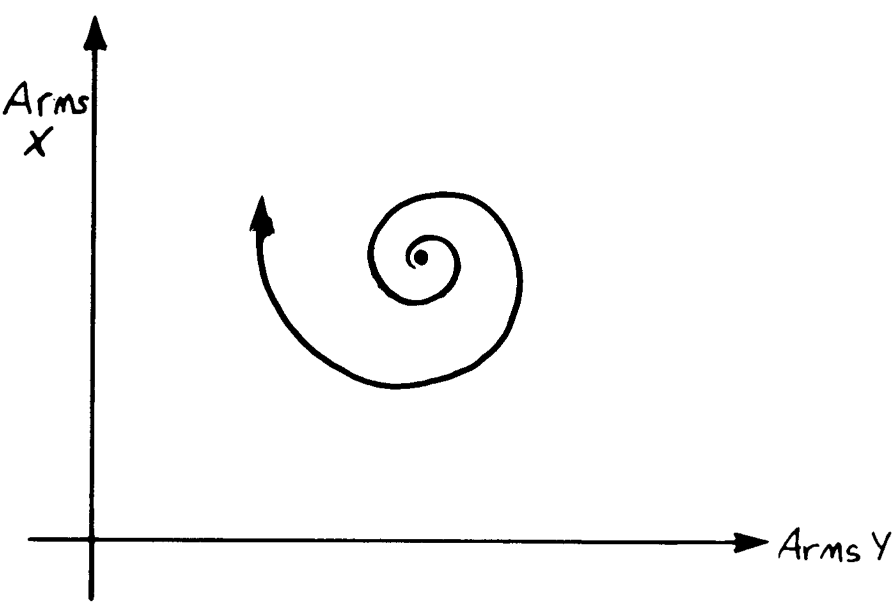

Figure 1. Richardson's spiral process.

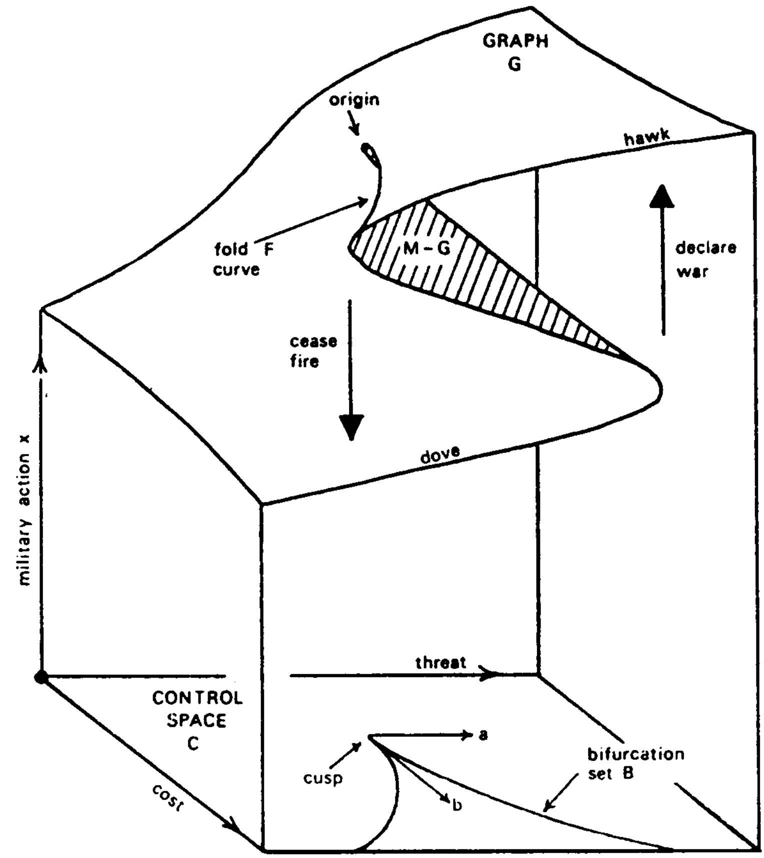

Figure 2. The cusp model.

race, completing the nonlinear version of Richardson's original model. It provides a map, in the two-dimensional space of sensitivities of each nation to armaments of the other, showing regions of different behaviors, such as hawks and hawks, hawksand doves, doves and hawks, and doves and doves, as shown in Figure 3. In the central region, labelled 4, there are four point attractors. In the two quadrants labelled 2, there are two point attractors. Surprisingly, in the north-west and south-east sectors of this map, Kadyrov found oscillating behavior. This might be significant in situations of codependence or addictive behavior.[3] A slightly different double cusp map was used by Callahan and Sashin in the treatment of anorexia nervosa.[4] Some other nonlinear adaptations of Richardson's model for the arms race have been studied by Mayer-Kress and Saperstein, who found chaotic behavior in their model.[5]

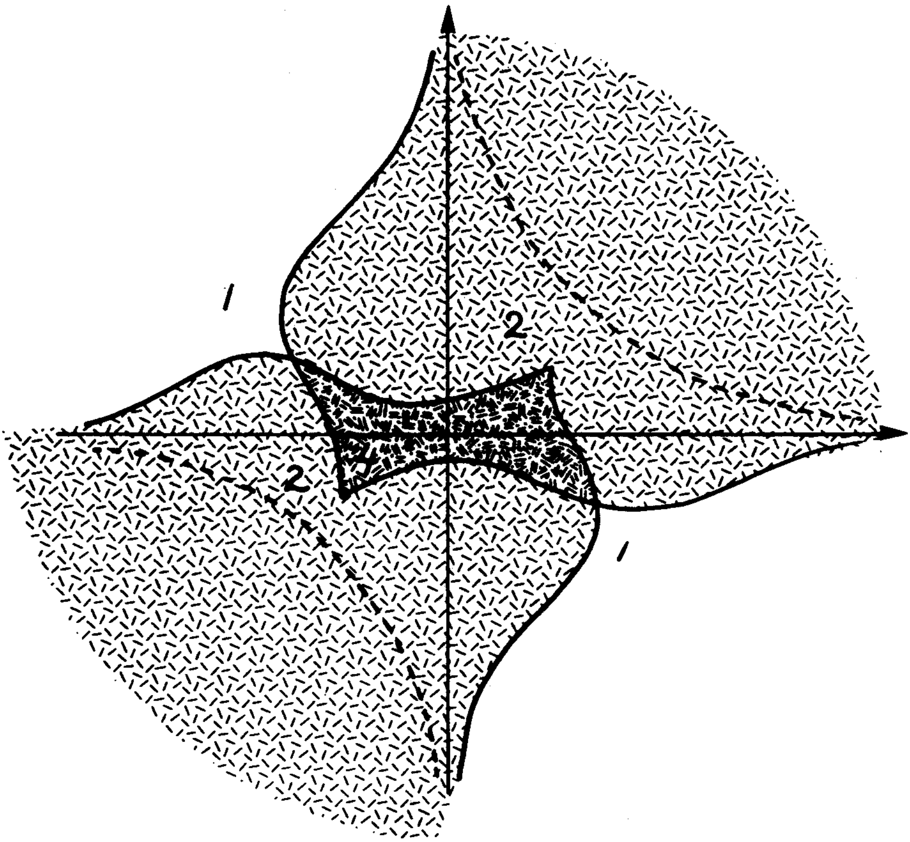

Figure 3. Kadyrov's map.

4. COOPERATION

Our goal in this section is to show that in a complex dynamical environment, a complex dynamical model may be used as a navigational aid, to improve the chances of achieving a mutually desired goal. Various examples of this navigational approach, basic to control theory, have been applied to politics (conflict resolution), medicine (surgical and pharmaceutical intervention), psychiatry (therapeutic strategies), and in many other areas. We may consider, as a typical application, the cooperation of two nations, or multi-national corporations, in trade, and use the Kadyrov model as an example. In this hypothetical situation, the control parameters in the double cusp model (the plane shown in Figure 3) would select strategies such as trade restrictions, tariffs, credit limits, and so on. The state of each partner, rather than stockpiles of weapons, will be inventories of products.[6] The two partners sit down to talk, and each must restrict their agreements according to the supervision of some group of stockholders or voters, which are subject to a process of schismogenesis into two camps, say conservatives and progressives. In the absence of a model, the complex dynamics of these four influence groups overwhelms and frightens the negotiators, and they may wish to risk little. They may at best make small adjustments in the status quo, and wait to observe the results and reactions.[7] But now let us suppose that a complex dynamical model is at hand, the Kadyrov model for example, and that confidence in it, in this application, is firmly established in experience. The trading partners then may simply consult their

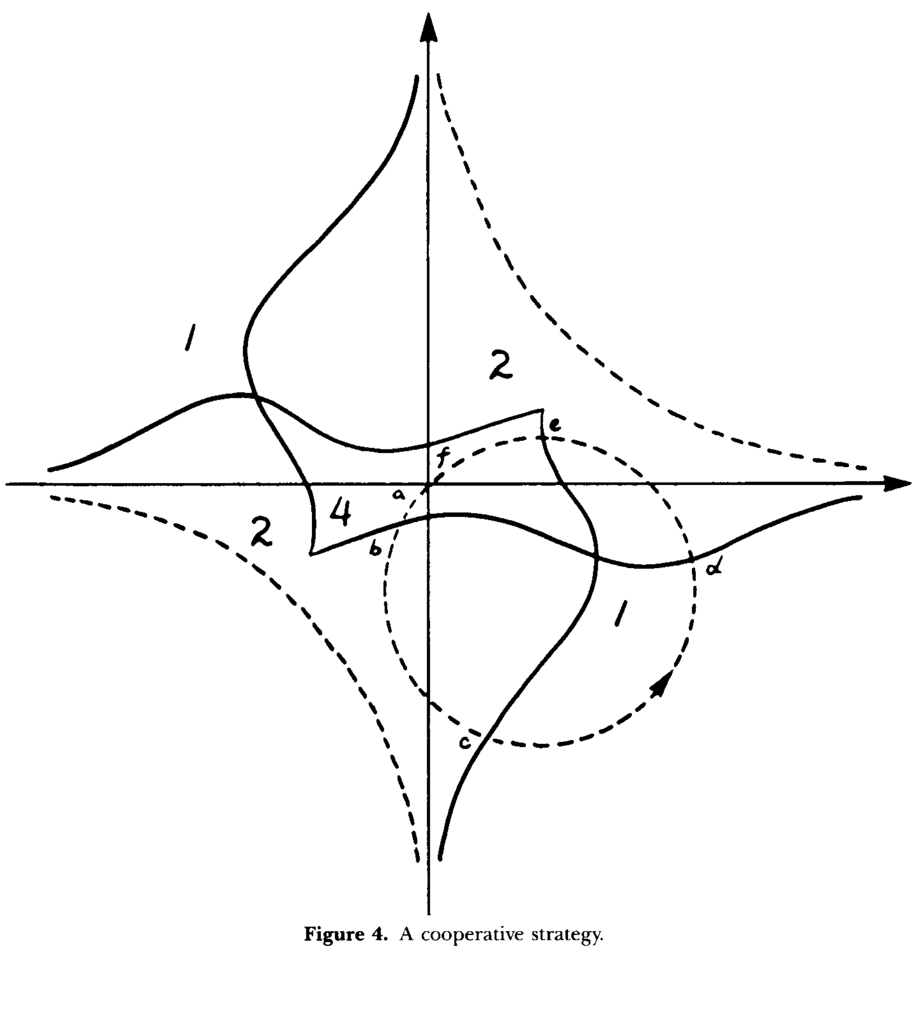

Figure 4. A cooperative strategy.

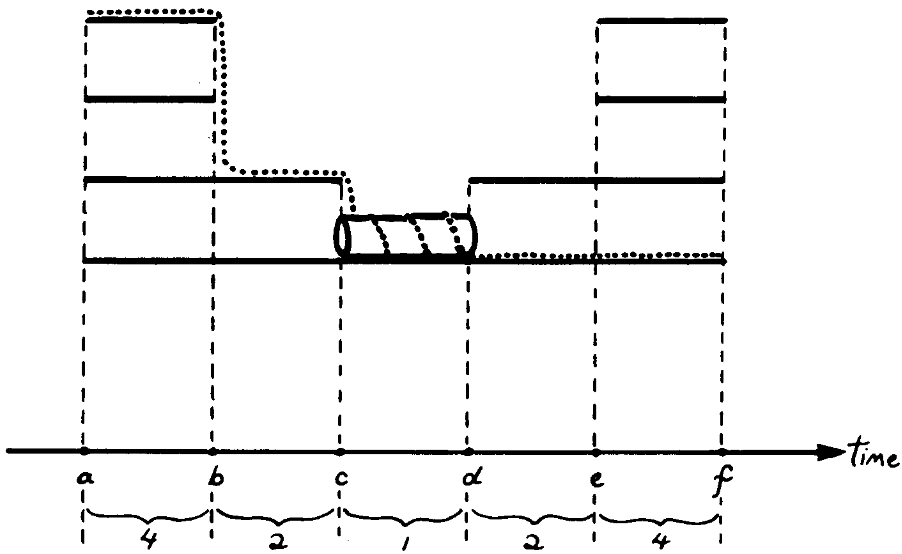

Figure 5. The bifurcation sequence.

data—quarterly statements, opinion polls, or whatever—and find their positions on the model map. This map applies to them jointly, not individually. They may then cooperate in choosing a strategy, a timed sequence of steps, from among the continuum of choices, with more courage than fear. One possibility is shown as the dotted curve in Figure 4. It is a short loop, to be traversed in a sequence of steps over a period of time, on a strict schedule. It is a closed loop in the space of strategies. And yet, while counter-intuitive, it achieves the mutually desired result with a minimum of cost. It leads the state of the joint system from the current state to a new state, after a sequence of bifurcations, shown in Figure 5. A similar approach has been used by Callahan and Sashin in the treatment of anorexia. It seems unlikely that this phased strategy shift would occur to the partners without a model. The oscillations in the Kadyrov model have been avoided as much as possible in this application, as they may be expensive states to maintain. However, if the subjects are lovers rather than trading partners, they may wish to maximize these vibrating states.

5. CONCLUSION

Here we rest our case. *Chaos*, the mathematics of complex dynamical systems, is in its infancy, and so are the social sciences. Joining forces under the banner of *Eros*, they may aid our future evolution in harmony with *Gaia*.

Acknowledgments

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Notes

1. See Bateson, 1972, p. 68.
2. For all of these cusp models, see Zeeman, 1977. For other psychological applications, see Postle, D.
3. See Abraham, 1991, for a description of this relatively inaccessible paper.
4. See Callahan, 1987.
5. See Saperstein.
6. The inspiration for this model is Morito, 1986, *Can This Be Love?*
7. This is the classical prisoner's dilemma, see Axelrod, 1984.

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