# Generalization of Soft Neutrosophic Rings and Soft Neutrosophic Fields 

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#### Abstract

In this paper we extend soft neutrosophic rings and soft neutrosophic fields to soft neutrosophic birings, soft neutrosophic N -rings and soft neutrosophic bifields and soft neutrosophic N -fields. We also extend soft neutrosophic ideal theory to form soft neutrosophic biideal and soft neutrosophic N -ideals over a neutrosophic biring


#### Abstract

and soft neutrosophic N -ring. We have given examples to illustrate the theory of soft neutrosophic birings, soft neutrosophic N -rings and soft neutrosophic fields and soft neutrosophic N -fields and display many properties of these.


Keywords: Neutrosophic biring, neutrosophic N-ring, neutrosophic bifield,neutrosophic N-field, soft set, soft neutrosophic biring, soft neutrosophic N -ring, soft neutrosophic bifield, soft neutrosophic N -field.

## 1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set, intuitionistic fuzzy set and interval valued fuzzy set. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N -semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophip groupoids, and neutrosophic bigroupoids and so on.

Molodtsov in [11] laid down the stone foundation of a richer structure called soft set theory which is free from the parameterization inadequacy, syndrome of fuzzy se theory, rough set theory, probability theory and so on. In many areas it has been successfully applied such as smoothness of
functions, game theory, operations research, Riemann integration, Perron integration, and probability. Recently soft set theory has attained much attention since its appearance and the work based on several operations of soft sets introduced in $[2,9,10]$. Some more exciting properties and algebra may be found in [1]. Feng et al. introduced the soft semirings[5]. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be seen in [6]. Some other new concept combined with fuzzy sets and rough sets was presented in [7, 8]. AygÄunoglu et al. introduced the Fuzzy soft groups [4].

Firstly, fundamental and basic concepts are given for neutrosophic birings, neutrosophic N-rings, neutrosohic bifields and soft neutrosophic N -fields. In the next section we presents the newly defined notions and results in soft neutrosophic birings, soft neutrosophic N-rings and soft neutrosophic bifields and soft neutrosophic N -fields. Various types of soft neutrosophic biideals and N -ideals of birings and N -rings are defined and elaborated with the help of examples.

## 2 Fundamental Concepts

In this section, we give a brief description of neutrosophic birings, neutrosophic N-rings, neutrosophic bifields and neutrosophic N -fields respectively.

Definition 2.1. Let ( $B N(\mathrm{R}), *, \circ$ ) be a non-empty set with two binary operations $*$ and $\circ .(B N(\mathrm{R}), *, \circ)$ is said to be a neutrosophic biring if $B N(\mathrm{Rs})=R_{1} \cup R_{2}$ where atleast one of $\left(\mathrm{R}_{1}, *, \circ\right)$ or $\left(\mathrm{R}_{2}, *, \circ\right)$ is a neutrosophic ring and other is just a ring. $R_{1}$ and $R_{2}$ are proper subsets of $B N(\mathrm{R})$.
Definition 2.2: Let $B N(\mathrm{R})=\left(\mathrm{R}_{1}, *, \circ\right) \cup\left(\mathrm{R}_{2}, *, \circ\right)$ be a neutrosophic biring. Then $B N(\mathrm{R})$ is called a commutative neutrosophic biring if each $\left(\mathrm{R}_{1}, *, \circ\right)$ and $\left(\mathrm{R}_{2}, *, \circ\right)$ is a commutative neutrosophic ring.

Definition 2.3: Let $B N(\mathrm{R})=\left(\mathrm{R}_{1}, *, \circ\right) \cup\left(\mathrm{R}_{2}, *, \circ\right)$ be a neutrosophic biring. Then $B N(\mathrm{R})$ is called a pseudo neutrosophic biring if each $\left(\mathrm{R}_{1}, *, \circ\right)$ and $\left(\mathrm{R}_{2}, *, \circ\right)$ is a pseudo neutrosophic ring.

Definition 2.4 Let $\left(B N(\mathrm{R})=R_{1} \cup R_{2} ; *, \circ\right)$ be a neutrosophic biring. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbiring of $B N(\mathrm{R})$ if

1) $T=T_{1} \cup T_{2}$ where $T_{1}=R_{1} \cap T$ and $T_{2}=R_{2} \cap T$ and
2) At least one of $\left(T_{1}, \circ\right)$ or $\left(T_{2}, *\right)$ is a neutrosophic ring.

Definition 2.5: If both $\left(\mathrm{R}_{1}, *\right)$ and $\left(\mathrm{R}_{2}, \circ\right)$ in the above definition 2.1 are neutrosophic rings then we call $(B N(\mathrm{R}), *, \circ)$ to be a strong neutrosophic biring.

Definition 2.6 Let $\left(B N(\mathrm{R})=R_{1} \cup R_{2} ; *, \circ\right)$ be a neutrosophic biring and let $(T, *, \circ)$ is a neutrosophic subbiring of $B N(\mathrm{R})$. Then $(T, *, \circ)$ is called a neutrosophic biideal of $B N(R)$ if

1) $T=T_{1} \cup T_{2}$ where $T_{1}=R_{1} \cap T$ and $T_{2}=R_{2} \cap T$ and
2) At least one of $\left(T_{1}, *, \circ\right)$ or $\left(T_{2}, *, \circ\right)$ is a neutrosophic ideal.
If both $\left(T_{1}, *, \circ\right)$ and $\left(T_{2}, *, \circ\right)$ in the above definition are neutrosophic ideals, then we call $(T, *, \circ)$ to be a strong
neutrosophic biideal of $B N(R)$.

Definition 2.7: Let $\left\{\mathrm{N}(\mathrm{R}), *_{1}, \ldots, *_{2}, \circ_{1}, \circ_{2}, \ldots, \circ_{N}\right\}$ be a non-empty set with two $N$-binary operations defined on it. We call $N(R)$ a neutrosophic $N-$ ring ( $N$ a positive integer) if the following conditions are satisfied.

1) $\mathrm{N}(\mathrm{R})=R_{1} \cup R_{2} \cup \ldots \cup R_{N}$ where each $R_{i}$ is a proper subset of $\mathrm{N}(\mathrm{R})$ i.e. $R_{i} \not \subset R_{j}$ or $R_{j} \not \subset R_{i}$ if $i \neq j$.
2) $\left(\mathrm{R}_{i}, *_{i}, \circ_{i}\right)$ is either a neutrosophic ring or a ring for $i=1,2,3, \ldots, N$.

Definition 2.8: If all the $N$-rings $\left(\mathrm{R}_{i}, *_{i}\right)$ in definition 2.7 are neutrosophic rings (i.e. for $i=1,2,3, \ldots, N$ ) then we call $\mathrm{N}(\mathrm{R})$ to be a neutrosophic strong $N$-ring.

Definition 2.9: Let
$\mathrm{N}(\mathrm{R})=\left\{\mathrm{R}_{1} \cup R_{2} \cup \ldots \cup \mathrm{R}_{N}, *_{1}, *_{2}, \ldots, *_{N}, \circ_{1},{ }_{2}, \ldots, \circ_{N}\right\}$ be a neutrosophic $N$-ring. A proper subset $P=\left\{\mathrm{P}_{1} \cup P_{2} \cup \ldots . \mathrm{P}_{N}, *_{1}, *_{2}, \ldots, *_{N}\right\}$ of $\mathrm{N}(\mathrm{R})$ is said to be a neutrosophic $N$-subring if $P_{i}=P \cap R_{i}, i=1,2, \ldots, N$ are subrings of $R_{i}$ in which atleast some of the subrings are neutrosophic subrings.

Definition 2.10: Let
$\mathrm{N}(\mathrm{R})=\left\{\mathrm{R}_{1} \cup R_{2} \cup \ldots \cup \mathrm{R}_{N}, *_{1}, *_{2}, \ldots, *_{N}, \circ_{1}, \circ_{2}, \ldots, \circ_{N}\right\}$
be a neutrosophic $N$-ring. A proper subset
$P=\left\{\mathrm{P}_{1} \cup P_{2} \cup \ldots \cup P_{N}, *_{1}, *_{2}, \ldots, *_{N}, \circ_{1}, \circ_{2}, \ldots, \circ_{N}\right\}$ where $P_{t}=P \cap R_{t}$ for $t=1,2, \ldots, N$ is said to be a neutrosophic $N$-ideal of $N(R)$ if the following conditions are satisfied.

1) Each it is a neutrosophic subring of

$$
R_{t}, t=1,2, \ldots, N
$$

2) Each it is a two sided ideal of $R_{t}$ for $t=1,2, \ldots, N$.

If $\left(\mathrm{P}_{i}, *_{i}, \circ_{i}\right)$ in the above definition are neutrosophic ideals, then we call $\left(\mathrm{P}_{i}, *_{i}, o_{i}\right)$ to be a strong neutrosophic N ideal of $N(R)$.

Definition 2.11: Let $(B N(\mathrm{~F}), *, \circ$ ) be a non-empty set with two binary operations $*$ and $\circ .(B N(\mathrm{~F}), *, \circ)$ is
said to be a neutrosophic bifiel if $B N(\mathrm{~F})=F_{1} \cup F_{2}$ where atleast one of $\left(\mathrm{F}_{1}, *, \circ\right)$ or $\left(\mathrm{F}_{2}, *, \circ\right)$ is a neutrosophic field and other is just a field. $F_{1}$ and $F_{2}$ are proper subsets of $B N(\mathrm{~F})$.
If in the above definition both $\left(\mathrm{F}_{1}, *, \circ\right)$ and $\left(\mathrm{F}_{2}, *, \circ\right)$ are neutrosophic fields, then we call $(B N(\mathrm{~F}), *, \circ)$ to be a neutrosophic strong bifield.

Definition 2.12: Let $B N(\mathrm{~F})=\left(\mathrm{F}_{1} \cup F_{2}, *, \circ\right)$ be a neutrosophic bifield. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbifield of $B N(\mathrm{~F})$ if

1. $T=T_{1} \cup T_{2}$ where $T_{1}=F_{1} \cap T$ and $T_{2}=F_{2} \cap T$ and
2. At least one of $\left(T_{1}, \circ\right)$ or $\left(T_{2}, *\right)$ is a neutrosophic field and the other is just a field.

Definition 2.13: Let $\left\{\mathrm{N}(\mathrm{F}), *_{1}, \ldots, *_{2}, \circ_{1}, \circ_{2}, \ldots, \circ_{N}\right\}$ be a non-empty set with two $N$-binary operations defined on it. We call $N(R)$ a neutrosophic $N$-field ( $N$ a positive integer) if the following conditions are satisfied.

1. $\quad \mathrm{N}(\mathrm{F})=F_{1} \cup F_{2} \cup \ldots \cup F_{N}$ where each $F_{i}$ is a proper subset of $\mathrm{N}(\mathrm{F})$ i.e. $R_{i} \not \subset R_{j}$ or

$$
R_{j} \not \subset R_{i} \text { if } i \neq j
$$

2. $\left(\mathrm{R}_{i}, *_{i}, \circ_{i}\right)$ is either a neutrosophic field or just a field for $i=1,2,3, \ldots, N$.
If in the above definition each $\left(\mathrm{R}_{i},{ }_{i}, \circ_{i}\right)$ is a neutrosophic field, then we call $N(R)$ to be a strong neutrosophic N -field.

Definition 2.14: Let
$\mathrm{N}(\mathrm{F})=\left\{\mathrm{F}_{1} \cup F_{2} \cup \ldots \cup F_{N}, *_{1}, *_{2}, \ldots, *_{N}, \circ_{1}, \circ_{2}, \ldots,{ }_{N}\right\}$ be a neutrosophic $N$-field. A proper subset
$T=\left\{\mathrm{T}_{1} \cup T_{2} \cup \ldots \cup \mathrm{~T}_{N}, *_{1}, *_{2}, \ldots, *_{N},{ }_{1},{ }_{2}, \ldots,{ }_{N}\right\}$ of $N(\mathrm{~F})$ is said to be a neutrosophic $N$-subfield if each $\left(T_{i}, *_{i}\right)$ is a neutrosophic subfield of $\left(\mathrm{F}_{i}, *_{i}, \circ_{i}\right)$ for $i=1,2, \ldots, N$ where $T_{i}=F_{i} \cap T$.

## 3 Soft Neutrosophic Birings

Definition 3.1: Let $(B N(\mathrm{R}), *, \circ)$ be a neutrosophic biring and $(F, A)$ be a soft set over $(B N(\mathrm{R}), *, \circ)$. Then
$(F, A)$ is called soft neutrosophic biring if and only if $F(a)$ is a neutrosophic subbiring of $(B N(\mathrm{R}), *, \circ)$ for all $a \in A$.

Example 3.2: Let $B N(\mathrm{R})=\left(\mathrm{R}_{1}, *, \circ\right) \cup\left(\mathrm{R}_{2}, *, \circ\right)$ be a neutrosophic biring, where $\left(\mathrm{R}_{1}, *, \circ\right)=(\langle\mathbb{Z} \cup I\rangle,+, \times)$ and $\left(\mathrm{R}_{2}, *, \circ\right)=(\mathbb{Q},+, \times)$. Let $A=\left\{a_{1}, a_{2}, a_{3}, a_{4}\right\}$ be a set of parameters. Then clearly $(F, A)$ is a soft neutrosophic biring over $B N(R)$, where

$$
\begin{gathered}
F\left(a_{1}\right)=\langle 2 \mathbb{Z} \cup I\rangle \cup \mathbb{R}, F\left(a_{2}\right)=\langle 3 \mathbb{Z} \cup I\rangle \cup \mathbb{Q} \\
F\left(a_{3}\right)=\langle 5 \mathbb{Z} \cup I\rangle \cup \mathbb{Z}, F\left(a_{4}\right)=\langle 6 \mathbb{Z} \cup I\rangle \cup 2 \mathbb{Z}
\end{gathered}
$$

Theorem 3.3: Let $F, A$ and $(H, A)$ be two soft neutrosophic birings over $B N(R)$. Then their intersection $F, A \cap H, A$ is again a soft neutrosophic biring over $B N(R)$.

Proof. The proof is straightforward.

Theorem 3.4: Let $F, A$ and $H, B$ be two soft neutrosophic birings over $B N(R)$. If $A \cap B=\phi$, then $F, A \cup H, B$ is a soft neutrosophic biring over $B N(R)$.

Proof. This is straightforward.
Remark 3.5: The extended union of two soft neutrosophic birings $F, A$ and $K, B$ over $B N(R)$ is not a soft neutrosophic ring over $B N(R)$.

We check this by the help of Examples.
Remark 3.6: The restricted union of two soft neutrosophic rings $F, A$ and $K, B \quad$ over $\langle R \cup I\rangle$ is not a soft neutrosophic ring over $\langle R \cup I\rangle$.

Theorem 3.7: The $O R$ operation of two soft neutrosophic rings over $\langle R \cup I\rangle$ may not be a soft neutrosophic ring over $\langle R \cup I\rangle$.

One can easily check these remarks with the help of Examples.

Theorem 3.8: The extended intersection of two soft neu trosophic birings over $B N(R)$ is soft neutrosophic
biring over $B N(R)$.
Proof. The proof is straightforward.
Theorem 3.9: The restricted intersection of two soft neutrosophic birings over $B N(R)$ is soft neutrosophic biring over $B N(R)$.

Theorem 3.10: The $A N D$ operation of two soft neutrosophic birings over $B N(R)$ is soft neutrosophic biring over $B N(R)$.

Definition 3.11: Let $F, A$ be a soft set over a neutrosophic biring over $B N(R)$. Then $(F, A)$ is called an absolute soft neutrosophic biring if $F(a)=B N(R)$ for all $a \in A$.

Definition 3.12: Let $(F, A)$ be a soft set over a neutrosophic ring $B N(R)$. Then $(F, A)$ is called soft neutrosophic biideal over $B N(R)$ if and only if $F(a)$ is a neutrosophic biideal of $B N(R)$.

Theorem 3.1.3: Every soft neutrosophic biideal $(F, A)$ over a neutrosophic biring $B N(R)$ is trivially a soft neutrosophic biring but the converse may not be true.

Proposition 3.14: Let $(F, A)$ and $(K, B)$ be two soft neutosophic biideals over a neutrosophic biring
$B N(R)$. Then

1. Their extended union $(F, A) \cup_{E}(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.
2. Their extended intersection $(F, A) \cap_{E}(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.
3. Their restricted union $(F, A) \cup_{R}(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.
4. Their restricted intersection $(F, A) \cap_{R}(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.
5. Their $O R$ operation $(F, A) \vee(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.
6. Their $A N D$ operation $(F, A) \vee(K, B)$ is again a soft neutrosophic biideal over $B N(R)$.

Definition 3.15: Let $(F, A)$ and $(K, B)$ be two soft neutrosophic birings over $B N(R)$. Then $(K, B)$ is called soft neutrosophic subbiring of $(F, A)$, if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic subbiring of $F(a)$ for all $a \in A$.

Theorem 3.16: Every soft biring over a biring is a soft neutrosophic subbiring of a soft
neutrosophic biring over the corresponding neutrosophic biring if $B \subseteq A$.

Definition 3.16: Let $(F, A)$ and $(K, B)$ be two soft neutrosophic birings over $B N(R)$. Then $(K, B)$ is called a soft neutrosophic biideal of $(F, A)$, if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic biideal of $F(a)$ for all $a \in A$.

Proposition 3.17: All soft neutrosophic biideals are trivially soft neutrosophic subbirings.

## 4 Soft Neutrosophic N-Ring

Definition 4.1: Let $\left(N(\mathrm{R}), *_{1}, *_{2}, \ldots, *_{N}\right)$ be a neutrosophic N -ring and $(F, A)$ be a soft set over $N(R)$ Then $(F, A)$ is called soft neutrosophic N-ring if and only if $F(a)$ is a neutrosophic sub N -ring of $N(R)$ for all $a \in A$.

## Example 4.2: Let

$N(\mathrm{R})=\left(\mathrm{R}_{1}, *, \circ\right) \cup\left(\mathrm{R}_{2}, *, \circ\right) \cup\left(\mathrm{R}_{3}, *, \circ\right)$ be aneutrosophic 3-ring, where
$\left(\mathrm{R}_{1}, *, \circ\right)=(\langle\mathbb{Z} \cup I\rangle,+, \times),\left(\mathrm{R}_{2}, *, \circ\right)=(\mathbb{C},+, \times)$ and $\left(\mathrm{R}_{3}, *, \circ\right)=(\mathbb{R},+, \times)$. Let $A=\left\{a_{1}, a_{2}, a_{3}, a_{4}\right\}$ be a set of parameters. Then clearly $(F, A)$ is a soft neutrosophic N -ring over $N(R)$, where
$F\left(a_{1}\right)=\langle 2 \mathbb{Z} \cup I\rangle \cup \mathbb{R} \cup \mathbb{Q}, F\left(a_{2}\right)=\langle 3 \mathbb{Z} \cup I\rangle \cup \mathbb{Q} \cup \mathbb{Z}$,
$F\left(a_{3}\right)=\langle 5 \mathbb{Z} \cup I\rangle \cup \mathbb{Z} \cup 2 \mathbb{Z}, F\left(a_{4}\right)=\langle 6 \mathbb{Z} \cup I\rangle \cup 2 \mathbb{Z} \cup \mathbb{R}$

Theorem 4.3: Let $F, A$ and $(H, A)$ be two soft neutrosophic N -rings over $N(R)$. Then their intersection $F, A \cap H, A \quad$ is again a soft neutrosophic N ring over $N(R)$.

Proof. The proof is straightforward.

Theorem 4.4: Let $F, A$ and $H, B$ be two soft neutrosophic N-rings over $N(R)$. If $A \cap B=\phi$, then $F, A \cup H, B \quad$ is a soft neutrosophic N -ring over $N(R)$.

Proof. This is straightforward.
Remark 4.5: The extended union of two soft neutrosophic N-rings $F, A$ and $K, B$ over $B N(R)$ is not a soft neutrosophic ring over $N(R)$.

We can check this by the help of Examples.
Remark 4.6: The restricted union of two soft neutrosophic N-rings $F, A$ and $K, B$ over $N(R)$ is not a soft neutrosophic N-ring over $B N(R)$

Theorem 4.7: The $O R$ operation of two soft neutrosophic N -rings over $N(R)$ may not be a soft neutrosophic N -ring over $N(R)$.

One can easily check these remarks with the help of Examples.

Theorem 4.8: The extended intersection of two soft neutrosophic N -rings over $N(R)$ is soft neutrosophic Nring over $N(R)$.

Proof. The proof is straightforward.
Theorem. The restricted intersection of two soft neutrosophic N-rings over $N(R)$ is soft neutrosophic N -ring over $N(\mathrm{R})$.

Proof. It is obvious.
Theorem 4.9: The $A N D$ operation of two soft neutrosophic N -rings over $N(R)$ is soft neutrosophic N -ring
over $N(R)$.
Definition 4.10: Let $F, A$ be a soft set over a neutrosophic N -ring over $N(R)$. Then $(F, A)$ is called an absolute soft neutrosophic N -ring if $F(a)=N(R)$ for all $a \in A$.

Definition 4.11: Let $(F, A)$ be a soft set over a neutrosophic N-ring $N(R)$. Then $(F, A)$ is called soft neutrosophic N -ideal over $N(R)$ if and only if $F(a)$ is a neutrosophic N -ideal of $N(R)$.

Theorem 4.12: Every soft neutrosophic N -ideal $(F, A)$ over a neutrosophic N -ring $N(R)$ is trivially a soft neutrosophic N -ring but the converse may not be true.

Proposition 4.13: Let $(F, A)$ and $(K, B)$ be two soft neutosophic N -ideals over a neutrosophic N -ring $N(R)$. Then

1. Their extended intersection $(F, A) \cap_{E}(K, B)$ is again a soft neutrosophic N -ideal over $N(R)$.
2. Their restricted intersection $(F, A) \cap_{R}(K, B)$ is again a soft neutrosophic N -ideal over $N(R)$.
3. Their $A N D$ operation $(F, A) \vee(K, B)$ is again a soft neutrosophic N -ideal over $N(R)$.

Remark 4.14: Let $(F, A)$ and $(K, B)$ be two soft neutosophic N -ideals over a neutrosophic N -ring $N(R)$. Then

1. Their extended union $(F, A) \cup_{E}(K, B)$ is not a soft neutrosophic N -ideal over $N(R)$.
2. Their restricted union $(F, A) \cup_{R}(K, B)$ is not a soft neutrosophic N -ideal over $N(R)$.
3. Their $O R$ operation $(F, A) \vee(K, B)$ is not a soft neutrosophic N -ideal over $N(R)$.

One can easily see these by the help of examples.
Definition. 4.15: Let $(F, A)$ and $(K, B)$ be two soft neutrosophic N-rings over $N(R)$. Then $(K, B)$ is called soft neutrosophic sub N -ring of $(F, A)$, if

1. $B \subseteq A$, and
2. $\quad K(a)$ is a neutrosophic sub N -ring of $F(a)$ for
all $a \in A$.

Theorem 4.16: Every soft N-ring over a N-ring is a soft neutrosophic sub N -ring of a soft
neutrosophic N -ring over the corresponding neutrosophic N -ring if $B \subseteq A$.

## Proof. Straightforward.

Definition 4.17: Let $(F, A)$ and $(K, B)$ be two soft neutrosophic N -rings over $N(R)$. Then $(K, B)$ is called a soft neutrosophic N -ideal of $(F, A)$, if

1. $B \subseteq A$, and
2. $\quad K(a)$ is a neutrosophic N -ideal of $F(a)$ for all $a \in A$.

Proposition 4.18: All soft neutrosophic N-ideals are trivially soft neutrosophic sub N -rings.

## 5 Soft Neutrosophic Bifield

Defintion 5.1: Let $B N(K)$ be a neutrosophic bifield and let $(F, A)$ be a soft set over $B N(K)$. Then $(F, A)$ is said to be soft neutrosophic bifield if and only if $F(a)$ is a neutrosophic subbifield of $B N(K)$ for all $a \in A$.

Example 5.2: Let $B N(K)=\langle\mathbb{C} \cup I\rangle \cup \mathbb{R}$ be a neutrosophic bifield of complex numbers. Let $A=\left\{a_{1}, a_{2}\right\}$ be a set of parameters and let $(F, A)$ be a soft set of $B N(K)$. Then $(\mathrm{F}, \mathrm{A})$ is a soft neutrosophic bifield over $B N(K)$, where

$$
F\left(a_{1}\right)=\langle\mathbb{R} \cup I\rangle \cup \mathbb{Q}, F\left(a_{2}\right)=\langle\mathbb{Q} \cup I\rangle \cup \mathbb{Q} .
$$

Where $\langle\mathbb{R} \cup I\rangle$ and $\langle\mathbb{Q} \cup I\rangle$ are the neutosophic fields of real numbers and rational numbers.

Proposition 5.3: Every soft neutrosophic bifield is trivially a soft neutrosophic biring.

Proof. The proof is trivial.
Definition 5.4: Let $(F, A)$ be a soft neutrosophic bifield over a neutrosophic bifield $B N(K)$. Then $(F, A)$ is called an absolute soft neutrosophic bifield if
$F(a)=B N(K)$, for all $a \in A$.

## Soft Neutrosophic N-field

Defintion 5.4: Let $N(K)$ be a neutrosophic N -field and let $(F, A)$ be a soft set over $N(K)$. Then $(F, A)$ is said to be soft neutrosophic N -field if and only if $F(a)$ is a neutrosophic sub N -field of $N(K)$ for all $a \in A$.

Proposition 5.5: Every soft neutrosophic N-field is trivially a soft neutrosophic N-ring.

Proof. The proof is trivial.
Definition 5.6: Let $(F, A)$ be a soft neutrosophic N -field over a neutrosophic N -field $N(K)$. Then $(F, A)$ is called an absolute soft neutrosophic N -field if
$F(a)=N(K)$, for all $a \in A$.

## Conclusion

In this paper we extend neutrosophicb rings, neutrosophic N-rings, Neutrosophic bifields and neutrosophic N-fields to soft neutrosophic birings, soft neutrosophic N -rings and soft neutrosophic bifields and soft neutrosophic N -fields respectively. The neutrosophic ideal theory is extend to soft neutrosophic biideal and soft neutrosophic N -ideal. Some new types of soft neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate soft neutrosophic biring, soft neutrosophic N -ring, soft neutrosophic bifield and soft neutrosophic N -field and many theorems and properties are discussed.

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