Spontaneous Localization Theories
with a Particle Ontology

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Abstract

Spontaneous localization theory is a quantum theory proposed by GianCarlo Ghirardi, together with Alberto Rimini and Tullio Weber in 1986. However, soon it became clear to Ghirardi that his work was more than just one theory: he actually developed a framework, a family of theories in which the wavefunction jumps, but where the ontology of the theory is underdetermined. After acknowledging that the wavefunction did not provide a satisfactory ontology, he assumed that matter was described by a continuous matter density field in three-dimensional space, whose evolution is governed by a stochastic wavefunction evolution. Alternatively, Bell assumed that the wavefunction would govern a spatiotemporal event ontology, dubbed 'flashes.' However, not much work has been done with the perhaps most obvious possibility, namely that physical objects are made of particles. This paper has two aims. First to explain the reason why people require spontaneous localization theory to be more than just a theory about the wavefunction. This is done by showing how the problem everyone in the foundation of quantum mechanics take to be the fundamental problem of quantum mechanics, namely the measurement problem, is a red herring. Then, the paper explores the possibility of spontaneous localization theories of particles. I argue that this discussion is not a mere exercise, as spontaneous localization theories of particles may be amenable to a relativistic extension which does not require a foliation, and because in general the peculiar type of indeterminism of spontaneous localization theories may help shedding new light on the nature of the tension between quantum theory and relativity.

1. Introduction

In this paper I wish to discuss spontaneous localization theories of particles within the primitive ontology framework. In the first part of the paper, I argue in favor of spontaneous localization theories with a primitive ontology, while in the second part I discuss the tenability and the superiority of a particle primitive ontology for this kind of theories. First, I discuss how the origin of the interpretational problems of quantum theory is not, as commonly maintained and as explained in section 2, the measurement problem. Indeed, the measurement problem is a red herring: even if one solves the measurement problem the theories so obtained are still problematical. I discuss in section 3 how the real problem stems from thinking of the wavefunction as describing physical objects. In line with the primitive ontology approach (POA), in section 4 I present the various proposals for spontaneous localization theories understood as theories about some microscopic ontology in three-dimensional space, in terms of a

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matter density field, or four-dimensional spacetime, in terms of flashes. In section 5, instead I move to possible spontaneous localization particle theories, and show how only one of the alternatives is worth pursuing. In section 6 I compare this theory with spontaneous localization mater density and flash theories, and finally I propose an approach that could lead to a relativistic spontaneous localization particle theory. I conclude arguing that the value of considering any of these theories does not reside in the simplicity of their ontology or law (as they are not simple when compared to alternatives such as the pilot-wave theory) but rather it lies in the lesson they may teach us about the compatibility between quantum theory and relativity.

2. The Traditional Problem of Realism and Quantum Mechanics

Since its birth quantum theory has been such an interpretative nightmare that many felt the lesson to be learned was to embrace instrumentalism. However, many others still searched for a realist interpretation of quantum mechanics, starting most famously Albert Einstein, Louis de Broglie and Erwin Schrödinger, continuing with David Bohm, High Everett, GianCarlo Ghirardi and John Stuart Bell. Most often than not, the problem of reconciling quantum theory with a realist description of the world is summarized mentioning the so-called *measurement problem*. This problem has been around since Schrödinger (1935) criticized the reading of standard quantum mechanics (the one of Bohr and Heisenberg) according to which the microscopic world has a 'blurred reality,' to be contrasted with the 'definite reality' one observes macroscopically. If the microscopic world is 'blurred' but completely described by the specification of a linearly evolving wavefunction, then this 'blurriness' would immediately spread to the macroscopic scale when we couple the microscopic system (in Schrödinger's example a radioactive source) to a macroscopic one (a cat). In other words, granting that radioactive nuclei can be 'blurred', we can measure whether a nucleus has decayed or not by hooking it up to a device which would kill a cat in case of decay. Since someone's death cannot be 'blurred,' one immediately sees that this interpretation is untenable. This conclusion is often reformulated as a problem for the view that a Schrödinger evolving wavefunction provides the complete description of a physical system: if every physical system is completely described such an object, because of the linearity of the Schrödinger equation we should observe macroscopic superpositions such as a cat which is both alive and dead. Since we do not observe them, this reading is empirically inadequate. Traditionally, three ways to get around the problem have been identified:[2] 1) deny that the wavefunction provides the complete description of each physical system; 2) deny that macroscopic superpositions are a problem; and 3) deny that the wavefunction evolves according to the Schrödinger equation. In the 1950s, building on some groundbreaking work done by de Broglie

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[2] See e.g. Bell (1987).

(1927), Bohm (1952) proposed a solution of this problem along route 1. In fact his theory, which many dub the pilot-wave theory or Bohmian mechanics, is often taken to be one in which there are particles and waves, and the particles' behavior is determined by the wave's behavior. Few years later, Everett (1957) proposed his 'relative state formulation' of quantum theory, which goes along route 2. Everett's theory later was developed into the so-called many-worlds theory, often characterized as accepting the macroscopic superpositions as real but suitably existing in other, undetectable, worlds which do not interact with the one we are in. In the 1980s, GianCarlo Ghirardi, Alberto Rimini and Tullio Weber (1986) added their solution going along the lines of route 3. In their theory, called among other names spontaneous localization theory or GRW theory, the wavefunction does not evolve according to the Schrödinger equation but it suitably collapses at random into one of the terms of the superposition, localizing in a small region of space in the case of macroscopic objects. All these theories are generally taken to be quantum theories that are amenable to a realist interpretation. The reason which is given for this is that they do not suffer from the measurement problem. In fact in the pilot-wave theory, we do not observe macroscopic superpositions because the complete description of the system is given by the specification of particles and wavefunction, and particles are always localized, just like cats, whether they are dead or alive. In the many-world theory the various terms of the superpositions 'live' in other worlds which are inaccessible to us and do not interact with us, and this explains why we do not encounter the alive counterpart of a dead cat. Finally, in the spontaneous localization theory the facts that the wavefunction localizes very fast for macroscopic objects explains why we never see macroscopic objects in superposition states, and why dead cates remain, perhaps unfortunately, dead.

3. The Real Trouble for the Quantum Realist

In the previous section I kept using the locutions 'traditionally,' 'often,' or 'generally.' Recently, a new approach to quantum theories, the so-called *primitive ontology approach* (POA), has been proposed,[3] and while this has not been explicitly stated anywhere, I think that the main lesson from it is that the tension between realism and quantum mechanics is not captured by the measurement problem. This implies that the pilot-wave theory is not a theory of waves and particles; the many-world theory and the spontaneous localization theories are not theories 'about' the behavior of the wavefunction. The main idea of the POA, I argue, is that the measurement problem is a red herring. That is, even if we solve the measurement problem, the tension between realism and quantum mechanics remains open. The real issue is instead the so-called *configuration space problem*: the wavefunction, whether it provides the complete description of a system or not, is not an object which is defined in three-dimensional

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[3] See Dürr, Goldstein, Zanghì (1997), Allori *et al*. (2008, 2011, 2014); Allori (2013a, 2013b, 2105a, 2015b, 2019).

space. Instead it is a function whose domain is the space of the configuration of particles, if there are particles as in the pilot-wave theory, or of 'particles' in the case of the other theories.[4]

The proponents of the POA have given several argument against the tenability or desirability of a non-three dimensional ontology such as the wavefunction, but I am not going to reproduce here.[5] Rather, let me focus on the reason why I think this approach implies that the measurement problem is not the real problem for the quantum realist. This problem is created by the existence of macroscopic superpositions, which arise from a linearly evolving entity, namely the wavefunction, which is taken to represent physical objects. However, there is nothing intrinsically strange in superpositions, either microscopic or macroscopic: they are natural for waves, and the wave ontology has been successfully used in physics before, as in the case of light, for instance. Superpositions are a problem only if we try to describe matter, as it is never found in such a state. So, one obvious solution would be to deny that matter is described by waves. However, historically, this is not what has been done. Instead, the theories presented in the previous section all maintain the wavefunction as part of the ontology.[6] This is so even in the case of the pilot-wave theory, in which it is granted that the wavefunction does not provide the complete description of physical systems. However, I argue that the problems are not over. In fact, the view that the pilot-wave theory is a theory in which the wave is material, just like the particles, is a theory of N particles in three-dimensional space, and a wave in $3N$ dimensional (configuration) space. And this is still problematical: what is the cat currently on my lap made of, particles of waves? As we said, to solve the measurement problem getting rid of macroscopic superpositions, one needs to get rid of the wave part. So, one thus could say that matter has a dual ontology: on the one hand, physical objects like cats are made of particles; on the other hand here is also another physical entity represented by the wavefunction, which is understood to be similar to electromagnetic fields, as another 'kind' of constituent of the world. Despite this, unlike electromagnetic fields the wavefunction is not defined in three-dimensional space, and this created another problem: how is this wave in configuration space supposed to interact with particles in three-dimensional space? Moreover, consider now the spontaneous localization theory. Before it localizes, for a brief but finite instant, the wavefunction is spread out in configuration space. That

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[4] That is, regardless of whether some coordinates $r_1, r_2, \dots, r_N$, where N is the number of particles thought to exist in the universe (roughly of the order of 10^{90}) can be interpreted as the position of real particles or not, the wavefunction is a function of a high-dimensional variable $q = (r_1, r_2, \dots, r_N)$.

[5] Se Allori (2013a, 2013b, 2015a, 2015b, 2018, 2019) and references therein for an exposition of these arguments.

[6] The reason for this is unclear, and I cannot fully explore this issue in this paper. Presumably however one could say that historically the theory developed and flourished after the proposal of the Schrödinger's equation, and so from that moment on it seemed unthinkable to not consider it as part of the theory.

is, the cat on my lap this wavefunction describes is, for a brief but finite instant, spread out configuration space, where the emphasis is not so much on 'spread out' but on 'configuration space.' It seems only slightly counterintuitive to think that cats have infinite three-dimensional tails: we think that cats have a matter distribution that do not extend in space to infinity, but we are wrong. This is not a big problem, however, as these tails are undetectable. [7] Instead, it seems much more troublesome to think that the cat, before the wavefunction collapse, was not in three-dimensional space but rather she was in the high dimensional configuration space. That is, the cat, which we would normally think of being described by a (soft) lump of matter now localized here on my lap, before localizing here was instead in *another* space with a large number of dimensions. Now consider the many-worlds theory. In this theory physical objects are 'made of' wavefunctions, which suitably 'splits' into 'different worlds' thus avoiding the macroscopic superpositions. However, in each world the object is described by a component of the superposition, which is something in configuration space, while the physical objects we experience are not in this space. Whatever we devise to account for why we perceive what we perceive will however not remove the fact that there is another space involved before this perception happens.

If one wants to stick to the idea that the wavefunction represents physical objects, after having solved the measurement problem, one has also to answer all these question.[8] In contrast, the POA takes a completely different route. Instead of trying to explain the connection between configuration space and three-dimensional space in the various theories, I take it, the proponents of the POA deny that the wavefunction is material to start with. What represents material objects, the so-called *primitive ontology* (PO), is instead something else. It's not important exactly what it is (a field, a particle, a string, a spatiotemporal event) aside from the fact that it is in three-dimensional space (or four-dimensional spacetime). In this way, there is no configuration space problem, as everything is in the same space. Moreover, there is no measurement problem, because either the PO does not superimpose, or the macroscopic superpositions are short lived and interference effects are undetectable. The former situation happens when one has a PO of particles, as in the pilot-wave theory, or, as we will see in the next section, a PO of flashes, as in some version of the spontaneous localization theory, while the latter happens in some version of the spontaneous localization theory.

Before moving on, let me pause for a second. To reject the wavefunction as material may seem like an outrageous proposal: abandon the very object that is taken to define quantum mechanics seems to be to deny quantum mechanics itself. That may be so: perhaps just to ditch quantum mechanics as we know it is the right thing to do. However, I think this is the wrong way of thinking about this move: the POA is

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[7] However, see section 4 for more on this.

[8] See Albert (2015), Ney (2017, forthcoming) and references therein for proposals to make sense of this.

thinking of quantum mechanics as an effective theory, which can be understood in terms of a more fundamental theory, just like thermodynamics can be understood in terms of statistical mechanics. As in thermodynamics one understand temperature and heat in terms of molecular motion, in the POA one understands the quantum behavior in terms of motion of objects in three-dimensional world. It just so happens that in doing that we use the wavefunction, very much in line with the practice and the spirit of physics before the advent of quantum mechanics.

Open questions in this approach are connected to the nature of the wavefunction. The wavefunction does not represent physical objects but rather is used to 'generate' the trajectories of the PO. Because of this, in the POA the wavefunction is usually taken to have a nomological role, even if it is debatable what is the best way of capturing this idea.[9]

4. Different Ontologies for Different Theories

The POA is a natural framework for the pilot-wave theory, as this theory has a natural interpretation as a theory with a PO of particles. However, the POA generalizes to the other theories as well by specifying a PO for each of them. So, in the POA the spontaneous localization theory and the many worlds theory are theories in which matter is made of whatever the PO is, and in which the evolution of the PO is governed respectively by a spontaneously localizing wavefunction, and a Schrödinger evolving one. In contrast with the pilot-wave theory, however, it is not obvious what the PO of these theories is supposed to be. Indeed, one can chose as one wishes: particles, fields, events. In fact, the PO quantum theory, like particles in classical mechanics, is

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[9] See Dürr, Goldstein, Zanghí (1997), Goldstein and Teufel (2000), Goldstein and Zanghí (2013), Allori (2018a) for a defense of the nomological approach. Since the wavefunction is part of the axioms of quantum theory, it can be naturally regarded as a Humean law (see Miller 2014, Esfeld 2014, Callender 2015, and Bhogal and Perry 2017). There are other ways in which someone could think of the wavefunction, broadly speaking, as nomological. One can think of the wavefunction as a property which expresses some non-material aspect of the particles (Monton 2013). Similarly, one can endorse a dispositional account where laws are understood in terms of dispositions, which in turn are described by the wave-function (Esfeld, Lazarovici, Hubert and Dürr 2014; Sàurez 2015). Arguably, since dispositions can be time dependent, the objection to the nomological view that laws of nature are time independent while the wavefunction evolves in time seems less compelling here. Having said that, I think these proposals are not very promising in that they rely on the notion of properties which are notoriously a rough nut to crack. As Esfeld (2014) has pointed out, there are several severe problems in trying to spell out what fundamental properties are, both in the classical and the quantum domain. On a different tone, let me notice that the objection that in theories with the wavefunction only there are two spaces involved and the relationship between these spaces is a mystery closely resembles one of the objection against Cartesian dualism: if mental states are not in three-dimensional space, how are they interacting with physical states? With this analogy in mind, one can argue that the answer of the proponents of the POA will be similar to the one of the reductive physicalist, presumably a functionalist: the wavefunction is whatever function it plays to generate the empirical data (see Allori 2019b for more on this).

postulated beforehand as the best compromise between simplicity and explanatory power. The theory is then constructed around it, adding various elements to it including the wavefunction, to successfully reproduce the empirical data, just like forces and potentials are added to Newtonian mechanics. This is clearly not what historically happened in the quantum domain, in which we got 'stuck' with the wavefunction. So, the PO quantum theories have been developed 'backwards,' by keeping the wavefunction and its evolution, and then trying to figure out what the PO should be. This of course means that the choice of the PO is underdetermined by the theory. However, this is not surprising, as it is merely a restatement of the well-known fact that there are many theories that fit the data. As a consequence, the spontaneous localization theory, as well as the many-worlds theory, are better seen as families of theories, rather than one single well definite theory: there is one theory for each choice of PO.

So, *what is the 'best' choice of PO for spontaneous collapse theories* and many-worlds theories? The answer is not straightforward for a variety of reasons. In this paper I will not discuss the many-world theory but only spontaneous collapse theories, even if presumably some of the considerations will also apply in that framework.[10] If one hadn't followed the literature, one might think that the natural PO for quantum theories could be the one of particles. In fact, a particle ontology seems to be the simplest, as it only takes a point to define a particle. Instead, the two formulations of the spontaneous localization theory with a PO which were historically considered are not particle theories. The first of these theories was proposed by John Stuart Bell (1987). In this theory the PO is directly into spacetime: matter is made of those events in spacetime in which the wavefunction happens to spatiotemporally localize. Thus, matter is spatially discontinuous, like the case of particles, but also temporally so that one can say that matter is made of flashes. The flashes are divided into families, with each family corresponding, intuitively, to a single particle. The wavefunction provides the conditional probability measure over the flashes, namely the probability that the next flash in a given family will be in a given spatiotemporal point. This theory is therefore known in the literature as GRWf, with an obvious notation. Bell chose this unfamiliar ontology not because it is the simples but because he noticed might help finding a relativistic invariant theory (see section 6 for more on this).

The other (prominent) spontaneous localization PO theory has been proposed by Ghirardi and collaborators (1995), who took matter to be described by a continuous three-dimensional matter field (which is defined in terms of the wavefunction). It is the most natural and the simplest choice of the PO in the following sense: assuming that the problem with the original theory was not the wave part but the configuration space

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[10] For more, see Allori (2019).

part, this theory solves the problem by putting the wave (and thus matter) in three-dimensional space.

However, these theories are very peculiar, as discussed by Peter Lewis and Michael Esfeld (this volume) among others. Lewis reminds us that the flash ontology is extremely counterintuitive, given that according to this theory matter is mostly empty: for a small macroscopic object made of roughly 10^{19} 'particles,' if each particle undergoes a collapse every 10^{16} seconds, there is a flash roughly once every 10^{-3} seconds, and nothing in between. Nevertheless, the gravitational and electromagnetic forces the object is subject to will be continuous in time. Moreover the matter density, being defined in terms of the wavefunction, inherits its tails. Because of this, macroscopic solid objects will have low-matter density tails which other macroscopic solid objects could cross without being subject to any actual interaction, contrary to expectations. In addition, as Esfeld points out, there is a tension between the matter density ontology and the quantum formalism, which is in terms of a finite number of particles. Also, the account of nonlocality may seem more mysterious in this theory than in others, as it implies that the matter density is instantaneously displaced across arbitrary distances.[11]

Regardless of whether or not it is possible to find satisfactory solutions to these challenges,[12] it is interesting to see whether other ontologies could do better. In this regard, the obvious choice would be to try with an ontology of particles. First, as

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[11] See Egg and Esfeld (2014), Esfeld and Deckert (2017).

[12] Here's a sketch of some possible responses. To the objection that flashes are counterintuitive one could reply that a satisfactory explanation can lead us far from common sense, so sometimes getting away from commonsensical explanation may be the right thing to do. For instance, while common sense suggests that matter is continuous, atomic theory has shown us that it is not the case: atomic theory, with its in-breath and in-depth explanatory power, is a better explanation of the behavior of matter than our common sense. So, we are justified in accepting atomic theory even if it pictures a world which is distant from what we initially thought. Moreover, the reply goes, in the case of GRWf abandoning common sense for an unfamiliar ontology makes the theory more compatible with relativity, as suggested by Bell (however, see section 6). Also, one could question the fact that the ontology and the explanation is really counterintuitive in a negative sense. It is true that the action of fields is continuous even if there are no flashes. However, this is a problem *only* if the field are intended as material, which in the POA is not necessarily the case: they could be taken to be not generated by the particles, but rather alike to nomological entities, similarly to what happens for the wavefunction in quantum theory (see Allori 2015 for a discussion of this). Also, see Esfeld (this volume) for a defense of GRWf. To the objection that the matter density has tails which could be crossed by other objects without any visible interaction, one could arguably maintain that this is counterintuitive merely when we look at things from a classical perspective: only because classically matter which encounters other matter interacts with it, it does not mean that it has to be the case in the quantum domain. Moreover, the fact that the quantum formalism is in terms of particles does not seem to force us to interpret it as a theory of particles, as one could presumably endorse the formalism of a continuous localization theories (CSL) which does not require particles (Esfeld, this volume). Finally nonlocality is a puzzle for all theories, not merely GRWm, so that it is unclear how serious the last objection actually is.

already noticed, there is a sense in which particles are the simplest ontology. Moreover, a particle ontology would increase the explanatory power of the theory: as underlined by Allori (2013b), Healey (2015), and Lewis (this volume), explanation in the quantum domain is parasitic on classical explanation. Since classical mechanics is about particles, and arguably we can explain the macroscopic feature of matter in terms of such an ontology, there is no in principle reason why this cannot be done also in the quantum domain, if the theory is a particle theory. Finally, as argued in Allori (2018b) and as discussed in section 6, a particle ontology would also help in solving the pessimistic meta-induction argument against scientific realism, since it would provide with an ontological continuity between the classical and the quantum domain.

5. A Particle Ontology for Spontaneous Localization Theories

What would a spontaneous localization theory of particles look like? The possibility of a particle ontology guided by a GRW-evolving wavefunction has been explored in the literature, however never in an exhausting and comprehensive way.[13] Among all the possibilities, I argue that only one survives scrutiny. Let us discuss them in turns.

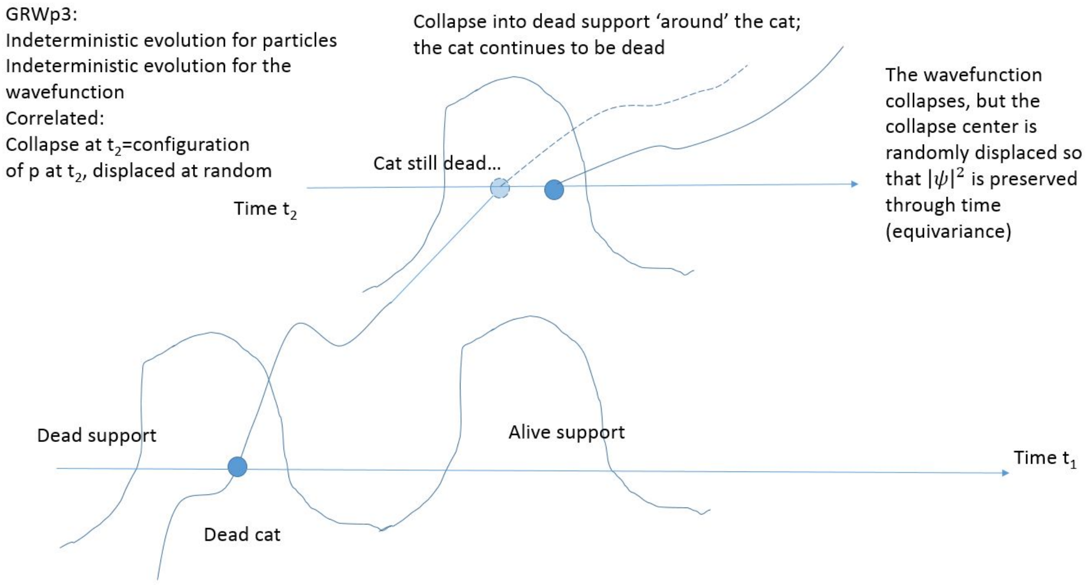

Figure 1

Bedingham (2011) was the first to propose a particle ontology for a GRW-evolving wave function. This theory has been later dubbed GRWp3 in Allori *et al.* (2014) and Allori (2019), given that it is the third GRW particle theory they analyze. In this theory both the particles and the wavefunction evolve stochastically. In particular, the wavefunction evolves in a GRW-fashion while the particles are guided by the same guidance equation of the pilot-wave theory. However, the wave-function localizes into

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[13] See Allori *et al.* (2008, 2014), Bedingham (2011), Allori (20019).

the *actual* position of the particle at that time (the localization time) but `displaced' at random (figure 1).

Two things need to be noticed. First, the localization of the collapse needs to be anchored to the evolution of the particles appropriately, and that is why it is in the particles configuration at the time of collapse. If one did not require this, the evolution of the wavefunction and the one of the particles would be uncorrelated. This would lead to a non-empirically adequate theory. In fact, in the POA the status and the behavior of a physical object is determined by the status and the behavior of the PO. The wavefunction is instead part of the dynamical law governing such behavior, so it should determine it appropriately. If the evolution of the PO and the wavefunction are not correlated (as in a theory dubbed GRWp1 in Allori *et al*. 2014), this does not happen (figure 2). In fact this could imply, somewhat dramatically, that a cat which has died, and was supposed to stay dead, could come back to life. To see this, assume the configuration of the cat's particles is under the 'dead' support of the wavefunction before the collapse. If so, the cat is, unfortunately, dead, and there's nothing that can be done about it. However, if we do not correlate the two evolutions, then it is possible that the wavefunction collapses into its 'alive' part. For that moment on, therefore it would be this part of the wavefunction which would guide the particles' motion, and this means that the cat could actually come back to life. While this may give some comfort to those who loved the cat, it is not what empirically happens. Instead, if one allows the center the localization of the wavefunction to be the actual position of the corresponding particle at the time of the collapse, then there is the right correlation between the two evolutions. In this way, if the cat is dead before the collapse (that is, the positions of its particles are under the dead support of the wavefunction), then the wavefunction collapses 'around' it, and a dead cat remains dead.

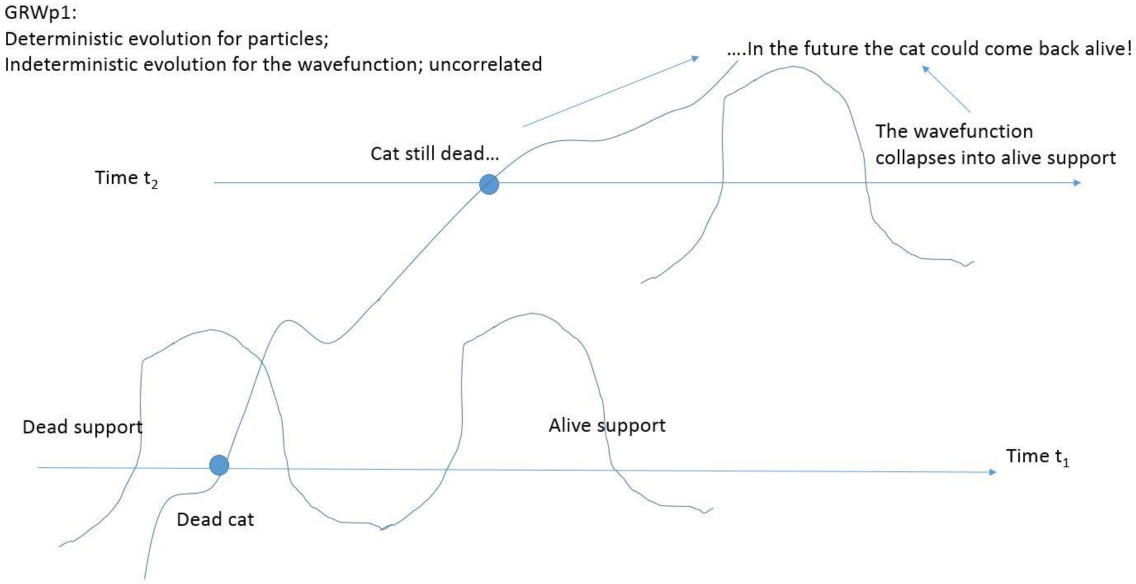

Figure 2

Secondly, it is interesting to notice that the stochastic evolution of the wavefunction does not combine with a deterministic evolution for the particles: in order to have an empirically adequate spontaneous localization particle theory, the particles have to evolve indeterministically as well in order to ensure the equivariance of the theory. In fact a random delocalization of the localization position is required in order to 'compensate' the wavefunction collapse so that the empirical distribution remains the one predicted by quantum theory, namely $|\psi|^2$.

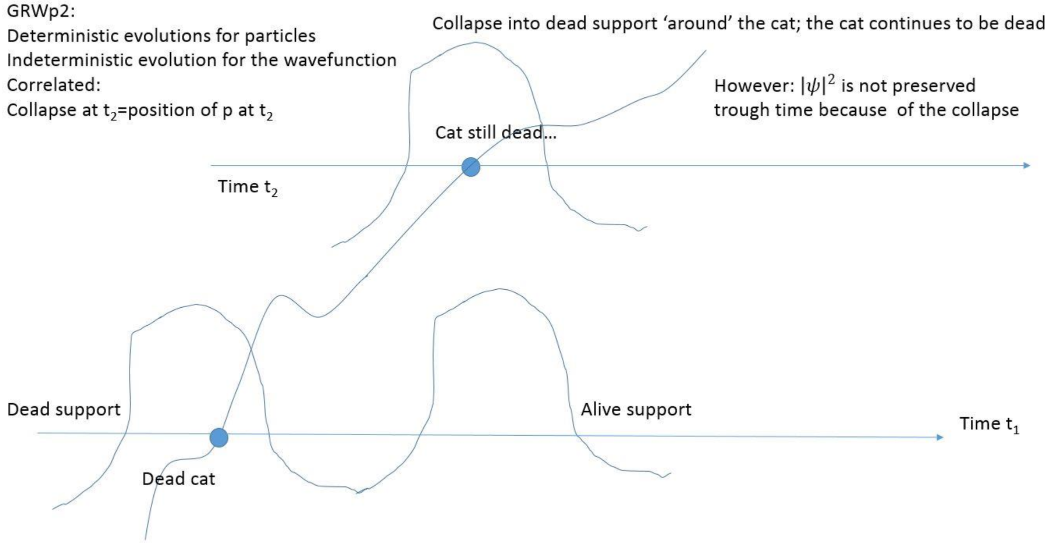

Figure 3

This theory, with a deterministic evolution of the particles, was originally[14] proposed by Allori *et al.* (2008), and they called it, rather obviously, GRWp. However, it was later recognized to be empirically inadequate and dubbed GRWp2 in Allori *et al.* (2014) (figure 3). The empirical inadequacy of this theory can be seen by noticing that without the displacement, the situation would be almost the same as in the case of the pilot-wave theory, where the wavefunction does not collapse but the particle position is effectively guided only by one of the terms of the superposition, namely the one under which the particles are (figure 4).

Thus, there is a sense in which it is the hallmark of being a GRW-type particle theory is to let both particles and the wavefunction jump.[15]

One may think that another possible implementation of this double indeterminism could be accomplished doing the opposite: instead of the wavefunction 'following' the

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[14] However, see Bohm and Hiley (1993), page 346.

[15] Other particle theories, aside from the pilot-wave theory, are stochastic mechanics (Nelson 1985), and Bell-type quantum field theories (Bell 1986, Dürr *et al.* 2004, 2005). In both theories the wavefunction evolves deterministically, in contrast with GRW-type particle theories. In the former the particles evolve according to a stochastic Markov process, while in the latter the evolution is also stochastic but the particles can also be created and destroyed.

particles and localize where the particles are, one can have the particles 'follow' the wavefunction and jump where the wavefunction localizes. That is, one takes the particles to move as in the pilot-wave theory between localizations, and at the time of collapse all particles jump in the point the wavefunction has collapsed (figure 5).

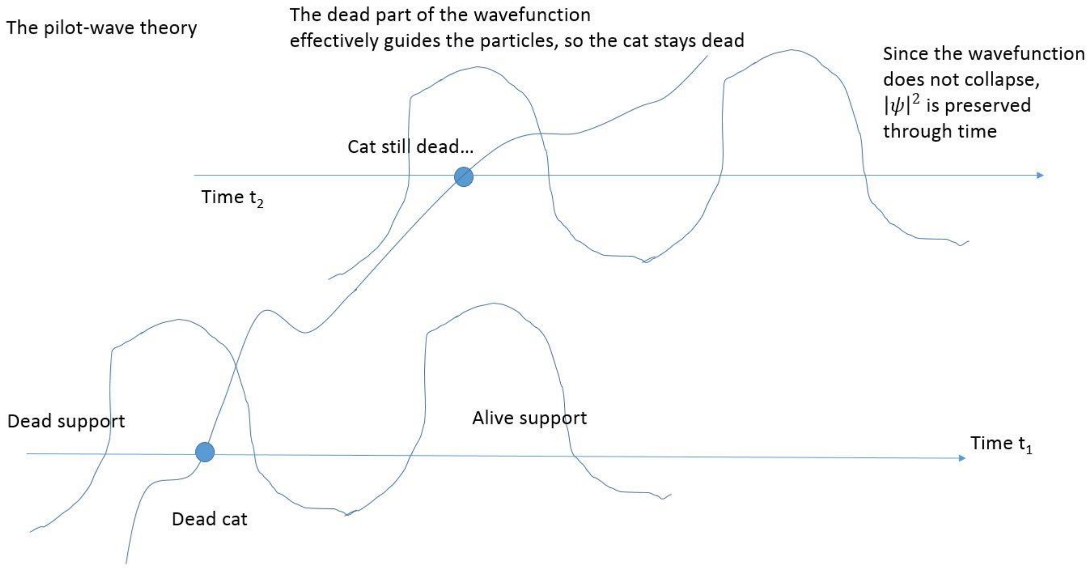

Figure 4

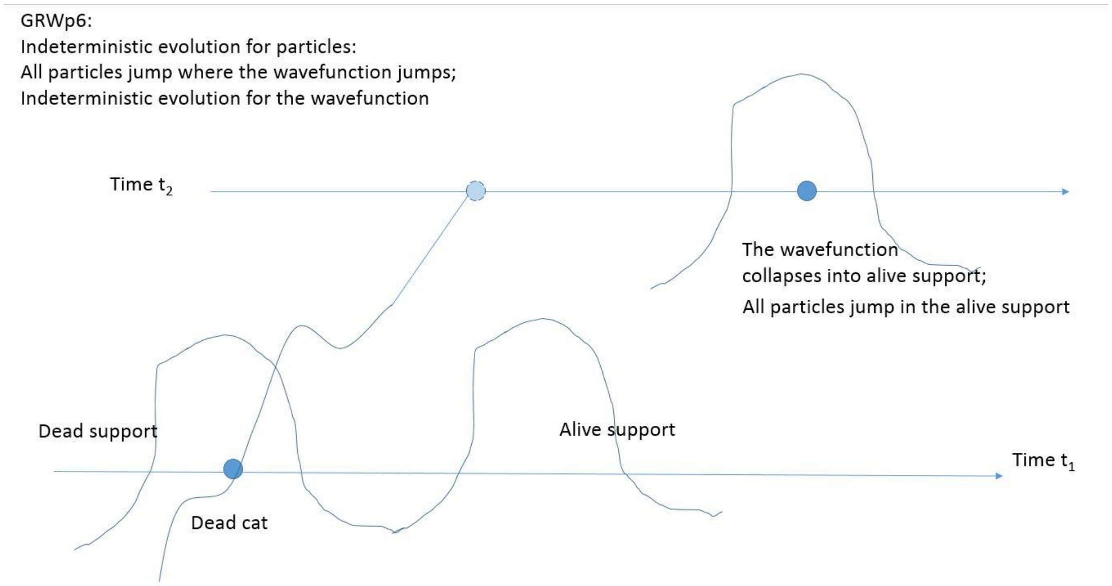

Figure 5

In this theory, dubbed GRWp6 in Allori *et al*. (2014) and Allori (2019), the wavefunction takes precedence on the particles, and this may already suggest it is going to be a problem, as in the POA the PO is, indeed, primitive. However, before explaining the major drawback of this move, let me first notice that, in order for this theory to get off the ground, one would need all particle to jump together. This is to guarantee that, again, a dead cat would stay dead. In fact if only one particle were to jump, then something similar to what described above is likely to happen again. Consider a situation where the cat is initially dead, and assume that the wavefunction collapses into the 'alive' portion of the wavefunction just after few collapses connected to a few particles. As a consequence, even if the wavefunction indicates 'alive,' most particles would still be 'under' the dead sector of the wavefunction. The 'alive' portion of the wavefunction will soon be dominant over the other part in guiding the evolution of the particles, therefore opening up for the possibility that the cat will end up alive again. This is what happens in a theory dubbed GRWp5 in Allori *et al*. (2014) (figure 6).

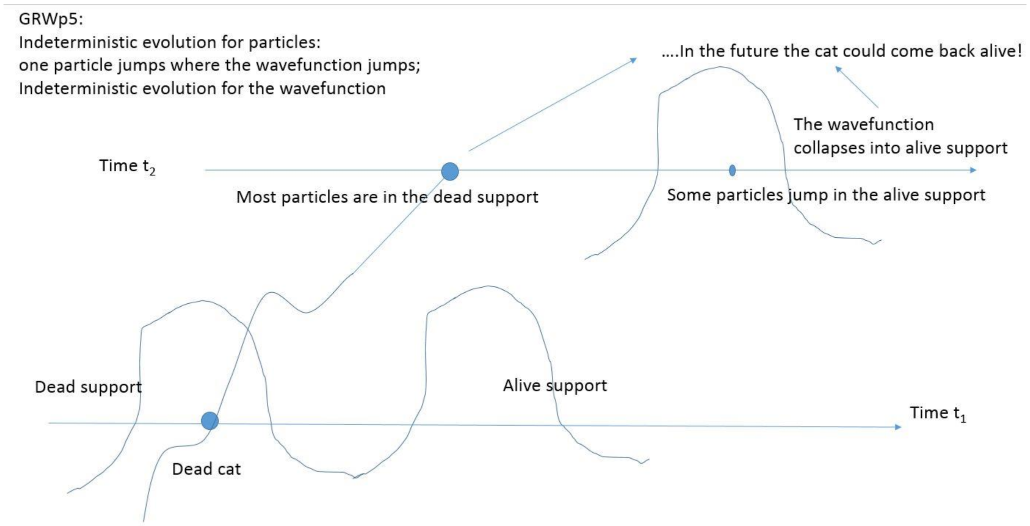

Figure 6

In any case, there is a problem, as GRWp6 closely resemble a (problematic) many-world theory. In fact this theory can be taken to represent a situation in which there is a world for each term of the superposition of the wavefunction. The problem is that, since all particles jump at the same time when the wavefunction gets localized from one term of the superposition to another, one effectively and instantaneously moves from one world to another. So, GRWp6 is a theory in which many worlds exist, even if not at the same time but one after the other. This makes the theory empirically incoherent, namely its truth undermines our empirical justification for believing it to be true.[16] In fact we could instantaneously move from one world in which there are dinosaurs to one in which there aren't any. This implies that our records of the past, including evidence to support the theory, are most likely false: we remember dinosaurs at time t, when we were in world 1, but at time $t + dt$ when we are in world 2 they have disappeared. Similarly, assume we gather some empirical evidence that justifies us in believing in GRWp6 at one time, when we are in world 1; but when we jump into world 2 a second

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[16] For more on empirical coherence, see Barrett (1999), Huggett and Wüthrich (2013).

later our memories of that very evidence is most likely false because the two terms of the superpositions describing world 1 and 2 are separated in configuration space and thus describe microscopically different state of affairs. Because of this, presumably, GRWp6 is not a viable theory. This does not happen in GRWp3 because in this theory the particles are 'in charge' and the wavefunction localizes where they actually are located.

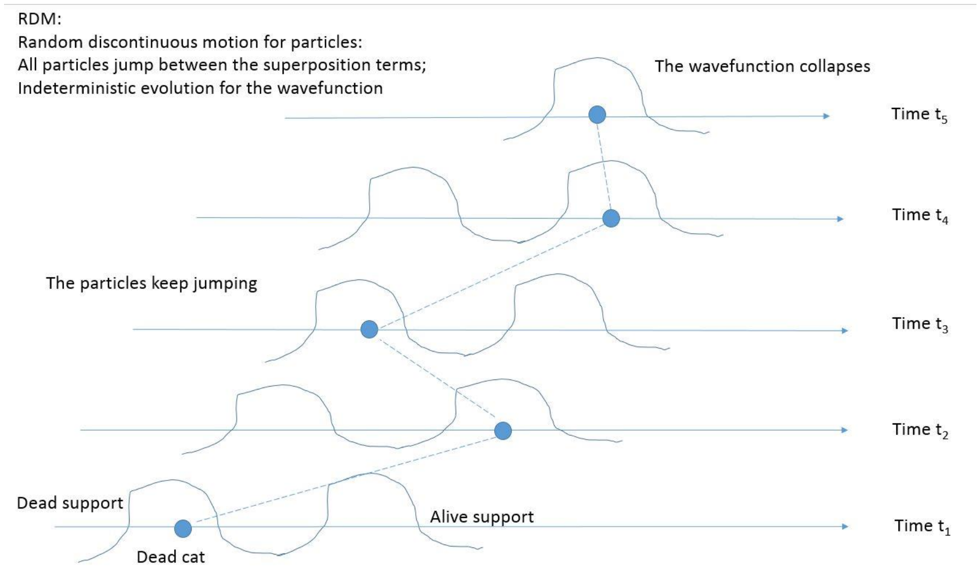

Figure 7

Another GRW-like particle theory has been explored by Shan Gao (2017). In his theory particles evolve in random discontinuous motion (RDM) guided by a GRW-evolving wavefunction (figure 7). Gao's idea is that the particles spend only an instant at each location, and jump between the different terms of the superposition of the wavefunction. This theory is different from GRWp3: in GRWp3 the evolution of the particles is deterministic before and after the localization of the wavefunction around the particles' configuration, while here the particles keep jumping between the different terms of the superpositions before the wavefunction localization. It is perhaps more similar to GRWp6, in that in both theories the wavefunction takes precedence over the PO. However, in GRWp6 the particles jump where the wavefunction localizes but they evolve deterministically before and after, while in RDM the particles keep jumping. Anyway, the collapse of the wavefunction is needed to guarantee that a macroscopic object, say a cat, would not be in a macroscopic superposition of, say, being alive and being dead. After the localization of the wavefunction the particles are confined to move only within the one term of the superposition remaining. Notice that, in order to avoid problems similar to the ones discussed above for GRWp3, one would need all particles to jump together. In any case, the theory seems to suffer from a severe form of empirical incoherence: in GRWp6 the particles were jumping between different words in between collapses; here instead the particles keep jumping between different words at every single instant, before 'setting' for one after the wavefunction collapse.

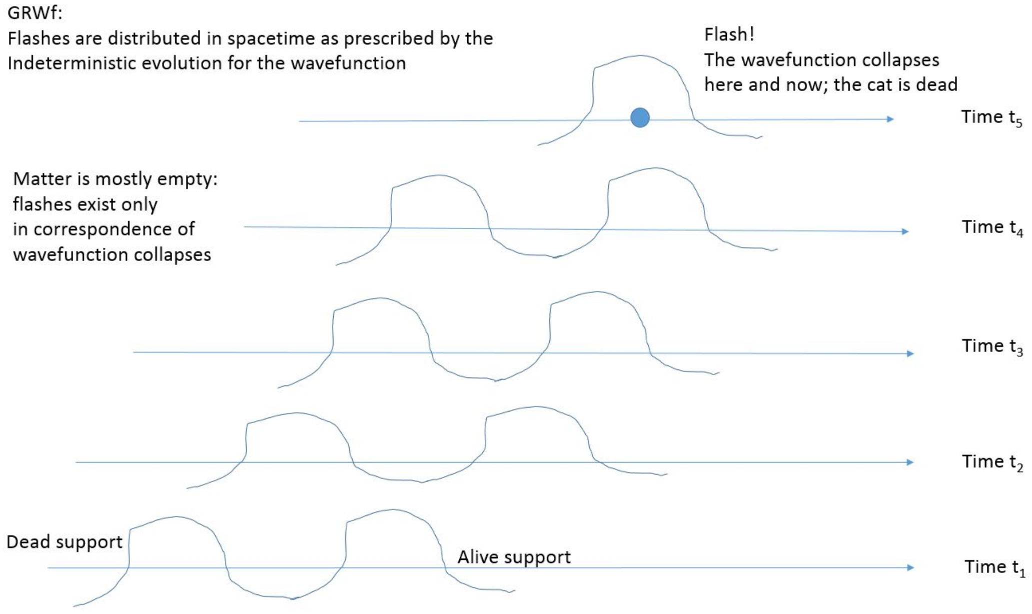

Figure 8

Interestingly, one may read RDM as a theory of flashes, namely of spatiotemporal events, given that particles are usually taken to have a continuous trajectories, unlike what happens in RDM. In any case, it seems interesting to compare RDM with GRWf (figure 8). As we said, in RDM the particles keep jumping at every instant, not only at localization times. In contrast, in GRWf each flash corresponds to one of the collapses of the wavefunction, and its space-time location is just the spacetime location of that collapse. So, as anticipated, in GRWf matter is mostly empty, because matter exists only at the points of collapse and at the instants of collapse. However, it is this feature which allows GRWf to avoid empirical incoherence: in between collapses we are not oscillating between two different words because in between collapses nothing exists.

6. Comparison among GRW-like Theories: Relativistic GRWp?

In the last section I have discussed how one can construct an empirically adequate theory of particles with a GRW-evolving wavefunction: its name in the literature is GRWp3. However, since this is the only particle theory which survives among the alternatives, I think it is more appropriate to just call it GRWp. As we have seen, this combination requires a stochastic evolution of the particles as well as one for the wavefunction. In this section, I will compare the various GRW-theories with different ontologies: GRWm, GRWp, GRWf. After having done that, I will sketch a proposal for a relativistic particle GRW theory.

When dealing with incompatible alternatives which are empirically equivalent, namely that cannot be set apart by empirical evidence, one usually invokes superempirical virtues to select one over the other. For instance one could consider parsimony, and argue that the theory which is most parsimonious is the most likely to be true. Assuming that parsimony is indeed a guide to truth, there is however the question of how to define parsimony univocally and objectively. Be that as it may, the matter density seems to score poorly on this criterion when compared with the alternative ontologies. In fact one could argue, as I hinted previously in the text, that the most parsimonious ontology is the one of particles. After all, they need merely a point to be specified, in contrast with the matter density, which is a continuous function. However, one could instead maintain that the flash ontology is the most parsimonious: the flashes are like particles, without trajectories (Esfeld, this volume). Nevertheless, as also pointed out in the last section, the flash ontology is more counterintuitive than the particle ontology. As a consequence, the explanatory power of the theory could suffer. Even if explanatory power, like parsimony, is not easy to define univocally, one could maintain that the needed to account for our mistaken intuitions (matter being discontinuous not only in space, like in the case of particles, but also in time in the case of flashes) is a burden for the flashy theory. So, parsimony and explanatory power seem to pull in different directions, and arguably cannot help breaking the tie between GRWf and GRWp, assuming we agree that GRWm scores poorly on these criteria.

Presumably, there are two other considerations that could help in theory selection. One has to do with scientific realism, and the other with relativity. Let us start with the former. As it was clear since the beginning, we are assuming scientific realism, namely we are assuming that these theories can tell us about the nature of reality. However, there is one serious objection to scientific realism: the pessimistic meta-induction argument. This argument goes against the no-miracle argument, which is the main argument for realism: the empirical success of a theory can, and should, be taken as evidence of its truth. The idea behind the pessimistic meta-induction argument is that the conclusion of the no-miracle argument does not follow: since past successful theories turned out to be false, it is unwarranted to believe that our current theories are true simply because they are successful (Laudan 1981). Since past theories were empirically successful but turned out to be false, it follows that our current theories, even if successful, are more likely to be false than true. One way to respond to the pessimistic meta-induction challenge is to argue that one should be realist about a restricted set of entities, not about the whole theory. Then, if one can show that the entities that are retained in moving from one theory to the next are the ones that are responsible for the empirical success of the theory, the pessimistic meta-induction argument is blocked. In this context, thus, since classically the ontology was the one of particles, a quantum theory with a particle ontology could solve the pessimistic meta-induction argument. Because of this, therefore, one should prefer GRWp over the alternatives GRWm and GRWf: it is the theory that makes scientific realism more plausible in the GRW framework. Allori (2018) has argued that flashes and matter density, in a suitable way, could defeat the pessimistic meta-induction as well, in contrast with GRW0, namely the GRW theory read as a theory about the wavefunction. However, one could maintain that the explanation for how GRWm and GRWf solve this problem diminish their simplicity and explanatory power when compared to GRWp: in GRWp it is obvious that the ontology is preserved from the classical to the quantum domain; this is not so in the case of GRWf and GRWm. So whatever story one has to tell to explain their solution of the pessimistic meta-induction will make the theory not as simple and as explanatory as GRWp.

Finally, another important criterion that could help break the tie between empirically equivalent theories is connected with relativistic invariance. All the GRW theories discussed so far are nonrelativistic, so a natural thought is that if one could extend one of them to the relativistic domain but not the others then that theory should be preferred. For a brief period of time this was indeed the case, as GRWf was the only one of them which had a relativistic extension. This theory, rGRWf, was proposed by Tumulka (2006) and provides the probability distribution of the flashes just like GRWf but now it does that with a Dirac evolving wavefunction. Fundamental is the fact that in the construction of the theory no mention of a preferred slicing of spacetime is mentioned, making the theory manifestly Lorentz covariant. This is contrast with relativistic invariant extensions of the pilot-wave theory, which require a preferred foliation.[17] Arguably, since in the traditional reading of relativity such a preferred foliation does not exist, people have been looking at rGRWf with great interest, as it avoids it entirely. However more recently a relativistic version of GRWm has been proposed by Bedingham *et al.* (2014). In this theory a Dirac evolving wavefunction defines the matter density field in a Lorentz covariant way, as it only depends on the metric structure, namely the past light-cone in every point. If this is the right way to think about relativistic invariance,[18] then relativistic invariance cannot be use to break the tie between GRWm and GRWf, as both rGRWm and rGRWm exist. However, one may wonder about GRWp: is it possible to construct a relativistic GRWp without the need of a foliation? One may think that the existence of rGRWm, in which there are trajectories for the matter density defined in terms of the wavefunction on the past light-cone, leaves open to the possibility of constructing a relativistic GRWp. One may instead think that this is not possible because just like in the pilot-wave theory in order to define the trajectories, even if discontinuous, one would need a preferred temporal frame. In this regard, it is interesting to consider the attempt by Goldstein and Tumulka

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[17] See Dürr *et al.*(1999), Dürr *et al.*(2014).

[18] For a criticism, see Barret (2014) and Esfeld and Gisin (2014), who argue that these theories, even if they are Lorentz invariant in terms of the overall histories of their PO, are unable to describe single events in a relativistic invariant way.

(2003) to build a pilot-wave theory without a foliation.[19] This theory, let's dub it GT from the names of the authors, specifies covariant particles trajectories with an equation which uses as surfaces of simultaneity the future light cones. In this way, no foliation is needed. The theory is strange, given that it has a microscopic arrow of time pointing towards the past, and also it is not empirically adequate, as it does not have any equivariant measure. However, one could observe the following. In section 5, we dismissed GRWp2, the theory in which the wavefunction jumps where the particles are, because it was not equivariant, while GRWp (in the text previously dubbed GRWp3), in which the particle position was randomly displaced, instead 'regained' equivariance. Perhaps, one could explore the possibility that the lack of equivariance of GT could be 'cured' as we did for GRWp2, and thus define a relativistic particle GRW theory as follows:

1. There are particles evolving according to a suitable pilot-wave-like guidance equation defined with future (or perhaps past) light-cones as simultaneity slices (as in GT),
2. There is a Dirac evolving wavefunction (as in rGRWf and GT),
3. The wavefunction collapses around where the particles are at the time of collapse displaced at random (as in GRWp).

Steps 1 and 2 would guarantee Lorentz covariance without a foliation, step 3 would instead make the theory equivariant. If this proposal could be made rigorous, then we could have a relativistic GRW theory of particles.

If so, then one could break the tie and select the preferred ontology for GRW-like theories as the one of particles: they are sufficiently simple, they imply less counterintuitive consequences than the alternatives, they help defeating the pessimistic meta-induction argument, and one (presumably) can construct a relativistic GRW theory without a foliation.[20]

7. Conclusion

In the first part of the papers (sections 2, 3 and 4) I have argued that the real problem with quantum theory is not the problem of superpositions, but rather the problem of considering the wavefunction in configuration space as representing physical objects. If so, one should always 'add' something to the wavefunction, regardless of whether one is considering the pilot-wave theory of the spontaneous localization theory. Then in the last part of the paper (sections 5 and 6) I have discussed GRW theories of particles, and I have compared them with the other spontaneous localization theories with different

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[19] Another interesting attempt of a relativistic pilot-wave theory without a foliation has been proposed by Sutherland (2008, 2017), which however involves retrocausation.

[20] Of course, if the rGRWp proposed above requires a microscopic inverse arrow of time, then this may be taken to diminish the explanatory power of the theory. However, since the theory has yet to be constructed this kind of considerations seem premature.

ontologies. I finally have argued that, if one were to successfully construct a relativistic spontaneous localization of particles along the lines I have sketched then this theory would score as the best among the alternatives.

However, if the argument I have advanced in the first part of the paper is sound, then one could wonder: what is the point of considering particle GRW (or many worlds!) theories at all, in this framework? In fact, if one grants that solving the measurement problem is not sufficient to dissolve the tension between realism and quantum theory, and that one also needs to solve the configuration space problem by postulating some PO in three-dimensional space, then one could also argue that solutions of the measurement problem which take route 2 and 3 (namely many-worlds and spontaneous localization as originally intended) have few chances of being taken seriously. One can turn them into PO theories, but they are doomed to fail because the pilot-wave-theory is *already* the simplest alternative. In other words, let us assume that one faces the configuration space problem *before* the measurement problem, rather than after, which one solves by postulating a given PO, may that be of particles, or fields, or strings, or spatiotemporal events. Then the measurement problem does not even arise, as particles never superimpose. If so, *one would hardly think of theories like the spontaneous localization theory as serious option*: the wavefunction solely appears in the law which governs the PO evolution, so why have a stochastic evolution for it if one can obtain perfectly experimentally adequate results with a deterministic one?

My proposal is that looking at theories like the spontaneous localization theory is valuable for a different reason. As we have seen in section 6, theories like rGRWf and rGRWm show genuine relativistic invariance, namely relativistic invariance without a foliation. This is not something that the pilot-wave theory possesses. So, as a matter of an historical accident, namely that people were led to (mistakenly) think that the measurement problem was the one to solve to make quantum theory compatible with scientific realism, they started to develop theories that they otherwise would not have even considered. That is, if the real problem were recognized by everyone to be the configuration space problem, then everyone would *also* agree that its simplest solution is given by the pilot-wave theory, with an ontology of particles and a deterministic evolution for the wavefunction. However, without the attempts to solve the measurement problems, presumably people would have just focused on trying to make the pilot-wave theory relativistic invariant, and they perhaps would not have developed theories such as rGRWf or rGRWm which, in contrast with the pilot-wave-theory, are genuinely relativistic. So, this historical accident led us astray in one sense but set us straight in another. That is, on the one hand the measurement problem led us astray because it made us consider theories which are, from the perspective of the solution of the configuration space problem, more complicated than needed. On the other hand it set us straight, because these more complicated theories also show some unique features that make them more amenable to a relativistic extension. In other words, it is valuable to look at these theories even if they are not the simplest solution of the configuration space problem because their stochastic laws may be helpful in solving the tension between quantum theory and relativity.

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