# The relativity of inertia and reality of nothing

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**Abstract**

The determination of inertia by matter is looked at in general relativity, where inertia can be represented by affine or projective structure. The matter tensor $T_{ab}$ seems to underdetermine affine structure by ten degrees of freedom, eight of which can be eliminated by gauge choices, leaving two. Their physical meaning—which is bound up with that of gravitational waves and the pseudotensor $t_{ab}$, and with the conservation of energy-momentum—is considered, along with the dependence of reality on invariance and of causal explanation on conservation.

## 1 Introduction

The indifference of mechanical phenomena and the classical laws governing them to translation, to absolute position has long been known. This 'relativity' extends (with appropriate restrictions) to the first derivative, velocity, but not to the second, acceleration, which—together with its 'opposite,'[1] *inertia*—has a troubling absoluteness. Disturbed by it Mach sought to make inertia relative, to bodies and their distribution, but never got as far as a genuine theory embodying his proposals. General relativity can be viewed as Einstein's attempt to implement them, and it is there that we consider the determination of inertia by matter. Following Einstein we take the energy-momentum tensor $T_{ab}$ to represent matter, in 3, where we briefly consider the issue of distant masses. Inertia on the other hand can be represented by affine or projective structure, as we see in 4. In 5 matter appears

[1] In the sense that motion is inertial when acceleration vanishes. This can also be understood, in more Aristotelian terms, as a contraposition of 'natural' (inertial) and 'violent' (accelerated) motion. Weyl [1] has a corresponding *Dualismus zwischen Führung* (guidance) *und Kraft* (force).

to underdetermine inertia by ten degrees of freedom, eight of which are made to 'disappear into the coordinates,' in 6. The status of the remaining (double) freedom is related to that of gravitational waves, whose physical reality is expressed by the controversial pseudotensor $t_{ab}$ and examined in 7. The troublesome energy conservation law, which involves $t_{ab}$, is looked at in 8. A thought experiment to reveal absolute motion with a Doppler effect using gravitational waves is proposed in the final section.

## 2 Background

Newton distinguished[2] between an "absolute" space he also called "true and mathematical," and the "relative, apparent and vulgar" space in which distances and velocities are measured. Absolute position and motion were not referred to anything. Leibniz identified unnecessary determinations, excess structure[3] in Newton's 'absolute' kinematics with celebrated arguments resting on the *principium identitatis indiscernibilium*: as a translation of everything, or an exchange of east and west, produces no observable effect, the situations before and after must be the same, for no difference is discerned. But there were superfluities even with respect to Newton's own dynamics (*cf.* [4] p. 178), founded as it was on the proportionality of force and acceleration. Galileo had already noted the indifference[4] of various *effetti* to inertial transformations; the invariance[5] of Newton's laws would more

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[2] [2]: "[...] convenit easdem [i.e. tempus, spatium, locum et motum] in absolutas & relativas, veras & apparentes, Mathematicas et vulgares distingui." [3] For a recent treatment see [3] pp. 76-80. [4] [5] *Giornata seconda*: "Riserratevi con qualche amico nella maggiore stanza che sia sotto coverta di alcun gran navilio, e quivi fate d'aver mosche, farfalle e simili animaletti volanti; siavi anco un gran vaso d'acqua, e dentrovi de' pescetti [...]: e stando ferma la nave, osservate diligentemente come quelli animaletti volanti con pari velocità vanno verso tutte le parti della stanza; i pesci si vedranno andar notando indifferentemente per tutti i versi; e voi, gettando all'amico alcuna cosa, non più gagliardamente la dovrete gettare verso quella parte che verso questa, quando le lontananze sieno eguali; e saltando voi, come si dice, a piè giunti, eguali spazii passerete verso tutte le parti. Osservate che avrete diligentemente tutte queste cose, benché niun dubbio ci sia che mentre il vassello sta fermo non debbano succeder così, fate muover la nave con quanta si voglia velocità; ché (pur che il moto sia uniforme e non fluttuante in qua e in là) voi *non riconoscerete una minima mutazione in tutti li nominati effetti, né da alcuno di quelli potrete comprender se la nave cammina o pure sta ferma* [...]" (my emphasis). [5] *Corollarium V*: "*Corporum dato spatio inclusorum iidem sunt motus inter se, sive spatium illud quiescat, sive moveatur idem uniformiter in directum sine motu circulari.* [...] Ergo per legem II æquales erunt congressuum effectus in utroque casu; & propterea manebunt motus inter se in uno casu æquales motibus inter se in altero. [...] Motus omnes eodem modo se habent in navi, sive ea quiescat, sive moveatur uniformiter in directum."

concisely express the indifference of all the *effetti* they governed.[6]

Modern notation, however anachronistic, can help sharpen interpretation. The derivatives

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} \qquad \ddot{\mathbf{x}} = \frac{d\dot{\mathbf{x}}}{dt}$$

are quotients of differences; already the position difference

$$\begin{aligned}\Delta\mathbf{x} &= \mathbf{x}(t+\varepsilon) - \mathbf{x}(t)\\ &= \mathbf{x}(t+\varepsilon) + \mathbf{u} - [\mathbf{x}(t) + \mathbf{u}]\end{aligned}$$

is indifferent to the addition of a constant $\mathbf{u}$. The velocity

$$\dot{\mathbf{x}} = \lim_{\varepsilon\to 0} \frac{\Delta\mathbf{x}}{\varepsilon}$$

is therefore unaffected by the three-parameter group $\mathcal{S}$ of translations $\mathbf{x} \mapsto \mathbf{x} + \mathbf{u}$ acting on the three-dimensional space $E$. The difference

$$\begin{aligned}\Delta\dot{\mathbf{x}} &= \dot{\mathbf{x}}(t+\varepsilon) - \dot{\mathbf{x}}(t)\\ &= \dot{\mathbf{x}}(t+\varepsilon) + \mathbf{v} - [\dot{\mathbf{x}}(t) + \mathbf{v}]\end{aligned}$$

of velocities is also indifferent to the addition of a constant velocity $\mathbf{v}$. The acceleration

$$\ddot{\mathbf{x}} = \lim_{\varepsilon\to 0} \frac{\Delta\dot{\mathbf{x}}}{\varepsilon}$$

is therefore invariant under the six-parameter group $\mathcal{S} \times \mathcal{V}$ which includes, alongside the translations, the group $\mathcal{V}$ of the inertial transformations $\mathbf{x} \mapsto \mathbf{x} + \mathbf{v}t$, $\dot{\mathbf{x}} \mapsto \dot{\mathbf{x}} + \mathbf{v}$ acting on the space-time $\mathbb{E} = E \times \mathbb{R}$.

Newton's second law[7] is 'covariant' with respect to the group $\mathcal{R} = \mathrm{SO}(\mathcal{S})$ of rotations $R : E \to E$, which turn ($F \mapsto RF$) the "lineam rectam qua vis illa imprimitur" with ($\ddot{\mathbf{x}} \mapsto R\ddot{\mathbf{x}}$) the "mutationem motus":

$$[F \sim \ddot{\mathbf{x}}] \mapsto [RF \sim R\ddot{\mathbf{x}}] \Leftrightarrow [F \sim \ddot{\mathbf{x}}].$$

Including the group $\mathcal{T}$ of temporal translations $t \mapsto t + a \in \mathbb{R}$, we can say the second law is indifferent[8] to the action of the ten-parameter Galilei group (see [6])

$$\mathcal{G} = (\mathcal{S} \times \mathcal{V}) \rtimes (\mathcal{T} \times \mathcal{R})$$

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[6] On this distinction and its significance in relativity see [4], where the *effetti* are called "factual states of affairs." [7] "Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur." [8] For Newton's forces are superpositions of fundamental forces $F = f(|\mathbf{x}_2 - \mathbf{x}_1|, |\dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1|, |\ddot{\mathbf{x}}_2 - \ddot{\mathbf{x}}_1|, \dots)$, covariant under $\mathcal{G}$, exchanged by pairs of points.

with composition

$$(\mathbf{u}, \mathbf{v}, a, R) \rtimes (\mathbf{u}', \mathbf{v}', a', R') = (\mathbf{u} + R\mathbf{u}' + a'\mathbf{v}, \mathbf{v} + R\mathbf{v}', a + a', RR'),$$

$\rtimes$ being the semidirect product. A larger group would undermine the laws, requiring generalisation with other forces.

Cartan undertook such a generalisation,[9] with new laws and forces, in 1923. The general covariance of his Newtonian formalism (with a flat connection) may seem to make inertia and acceleration relative, but in fact the meaningful acceleration in his theory is not $d^2x^a/dt^2$, which can be called *relative*[10] (to the coordinates), but the *absolute*

$$A^a = \frac{d^2x^a}{dt^2} + \sum_{b,c=1}^{3} \mathit{\Gamma}^a_{bc} \frac{dx^b}{dt}\frac{dx^c}{dt} \tag{1}$$

($a = 1, 2, 3$ and the time $t$ is absolute). Relative acceleration comes and goes as coordinates change, whereas absolute acceleration is generally covariant and transforms as a tensor: if it vanishes in one system it always will. The two accelerations coincide with respect to inertial coordinates, which make the connection components

$$\mathit{\Gamma}^a_{bc} = \langle dx^a, \nabla_{\partial_b} \partial_c \rangle$$

vanish, where $\langle \alpha, v \rangle$ is the value of the covector $\alpha$ at the vector $v$, and $\partial_a = \partial/\partial x^a$ is the partial derivative operator representing the (basis) vector tangent to the $a$th coordinate line. The absolute acceleration of inertial motion vanishes however it is represented—the connection is there to cancel the acceleration of noninertial coordinates.

So far, then, we have two formal criteria of inertial motion:

- $\ddot{\mathbf{x}} = \mathbf{0}$ in Newton's theory
- $A^a = 0$ in Cartan's.

[9] In [7]. Cartan's theory is dealt with extensively in [8]. [10] In [9] there appears to be a confusion of the two accelerations as they arise—in much the same way—in general relativity. The acceleration $d^2x^a/d\tau^2 \neq 0$ Baker sees as evidence of the causal powers possessed by an ostensibly empty spacetime with $\Lambda \neq 0$ is merely *relative*. Even with $\Lambda \neq 0$ free bodies describe geodesics, which are wordlines whose *absolute* acceleration vanishes. The sensitivity of projective structure to the cosmological constant would seem to be more meaningful, and can serve to indicate similar causal powers.

But Newton's criterion doesn't really get us anywhere, as the vanishing acceleration has to be referred to an inertial frame in the first place; we shall return to Cartan's in a moment.

Einstein ([10] p. 770) and others have appealed to the *simplicity of laws* to tell inertia apart from acceleration: inertial systems admit the simplest laws. Condition $\ddot{\mathbf{x}} = \mathbf{0}$, for instance, is simpler than $\ddot{\mathbf{y}}+\mathbf{a} = \mathbf{0}$, with a term $\mathbf{a}$ to compensate the acceleration of system $y$. But we have just seen that Cartan's theory takes account of possibile acceleration *ab initio*, thus preempting subsequent complication—for accelerated coordinates do not appear to affect the syntatical form (*cf.* [4] p. 186) of (1), which is complicated to begin with by the connection term. One could argue that the law simplifies when that term disappears, when the coefficients $\Gamma^a_{bc}$ all vanish; but then we're back to the Newtonian condition $\ddot{\mathbf{x}} = \mathbf{0}$. And just as that condition requires an inertial system in the first place, Cartan's condition $A^a = 0$ requires a connection, which is equivalent: it can be seen as a convention stipulating how the three-dimensional simultaneity surfaces are 'stitched' together by a congruence of (mathematically) arbitrary curves *defined* as geodesics. The connection would then be determined, *a posteriori* as it were, by the requirement that its coefficients vanish for those inertial curves. Once one congruence is chosen the connection, thus determined, provides all other congruences that are inertial with respect to the first. So in fact a single congruence, providing a standard of absolute rest, overdetermines the connection, which represents inertia in general and hence puts all other inertial motions on the same footing.

We should not be too surprised that purely formal criteria are of little use on their own for the identification of something as physical as inertia. But are more physical, empirical methods not available? Can inertial systems not be characterised[11] as free and far from everything else? Even if certain bodies may be isolated enough to be almost entirely uninfluenced by others, the matter remains troublesome. For one thing we have no direct access to such approximately free bodies; everything around us gets pulled and accelerated. And the absence of gravitational force is best assessed with respect to an inertial system, which is what we were after in the first place.

[11] [10] p. 772: "[...] ein G a l i l e i sches Bezugsystem, d. h. ein solches, relativ zu welchem [...] ein von anderen hinlänglich entfernte Masse sich geradlinig und gleichförmig bewegt."

Various passages[12] in the *scholium* on absolute space and time show that Newton proposed to tell apart inertia and acceleration[13] through *causes*, *effects*, *forces* (*cf.* [11]). In the two experiments described at the end of the *scholium*—involving the bucket and the rotating globes—there is an interplay of local causes and effects: the rotation of the water causes it to rise on the outside; the forces applied to opposite sides of the globes cause the tension in the string joining them to vary. Einstein also speaks of cause and effect in his analysis of the thought experiment described on p. 771 of [10], in which he brings together elements of Newton's two experiments: rotating fluid, two rotating bodies. Two fluid bodies of the same size and kind, $S_1$ and $S_2$, spin with respect to one another around the axis joining them while they float freely in space, far from everything else and at a considerable, unchanging distance from each other. Whereas $S_1$ is a sphere $S_2$ is ellipsoidal.

Einstein's analysis betrays positivist zeal and intolerance of metaphysics. Newton, who could be metaphysically indulgent to a point of mysticism, might—untroubled by the absence of a manifest local cause—have been content to view the deformation of $S_2$ as the effect of an absolute rotation it would thus serve to reveal. Einstein's epistemological severity makes him more exacting; he wants the cause;[14] seeing no *local* cause, within the system,[15] he feels obliged to look

[12] "Distinguuntur autem quies & motus absoluti & relativi ab invicem per proprietates suas & causas & effectus"; "Causæ, quibus motus veri & relativi distinguuntur ab invicem, sunt vires in corpora impressæ ad motum generandum"; "Effectus, quibus motus absoluti & relativi distinguntur ab invicem, sunt vires recedendi ab axe motus circularis"; "Motus autem veros ex eorum causis, effectibus, & apparentibus differentiis colligere, & contra ex motibus seu veris seu apparentibus eorum causas & effectus, docebitur fusius in sequentibus." [13] Newton seems to speak of mere 'motion'—*motus*—but his kinematical terminology (*cf. lex II*) is ambiguous and confusing; he clearly means acceleration. [14] He speaks (p. 771) of the *Kausalitätsgesetz*, and asks "Aus welchem Grunde verhalten sich die Körper $S_1$ und $S_2$ verschieden?" The *Grund*, the *Ursache* has to be a *beobachtbare Erfahrungstatsache*, a *beobachtbare Tatsache* from the *Erfahrungswelt* and not something as *erkenntnistheoretisch unbefriedigend* as the *sensorium Dei* or a *spatium absolutus*, which would be a "*bloß fingierte* Ursache, keine beobachtbare Sache. Es ist also klar, daß die N e w t o nsche Mechanik der Forderung der Kausalität in dem betrachteten Falle nicht wirklich, sondern nur scheinbar Genüge leistet, indem sie die bloß fingierte Ursache [absolute space] für das beobachtbare verschiedene Verhalten der Körper $S_1$ und $S_2$ verantwortlich macht." [15] P. 772: "Das aus $S_1$ und $S_2$ bestehende physikalische System zeigt für sich allein keine denkbare Ursache, auf welche das verschiedene Verhalten von $S_1$ und $S_2$ zurückgeführt werden könnte."

elsewhere[16] and finds an *external* one in distant masses[17] which rotate with respect to $S_2$. General relativity, which he goes on to formulate, does away with absolute inertia (to some extent at any rate) by spelling out its dependence on matter.

In 1918 Einstein goes so far as to claim[18] that inertia[19] is entirely determined by matter, which he uses $T_{ab}$ to represent.[20] He explains in a footnote[21] that this *Machsches Prinzip* is a generalisation of Mach's requirement [13] that inertia be derivable from interactions between bodies.[22]

# 3 Matter

Mach's principle[23] seems to be something along the lines of *matter determines inertia*. But what is matter? Einstein, we have seen, uses $T_{ab}$ to characterise it. But the coordinate-dependence of the pseudotensor $t_{ab}$, to which we shall return, allows the assignment of mass-energy to just about any region of spacetime, however flat or empty; and matter without mass, or mass away from matter, is hard to conceive. Is matter everywhere, then? Potentially everywhere? If we spread matter too liberally, and attribute materiality to regions where $T_{ab}$ (or even $R^a_{bcd}$) vanishes, we hardly leave the relationalist and substantivalist any room to differ. Their debate has been called [18] outmoded; the surest way to hasten its complete demise is to impose agreement, by a dubious appeal to a dubious object, which

[16] P. 772: "Die Ursache muß also *außerhalb* dieses Systems liegen." [17] P. 772: "Man gelangt zu der Auffassung, daß die allgemeine Bewegungsgesetze, welche im speziellen die Gestalten von $S_1$ und $S_2$ bestimmen, derart sein müssen, daß das mechanische Verhalten von $S_1$ und $S_2$ ganz wesentlich durch ferne Massen mitbedingt werden muß, welche wir nicht zu dem betrachteten System gerechnet hatten. Diese fernen Massen [...] sind dann als Träger prinzipiell beobachtbarer Ursachen für das verschiedene Verhalten unserer betrachteten Körper anzusehen; sie übernehmen die Roller der fingierten Ursache [...]." [18] [12] p. 241: "*Machsches Prinzip:* Das $G$-Feld is *restlos* durch die Massen der Körper bestimmt." [19] In fact he speaks (p. 241) of the "$G$-Feld," "Den durch den Fundamentaltensor beschriebenen Raumzustand [...]," by which inertia is represented: "Trägheit und Schwere sind wesensgleich. Hieraus und aus den Ergebnissen der speziellen Relativitätstheorie folgt notwendig, daß der symmetrische „Fundamentaltensor" ($g_{\mu\nu}$) die metrischen Eigenschaften des Raumes, das Trägheitsverhalten der Körper in ihm, sowie die Gravitationswirkungen bestimmt." [20] Pp. 241-2: "Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Tensor ($T_{\mu\nu}$) beschrieben wird, so besagt dies, daß das $G$-Feld durch den Energietensor der Materie bedingt und bestimmt sei." [21] P. 241: "Den Namen „Machsches Prinzip" habe ich deshalb gewählt, weil dies Prinzip eine Verallgemeinerung der Machschen Forderung bedeutet, daß die Trägheit auf eine Wechselwirkung der Körper zurückgeführt werden müsse." [22] See [14] on "Einstein's formulations of Mach's principle." [23] [15] is full of excellent accounts; see also [16] and [17].

can fill the whole universe with slippery coordinate-dependent matter that disappears in free fall and reappears under acceleration. To allow the most freedom for disagreement, to preserve an old and salutary debate, threatened and confused by new categories that have undermined traditional distinctions, we will keep demarcations as clean as possible and take matter to exist only where $T_{ab}$ does not vanish. But desirable as their survival may be, the debate and the positions contraposed in it will largely remain implicit here, seldom emerging from a background where they nonetheless maintain a pervasive presence.

Mach and Einstein both speak of *distant* masses. But Einstein's equation $G_{ab}(P) = T_{ab}(P)$ seems to express a determination—be it excessive, exact or insufficient—of inertia at point $P$ by the matter there; inertia would thus be governed by 'local' and not distant matter. The matter tensor[24]

$$T^{ab}(P) = \rho(P)V^a \otimes V^b,$$

for instance, describing a 'dust' with density $\rho$ and four-velocity $V^a$, would (directly) determine inertia at $P$, not at other points far away. So what about distant masses? Much as in electromagnetism, the 'continuity' of $\rho$ is deceptive. Once the scale begins to give a semblance of continuity to the density $\rho$, almost all the celestial bodies contributing to the determination of $\rho(P)$ will be very far, on any familiar scale, from $P$.

General relativity is furthermore a field theory, and fields are smooth, holistic entities, which undulate, drag, propagate and so forth,[25] but we come to that—to the extent we will at all—in 6.

# 4 Inertia

We have seen that Einstein identifies inertia with the metric $g$, which in general relativity, where $\nabla g$ vanishes, corresponds to affine structure $\nabla = \Pi_0$, with twenty degrees of freedom (in the absence of torsion). This gives the *parametrised* geodesics

$$\sigma_0 : (a_0, b_0) \to M$$
$$s_0 \mapsto \sigma_0(s_0)$$

[24] Following Ehlers we will let context determine whether indices indicate the valence of the tensor, as they do here, or single out a particular component. [25] The dependence of inertia on *all* masses—"an der Ezeugung des $G$-Feldes werden alle Massen der Welt teilhaben" ( [12] p. 243)—is best seen in the initial-value formulation; see [19], for instance, and the many lists of references it contains.

through $\nabla_{\dot{\sigma}_0}\dot{\sigma}_0 = 0$, and represents the 'inertia' of the parameter, hence of time, along with that of matter. ($M$ is the differential manifold representing the universe.)

General relativity offers another candidate for inertia, namely projective structure[26] $\Pi$, which gives the 'generalised geodesics'[27]

$$\sigma : (a, b) \to M$$
$$s \mapsto \sigma(s),$$

through

$$\nabla_{\dot{\sigma}}\dot{\sigma} = \lambda\dot{\sigma} = -\left(\frac{ds_0}{ds}\right)^2 \frac{d^2 s}{ds_0^2}\dot{\sigma}.$$

The less comprehensive inertia represented by projective structure is therefore purely 'material'—rather than 'materio-temporal.'

A particular connection $\Pi_\alpha$ in the projective class $\{\Pi_\alpha\} \leftrightarrow \Pi$ is singled out by a one-form $\alpha$, which fixes the parametrisations $s$ of all the generalised geodesics $\sigma$. So projective structure has twenty-four degrees of freedom, four $\alpha_a = \langle\alpha, \partial_a\rangle$ more than affine structure. We can write

$$\langle dx^a, \Pi_{\alpha\,\partial_b}\partial_c\rangle = \mathit{\Gamma}^a_{bc} + \delta^a_b\alpha_c + \delta^a_c\alpha_b,$$

where the $\mathit{\Gamma}^a_{bc}$ are the components of the Levi-Civita connection. The most meaningful part of the added freedom would appear to be the 'acceleration' $\lambda = -2\langle\alpha, \dot{\sigma}\rangle$ of the parameter $s$ along the generalised geodesic $\sigma$ determined by $\Pi_\alpha$.

We can now try to quantify the underdetermination of inertia by matter.

# 5 Freedom

The relationship between affine structure and curvature is given by

$$B^a_{bcd} = 2\mathit{\Gamma}^a_{b[d,c]} + \mathit{\Gamma}^e_{bd}\mathit{\Gamma}^a_{ec} - \mathit{\Gamma}^e_{bc}\mathit{\Gamma}^a_{ed},$$

which has ninety-six ($6 \times 4^2$) independent quantities, eighty if the connection is symmetrical, only twenty if it is metric, in which case $B^a_{bcd}$ becomes the Riemann tensor $R^a_{bcd}$. Einstein's equation expresses the equality of the matter tensor $T_{ab}$ and Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab},$$

[26] The distinction is due to Weyl [20]. See [21] for a more modern treatment. [27] Or alternatively the unparametrised geodesics, in other words just the image $\mathfrak{I}(\sigma) = \mathfrak{I}(\sigma_0) \subset M$.

where the Ricci scalar $R$ is the contraction $g^{ab}R_{ab}$ of the Ricci tensor $R_{ab} = R^c_{acb}$. Many Riemann tensors therefore correspond[28] to the same Ricci and Einstein tensors. By removing the ten freedom degrees of a symmetric index pair, the contraction $R_{ab} = R^c_{acb}$ leaves the ten independent quantities of the Ricci tensor; the lost freedoms end up in the Weyl tensor

$$C_{abcd} = R_{abcd} - g_{a[c}R_{d]b} + g_{b[c}R_{d]a} + \frac{1}{3}Rg_{a[c}g_{d]b},$$

which describes tidal effects. Matter would therefore seem to underdetermine affine structure by ten degrees of freedom, projective structure by fourteen. Much of the underdetermination is bound up with a gauge freedom whose physical meaning has been amply discussed, in [22], [16], [23], [24], [25] (pp. 19-23) and elsewhere.

To see how gauge choices eliminate eight degrees of freedom we can look at gravitational waves.[29]

# 6 Waves

The weak perturbation $h_{ab} = g_{ab} - \eta_{ab}$ would first (being symmetrical) appear to maintain the ten freedoms of the Weyl tensor. It is customary to write

$$\gamma_{ab} = h_{ab} - \frac{1}{2}\eta_{ab}h,$$

where $h$ is the trace $h^a_a$. A choice of coordinates satisfying the four conditions $\partial_b \gamma_{ab} = 0$ (four continuity equations for the 'perturbation fluids' $\gamma_{0b}, \ldots, \gamma_{3b}$) allows us to set $\gamma_{a0} = 0$, which does away with the four 'temporal' freedoms. There remains a symmetric 'purely spatial' matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{21} & h_{31} \\ 0 & h_{21} & h_{22} & h_{32} \\ 0 & h_{31} & h_{32} & h_{33} \end{pmatrix}$$

(for now $\gamma_{ab} = h_{ab}$) with six degrees of freedom. We can also require $h^a_a$ to vanish, which eliminates another freedom, leaving five. To understand the fates of these

[28] This correspondence is complicated by the presence in the Einstein tensor of the metric—without which even $T_{ab}$ has little meaning. [29] For a recent and readable account see [26].

remaining freedoms we can consider the plane harmonic $h_{ab} = \mathrm{Re}\{A_{ab}e^{i\langle k,x\rangle}\}$ obeying $\Box h_{ab} = 0$. If the wave equation were

$$\Box_{\mathbf{c}} h_{ab} = (\partial_0^2 - \mathbf{c}^2\Delta)h_{ab} = 0$$

instead, with arbitrary $\mathbf{c}$, the wave (co)vector $k$ would have four independent components $k_a = \langle k, \partial_a\rangle$:

- the direction $k_1 : k_2 : k_3$, in other words $\mathbf{k}/|\mathbf{k}|$ (two)
- the length $|\mathbf{k}| = \sqrt{k_1^2 + k_2^2 + k_3^2}$ (one)
- the frequency $\omega = k_0 = \langle k, \partial_0\rangle = \mathbf{c}|\mathbf{k}|$ (one).

Since $\mathbf{c} = 1$ is a natural constant, condition $\Box h_{ab} = 0$ reduces them to three, by identifying $|\mathbf{k}|$ and $\omega$, which makes the squared length

$$\langle k, k^\sharp\rangle = k_0 k^0 - |\mathbf{k}|^2 = \omega^2 - |\mathbf{k}|^2$$

vanish. And even these three degrees of freedom disappear into the coordinates if the wave is made to propagate along the third spatial direction, leaving two $(5 - 3)$ freedoms, of polarisation. For such an alignment, bearing in mind the three orthogonality relations[30]

$$\sum_{b=1}^{3} A_{ab}k^b = 0,$$

gets rid of the components $h_{3b}$, leaving a traceless symmetric matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{21} & 0 \\ 0 & h_{21} & -h_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

with two independent components, $h_{11} = -h_{22}$ and $h_{12} = h_{21}$.

The above gauge choices therefore eliminate eight degrees of freedom:

- the four 'temporal' coordinates $h_{a0}$ eliminated by the conditions $\partial_b\gamma_{ab} = 0$
- the freedom eliminated by $\gamma_a^a = 0$
- the three freedoms of $\mathbf{k}$ eliminated by the conditions $A_{ab}k^b = 0$.

[30] Which follow from $\partial_b h_{ab} = 0$ and situate the polarisation tensor $A_{ab}$ in the plane $\mathbf{k}^\perp \subset k^\perp$ orthogonal to the three-vector $\mathbf{k} \in k^\perp$.

# 7 Invariance

We are left, then, with the double freedom of polarisation. But is it really there? Those who, in the tradition of Leibniz and Mach, hope to account for motion and inertia, for the structure of spacetime in terms of the bodies in it, may prefer to dismiss the freedom as an empty mathematical fiction without physical consequence. One can say it is just as meaningful as gravitational waves, whose reality, while we await unambiguous detection, will have to rest on theoretical considerations—with a theoretical ascription of mass-energy, for instance. As it happens general relativity does, as we have seen, allow the attribution of mass-energy to the gravitational field, to gravitational waves, through the pseudotensor[31]

$$t^a_b = \frac{1}{2}\delta^a_b g^{mn}\Gamma^l_{mr}\Gamma^r_{nl} - g^{mn}\Gamma^a_{mr}\Gamma^r_{nb}.$$

The trouble is that $t^a_b$, and anything it may represent, is very much a 'matter of opinion' (*cf.* [22] p. 519): in free fall, where the connection vanishes, $t^a_b$ will too, whereas accelerated observers see mass-energy.

Is the physical meaning of $t^a_b$ really compromised by its susceptibility to disappear, and reappear under acceleration?

General relativity has been at the centre of a tradition linking invariance and reality, conspicuously associated with Hilbert [27] (see [28]), Cassirer [29], Meyerson [30], Weyl [31], certainly with Einstein.[32] Roots can be sought as far back as Democritus, who is said to have claimed that "sweet, bitter, hot, cold, colour" are mere opinion, "only atoms and void"—concerning which there must be better agreement—"are real"; or more recently (1872) in Felix Klein's *Erlangen programme*, which based geometrical relevance on invariance under the groups he

[31] This convenient form is assumed with respect to coordinates satisfying $\sqrt{-g} = 1$. [32] [32] p. 5: "Verschiedene Menschen können mit Hilfe der Sprache ihre Erlebnisse bis zu einem gewissen Grade miteinander vergleichen. Dabei zeigt sich, daß gewisse sinnliche Erlebnisse verschiedener Menschen einander entsprechen, während bei anderen ein solches Entsprechen nicht festgestellt werden kann. Jenen sinnlichen Erlebnissen verschiedener Individuen, welche einander entsprechen und demnach in gewissem Sinne überpersönlich sind, wird eine Realität gedanklich zugeordnet. Von ihr, daher mittelbar von der Gesamtheit jener Erlebnisse, handeln die Naturwissenschaften, speziell auch deren elementarste, die Physik. Relativ konstanten Erlebnis-komplexen solcher Art entspricht der Begriff des physikalischen Körpers, speziell auch des festen Körpers." Eight pages on: "Offenbar haben in der euklidischen Geometrie nur solche (und alle solche) Größen eine objektive (von der besonderen Wahl des kartesischen Systems unabhängige) Bedeutung, welche sich durch eine Invariante (bezüglich linearer orthogonaler Koordinaten) ausdrücken lassen. Hierauf beruht es, daß die Invariantentheorie, welche sich mit den Strukturgesetzen der Invariante beschäftigt, für die analytische Geometrie von Bedeutung ist."

used to classify geometries. Bertrand Russell, in his version of neutral monism,[33] identified objects with the class of their appearances from different points of view—not really an association of invariance and reality, but an attempt to transcend the misleading peculiarities of individual perspectives nonetheless. Levi-Civita,[34] Schrödinger and Bauer, who embraced the association of invariance and reality they rightly took to be so central to relativity, understandably questioned (see [40]) the reality of $t^a_b$. Schrödinger[35] noted that appropriate coordinates would make $t^a_b$ vanish identically in a curved spacetime (containing only one body however); Bauer[36] that appropriate coordinates would give energy-momentum to flat regions.

The requirement of invariance can even be motivated in terms of a 'consistency' of sorts.[37] Suppose observer $\Omega$ with four-velocity $V$ attributes speed[38] $w = |\mathbf{w}| = |P_{V^\perp}W|$ to body $\beta$ with four-velocity $W$, while $\Omega'$ moving at $V'$ sees speed $w' \neq w$ (all of this around the same event). The short statements

- $\beta$ has speed $w$
- $\beta$ has speed $w'$

are contradictory. Consistency can of course be restored with longer statements specifying perspective, but the tension between the short statements is not without significance—if the number were a scalar even they would agree. Similar considerations apply, *mutatis mutandis*, to covariance; one would then speak of form or syntax being the same, rather than of numerical equality.

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[33] Accounts can be found in [33], [34] and [35]. See also [36] p. 14. [34] [37] p. 382: "L'idea di un tensore gravitazionale fa parte della grandiosa costruzione di Einstein. Però la definizione propostane dall'Autore non può risguardarsi definitiva. Anzi tutto, dal punto di vista matematico, le fa difetto quel carattere invariantivo che dovrebbe invece necessariamente competerle secondo lo spirito della relatività generale." [35] [38] pp. 6,7: "Dieses Ergebnis scheint mir [...] von ziemlicher Bedeutung für unsere Auffassung von der physikalichen Natur des Gravitationsfeldes. Denn entweder müssen wir darauf verzichten, in den durch die Gleichung (2) definierten $t^\alpha_\sigma$ die Energiekomponenten des Gravitationsfeldes zu erblicken; damit würde aber zunächst auch die Bedeutung der „Erhaltungssätze“ [...] fallen und die Aufgabe erwachsen, diesen integrierenden Bestandteil der Fundamente neuerdings sicher zu stellen. – Halten wir jedoch an den Ausdrücken (2) fest, dann lehrt unsere Rechnung, daß es wirkliche Gravitationsfelder (d.i. Felder, die sich nicht „wegtransformieren“ lassen) gibt, mit durchaus verschwindenden oder richtiger gesagt „wegtransformierbaren“ Energiekomponenten; Felder, in denen nicht nur Bewegungsgröße und Energiestrom, sondern auch die Energiedichte und die Analoga der M a x w e l lschen Spannungen durch geeignete Wahl des Koordinatensystems für endliche Bezirke zum Verschwinden gebracht werden können." [36] [39] p. 165: "Ihre physikalische Bedeutung erscheint somit mehr als zweifelhaft." [37] The relevance of consistency was pointed out by Pierluigi Graziani. [38] The operator $P_{V^\perp}$ projects onto the three-dimensional simultaneity surface $V^\perp$ orthogonal to $V$.

Consistency has long been considered necessary for *mathematical* existence, for Poincaré [41] it was practically sufficient. Here physical reality is at issue, rather than mathematical existence. But $t^a_b$ is undeniably a mathematical object, whose existence is complicated by a kind of inconsistency; and physical reality presupposes existence.

In January 1918 Einstein upheld[39] the physical reality of $t^a_b$ and of gravitational waves, claiming that even objects without the transformation properties of tensors can be physically significant. In February he responded [43] to Schrödinger's objection, arguing that with more than one body the stresses $t^a_b$ $(a, b = 1, 2, 3)$ transmitting gravitational interactions would not vanish: Take two bodies $M_1$ and $M_2$ kept at a constant distance by a rigid rod $R$ aligned along $\partial_1$. $M_1$ is enclosed in a two-surface $\partial V$ which leaves out $M_2$ and hence cuts $R$ (orthogonally, for simplicity). Integrating over the three-dimensional region $V$, the conservation law $\partial_a U^a_b = 0$ (where $U^a_b = T^a_b + t^a_b$) yields[40]

$$\frac{d}{dx^0} \int_V U^0_b d^3V = \int_{\partial V} \sum_{a=1}^{3} U^a_b d^2\Sigma_a :$$

any change in the total energy $\int U^0_b d^3V$ enclosed in volume $V$ would be due to a flow, represented on the right-hand-side, through the boundary $\partial V$. Since the situation is stationary and there are no flows, both sides of the equation vanish, for $b = 0, 1, 2, 3$. Einstein takes $b = 1$ and uses

$$\int_{\partial V} \sum_{a=1}^{3} U^a_1 d^2\Sigma_a = 0.$$

He is very concise, and leaves out much more than he writes, but we are presumably to consider the intersection $R \cap \partial V$ of rod and enclosing surface, where it seems that $\partial_1$ is orthogonal to $\partial_2$ and $\partial_3$, which means the off-diagonal components $T^2_1$ and $T^3_1$ vanish, unlike the component $T^1_1$ along $R$. Since

$$-\int_{\partial V} \sum_{a=1}^{3} t^a_1 d^2\Sigma_a,$$

[39] In [42], where one reads (p. 167) that: "[Levi-Civita] (und mit ihm auch andere Fachgenossen) ist gegen eine Betonung der Gleichung $[\partial_\nu(\mathfrak{T}^\nu_\sigma + \mathfrak{t}^\nu_\sigma) = 0]$ und gegen die obige Interpretation, weil die $\mathfrak{t}^\nu_\sigma$ keinen T e n s o r bilden. Letzteres ist zuzugeben; aber ich sehe nicht ein, warum nur solchen Größen eine physikalische Bedeutung zugeschrieben werden soll, welche die Transformationseigenschaften von Tensorkomponenten haben." [40] We have replaced Einstein's cosines with the notation used, for instance, in [44].

must be equal to something like $T^1_1$ times the sectional area of $R$, the gravitational stresses $t^1_1$, $t^2_1$, $t^3_1$ cannot all vanish identically. The argument is contrived and full of gaps, but the conclusion that gravitational stresses between two (or more) bodies cannot be 'transformed away' seems valid.

Then in May Einstein attempted a more comprehensive response,[41] claiming—with a subtlety and logical flexibility verging on inconsistency,[42] which secured widespread and enduring assent (see [46])—that the integral conservation law depends less on the contingencies of particular representation than his opponents suggested. We shall come to his argument presently.

# 8 Conservation

We have already seen that the pseudotensor $t^a_b$ is related to the conservation of energy, which is just as problematic.[43] While the covariant divergence $\nabla_a T^a_b$ always vanishes, the ordinary divergence $\partial_a T^a_b$ only does in free fall (where it coincides with $\nabla_a T^a_b$), and otherwise registers the gain or loss seen by an accelerated observer. If such variations are to be viewed as exchanges with the environment and not as definitive gains or losses, account of them can be taken with $t^a_b$, which makes $\partial_a(T^a_b + t^a_b)$ vanish by compensating the difference (*cf.* [28] p. 136). But a good conservation law has to admit integration,[44] that complicates matters by involving a distant comparison of direction, which cannot be invariant.

Nothing prevents us from comparing the values of a genuine scalar at distant points. But we know the density of mass-energy is not invariant, and transforms according to

$$(\rho, \mathbf{0}) \mapsto \frac{\rho}{\sqrt{1 - |\mathbf{v}|^2}}(1, \mathbf{v}),$$

where $\mathbf{v}$ is the three-velocity of the observer. So the invariant object is not the mass-energy density, but the energy-momentum density, which *is manifestly directional*. And how are distant directions to be compared? Comparison of com-

[41] In [45], where he laments (p. 447) that: "Diese Formulierung [of conservation] stößt bei den Fachgenossen deshalb auf Widerstand, weil $(\mathfrak{U}^\nu_\sigma)$ und $(\mathfrak{t}^\nu_\sigma)$ keine Tensoren sind, während sie erwarten, daß alle für die Physik bedeutsamen Größen sich als Skalare und Tensorkomponenten auffassen lassen müssen." [42] Certainly with the letter of [32], but even with the spirit—perhaps the letter too—of his whole relativistic programme. [43] For more on the shortcomings of the conservation law see [46]. [44] *Cf.* [45] p. 449: "Vom physikalischen Standpunkt aus kann diese Gleichung nicht als vollwertiges Äquivalent für die Erhaltungssätze des Impulses und der Energie angesehen werden, weil ihr nicht Integralgleichungen entsprechen, die als Erhaltungssätze des Impulses und der Energie gedeutet werden können."

ponents is not invariant: directions equal with respect to one coordinate system may differ in another. Comparison by parallel transport will not depend on the coordinate system, but on the path followed.

Einstein tries to get around the problem in [45], in which he attributes an energy-momentum $J$ to the universe. He legitimates $J$ by imposing a kind of 'general' (but in fact rather limited) invariance on each component $J_a$, defined as the spatial integral

$$J_a = \int U^0_a d^3V$$

of the combined energy-momentum $U^0_a = T^0_a + t^0_a$ of matter and field. To impose it he first assumes the fields $T^a_b$ and $t^a_b$ vanish outside a forever $(\forall x^0)$ circumscribed spatial region $B$, outside of which the coordinates are to remain Minkowskian (which implies flatness). He then uses the temporal constancy $dJ_a/dx^0 = 0$ of each component $J_a$, which follows from $\partial_a U^a_b = 0$, to prove that $J_a$ has the same value $(J_a)_1 = (J_a)_2$ on both three-dimensional simultaneity slices[45] $x^0 = t_1$ and $x^0 = t_2$ of coordinate system $K$, and value $(J'_a)_1 = (J'_a)_2$ at $x'^0 = t'_1$ and $x'^0 = t'_2$ in another system $K'$. A third system $K''$ coinciding with $K$ around the slice $x^0 = t_1$ and with $K'$ around $x'^0 = t'_2$ allows the diachronic comparison of $K$ and $K'$. The invariance of each component $J_a$ follows from $(J_a)_1 = (J'_a)_2$, and holds under transformations that

- change nothing outside[46] $B \times \mathbb{R}$
- preserve the (Minkowskian) orthogonality[47] $\partial_0 \perp \text{span}\{\partial_1, \partial_2, \partial_3\}$.

Having established that, Einstein views the world as a body immersed in an otherwise flat spacetime, whose energy-momentum $J_a$ is covariant under the transformation laws—Lorentz transformations—considered appropriate (despite [48]) for such an environment. Unusal mixture of invariances: four components, each one 'generally' invariant, which together make up a Lorentz-covariant four-vector.

A conservation law whose co-invariance is so artificial and restricted can make one even wonder (see [46]) about the *generation* of gravitational waves: if a belief in the production of radiation rests on the conservation of energy, how can that belief remain indifferent to the shortcomings of the conservation law?

[45] For a recent treatment see [47]. [46] Where $B$ 'ages' along the field $\partial_0$. [47] For so we understand Einstein's condition: "Wir wählen im folgenden das Koordinatensystem so, daß alle Linienelemente $(0, 0, 0, dx_4)$ zeitartig, alle Linienelemente $(dx_1, dx_2, dx_3, 0)$ raumartig sind; dann können wir die vierte Koordinate in gewissem Sinne als „die Zeit“ bezeichnen" (p. 450).

Everything suggests the binary star PSR 1913 +16 loses kinetic energy as it spirals inwards. If the kinetic energy is not to disappear without trace, it has to be radiated. Since its disappearance is ruled out by conservation, the very generation of gravitational waves must be subject to the doubts surrounding conservation (*cf.* [9], [46]). If the conservation law is suspicious enough to make us wonder whether the lost energy is really radiated into the freedom degrees of the gravitational field, why take that freedom—the underdetermination of inertia by matter—seriously? Couldn't it be no more than a purely decorative gauge, without reality or physical meaning?

Suppose a gravitational wave detector seems to reveal a signal that stands out well against the noise and appears to come from the decaying binary star. There would be a kind of correlation between signal and star—Meyerson [49] would at least speak of *légalité*, of lawlike regularity. For there to be *causalité*, and an *explanation*, he would require the transmission of a 'vehicle,' an 'agent,' a substance maintaining a recognisable identity between star and detector. And for the unambiguous preservation of that identity he would demand invariant *conservation*, without which there would be legality without causality.

Meyerson's association of conservation and causality is not without its relevance here; for even with an apparent detection—which in any case will not be without its ambiguities—our belief in gravitational waves cannot be immune to the uncertainties concerning conservation. Mere legality without a causal explanation founded on invariant conservation can leave a scepticism easily aggravated by the abundant noise in which the signal may or may not be buried.

So the exact physical meaning of the (double) underdetermination of affine structure by matter remains unclear and awaits experimental elucidation, which could be imminent. Leibniz or Mach, to uphold a thoroughgoing relativity of motion, might meanwhile insist on the disturbing limitations of the conservation law and even dismiss gravitational waves, perhaps their very generation, as mere *opinion*, claiming that reality must rest on invariance. We have to remember that if inertia is taken to concern only matter and not time as well, and is accordingly represented by projective rather than affine structure, there would remain another four degrees underdetermination to dismiss as physically meaningless. But as their significance is far from clear, the gauge choice $\alpha = \mathbf{0}$ (which takes us back to affine structure) is unlikely to trouble anyone.

# 9 Absolute motion

To conclude we can return to absolute motion, and to Newton. Let us say that relative motion is motion referred to some*thing*—where by ‘thing’ we mean a material object that has mass whatever the state of motion of the observer (materiality, again, *is not an opinion*). Otherwise motion will be *absolute*.

Suppose an empty flat universe is perturbed by the plane harmonic

$$h_{ab} = \mathrm{Re}\{A_{ab}e^{i\langle k,x\rangle}\}.$$

Changes in the frequency $\omega$ measured by a roving observer would indicate absolute motion, and allow a reconstruction, through $\omega = \langle k, v\rangle$, of the observer's absolute velocity $v$.

Is this undulating spacetime absolute, substantival,[48] Newtonian? It is absolute to the extent that according to the criterion adopted, it admits absolute motion. But its absoluteness precludes its substantival reification, which would make the motion relative to some*thing* and hence not absolute. Newton, though no doubt approving on the whole, would disown it, for “Spatium absolutum [...] semper manet similare et immobile [...],” and our undulating spacetime is neither ‘similar to itself’ ($R^a_{bcd}$ varies) nor immobile.

We may remember that Newton spoke of revealing absolute *motus* through its causes and effects, through forces. Absolute *motion* is precisely what our thought experiment would reveal, and through forces, just as Newton wanted: the forces, for instance, registered by a (most sensitive) dynamometer linking the masses whose varying tidal oscillations give rise to the described Doppler effect.

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