Neutrosophic Triplet Normed Ring Space

Mehmet Şahin[1], Abdullah Kargın[2]

[1] Department of Mathematics, Gaziantep University, Gaziantep, 27000, Turkey. E-mail: mesahin@gantep.edu.tr

[2] Department of Mathematics, Gaziantep University, Gaziantep, 27000, Turkey. E-mail: abdullahkargin27@gmail.com

Abstract. In this article, a notion of neutrosophic triplet (NT) normed ring space is given and properties of NT normed ring spaces are studied. We demonstrate that NT normed ring is different from the classical one. Also, we show that a neutrosophic triplet normed ring can be a neutrosophic triplet norm when certain conditions are met.

Keywords: Neutrosophic triplet set, neutrosophic triplet ring, neutrosophic triplet normed ring.

1 Introduction

Neutrosophy is a branch of philosophy, introduced by Smarandache in 1980, which studies the origin, nature and scope of neutralities, as good as their interactions with distinctive ideational spectra. Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set and neutrosophic facts in [1]. Neutrosophic logic is a general framework for unification of many existing logics such as fuzzy logic which is introduced by Zadeh in [2] and intuitionistic fuzzy logic which is introduced by Atanassov in [3]. Fuzzy set has best measure of membership; intuitionistic fuzzy set has most effective degree of membership and degree of non-membership. Thus; they do not explain the indeterminacy states. But neutrosophic set has degree of membership (t), degree of indeterminacy (i) and degree of non-membership (f) and define the neutrosophic set on three components (t, i, f). A lot of researchers have been dealing with neutrosophic set theory in [4-22]. Recently; Broumi, Bakali, Talea and Smarandache studied the single valued neutrosophic graphs in [23] and interval valued neutrosophic graphs in [24]. Liu studied the aggregation operators based on Archimedean t-conorm and t-norm for the single valued neutrosophic numbers in [25]. Additionally, Smarandache and Ali introduced NT theory in [26] and NT groups in [27, 28]. The NT set is completely different from the classical one, since for each element "a" in NT set N together with a binary operation *; there exist a neutral of "a" called neut(a) such that a*neut(a)=neut(a)*a=a and an opposite of "a" called anti(a) such that a*anti(a)=anti(a)*a=neut(a). Where, neut(a) is different from the classical algebraic unitary element. A NT is of the form <a, neut(a), anti(a)>. Also, Smarandache and Ali studied the NT field in [29] and the NT ring in [30]. Recently, some researchers have been dealing with NT set thought. For instance, Şahin and Kargın introduced NT metric space, NT vector space and NT normed space in [31]. Şahin and Kargin studied NT inner product in [32].

Normed ring is an algebraic structure. Some of the properties of the normed rings are similarly to some of the properties of the classical norms, but the normed rings also have their own characteristic properties. Shilov introduced the notion of commutative normed ring in [33] and Jarden introduced the notion of normed ring in [34]. Recently Ulucay, Şahin and Olgun introduced normed rings with soft set theory in [35].

In this paper, we introduced NT normed ring space and we give properties of NT normed ring space. In section 2, we give some preliminary results and definition for NT structures. In section 3, NT normed ring space is defined and some properties of a NT normed ring space are given. It is show that NT normed ring is different from the classical normed ring. Also, it is show that if certain conditions are met, every NT normed ring can be a NT metric and NT norm at the same time. Furthermore, the convergence of a sequence and a Cauchy sequence in a NT normed ring space are defined. In section 4, conclusions are given.

2 Preliminaries

Definition 2.1. [27] Let N be a set together with a binary operation *. Then, N is called a NT set if for any a∈ N, there exists a neutral of "a" called neut(a), different from the classical algebraic unitary element, and an opposite of "a" called anti(a), with neut(a) and anti(a) belonging to N, such that

a*neut(a) = neut(a)* a=a

a*anti(a)= anti(a)* a=neut(a).

The elements a, neut(a) and anti(a) are collectively called as neutrosophic triplet, and we denote it by (a, neut(a), anti(a)). Here, we mean neutral of a and apparently, "a" is just the first coordinate of a NT and it is not a neutrosophic triplet. For the same element "a" in N, there may be more neutrals to it neut(a)'s and more opposites of it anti(a)'s.

Theorem 2.2. [27] Let (N,*) be a commutative NT group with respect to * and a, b ∈ N;

i) neut(a)*neut(b)= neut(a*b);
ii) anti(a)*anti(b)= anti(a*b);

Definition 2.3. [29] Let (NTF,*, #) be a NT set together with two binary operations * and #. Then (NTF,*, #) is called NT field if the following conditions hold.

i. (NTF,*) is a commutative NT group with respect to *
ii. (NTF, #) is a NT group with respect to #.
iii. a#(b*c)= (a#b)*(a#c) and (b*c)#a = (b#a)*(c#a) for all a, b, c ∈ NTF.

Definition 2.4. [30] The NT ring is a set endowed with two binary laws (M,*, #) such that,

a) (M, *) is a commutative NT group; which means that:
- (M, *) is a commutative neutrosophic triplets with respect to the law * (i.e. if x belongs to M, then neut(x) and anti(x), defined with respect to the law *, also belong to M)
- The law * is well – defined, associative, and commutative on M (as in the classical sense);

b) (M, *) is a set such that the law # on M is well-defined and associative (as in the classical sense);
c) The law is distributive with respect to the law * (as in the classical sense)

Theorem 2.5. [31] Let (N,*) be a NT group with no zero divisors and with respect to *. For a ∈ N,

i) neut(neut(a))= neut(a)
ii) anti(neut(a))= neut(a))
iii) anti(anti(a))= a
iv) neut(anti(a))= neut(a)

Theorem 2.6. [31] Let (NTV, *, #) be a NT vector space on a NT field. If (NTV, *, #) is satisfies the following condition, (NTV, *, #) is also a NT field;
1) a#b∈ NTV; for all a, b∈ NTV;
2) a#(b#c) = (a#b)#c; for all a, b, c ∈ NTV;
3) a#(b*c) = (a#b)*(a#c) and (b*c)#a = (b#a)*(c#a); for all a, b, c ∈ NTV.

Definition 2.7. [31] Let (N,*) be a NT set and let x*y ∈ N for all x, y ∈ N. If the function d:NxN→$\mathbb{R}^+$∪{0}satisfies the following conditions; d is called a NT metric. For all x, y, z ∈ N;

a) d(x, y)≥0;
b) If x=y; then d(x, y)=0
c) d(x, y)= d(y, x)
d) If there exists any element y ∈N such that;
d(x, z)≤ d(x, z*neut(y)), then
d(x, z*neut(y)) ≤ d(x, y)+ d(y, z).
Furthermore, ((N,*), d) is called NT metric space.

Definition 2.8. [31] Let (NTV, $*_2$, $\#_2$)) be a NT vector space on (NTF, , $*_1$, $\#_1$) NT field. If ‖.‖ : NTV →$\mathbb{R}^+$+∪{0} function satisfies following condition; ‖.‖ is called NT normed on (NTV, ,$*_2$, $\#_2$). Where;

f: NTF X NTV →$\mathbb{R}^+$∪{0}, f(α,x)= f(anti(α), anti(x))
is a function and for every x, y ∈ NTV and α ∈ NTF;
a) ‖x‖ ≥0;
b) If x=neut(x), then ‖x‖ =0
c) ‖α$\#_2$ x‖ = f(α,x).‖x‖
d) ‖anti(x)‖= ‖x‖
e) If there exists any element k ∈ NTV such that ‖x $*_2$ y‖ ≤ ‖x $*_2$ y $*_2$ neut(k)‖ then;

$\|x *_2 y *_2 neut(k)\| \leq \|x\| + \|y\|$.
Furthermore, ((NTV, $*_2$, $\#_2$), $\|.\|$) is called a NT normed space on (NTF, ,$*_1$, $\#_1$) NT field.

Definition 2.9. [34] Let R be an associative ring with 1. A norm on R is a function $\|.\|$: R → R that satisfies the following conditions for all a, b ∈ R.

a) $\|a\| \geq 0$ and $\|a\| = 0$ if and only if
   a = 0; further $\|1\| = \|-1\| = 1$
b) There is an x ∈ R with $0 < \|x\| < 1$
c) $\|a.b\| \leq \|a\|.\|b\|$
d) $\|a+b\| \leq \max\{\|a\|, \|b\|\}$

3 Neutrosophic Triplet Normed Ring Space

Definition 3.1. Let (NTR,*,#) be a NT ring. If $\|.\|$: NTR $\rightarrow \mathbb{R}^+ \cup \{0\}$ function satisfies following condition; $\|.\|$ is called NT normed ring on (NTR,* ,#). For x, y, z ∈ NTR,

a) $\|x\| \geq 0$;
b) If x=$neut_*(x)$, then $\|x\| = 0$. Where, $neut_\#(x)$) is neutral of x with respect to #.
c) There is a x ∈ NTR such that
   $\|neut_*(x)\| \leq \|x\| \leq \|neut_\#(x)\|$.Where, $neut_\#(x)$ is neutral of x with respect to # and $neut_*(x)$ is neutral of x with respect to * .
d) $\|anti_*(x)\| = \|x\|$. Where, $anti_*(x)$ is anti of x with respect to *.
e) If there exists a element k ∈ NTR such that $\|x\# y\| \leq \|x\# y\# neut_\#(k)\|$; then
   $\|x\# y\# neut_\#(k)\| \leq \|x\|.\|y\|$
f) If there exists a element k ∈ NTR such that $\|x* y\| \leq \|x* y* neut_*(k)\|$; then
   $\|x* y* neut_*(k)\| \leq \max\{\|x\|,\|y\|\}$

Furthermore, ((NTR,* ,#) $\|.\|$) is called NT normed ring space.

Example 3.2. Let X = {1, 2}, and P(X) be power set of X. From Definition 2.4; (P(X), *, ∩) is a NT ring. Where,

$$A*B = \begin{cases} B\backslash A, & s(A)<s(B) \wedge B\supset A \wedge A'=B \\ A\backslash B, & s(A)>s(B) \wedge A\supset B \wedge B'=A \\ (A\backslash B)', & s(A)>s(B \wedge A\supset B \wedge B' \neq A \\ (B\backslash A)', & s(A)<s(B) \wedge B\supset A \wedge A' \neq B \\ X, & s(A)=s(B) \wedge A\neq B \\ \emptyset, & A=B \end{cases}$$

The NT with respect to *;
neut(∅)=∅, anti(∅) = ∅; neut({1})= {1, 2},
anti({1}) ={2}; neut({2}) = {1, 2},
anti({2}) ={1}; neut({1, 2}) = ∅, anti({1,2}) ={1, 2};

The NT with respect to ∩; neut(A) =A and anti(A) = B. Where, B⊇A and s(A) is number of elements in A ∈ P(X) and A' is complement of A ∈P(X). Now we show that (P(X), *, ∩), $\|,\|$) is a NT normed ring space such that $\|A\| = s(A)$.
a) $\|A\| = s(A) \geq 0$
b) Since neut(∅)=∅, $\|\emptyset\| = s(\emptyset) = 0$
c) For ∅ ∈P(X), $neut_*(\emptyset) = \emptyset$, $neut_\cap(\emptyset) = \emptyset$; then $\|neut_*(x)\| \leq \|x\| \leq \|neut_\cap(x)\|$
d) Since anti(∅) = ∅, anti({1})={2}, anti({2})={1},
   anti({1,2})={1, 2};
   $\|\emptyset\| = \|\emptyset\|$, $\|\{1\}\| = \|\{2\}\|$, $\|\{1, 2\}\| = \|\{1, 2\}\|$. Thus; $\|anti(A)\| = \|A\|$ for A ∈P(X).
e) Since neut(∅)=∅, anti(∅)=∅;
   neut({1})= {1, 2}, anti({1})={2};
   neut({2})= {1, 2}, anti({2})={1};
   neut({1, 2})= ∅, anti({1,2})={1, 2};

For $\emptyset$ and $\emptyset$;
$\| \emptyset * \emptyset\| = \|\emptyset\| = 0 \leq \| \emptyset * \emptyset * neut(\{1\})\| = \|\{1,2\}\| = 2$
Thus; $\| \emptyset * \emptyset\| = 0 \leq \|\emptyset\| + \|\emptyset\| = 0$

For $\emptyset$ and $\{1\}$;
$\| \emptyset*\{1\}\| = \|\{2\}\| = 1 \leq \| \emptyset*\{1\}*neut(\{2\})\| = \|\{1\}\| = 1$. Thus; $\| \emptyset*\{1\}\| = 1 \leq \|\emptyset\| + \|\{1\}\| = 1$

For $\emptyset$ and $\{2\}$;
$\| \emptyset*\{2\}\| = \|\{1\}\| = 1 \leq \| \emptyset*\{2\}*neut(\{1\})\| = \|\{2\}\| = 1$. Thus; $\| \emptyset*\{2\}\| = 1 \leq \|\emptyset\| + \|\{2\}\| = 1$

For $\emptyset$ and $\{1,2\}$;
$\| \emptyset*\{1,2\}\| = \|\{1,2\}\| = 2 \leq \| \emptyset*\{1,2\}*neut(\{\emptyset\})\| = \|\{1,2\}\| = 2$.
Thus; $\| \emptyset*\{1, 2\}\| = 2 \leq \|\emptyset\| + \|\{1,2\}\| = 2$

For $\{1\}$ and $\{1\}$;
$\| \{1\}*\{1\}\| = \|\emptyset\| = 0 \leq \| \{1\}*\{1\}*neut(\{2\})\| = \|\{1,2\}\| = 2$.
Thus; $\| \{1\}*\{1\}\| = 0 \leq \|\{1\}\| + \|\{1\}\| = 2$

For $\{1\}$ and $\{2\}$;
$\| \{1\}*\{2\}\| = \|\emptyset\| = \| \{1\}*\{2\}*neut(\{2\})\|$
$= \|\{1,2\}\| = 2$.
Thus; $\| \{1\}*\{2\}\| = 0 \leq \|\{1\}\| + \|\{1\}\| = 2$

For $\{1\}$ and $\{1,2\}$;
$\| \{1\}*\{1,2\}\| = \|\{2\}\| = 1 \leq \| \{1\}*\{1,2\}*neut(\{2\})\|$
$= \|\{1\}\| = 1$.
Thus; $\| \{1\}*\{1,2\}\| = 0 \leq \|\{1\}\| + \|\{1,2\}\| = 3$

For $\{2\}$ and $\{2\}$;
$\| \{2\}*\{2\}\| = \|\emptyset\| = 0 \leq \| \{2\}*\{2\}*neut(\{1\})\|$
$= \|\{1,2\}\| = 2$.
Thus; $\| \{2\}*\{2\}\| = 0 \leq \|\{2\}\| + \|\{2\}\| = 2$

For $\{2\}$ and $\{1,2\}$; $\| \{2\}*\{1,2\}\| = \|\{1\}\| = 1 \leq \| \{2\}*\{1,2\}*neut(\{1\})\| = \|\{2\}\| = 1$.
Thus; $\| \{2\}*\{1,2\}\| = 1 \leq \|\{2\}\| + \|\{1,2\}\| = 3$

f) Since A, B$\in$P(X) and neut($\{1\}$) = $\{1, 2\}$ = X; $A\cap B\cap X = A \cap B$. Thus; $\|A\cap B\| = \| A\cap B\cap neut(\{1\})\|$. Now we show that;
$\|A\cap B\| = \| A\cap B\cap neut(\{1\})\| \leq \max\{\|A\|,\|B\|\}$.

For $\emptyset$ and $\emptyset$;
$\| \emptyset\cap\emptyset\| = 0 \leq \max\{\|\emptyset\|,\|\emptyset\|\} = \emptyset$

For $\emptyset$ and $\{1\}$;
$\| \emptyset\cap\{1\}\| = 0 \leq \max\{\|\emptyset\|,\|\{1\}\|\} = 1$

For $\emptyset$ and $\{2\}$;
$\| \emptyset\cap\{2\}\| = 0 \leq \max\{\|\emptyset\|,\|\{2\}\|\} = 1$

For $\emptyset$ and $\{1,2\}$;
$\| \emptyset\cap\{1,2\}\| = 0 \leq \max\{\|\emptyset\|,\|\{1,2\}\|\} = 2$

For $\{1\}$ and $\{1\}$;
$\|\{1\} \cap\{1\}\| = 1 \leq \max\{\|\{1\}\|,\|\{1\}\|\} = 1$

For $\{1\}$ and $\{2\}$;
$\|\{1\} \cap\{2\}\| = 0 \leq \max\{\|\{1\}\|,\|\{2\}\|\} = 1$

For $\{1\}$ and $\{1, 2\}$;
$\|\{1\} \cap\{1,2\}\| = 0 \leq \max\{\|\{1\}\|,\|\{1,2\}\|\} = 2$

For {2} and { 2};
$\|\{2\} \cap \{2\}\| = 1 \leq \max\{\|\{2\}\|, \|\{2\}\|\} = 1$

For {2} and {1, 2};
$\|\{2\} \cap \{1,2\}\| = 0 \leq \max\{\|\{2\}\|, \|\{1,2\}\|\} = 2.$

For {1,2} and {1, 2}; $\|\{1,2\} \cap \{1,2\}\| = 2$
$\leq \max\{\|\{1,2\}\|, \|\{1,2\}\|\} = 2$

Thus; (P(X), *, ∩), ‖,‖) is a NT normed ring space.

Corollary 3.3. It is clear that NT normed ring spaces are generally different from classical normed ring spaces since for neut(x) different from classical unit element. However; if certain conditions are met; every classical normed space can be a NT normed space at the same time.

Proposition 3.4. Let (NTR, *, #), ‖.‖) be a NT normed ring space.

a) If there exists a element k ∈ NTR such that
$\|x * y\| \leq \|x * y * neut(k)\|$ and $\|x\| < \|y\|$ then $\|x * y\| = \|y\|$.
b) If there exists a element k ∈ NTR such that $\|x * y\| \leq \|x * y * neut(k)\|$ then $\|x * y\| \leq \|x\| + \|y\|$.

Proof.
a) From Definition 3.1, now that there exists a element k ∈ NTR such that
$\|x * y\| \leq \|x * y * neut(k)\|$, it is clear that
$\|z\| \leq \|z * neut(k)\|$ for $x * y = z$, z∈NTR (i)
Furthermore, from Definition 3.1, now that there exists a element k ∈ NTR such that
$\|x * y\| \leq \|x * y * neut(k)\|$, $\|x * y\| \leq \max\{\|x\|, \|y\|\}$.
For $\|x\| < \|y\|$, it is clear that $\|x * y * neut(k)\| \leq \|y\|$.
Assume that $\|x * y * neut(k)\| \lneq \|y\|$.
From (i), we can take x = k such that $\|y\| \leq \|y * neut(x)\|$.
Thus;
$\|x * y\| = \|y\|$.
b) From definition 1, now that there exists a element k ∈ NTR such that
$\|x * y\| \leq \|x * y * neut(k)\|$, it is clear that
$\|x * y\| \leq \max\{\|x\|, \|y\|\}$ (ii)
Furthermore,
$\max\{\|x\|, \|y\|\} \leq \|x\| + \|y\|$ (iii)
From (ii) and (iii), it is clear that,
$\|x * y\| \leq \|x\| + \|y\|$.

Theorem 3.5. Let (NTR, * ,#), ‖.‖) be a NT normed ring space. If $\|.\|$: NTR → $\mathbb{R}^+ \cup \{0\}$ function and (NTR,* ,#), ‖.‖) satisfy following conditions; it is called NT normed space on (NTR,* ,#), ‖.‖). For x, y, z ∈ NTR,

a) (NTR,* ,#) be a NT vector space.
b) $\|x \# y\| = \|x\| . \|y\|$

Proof.
Now that (NTR,* ,#)) is a NT ring,
a#b ∈ NTR; for all a, b∈ NTV;
a#(b#c)=(a#b)#c; for all a, b, c ∈ NTR;
a#(b*c)= (a#b)*(a#c) and
(b*c)#a = (b#a)*(c#a); for all a, b, c ∈ NTR.

From a), (NTR,* ,#) is a NT vector space. Thus, (NTR,* ,#) be a NT field. Now we show that (NTR,* ,#), ‖.‖) is a NT normed space.
For x, y, k ∈ NTR,
a) From Definition 3.1, $\|x\| \geq 0$
b) From Definition 3.1, if $x = neut_*(x)$, then $\|x\| = 0$
c) Now that (NTR,* ,#) is a NT field and from b),

‖x# y‖ =‖x‖.‖y‖. Then we can take
f: NTR X NTR $\rightarrow \mathbb{R}^{+}\cup\{0\}$, f(x, y)= ‖x‖. Where, it is clear that f(x, y) = ‖x‖ = ‖$anti_*$(x)‖ = f($anti_*$(x), $anti_*$(y)).
Thus; ‖x# y‖ = f(x, y).‖y‖
d) From Definition 3.1, ‖$anti_*$(x)‖= ‖x‖
e) From Definition 3.1, and Proposition 3.4, it is clear that If there exists a element k ∊ NTR such that

‖x*y‖ ≤ ‖x*y*neut(k)‖;

then

‖x*y*neut(k)‖ ≤ ‖x‖+‖y‖.

Thus; (NTR,* ,#), ‖. ‖) is a NT normed space.

Proposition **3.6.** Let ((NTR,* ,#), ‖.‖) be a NT normed ring space. If ‖neut(x)‖ = 0, for x ∈ NTR, then the function d: NTR x NTR→ $\mathbb{R}$ defined by d(x, y) = ‖x* anti(y)‖ provides NT metric space conditions.

Proof.
Let x, y, z ∈ NTR. From the Definition 3.1,
1) d(x, y) = ‖x ∗ anti(y)‖)≥0;
2) If x = y then; d(x, y) = ‖x ∗ anti(y)‖= ‖y ∗ anti(y)‖ = ‖neut(x)‖ = ‖neut(y)‖. Now that ‖neut(x)‖ = 0, d(x, y) = 0.
3) Now that ‖anti(x)‖ =‖x‖, we have d(x, y) = ‖x ∗ anti(y)‖ =‖anti(x ∗ anti(y))‖. From the Theorem 2.2 and Theorem 2.5, we have
d(x, y)= ‖anti(x ∗ anti(y))‖ = ‖anti(x) ∗ anti(anti(y))‖ = ‖anti(x) ∗ y‖ = d(y, x).
4) For any k ∊ NTV; suppose that
d(x, z) =‖x ∗ anti(z)‖≤‖x ∗ anti(z) ∗ $neut$(k)‖=
d(x, $z*_2$neut(k)), then
‖x ∗ anti(z)‖≤
‖x ∗ anti(z) ∗ $neut$(k)‖=
‖x ∗ anti(z) ∗ k ∗ $anti$(k)‖.
Now that NTV is a commutative group with respect to "*", we have
‖x ∗ anti(z) ∗ k ∗ $anti$(k)‖=
‖(x ∗ $anti$(k)) ∗ (anti(z) ∗ k)‖≤
max{ ‖x ∗ $anti$(k)‖,‖k ∗ $anti$(z))‖ } ≤
‖x ∗ $anti$(k)‖+‖k ∗ $anti$(z))‖.
Thus, if d(x, z) ≤ d(x, $z*_2$neut(k)), then d(x, $z*_2$neut(k)) ≤ d(x, k)+ d(k, z).

Definition 3.7. Let (NTR,* ,#), ‖. ‖) be a NT normed ring space, $\{x_n\}$ be a sequence in this space. For all ε>0, for all n ≥M such that

‖x ∗ anti(x_n})‖< ε

if there exists a M∊ ℕ; $\{x_n\}$ sequence converges to x. It is denoted by

$\lim_{n\to\infty} x_n$= x or x_n→ x

Definition 3.8. Let (NTR,* ,#), ‖. ‖) be a NT normed ring space, $\{x_n\}$ be a sequence in this space. For all ε>0 such that for all n, m ≥M

‖x_n ∗ anti($\{x_m\}$)‖< ε

If there exists a M ∊ ℕ, then $\{x_n\}$ sequence is called Cauchy sequence.

Definition 3.9. Let (NTR,* ,#), ‖. ‖) be a NT normed ring space, $\{x_n\}$ be a sequence in this space. If each $\{x_n\}$ Cauchy sequence in this space is convergent to d reduced NT metric; (NTR,* ,#), ‖. ‖) is called complete NT normed ring space.

Theorem 3.10. Let (NTR, *, #), ‖. ‖) be a NT normed ring space, $\{x_n\}$ be a sequence in this space. If there exist a M ∊ ℕ such that ‖x_m ∗ anti($\{x_{m+1}\}$)‖< ε for all n, m ≥M and there exist a k ∊ NTR such that ‖x* y‖ ≤ ‖x* y*neut(k)‖, then $\{x_n\}$ is a Cauchy sequence.

Proof.
Let n>m≥ M. From Definition 3.1, now that there exist a k ∊ NTR such that
‖x* y‖ ≤ ‖x* y*neut(k)‖,
‖x_n ∗ anti(x_m)‖< ‖x_n ∗ anti($\{x_m\}$) ∗ neut(x_{n-1})‖ (iv)
Thus, from definition 3.1,
‖x_n ∗ anti(x_m)‖<
‖x_n ∗ anti($\{x_m\}$) ∗ neut(x_{n-1})‖ =

$\|x_n * \text{anti}(x_m) * (x_{n-1}) * \text{anti}(x_{n-1})\| \leq$
$\max\{\|x_n * \text{anti}(x_{n-1})\|, \|(x_{n-1}) * \text{anti}(x_m)\|\}$.
Similarly from (iv),
for $\|(x_{n-1}) * \text{anti}(x_m)\|$;
$\|(x_{n-1}) * \text{anti}(x_m)\| \leq \max\{\|x_{n-1} * \text{anti}(x_{n-2})\|, \|(x_{n-2}) * \text{anti}(x_m)\|\}$
:
:
for $\|(x_{m+2}) * \text{anti}(x_m)\|$;
$\|(x_{m+2}) * \text{anti}(x_m)\| \leq \max\{\|x_{m+2} * \text{anti}(x_{m+1})\|, \|(x_{m+1}) * \text{anti}(x_m)\|\}$. Thus,
$\|x_n * \text{anti}(x_m)\| \leq$
$\max\{\|x_{n-1} * \text{anti}(x_{n-2})\|, \|(x_{n-2}) * \text{anti}(x_{n-3})\|, \ldots, \|(x_{m+1}) * \text{anti}(x_m)\|\}$
and now that $\|x_m * \text{anti}(\{x_{m+1}\})\| < \varepsilon$, it is clear that $\|x_n * \text{anti}(x_m)\| < \varepsilon$. Therefore; $\{x_n\}$ is a Cauchy sequence.

4 Conclusion

In this paper; we introduced NT normed ring space. We also show that NT normed ring different from the classical one. This NT notion has several extraordinary properties compared to the classical one. We also studied some interesting properties of this newly born structure. We give rise to a new field or research called NT structures.

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