Relativistic Pilot-Wave Theories as the Rational Completion of Quantum Mechanics and Relativity[1]

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Abstract

Einstein thought that quantum mechanics was incomplete because it was nonlocal. In this paper I argue instead that quantum theory is incomplete, even if it is nonlocal, and that relativity is incomplete because its minimal spatiotemporal structure cannot naturally accommodate such nonlocality. So, I show that relativistic pilot-wave theories are the rational completion of quantum mechanics as well as relativity: they provide a spatiotemporal ontology of particles, as well as a spatiotemporal structure able to explain quantum correlations.

Keywords: quantum mechanics; relativity; pilot-wave theory; spacetime; nonlocality

1. Introduction

In this paper I wish to argue that the best way of reconciling quantum theory and relativity is to think of them both as incomplete. The 'spirits' of these two theories are in tension: the 'spirit' of quantum theory is nonlocality, but it is developed in a theory without a fundamental spacetime; while the 'spirit' of relativity is spacetime in which interactions propagate locally in a Lorentz invariant framework. So, how should we proceed in our research toward a relativistic quantum theory?

I argue that quantum theory, *contra* to popular opinion, is incomplete because it lacks a microscopic spatiotemporal ontology to explain the phenomena constructively: macroscopic bodies are composed of the microscopic fundamental entities, and their properties are derived by the microscopic dynamics. The pilot-wave theory completes quantum mechanics by specifying a spatiotemporal ontology of particles. Nonetheless, its nonlocality clashes with relativity. This is (part of) the reason why people have gained interest in alternative quantum theories, like for instance the many-worlds theory, whose nonlocality is more controversial. However, many have argued that the experimental violation of Bell's inequality has unquestionably shown that all quantum theories are nonlocal.[3] If so, the nonlocality of the pilot-wave theory cannot count against it. Quantum nonlocality indeed suggests a natural fix: a preferred frame. If so, also relativity turns out to be incomplete, as it lacks the (natural) spatiotemporal structure necessary to account for nonlocality. I conclude by arguing that the simplest and most straightforward type of theory which best combines relativistic

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[3] However, see later, especially footnote 10.

spatiotemporal structure and quantum nonlocality is a relativistic pilot-wave theory, in which there is a microscopic spatiotemporal ontology to constructively explain the phenomena and there is a Lorentz invariant dynamical preferred spatiotemporal foliation implementing the nonlocal interaction. That is, relativistic pilot-wave theories are the rational completion of both relativity and quantum mechanics.

Here is a roadmap of the paper. In section 2, I start with quantum theory. I briefly argue that the real problem with quantum theory is not the measurement problem but rather the absence of a spatiotemporal ontology. I then discuss locality, its desirability, and the charge, originally advanced by Einstein, that quantum theory is nonlocal. I conclude by showing how, given the experimental violation of Bell's inequality, the 'spirit' of quantum theory is nonlocality: no theory which reproduces the quantum predictions can be local. In section 3, I present the pilot-wave theory as a theory of particles which interact nonlocally. Defined in this way, the theory straightforwardly has none of the previous problems, and, in accordance with the violation of Bell's inequality, it is nonlocal. I then turn in section 4 to relativity, I explore the possible 'spirits' of relativity, over and above the Lorentz invariance of the laws, and I argue that it is fundamentally a theory about spacetime. In section 5 I analyze the various proposed relativistic quantum theories, first relativistic pilot-wave theories, and then relativistic spontaneous localization theories. They all have Lorentz invariant laws, and they are all nonlocal, but while the majority of relativistic pilot-wave theories add a preferred foliation to the relativistic spatiotemporal structure, relativistic spontaneous collapse theories are foliation-free. Nonetheless, I show that, in addition to being counterintuitive and *ad hoc*, they threaten the notions of influence upon which the desirability of locality was based. In this way, relativistic spontaneous collapse theories show the cost of insisting about relativity being a complete theory. In section 6, I argue that the best way of thinking about relativity given the nonlocality of nature is as an incomplete theory with an important heuristic value: it is a principle theory which puts constraints *on theories*, rather than on phenomena.

2. Quantum Theory and Its Problems

Quantum theory is unproblematic if understood from an antirealist perspective as it provides incredibly accurate and precise predictions. However, it is unclear what it tells us about the world. This is usually explained presenting the measurement problem, which is the problem of macroscopic superpositions.

2.1 Unobserved Macroscopic Superpositions

According to quantum mechanics as seen in physics books the complete state of any physical system is given by the quantum state, represented by the wavefunction ψ, evolving according to the Schrödinger equation which is linear. As such, this theory is notoriously problematic, because the sum of two solutions of a linear equations is still a solution, and this leads to unobserved macroscopic superpositions. This is the so-called problem of the Schrödinger cat, otherwise known as the measurement problem. The standard solution of this problem is given by the collapse rule: when a measurement is performed, the superposition randomly and instantaneously collapses in one of its terms. However, this raises questions about what a measurement is, and why it is not another physical process. To avoid these questions, people have proposed more precise quantum theories, among which the pilot-wave or de Broglie-Bohm theory.[4]

2.2 Absence of a Spatiotemporal Ontology

Having said that, I think that a deeper problem with quantum theory is that it does not have a spatiotemporal ontology. The wavefunction in standard quantum theory could be naturally understood as a field, analog to electromagnetic fields. However, unlike electromagnetic fields, the wavefunction is a field in a high dimensional space. As such, the wavefunction cannot straightforwardly be seen as a wave oscillating in space.[5] Moreover, if one favors constructively explaining the behavior of macroscopic objects as composed of microscopic entities, then one arguably needs to require that both macro and micro objects live in the same space, namely three-dimensional space.[6] In addition, it has been argued that theories in which the fundamental field describing matter are not in spacetime are problematical, as they obscure the role of spacetime, especially when discussing the compatibility with relativity.[7] As we will see in the last subsection, the pilot-wave theory does not have this problem because it is a theory of particles.

2.3 Nonlocality as the 'Spirit' of Quantum Theory

In addition, as emphasized by Einstein, quantum theory with collapse is nonlocal. He took this nonlocality to show that the theory is incomplete: if quantum theory were complete then it would be nonlocal, and nonlocality is not acceptable. Locality is the idea that influences travel at finite velocity. It is always assumed that nature is local for two main reasons. First, since causes precede effects in time, interaction between two objects takes time. Moreover, locality seems necessary to treat systems as effectively isolated: we assume that what happens in a certain spatial region is substantially influenced only by objects which are close by. Notice that Newtonian mechanics violates locality: the gravitational and the electromagnetic forces act instantaneously on matter. For instance, lifting my arm now instantly influences the motion of

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[4] De Broglie (1924), Bohm (1952).

[5] This problem was originally pointed out by de Broglie, Lorentz, Einstein, and Schrödinger. In 1927 de Broglie justified his introduction of particles because "it seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist" (Bacciagaluppi and Valentini, 2009, p. 346). Lorentz wrote to Schrödinger: "If I had to choose now between your wave mechanics and the matrix mechanics [referring to the quantum formalism proposed by Heisenberg earlier that year], I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates x, y, z. (Prizbram 1967). Similar concerns come from Einstein: "The field in a many-dimensional coordinate space does not smell like something real" (Howard 1990). Schrödinger agreed: "of course this use of the q-space (configuration space, *noa*) is to be seen only as a mathematical tool, […] ultimately [...] the process to be described is one in space and time" (Bacciagaluppi and Valentini, 2009, p. 447). The so-called wavefunction realists do not find these arguments compelling; nonetheless, they do not deny that spacetime and a spatiotemporal ontology need to suitably emerge (see Ney 2021, and references therein).

[6] Allori (2013), Maudlin (2019).

[7] Wallace and Timpson (2010) and, in different terms, by Myrvold (2015).

Saturn. Moreover, relativity imposes the constraint that the interaction cannot propagate faster than light. However, the concept of field was later introduced to take care of the first problem: it is a mediator of the interaction, present in every point at each instant. Also, in classical mechanics the intensity of the forces decreases as the inverse of the distance squared, so that the forces generated by distant objects can be neglected and we can treat distant objects as *effectively* isolated, bypassing the second problem.

Einstein argued that the wavefunction collapse is nonlocal, making quantum theory unacceptable. He proposed various incompleteness arguments which rely on the assumption of locality, and which culminated in the famous EPR paper (1935). In the version discussed by Bohm (1951), consider two particles in a spin singlet state flying in opposite directions, where two experimenters are ready to measure their spin properties. According to standard quantum theory, an individual particle in such a pair does not possess a definite spin property in any direction until this property is measured. If quantum theory were complete, then a spin measurement along some direction on one particle would collapse the wavefunction, instantaneously determining the spin measurement outcome for the other particle, regardless of their mutual distance. However, this straightforwardly violates locality. Collapse is instantaneous as the action of the classical forces, but there is no three-dimensional field mediating the interaction, as the wavefunction is not in spacetime. Moreover, unlike classical fields, the strength of the interaction is unaffected by distance. It is this last feature which prevents us from assuming quantum systems to be isolated: they are instead entangled.

Thus, EPR concluded, the only other way of accounting for these correlations is the assumption of 'pre-existing' values of spin properties along some direction which are revealed by spin measurements, but which quantum theory does not specify ('hidden variables'). That is, EPR thought that one could (and should!) locally complete quantum mechanics to make this nonlocality go away. But EPR were empirically wrong. Bell (1964) constructed an EPR-like hidden variable theory and could write a mathematical constraint on this theory's results (namely Bell's inequality), which does not hold for quantum theory.[8] One can then set up a crucial test, which was later performed, and its results were compatible with quantum mechanics,[9] thereby falsifying any EPR-like hidden variable theory aimed at explaining the perfect correlations locally. In other words, the EPR- correlations can only be explained by a nonlocal interaction: any theory matching the quantum predictions has to be nonlocal.[10] In light of this, it seems fair to say that, rather than a problem, nonlocality is the true novel feature of quantum theory, and that it embodies its 'spirit.' Thus, quantum theory is a theory of matter moving in spacetime, interacting nonlocally.

[8] Notice that the derivation of Bell's inequality is more general than that; see notably Goldstein *et al.* (2011).

[9] Aspect (1982).

[10] For more on Bell's proof of nonlocality, see Goldstein *et al.* (2011). See also Maudlin (1994) for a general discussion on the interplay between nonlocality and relativity, as well as Dürr and Lazarovici (2020). Importantly, I am leaving aside approaches which reject the nonlocality conclusion by questioning the hypothesis of statistical independence (superdeterminism) as well as retrocausal approaches.

3 The Pilot-Wave Theory

The pilot-wave theory, which is about to be described in this section, has no problem of superpositions, it has a spatiotemporal ontology, but it is nonlocal. These features would make it a perfectly suitable candidate for a fundamental physical theory in the eyes of any physicist in the pre-quantum era, if it were not for its nonlocality. Nonetheless, now we know more: the experimental violation of Bell's inequality has shown that we cannot do better than this: nonlocality is the spirit of quantum theory. Accordingly, some have advertised the pilot-wave theory as the rational completion of quantum mechanics:[11] it is the simplest, most explanatory spatiotemporal theory in which the interaction is straightforwardly nonlocal, as required by the empirical violation of Bell's inequality.

3.1 Ontology and Nomology

The pilot-wave theory has a clear spatiotemporal ontology: it is a theory of microscopic point-particles which can be understood as composing macroscopic objects, as in the classical case. However, contrary to classical mechanics, particles evolve according to an equation which constrains their velocities: $\frac{d}{dt}x(t) = \frac{\hbar}{m}\nabla S(x,t)$, where $\hbar = h/2\pi$ is the reduced Planck constant, and S determines a guiding field for the particles, and is the phase of the wavefunction: $\psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)}$. This ψ function evolves according to the Schrödinger equation: $\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x,t) + V\psi(x,t)$. For a system of N particles the wavefunction is a function of their positions: $\psi = \psi(x_1, x_2, \dots, x_n, t)$, namely a function is the configuration space of these particles. For the reasons sketched in the previous section, the wavefunction describes the *interaction* between particles. That is, it is part of the nomology: the wavefunction and its evolution equation need to be specified to properly define the theory, just like the law of the forces are needed to properly define classical mechanics.[12]

3.2 Nonlocality

The pilot-wave theory is explicitly nonlocal: for N particles, the wavefunction depends on the configuration of all particles at the same instant. This means that poking one particle affects the behavior of all the others, instantly, no matter how far apart. The effective collapse of the wavefunction is an explicit example of nonlocality in the pilot-wave theory. In this theory, the wavefunction for all particles (namely, the wavefunction of the universe) $\Psi(x_1, \dots, x_N)$ always evolves deterministically according to the Schrödinger equation, so no collapse ever happens. The evolution of sub-systems, like, say, one with three particles with positions x_1, x_2, x_3, is effectively determined by its conditional wavefunction. This is computed by fixing the position variables in $\Psi(x_1, \dots, x_N)$ with the real positions (denoted using capital letters) of all the particles except x_1, x_2, x_3: $\psi(x_1, x_2, x_3) = \Psi(x_1, x_2, x_3, X_4, \dots, X_N)$.[13] Under certain circumstances, sub-

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[11] Bricmont (2016).

[12] For a recent proposal on how to think about the wavefunction along these lines, see Allori (2021). Alternatives views on the nature of the wavefunction in the pilot-wave theory, include the ones discussed in Albert (1996), Norsen (2010), Esfeld *et al.* (2014), Suàrez (2015), Hubert and Romano (2018).

[13] See Dürr, Goldstein and Zanghì (1992).

systems may have a Schrödinger evolving conditional wavefunction, but not when the sub-system is being measured: after the interaction with the measuring device, the sub-system conditional wavefunction ψ *effectively* collapses into one of the terms of the superpositions of Ψ.[14] So, the pilot-wave theory explains the effective collapse of the wavefunction granting that the particles nonlocally interact.

3.3. The Role of Hidden Variables

The pilot-wave theory is a hidden variable theory in the sense that it completes the description of the state of any physical system by adding to the wavefunction the specification of the particles configurations. Nonetheless, it is not a hidden variable theory as EPR would have wanted. As reconstructed in Bricmont *et al.* (2022), EPR needed hidden variables such as spin properties to locally explain the perfect correlations. However, these properties had to be found to be the same in all possible measurements of them. That is, spin values had to be *non-contextual*: the value of a non-contextual property is revealed by the measurement, and it does not depend on the way in which it is measured. All properties we think of as genuine properties are non-contextual. The violation of Bell's inequality falsifies these theories, in which the hidden variables explain the correlations locally. Instead, the pilot-wave theory correctly violates Bell's inequality, just like quantum theory: in this theory the correlations are explained by what Einstein thought unthinkable, namely nonlocality. Notice that in order for the pilot-wave theory to violate Bell's inequality, and thus make the same prediction of quantum theory, it cannot have any hidden variables *a-la* EPR (otherwise the theory would have been falsified). That means that in the pilot-wave theory spin properties are contextual: spin 'measurements' do not reveal the spin values pre-measurement, so they are not really measurements of anything. [15] Measurements should be processes in which the system-detector interaction is negligible so that the outcome faithfully reproduces the value of the property before it is measured. Instead, a spin 'measurement' is a process in which the system-device interaction significantly changes the system's property value so that the outcome does not reflect any property of the system before the interaction. Accordingly, there is no spin property in the pilot-wave theory. The only real hidden variables which can be revealed by measurements are particles positions. Notice that the particles positions are hidden variables which are not introduced to explain the observed correlations of EPR-type of experiments; the nonlocality of the interaction explains them. Rather, as anticipated above, particles positions are needed to ground the theory to spacetime, providing quantum theory with a spatiotemporal ontology.[16]

4. The 'Spirit' of Relativity

Some argue that the type of nonlocality in the pilot-wave theory makes this theory particularly unsuited for a relativistic extension. To properly assess these claims, one needs to better understand what a relativistic extension requires.

[14] For more on this, and on the conditional wavefunction in the pilot-wave theory, see, notably, Norsen (2016).
[15] See Bricmont (2024).
[16] For more on this, see Allori (2013).

Relativity was originally formulated in terms of two principles: the constancy of the velocity of light c in all inertial reference frames and the relativity principle, according to which physical laws should have the same form in all inertial frames. Together these two principles imply that the description of the phenomena in two inertial frames are connected by Lorentz transformations.

4.1 Relativity and Locality

The first principle, about the velocity of light, comes from the observation that in electromagnetism the velocity of light is constant, independent of the reference frame. As anticipated, when combining this principle with the second principle, it follows that this constant is the maximum possible velocity. In this respect, this encodes the idea of locality that interaction travels, and it restricts it by stipulating that it never travels faster than light. Thus, no interaction can be instantaneous. As we have seen, locality is not a prerogative of relativity, but a more a generally desirable feature of physics: for instance, electrodynamics is local, as the interaction is mediated by the fields travelling at the velocity of light. But, as we also have seen, the violation of Bell's inequality makes locality empirically not an option.[17] So, given quantum nonlocality, the idea of relativity as prescribing a limit to the velocity of interaction has to go. Aspect's experiments falsify this: some influences are instantaneous.

4.2 Relativity and Lorentz Invariance

Turning to the second principle, the idea behind it is very general: it is the requirement that the description we provide of the phenomena is as independent as possible from our point of view. The principle of Galilean relativity is the expression of the principle of relativity within classical mechanics. It states that the form of the laws is the same in all (inertial) frames, which for classical mechanics translates as: laws in different frames are connected by Galilean transformations. Or: laws are invariant under Galilean transformations. In special relativity, the relativity principle still holds: the description of the phenomena should be as observer-independent as possible. However, when combined with the light principle, it leads to the Lorentz transformations, rather than the Galilei ones. Thus, the relativity principle is equivalent of stating that physical laws should be invariant under Lorentz transformations. Einstein wrote: "The universal principle of the special theory of relativity is contained in the postulate: The laws of physics are invariant with respect to Lorentz transformations (for the transition from one inertial system to any other arbitrarily chosen inertial system)."[18] Is something else required from a theory to be 'genuinely' relativistic? Is there a deeper 'spirit' of relativity, aside from Lorentz invariance?

4.3 Relativity as a Theory of Space-Time Structure

Another formulation of relativity, which does not explicitly use the two principles but is nonetheless equivalent, is in terms of Minkowski spacetime. Since c is constant, one can consider ct as an additional spatial coordinate in a newly combined 'spacetime' manifold. This is Minkowski spacetime, which is a four-dimensional space which unifies three-dimensional

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[17] For more on relativity and nonlocality, see Maudlin (1994) and Dürr and Lazarovici (2020).

[18] Einstein (1949).

space and time. In it, each point has coordinates given by the quadruple $(x, y, z, ct) = (\boldsymbol{x}, ct)$. Two events in spacetime, A and B, may be represented in coordinate systems S as well as coordinate system S', moving uniformly at velocity v with respect to S. S and S' are connected by Lorentz transformations and it turns out that if the axes of S are orthogonal with one another, the ones of S' instead meet at acute or obtuse angles. Nonetheless, given the second principle, they are equivalent, and given the first principle, light travels in both along the diagonal lines.

As direct consequences of Lorentz transformations, one has a new transformation for velocities, effects like time dilation and length contraction. Moreover, one could show that if signals could travel faster than light one would encounter causal paradoxes in which a signal arrives before it is being sent. So, faster than light motion is impossible. In addition, it follows from the Lorentz transformations in terms of coordinate differences, that two events which are simultaneous in S are not necessarily simultaneous in S'. Since all frames are equivalent, the notion of absolute simultaneity is lost. This is the so-called *relativity of simultaneity*. In Minkowski spacetime the trajectory of an object moving at the speed of light is a 'light-like curve.' One therefore can imagine that for any point P, spacetime divides into three regions delineated by the light ray trajectories originating from and arriving at P. They form the surface of a double cone (the 'light-cone') whose vertex is P. These three regions are the absolute past of P (the lower part of the light-cone), the absolute future (the upper light-cone), and the points elsewhere. An object in P can only come from the past light-cone points, and it can only evolve into the future light-cone points. Light travels on the surface of the light-cone, while particles are confined to time-like worldlines, inside the light cone. Two points in the elsewhere region (outside the light-cone) would have to travel faster than light to reach one another. Since nothing propagates faster than light, two such points, called space-like separated, cannot interact. Minkowski spacetime has a metric, specifying the distance between spatiotemporal points, or events. This is given by the interval $I = \sqrt{(\boldsymbol{x}_2 - \boldsymbol{x}_1)^2 - c^2(t_2 - t_1)^2}$. Accordingly, the surfaces containing points at equal distance form P are hyperboloids. One can slice spacetime into instantaneous snapshots called space-like hypersurfaces. Events on a given snapshot are simultaneous, namely they happen at the same time. However, as anticipated, observers in relative motion will disagree about the simultaneity of two events. That means that there are different possible ways of slicing spacetime into instantaneous surfaces, one for each observer. These different 'slicings' are titled with respect to one another, ultimately because of the Minkowski metric, which has hyperboloids as same distance surfaces. The upshot is that no slicing is 'the' correct one: there are simultaneous events only with respect to a particular slicing. This is the relativity of simultaneity: there is no preferred spatiotemporal slicing.

Especially given its Minkowski formulation, relativity seems straightforwardly a theory of spacetime. Because of this, any theory containing a non-spatiotemporal object, such as the wavefunction, seems already in tension with relativity. In other words, as anticipated, one cannot see the wavefunction as a spatiotemporal mediator field like one could do in the case of electrodynamics. In this reading, only a theory in which matter and its interaction are represented by spatiotemporal entities (emerging or fundamental) could qualify as a candidate for genuinely relativistic status. Nonetheless, the requirement of a theory to be framed in spacetime is clearly not enough to qualify a theory as relativistic, as obviously one can imagine spatiotemporal theories which have nothing to do with relativity. A better idea is to think that a truly relativistic theory only uses relativistic spacetime structure as encoded in Minkowski spacetime, such as the light-cone. Notice that a global slicing of spacetime, as the ones we will see in the next section, would be a structure over and above what one finds in Minkowski spacetime.

5. Relativistic Nonlocal Theories

So, to summarize the discussion the previous sections, a quantum theory would be nonlocal, and it would be relativistic if it were Lorentz invariant. Moreover, ideally, the nonlocal interaction would have to happen *via* a Minkowski spatiotemporal structure. The following two sections briefly discuss several relativistic pilot-wave and spontaneous collapse theories and their features. It is worth noticing that, while they are certainly notable proposals, none of them is (yet) fully capable of describing an interacting picture.[19]

5.1 Relativistic Pilot-Wave Theories

Some think that the nonlocality of the pilot-wave theory implies that the pilot-wave theory fundamentally violates Lorentz invariance. Accordingly, in such a theory Lorentz invariance would be an emergent symmetry of the observational level.[20] Nonetheless, Lorentz invariant extensions of the pilot-wave theory have been proposed: some have a Dirac evolving wavefunction, others a Klein-Gordon evolving wavefunction.[21] All these theories have therefore deterministic evolution both for the wavefunction and the particles.[22] They all are explicitly nonlocal, as they should, and since each particle configuration in the wavefunction is taken at the same time, they all require a notion of *absolute simultaneity*. So, while the Lorentz invariance of the law can be taken care of by having a suitably evolving wavefunction, one will also need a global spacetime structure to implement nonlocality: a preferred foliation of spacetime. A possibility is to simply postulate one.[23] Otherwise, the foliation may be suitably defined in terms of the wavefunction. [24] In both cases (postulated or derived foliation), the preferred spatiotemporal foliation is not a static object but evolves dynamically according to a Lorentz invariant law. dynamical and Lorentz invariant itself.

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[19] See, e.g., Dürr and Lazarovici (2020), chapters 11 and 12.

[20] Bohm and Hiley (1993), Holland (1993), Valentini (1997).

[21] For all these theories, see Tumulka (2018) and references therein.

[22] Proposals which include particles creation and annihilation will have a stochastic particle evolution, see Tumulka (2018) and references therein. Nonetheless, one could describe particle creations and annihilations even with a deterministic dynamics (see Oldofredi 2020 and references therein). Galvan (2015) has proposed a Lorentz invariant pilot-wave theory with formally no preferred foliation, which however possesses an absolute synchronization. He argues that this makes his theory more compatible with relativity than the alternatives, while I remain unconvinced, as synchronization is still a global notion (see Drezet 2019 for a similar model).

[23] Dürr *et al*. (1999).

[24] Dürr *et al*. (2014).

Notice that such preferred foliation of spacetime is not *ad hoc*: it is in principle undetectable, but there is an explanation of why that is the case, rooted on our lack of access to the particle configurations.[25] Nonetheless, because the preferred foliation is a global spatiotemporal slicing which is not part of the Minkowski spatiotemporal structure, many have looked for alternatives, like spontaneous localization theories, to see whether they can do better.[26]

5.2 Relativistic Stochastic Theories

The first working spontaneous localization theory, also called GRW theory, was originally proposed to build the collapse in the dynamics of the wavefunction, which now becomes nonlinear and indeterministic.[27] In this theory the wavefunction evolves according to the Schrödinger equation up to a random time, at which it collapses around a random location with a given accuracy (determined by free parameters, which are seen as new constant of nature). One could turn this into a theory about some spatiotemporal ontology, by suitably postulating one and adding it to the theory. Various proposals have been put forward and respectively dubbed GRWf (for an ontology of spatiotemporal events, or flashes),[28] GRWm (for a matter field ontology),[29] and GRWp (for particles).[30] In these theories the wavefunction governs the spatiotemporal ontology in a stochastic way. Several relativistic extensions have been suggested, most notably rGRWf and rGRWm.[31] They have in common that they are all Lorentz invariant, nonlocal and only use relativistic spatiotemporal structure.

6. (Nonlocal) Quantum Relativity?

Within their limitations, all the proposals discussed in the previous section are nonlocal and Lorentz invariant. However, while relativistic pilot-wave theories are linear and deterministic, but they require a preferred spatiotemporal global structure (the foliation), relativistic GRW-type theories are nonlinear and stochastic but also foliation-free. If requiring exclusively a relativistic spatiotemporal structure is necessary for labelling a theory 'genuinely relativistic,' then only GRW-type theory qualify. Nonetheless, in this section I am going to argue that, given nonlocality, it would be better to proceed with a theory which modifies both theories as minimally as possible, instead of being as faithful as possible to relativity, especially if that implies modifying quantum theory in an unnatural way. This leads to identify relativistic pilot-wave theories as the best balance between nonlocality and Lorentz invariant spatiotemporal structure.

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[25] Bricmont (2016). Nonetheless, many physicists think that it would be better if one could empirically observe the foliation: "From our perspective, for as long as we are confined to a state of statistical equilibrium that hides the underlying non-locality from direct view, it seems probable that the argument [non-locality vs. relativistic space-time] will continue to be unresolved" (Valentini 2008).

[26] See Bell (1987), Tumulka (2006). See also e.g., Ghirardi (2012), Myrvold (2021), and references therein.

[27] Ghirardi, Rimini, and Weber (1986).

[28] Bell (1987).

[29] Benatti *et al.* (1995).

[30] Allori (2020) and references therein.

[31] Respectively in Tumulka (2006, 2021) and Bedingham *et al.* (2013).

6.1 *Combining Quantum Theory and Relativity*

To combine quantum theory and relativity we need to combine their essential features. I have argued earlier that quantum theory is a theory of matter moving in spacetime and interacting nonlocally. So, in short, the spirit of quantum mechanics is the nonlocality of interaction. I also have argued earlier that relativity is a theory about spacetime and its structure, whose fundamental symmetry is Lorentz invariance. So, in short, the spirit of relativity is Lorentz invariant spatiotemporal structures. Thus, the former is a theory of matter, and the latter is a theory of the arena in which matter interacts. To combine these spirits, we need to build a theory of matter evolving in spacetime interacting nonlocally whose fundamental symmetry is Lorentz invariance.

6.2 *Minimal Modification of Relativity but Unnatural Modification of Quantum Theory*

Relativistic GRW-type theories provide such a theory by being as faithful as possible to relativity. That is, they are nonlocal, but they only use relativistic spatiotemporal structure (no global structure). However, what are the costs of keeping only relativistic spatiotemporal structure to accommodate quantum nonlocality?

Notice that while it would perhaps be simpler or better to have nonlocal, Lorentz invariant, foliation-free theories which are also deterministic, I have argued elsewhere that only stochastic theory can be (nonlocal, Lorentz invariant and) foliation-free. I have also argued that this stochasticity leads to the dissolution of the notion of locality.[32] In fact, without continuous worldlines for the spatiotemporal ontology it is difficult to say in what sense an event has caused another. This threatens the very notion of influence or interaction: how can we make sense of an object influencing another one if we cannot even identify the cause and the effect?[33] In turn, without the notion of influence it is difficult to make sense of the notion of locality, since locality is defined as influence travelling at finite velocity. Relatedly, GRW-type theories display 'supernonlocal' behavior. That is, due to the stochasticity of their law, they allow for nonlocal transfer of matter even in single 'particle' systems. Consider a single 'particle' confined in a box.[34] Inserts a barrier in the box, splitting the wavefunction, separate the two half-boxes and then open one of them. In GRWm, say, given that the system is microscopic (only one 'particle'), the matter field is spread out in both half-boxes until one box is opened, at which point the wavefunction instantaneously (thus nonlocally) localizes.[35] Nothing like this happens in the pilot-wave theory, where to see nonlocal effects, one needs at least two particles. This is problematical because locality, or the lack thereof, is a feature of interacting systems of two or more particles. Here instead even single-particle systems display nonlocality, and it is unclear how to interpret it. Nonlocality is not about influence anymore, as there is no interaction within one system. So, this is another example that the stochasticity of the law, responsible of this

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[32] Allori (2022).

[33] For a similar point, see Esfeld and Gisin (2014). This has been questioned by Myrvold (2021).

[34] The scary quotes are needed because, strictly speaking there may be no fundamental particles in these theories, which may have a matter field or a flash ontology. Nonetheless, we can speak about a single 'particle' because the system behaves as if it were composed by a single particle-like entity.

[35] A similar scenario holds for GRWf and GRWp; for details, see Allori (2022).

supenonlocality, undermines the notion of influence upon which the desirability of locality was based. Thus, while these theories only use relativistic spatiotemporal structure, they have to implement nonlocality with stochastic laws dissolving the intuitive notion of interaction.

Moreover, these theories are empirically inequivalent to quantum mechanics, predicting for instance that energy increases. Even if one could accommodate this theoretically, the failure of energy conservation may be rightfully considered a drawback of this theory. Even if crucial tests to determine which theory is correct are underway, GRW-type theories have free parameters which are adjusted after every single experiment to avoid falsification. This constant adjustment makes these theories rather *ad hoc*, if not fine-tuned.

Therefore, to summarize, in my opinion it is useful to explore GRW-type theories in order to probe the consequences of combining quantum nonlocality with a minimal relativistic spatiotemporal structure. The result however is that a nonlocal theory without any additional relativistic spatiotemporal structure leads to the dissolution of the natural notion of nonlocality. Moreover, this can only be accomplished by a nonlinear and stochastic theory which modifies the beauty of the Schrödinger evolution, with *ad hoc* parameters, which predicts, among other things, that energy is not conserved. In other words, any quantum (i.e., nonlocal) theory which tries to use only a minimal relativistic spatiotemporal structure, namely any theory which does not modify relativity (aside from being nonlocal) ends up unnaturally modifying quantum theory. If this is what 'genuinely relativistic' necessitates (Lorentz invariance and only Minkowski spacetime structure), then it seems better to forget about it.

6.3 Minimal Modification of Both Theories

The alternative is given by the Lorentz invariant pilot-wave theories, which are deterministic and linear. In these theories quantum mechanics is naturally completed by the specification of particles positions and relativity is completed by the specification of a preferred spatiotemporal slicing. These theories are not *ad hoc*, not stochastic, not nonlinear, and they have not so hard-to-swallow predictions. The explicit nonlocality of the pilot-wave theory clearly embodies the spirit of quantum theory, and it also clearly suggests how to amend relativity in light of nonlocality: add a preferred spatiotemporal foliation. That is, relativistic pilot-wave theories provide the best balance between the quantum and the relativistic spirit, respectively nonlocality and Lorentz invariant spatiotemporal structure, by slightly modifying them both: add particles to quantum theory and add a preferred foliation to relativity. All this by remaining deterministic and not *ad hoc*.

As emphasized above, many people think the preferred foliation is in contrast with the spirit of relativity: a preferred foliation reintroduced the notion of absolute simultaneity which is not part of the Minkowski spacetime structure and which is not required by Lorentz invariance. However, since we are trying to put together quantum theory with relativity, Lorentz invariance cannot be the only constraint, as we also need to accommodate nonlocality. In other words, relativity was formulated with the idea that interaction travels, and that Lorentz invariance is a constraint to its maximum velocity. Thus, it was natural to suppose that there was no other structure than the one dictated by Lorentz invariance. However, now the situation has changed: we know about quantum nonlocality and the natural way to accommodate it is to have an additional structure to Minkowski spacetime. This is not going to be a redundant structure, as it is needed to account for nonlocality which we know is part of the structure of the world. There are no simpler options than doing that. As we have seen, in GRW-type theories we may drop this preferred foliation at the costs of unnaturally modifying quantum theory in an artificial way. But why would we do that? *Why protect relativity at all costs if we already know that we need to modify it already, due to nonlocality?* Why modify only quantum theory? Wouldn't it be best to modify minimally both theories instead? So, to me the choice is clear: relativistic pilot-wave theories provide the best balance between quantum mechanics and relativity, and thus they are the way to go. Relativistic pilot-wave theories provide the minimal modification of both theories to accommodate all the experimental data, including nonlocality. They are theories of matter evolving in spacetime interacting nonlocally, whose fundamental symmetry is Lorentz invariance. These theories add a spatiotemporal ontology to quantum theory, and a preferred spatiotemporal foliation to relativistic spacetime. By doing this, they put all the puzzles pieces together: the lack of a spatiotemporal ontology of quantum theory is solved by adding particles which also allow to recognize relativity as a theory of spacetime; quantum nonlocality is explicitly acknowledged in terms of the wavefunction and it suggests the missing piece of relativity, namely a preferred foliation. Both quantum theory and relativity are incomplete, and they are both naturally and minimally completed by relativistic pilot-wave theories which therefore seem to me tailor-made, or just what the doctor ordered.[36]

6.4 How to Understand Relativity

If we go in this direction, it does not mean that relativity is irrelevant: laws are Lorentz invariant, and all the relativistic spatiotemporal structures (light-cones, Minkowski metric, etc.) will still be there. Both relativity principles still hold. The relativity principle, which is a guide to what a good theory should look like, is still true, as the theories are Lorentz invariant. However, spacetime is not as invariant as we thought it would be: there is a preferred frame. But one cannot eliminate what is needed to explain the empirical data: as in the case of classical mechanics one needs a preferred frame to account for the effects of accelerated motion, so here one needs a preferred frame to deal with nonlocality.

The only other implication of having a preferred spatiotemporal foliation, in addition to straightforwardly accounting for nonlocality, is that the notion of absolute simultaneity comes back into the picture. But, as discussed above, relativity of simultaneity is a consequence of Lorentz invariance alone, while now we also have the nonlocality constraint. As a consequence, there seems to be little reason in insisting on requiring that simultaneity is relative. This seems to be the case, especially considering that it is a counterintuitive feature of relativity which does not seem to contribute to its overall explanatory power, being a consequence, rather than a presupposition, of (unconstrained) Lorentz invariance.

[36] This is what Detlef Dürr used to say.

So, in my opinion, the best way of thinking of relativity is as an incomplete theory of spacetime structure, which relativistic pilot-wave theories suitably complete. Arguably, Einstein might not have been too surprised or upset to discover that relativity is not the last word. In fact, he originally believed that relativity was a principle theory, and in his view principle theories require a deeper explanation in terms of a constructive theory.[37] Principle theories (like thermodynamics) explain macroscopic phenomena constraining them in terms of principle, while constructive theories (like kinetic theory) explain them in terms of their microscopic components and their dynamics, and give a deeper understanding of why the principles hold. If relativity is a principle theory, the corresponding constructive theory would be like the one proposed by Lorentz, according to which relativistic effects are explained by actual contraction of matter. Einstein rejected it, not because he thought that it was wrong-headed but rather because he thought we lacked a satisfactory theory of matter.[38] Relativistic pilot-wave theories do not currently provide a constructive explanation of the principles of relativity or their effects. Nonetheless, what we know about matter interaction, which Einstein did not, is that it is nonlocal. So, something needs to be modified in relativity theory to account for it, regardless of whether it is a principle theory or not.

Perhaps one may think of relativity as a principle theory constraining acceptable *theories*, rather than phenomena: theories should at least be Lorentz invariant and have a Minkowski spatiotemporal structure, without specifying whether they need to have something else or not. For what it is worth, this seems compatible with what Einstein wrote: "General laws of nature are co-variant with respect to Lorentz transformations. This is a definite mathematical condition that the theory of relativity demands of a natural law, and in virtue of this, the theory becomes a valuable heuristic aid in the search for general laws of nature."[39]

7 Final Considerations

If what is argued here is correct, then the situation is somewhat bitterly ironic. In fact, Einstein's most famous argument for the incompleteness of quantum theory is that if it were complete, it would be nonlocal; instead, quantum theory is incomplete *and* nonlocal. Moreover, relativity turns out to be incomplete too, because it cannot deal with quantum nonlocality. Lorentz invariant extensions of the pilot-wave theory, I have maintained, provide the current best option at completing both theories: the ground quantum theory in spacetime and they account for nonlocality by adding the needed spatiotemporal structure to relativity.

Naturally, there are many open questions. Quantum nonlocal interactions are instantaneous. What implication does it have for the notion of interaction? As we saw, the idea of locality captures the idea that interaction travels: an object takes some time to 'notify its presence' to another object. Since instead we have nonlocality, we are back to Newton's problem that lifting my arm instantly influences the motion of Saturn. As we saw, in the classical case we can

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37 Einstein (1919).

38 Pauli and Eddington were also supporters of constructive explanation in relativity (see Brwon and Pooley 2004, and references therein).

39 Einstein (1920).

alleviate the problem because we can use fields to mediate the interaction, which then becomes not instantaneous, and the strength of the interaction decreases with distance so that we can ignore the effects of objects which are far away. In the quantum case instead, as anticipated, we do not have either: no spatiotemporal mediator, and no decrease of strength with distance, even if the interaction of the system with its environment (decoherence) effectively destroys entanglement, giving rise to a seemingly local universe. Could (or should) we still think that interaction travels under these constraints? If it does, it travels at speed greater than the velocity of light, which in turn means that light is not the fastest. If instead we give up the idea that interaction is something that travels, then what are the consequences and how else should we understand interaction? As a side remark, let me notice that if we do abandon the notion of travelling interaction, we also lose the need of having fields as mediators, which would make the overall ontology of the theory simpler. Be that as it may, how should we think of entangled particles? One possibility could be to think of them not as two particles interacting nonlocally, but as nonseparable entities. While the notion of locality has to do with interaction, the notion of separability has to do with the way matter is. That is, if the properties if the whole are completely determined by the properties of its parts, then we talk about separability.[40] But in this context matter is made of particles, so in virtue of what is a pair of entangled particles nonseparable? Another possible reaction is to simply maintain that the law governing the behavior of matter is nonlocal, and such law does not 'propagate' but rather it is what primitively connects a pair of entangled particles. Presumably, one can think of many more ways to make sense of nonlocality, and I do not know which attitude will provide itself to be most fruitful. But, since the word is nonlocal, everyone, not just those who endorse the pilot-wave theory, needs to start asking this type of question.

[40] Howard (1985).

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