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The Metaphysical Subject and Logical Space: Solipsism and Singularity in the *Tractatus*

Abstract: This essay presents a heterodox reading of the issue of solipsism in Wittgenstein's *Tractatus Logico-Philosophicus* (TLP), out of which the whole of the TLP can be re-read. Inspired by, though not dependent on, the themes of virtuality and singularity found in Deleuze's 'transcendental empiricism' (presented as a Wittgensteinian 'immaculate conception'), Wittgenstein's concept of 'logical space' is here complexly related to the paradoxes of the 'metaphysical subject' and 'solipsism,' within which the strictures of sense are defined, and through which the logico-pictorial scaffolding of the TLP precipitates its own systematic dissolution. It shows how nonsense envelopes not only not idle chatter, and metaphysical confusion, but sense itself.

Keywords: Deleuze, Wittgenstein, sense, nonsense, phase space, Hintikka, realism, idealism, transcendental, the mystical

1 Introduction

Discussions of Wittgenstein's notion of solipsism in the *Tractatus* (TLP) have generally focused either on interpreting it in terms of *idealism* (usually in light of Schopenhauer or Kant) or in terms of *realism* (broadly in relation to the linguistic correspondence theories of truth in Russell and Frege), or some combination of the two.[1] In a sense this is obvious, since it is these two camps which chiefly influenced Wittgenstein's thinking about philosophy up to the writing of the *Tractatus*. Nevertheless, these discussions have not furnished a reason for the explicitly *point-like* character of the *metaphysical subject*, its relation to realism and the limits of language and the world. By bringing back in earlier parts of the TLP, we can make more concrete sense out of Wittgenstein's notion of solipsism not just in terms of his philosophical influences, but, importantly, of its place within the TLP itself.

By understanding the relation between the metaphysical subject and logical space, we can explain the quasi-equivocal nature of the remarks on solipsism—their bearing on the said/shown distinction and solipsism's collapse with realism—and we can also give substantive context to some conclusions of earlier commentators (Hintikka, Pears, Harré, Hamilton, Pârvu, etc.).

This will be done in three stages. First, by preparing the ground for our discussion in investigating Jaako Hintikka's interpretation, one of the few that tries to configure Wittgenstein's solipsism within the broader claims of the TLP. Second, by articulating the concept of *logical space* in analogy to the *phase space*

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of statistical mechanics: as a non-actual, quasi-geometrical state (of affairs) space in which something like 'reality' (*Wirklichkeit*, *Realität*) or the 'world' is described by time-series which chart-out trajectories in this logical space. This brings into the picture some of Wittgenstein's other influences—Heinrich Hertz and Ludwig Boltzmann. Lastly, because the 'world' can be defined in relation to logical space, this allows us to work back to Wittgenstein's discussion of solipsism and gain traction on the point-like character of the Tractarian subject. Our contention is that the metaphysical subject behaves like a singularity in logical space—a problematic point where the ordinary coordination functions of the space (mirroring, correspondence, truth-functionality) are undefined, blow up, or become degenerate; and this because the subject-point situates the boundary of logical space in a unique way (i.e., in a way distinct from logical propositions, tautology and contradiction).

2 Hintikka's complex subject

Without a doubt, what Wittgenstein does with the notion of solipsism is heterodox, and perhaps philosophically unique.[2] Accordingly, Jaakko Hintikka presents a formidable, if brief, positive account of Wittgenstein's conception of solipsism—*positive* insofar as it is supposed that the propositions of the *Tractatus* are not *mere* nonsense, that they have assertoric content. His major contention is that Wittgenstein's conception of solipsism remains confused unless we recognize in what ways it differs radically from traditional solipsism and the problems related to it (e.g., Cartesian skepticism, Berkeleyan idealism, the problem of 'other minds', etc.).[3] Instead of saying that language imprisons me within a private discourse, which I alone can understand and can articulate only to myself, Hintikka rightly reads Wittgenstein's claim that "the limits of my language mean the limits of my world" in terms of language in general.[4] This is primarily how Wittgenstein's solipsism differs from the traditional view: the language which bounds 'my' world is publicly available, guaranteed by the necessary structural relationship which inheres in logic between language and the world (what Wittgenstein calls in one instance, "*dem großen Spiegel*"),[5] whose limit is given—in the case both of empirical reality and of the elementary propositions—by the totality of objects[6]. It is through this logical relationship, in its immanent limits, that Wittgenstein's concept of solipsism must be elaborated.

What is at stake in so construing the limits (*Grenzen*) of language is Wittgenstein's notion of the 'metaphysical subject'. If the subject is not in the world, but its limit[7], then we must cede, accordingly, the subject to the limits of language so construed. What makes it solipsistic is that this limit is not transgressed; one must remain on *this* side of the limit.[8] The limit, then, according to Hintikka, is not *said* but *shown* in the totality (*Gesamtheit*) of elementary propositions. Therefore, for Hintikka, in order that the metaphysical subject is associated with this limit it must be a complex—namely, the entirety of possible thoughts

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2 A full account of Wittgenstein's solipsism would need to take into account the changes of his view from the *Notebooks*, as well as its relationship to some of Wittgenstein's early philosophical influences, which are almost entirely sublimated in the *Tractatus*. Chief among these influences on his notion of solipsism is Schopenhauer's idealism. Illuminating this connection has been done thoroughly by Hacker, in *Insight and Illusion,* 81-107, though we agree with Tang that Wittgenstein's use of Schopenhauer's nomenclature does not necessarily indicate any theoretical agreement. Also Hacker's strategy for introducing the realist and idealist strands of the TLP has the unfortunate effect of splitting it up into its empirical and transcendental parts in a somewhat incommensurable way.
3 See Hintikka, "Wittgenstein's 'Solipsism'", 89: "If I can say that something is mine, it immediately follows that this something can possibly be yours too."; and 91: "Hence, it appears rather far-fetched to see in Wittgenstein's 'solipsism' an anticipation of the troubles of the logical positivists about 'other minds'."
4 Perhaps it would be more precise to say Hintikka's interpretation involves *necessary* or *logical* language. Ibid., 89.
5 "The great mirror," Wittgenstein, *Tractatus Logico-Philosophicus*, 5.511. Hereafter all references to the *Tractatus* will be abbreviated by TLP, and cited according to its numbered remarks: e.g., TLP 5.511.
6 The status of Tractarian objects will become important in more than one way below. Hintikka, "Wittgenstein's 'Solipsism'", 89.
7 TLP 5.641.
8 TLP 5.61.

(*Gedanken*) given by the totality of elementary propositions.[9]

From this interpretation, we draw some consequences from the view of the subject-complex, problematize it with what has been occluded in Hintikka's use of the *Tractatus*, and propose an alterative to the complex for the function of the limit.

First of all, the real (*wirklich*) totality of elementary propositions (the subject-complex) cannot be constructed a priori[10]—a totality which, in any case, may even be infinite.[11] Given that it cannot be constructed in advance, even if we agree that the subject is the limit of the world in relation to language in this particular sense, it seems hard to see what conceptual value this takes for the subject. The empirical limits of language (i.e., a given empirical totality of propositions) cannot but be accidental for Wittgenstein[12] and hence cannot count in terms of the metaphysical limit-subject in a logically significant way.[13] Since the totality of elementary propositions cannot be given a priori, and could only *accidentally* be given a posteriori (empirically), it is difficult to see in what sense the subject, as identical with this totality, is effectively limited—even if minimally.[14] That is to say, the empirical appearance of propositions seems not to make manifest the logic of the fact of totality, without some additional qualification, caveat, or conceptual linkage. Here is a stale mate: on the one hand, there is no empirical, psychological subject for Wittgenstein, and on the other, the metaphysical subject (*à la* Hintikka) is not manifest without some kind of construction of the totality—it remains to be seen how we could run up against these limits, under the auspices of the complex, except through an in principle argument which is absent from Hintikka's view.

Secondly, what is occluded in Hintikka's account of the metaphysical subject's association with the complex of the totality of elementary propositions is that Wittgenstein allies the subject not only with the limits of language and world, but also ascribes to it a point—conceives it just as this point—and seems to claim that the soul (synonymous with the subject) is not thinkable in composite terms. What this problematic point is, and its relation to the limits of language and world, we leave to the reader for now. However, we must investigate Hintikka's interpretation of the remarks in the *Tractatus* regarding propositional attitudeson which hinge his notion of the subject-complex[15].

Hintikka is right to insist that when Wittgenstein states that "'A believes that p', 'A has the thought p', 'A says p' are of the form "'p' says p'"[16] this 'p' refers to a propositional sign which pictures a fact p,[17] however he errs when he then associates the complex of the proposition with Wittgenstein's concept of the metaphysical subject (and jumps from this complex to the complex of the totality of propositions): Wittgenstein's point is to show that there is no logically valid way to conceive of a complex subject which "thinks or entertains ideas,"[18] since *any* complex cashes out only in terms of logical relations between propositions (or between names of objects in elementary propositions), not in terms of some thinking substance which maintains itself over above language, as the notation of the propositional attitudes makes it appear. The subject (the

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9 See Hintikka, "Wittgenstein's 'Solipsism'", 90. For Hintikka, Wittgenstein takes *Gedanke* in the Fregean (viz., public, objective) sense, as opposed to the private *Vorstellung*. The relationship between Frege and Wittgenstein for 'thought' and 'thinking' is nonetheless contentious: see, especially, Floyd, "Uncaptive Eye".

10 See TLP 5.55. Although the necessity of the totality can be derived a priori on logical grounds, cf. TLP 4.2211.

11 This is clearly dependent on the number of objects; see again TLP 4.2211 and 5.55.

12 TLP 6.1231-6.1232, 6.41, etc.

13 Cf. TLP 2.012: "In logic nothing is accidental..."; 6.1232: "The general validity of logic might be called essential, in contrast to the accidental general validity of such propositions as 'All men are mortal'; 6.41: "The sense of the world must lie outside the world. [...] For all that happens and is the case is accidental..."

14 Both Pears and Floyd, reach the conclusion that these limits are not 'true' limits, though for very different reasons. Floyd sees it as a methodological effect: denuding us of the traditional metaphysical presuppositions about limits to thought is part of getting us to see with an 'uncaptive eye'; Pears, on the other hand, sees it merely as a theoretical consequence of the fact the subject is not part of the world, and thus that "...solipsism is not genuinely restrictive because it is a useless reference point", "Wittgenstein's Treatment of Solipsism in the Tractatus", 70. I am more inclined to the latter, although his account remains too negative in its determination, and strips even the *showing* of its truth from solipsism, not only our ability to *say* what it means.

15 TLP 5.541-5.5421.

16 TLP 5.541.

17 Hintikka, "Solipsism", 90.

18 TLP 5.631. The German reads: *"Das denkende, vorstellende, Subjekt gibt es nicht."*

soul) for Wittgenstein is not complex, it is unavoidably non-composite, "[i]ndeed a composite soul would no longer be a soul."[19] But, at the same time, it must clearly also constitute the limit of the world.

Finally, the most outstanding omission of Hintikka's view occurs at the end of the remark relating solipsism and realism, which embodies the crux of Wittgenstein's solipsistic view. There the contention that the subject is point-like seems unmistakable. This conception also provides the conceptual ground relating Wittgenstein's solipsism to the traditional (Cartesian) view most clearly—namely, that the subject is without extension: "[t]he I of solipsism shrinks to a point without extension, and there remains the reality [*Realität*] co-ordinated with it."[20]

Such a point-subject, insofar as it must also be construed in terms of the limit, in my view, can only be understood as a *singularity*—a problematic or remarkable point—within the metaphysics of the logical space established by the *Tractatus*. This can be shown by paying attention to the status of 'points' in the *Tractatus*; e.g. in relation to names, to tautology, and to the subject, and by paying attention to the semantic function of the 'limit' in Wittgenstein's text (again for example in relation to tautology, etc.).[21] Finally, by deploying these examples from the text along side a delimitation of the mathematical concept of the singularity, as well as its ramifications in the concept of logical space, a positive account of Wittgenstein's solipsism is given which takes into account both the *point* and the *limit*—one that also firmly establishes what kind of 'subject' the *Tractatus* provides.

3 Logical space and phase space

The term 'logical space' occurs in the first page of the TLP: "The facts [*Tatsachen*] in logical space are the world".[22] But we also know that the world is not co-extensive with logical space, since "the world is all that is the case"[23] and because logical space does not determine what is the case but rather, so to speak, *the form of the world*. That objects imply, essentially, their possible constituency in states of affairs (*Sachverhalten*)[24] is manifest through the form-spaces of objects, which are specified by what is the case, or reality (*Wirklichkeit*): "A spatial object must be situated in infinite space (a spatial point in an argument-place.) A speck in the visual field, though it need not be red, must have some colour: it is, so to speak, surrounded by colour-space."[25] Logical space, then, is the coordination-space of the possible specifications of the form-spaces of objects, that is to say, of the possible concatenation of objects (into states of affairs): "The possibility of occurring in states of affairs is the form of the object"[26]; "Each thing is, as it were, in a space of possible states of affairs."[27]

As we know, this is the ground from which the picture theory of language is built.[28] But in order to understand how the proposition 'touches' reality, we must elaborate what the relation is between the world, logic, and language in outline.

It is assumed by many commentators on the TLP that logic and language are more or less synonymous. There is language on the one side, made up of facts called 'propositions,' that picture other facts, called 'situations' (*Sachlagen*) or 'states of affairs' (*Sachverhalten*). Logic, on this view, is then that which provides rules for the justified implementation of propositional signs required for sense, or for propositions to be truth-apt. This is broadly the picture of language we get from Frege and Russell. However, Wittgenstein's

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19 Ibid., 5.5421.
20 Ibid., 5.64.
21 In relation to names and tautology respectively see, e.g., TLP 3.1431 and 5.143.
22 TLP 1.13.
23 Ibid., 1.
24 Ibid., 2.011-2.0122.
25 Ibid., 2.0131.
26 Ibid., 2.0141.
27 Ibid., 2.013.
28 For a concise introduction to the picture theory in the TLP, see, Kenny, "The Picture Theory of the Proposition" in *Wittgenstein*, 44-56. Although introductory, Kenny's elaboration of the theory has the strength of *explicitly* avoiding interpreting 'pictorial form' on the basis of resemblances.

injunction is to ask after the *necessity* of logic—to ask, why there are formal sense-conditions on the implementation of signs at all. If they exist, then there must be limiting conditions (forms) related to that which the proposition pictures, otherwise there is no reason why propositions would ever have sense-efficacy with respect to the picturing of facts in the first place. Hence, there is not language and logic, on the one side, and the world and the facts on the other. Instead, *logic is that which inheres between the proposition and the state of affairs.* If a proposition does not satisfy the logico-pictorial form of that which it supposedly pictures, it is not a linguistic fact, but a fact among others. (Propositional signs divorced from their symbols are, so to speak, simply 'material' facts—physical marks on a substrate.) Logical space is the domain of formal inherence conditions between language and the world—it does not belong to language as such. As we will see, the inherence of logic has widespread metaphysical and epistemological consequences: consequences that will come to ground the said/shown distinction.

Logical space is the domain of inherence conditions between language and the world, but language is nonetheless special (though not because logic belongs solely to language). Given that valid propositions are not simply true by virtue of their validity, but are truth-functional, language does not pertain merely to what is the case—as the world, reality, must—but to its possibility. And thus the articulation of logical space is given in propositional form. Language can express *possible* states of affairs—this is a requirement of its truth-functionality. The fact that it can do this, for Wittgenstein, means it is co-extensive with logical space, in principle. But logical space articulates the possibility of the world—whatever *could* be the case— its limits. Therefore, the limits of language mean the limits of the world.

The purport of Wittgenstein's concept of 'logical space' cannot be fully made intelligible without an understanding of the isomorphism it maintains with the concept of 'phase space' in statistical mechanics, since in many ways they serve analogous functional roles.[29]

In the same way that a phase space is a quasi-geometrical space of possible states of a dynamical system, whose axes are the degrees of freedom of the system (e.g. possible positions and momenta of an ensemble of gas particles in a volume), logical space is a space of possible states (of affairs), whose 'axes' are the form-spaces of objects, the possibilities of their connection in a state of affairs. Harré confirms this in his foray into the extra-philosophical dimensions of the TLP:

> As Boltzmann and Hertz emphasize the propositions of physics are differential equations, the domains of which are manifolds of numbers, representing possibilities which might or might not be realized by the development of real systems, represented by particular sets of values of the parameters that define their possible states. Physics, too, handles this routinely by the construction of phase-spaces, to represent all possible states of a system... What else is a truth-table representation of a proposition but a phase space in ultimate logical terms?[30]

Truth tables may be "displays of the detailed topography of logical space,"[31] but of course there is no developed formal or geometrical account of logical space in the TLP, so our analogy must here remain conceptual.[32] Nevertheless, there has for some time now been an intermittent literature on the role of science,

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29 Whether or not Wittgenstein was conscious of or intended this analogy is of little concern here, since we are only interested in thinking through Wittgenstein, not trying to ascertain his motivations, nor what L.W., the man, had in mind, but rather what the ideas and text of the TLP afford us. Nevertheless, even though Boltzmann himself never used the term 'phase space' in his own writing, the content of the idea was apparent in his writings from the 1870's on, and Wittgenstein we know was familiar with the ideas of Boltzmann, especially through Hertz's *Principles of Mechanics* and Boltzmann's *Populäre Schriften* (for the latter see, Boltzmann, *Theoretical Physics and Philosophical Problems*). For the biographical evidence see, Ray Monk, *The Duty of Genius*, 26-7. Monk also claims that the Wittgenstein family had plans for the young Ludwig to study with Boltzmann in Vienna before the latter's suicide of 1906. He went instead went to Manchester. Hamilton in her *"Darstellungen in the Principles of Mechanics and the Tractatus"* also corroborates this. For the history of the emergence of the formal concepts involved and the invention of the term 'phase space,' see Nolte, "The Tangled Tale of Phase Space", 33-38.
30 Harré, "Wittgenstein: Science and Religion", 218.
31 Harré, "The *Tractatus* and the German Interpretation of Physics", cited in Pârvu, "'*Meine Grundgedanke ist*...' The Structural Theory of Representation as the Metaphysics of Wittgenstein's *Tractatus Logico-Philosophicus*", 268.
32 There have been, nevertheless, explicit formalizations the concept of logical space into a genuine quasi-geometrical space. These, necessarily extend beyond the bounds of strict scholarship on the TLP. See, e.g. Turner, *The Facts in Logical Space*.

and of Wittgenstein's scientific training, in his philosophy.[33] To understand the connection to logical space, it is worth drawing attention to several works addressing the scientific heritage of the Tractarian conceptual taxonomy, which aid our delimitation of the notion of possibility in the TLP. Through the works of Mark Wilson, Kelly Hamilton, and Brad Kallenberg, among others, we can see the influence of Wittgenstein's relation to science on his philosophical ideas, not only within the TLP but throughout his corpus.[34]

Wilson's investigation of 'Reuleaux developments' and Reuleaux's conception of 'machine essences' provide important features of the Tractarian view of logical states of possibility and insight into the methodological role idealization plays in Wittgenstein's view of science.[35] Although there is a great deal to admire in Wilson's essay on its own merits, he also misapprehends Wittgenstein's relation to idealization (or the critical tradition of idealism more generally) by privileging, at the outset, a mechanistic vantage from which he performs his analyses.[36] Not taking Wittgenstein on his own terms stops Wilson, ultimately, from getting a more penetrating vision of the relation between science and philosophy in Wittgenstein, especially as it pertains to the metaphysical project of the TLP.

Hamilton's comparative study of a relational concept of the object in Hertz's *Principles of Mechanics* and the *Tractatus* remains important for our own delimitation of logical space. Through it, Hamilton stipulates an important correspondence between the two thinkers and their biographies, as it bears on their parallel conceptions of the 'object': one that is an eminently relational, ontologically primitive, but plural condition of possibility of intelligible structure (whether physical or logical). With this correspondence she gives a nuanced account of the relationships of priority between possibility and actuality (that is, between what she distinguishes as 'form' and 'structure') in the TLP. Despite the integrity she brings to Hertz and Wittgenstein on the object, she fails to countenance the point-like character of Tractarian objects (something shared in common the point particles of Hertz's physical formalism), and her excavation fails to cradle the Tractarian concept of the object within the broader logical and metaphysical coordinates of logical space (or of a configuration space capable of articulating the forms of objects by virtue of their constituency in states of affairs).[37]

In relation both to the philosophical and scientific influences on the TLP, the most important synthetic interpretation is Ilie Pârvu's,[38] who conceives of the TLP as a theory of 'structural representation,'[39] one that introduces a historically novel metaphysical project, pushing transcendental philosophy beyond

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33 Of course this literature reaches as far back as the Vienna Circle interpretation of the TLP, but the discourse of the stripe that holds our attention here begins from somewhat different motivations, and stems (in Pârvu's estimation) from Griffin's 1964 monograph, *Wittgenstein's Logical Atomism*.

34 See, Wilson, "Wittgenstein: *Physica Sunt Non Leguntur*"; Hamilton, "*Darstellungen*"; Kallenberg, "Rethinking Fideism Through the Lens of Wittgenstein's Engineering Outlook."

35 Alongside Wilson's work, Kallenberg in "Rethinking Fideism" has also interestingly shown how specific engineering concepts (the method of projection, dynamical similarity, satisfactoriness) generally affect Wittgenstein's (later) approach to philosophical problems, and specifically his treatment of questions of knowledge and belief.

36 On the one hand, Wilson is fairly straightforward about his disagreement with Wittgenstein's view and his appraisal of science. On the other hand, he rather flippantly conflates the Romantic (Coleridgean) view of scientific enterprise, with that of the critical view of ontological claims held by Hertz, et al., as with that of Wittgenstein's apparent dismissals of scientific discourse as philosophically irrelevant. These claims require more careful attention then are given by Wilson. That said, Wilson's interest in the difference of focus between Reuleaux's work and Hertz's—i.e., in the ends of "machine design" rather than "behavioral description per se"—is well worth further probing. His notion of 'distributed normativity,' especially, and its use in the analysis of ends and means distinctions, is of emphatic interest and very general philosophical import.

37 For all of the above see, Hamilton, "*Darstellungen*". Curiously, while Wilson sees Hertz's mechanics as overly ideal in relation to its claims (within which the Hertzian 'object' represents the unobservable in science *par excellence*), Hamilton understands the critical import, and theoretical dimensions of the object in both Hertz and Wittgenstein to stem from their *resistance* to abstraction, and their love of the concrete. How both of these can be entertained is not easily explicable. Though we are more tempted by Hamilton's proposal in understanding Wittgenstein's intellectual biography, both Wilson and Hamilton have, so to speak, partial views of the same problem regarding the status of the transcendental in Wittgenstein's corpus and its relation to naturalism. Part of a solution is hinted at through Pârvu below.

38 See Pârvu, "'*Meine Grundgedanke ist...*'" In line with the above as it relates to the "German interpretation of physics," see also Harré, "Wittgenstein: Science and Religion."

39 On the notion of 'structural representation' see Swoyer, "Structural Representation and Surrogative Reasoning."

the purview of Kant. Despite its brevity, Pârvu's interpretation points towards original work that may be fruitfully made by extending the insights extracted from the TLP. Thus, the essay sets the transcendental in a novel 'structuralist' direction that has direct bearing on Wittgenstein's 'metaphysical subject': it points us to the relation between the real and ideal elements of the TLP and the concrete character of the subject it appeals to.

For now, let it suffice to outline Pârvu's relevance to the problem at hand. His strategy of 'rational generalization' employs two stages in which the concept of logical space is naturally implicated with phase space. The first involves the extension of the elements implicit in some ordinary fact (e.g., in the observation of some experimental phenomenon, the existence of some state of affairs). This first stage he calls *ontological* or *logical*. Within it the objects of some propositions (the propositions of natural science) are ontologically extended by virtue of their appearance in the regularity, form, or lawfulness of that in which they appear.[40] It translates the ontology from actually observed phenomena to extended objects, which are postulated as hypothetical or merely possible.[41] The second stage is *transcendental* or *metaphysical*, through it these mere possibles become the conditions of possibility for some actual state given in the extension of the theory (they move from ideal-extensions, or hypothetical entities, to that from which the initial actuality must issue). In the metaphysical second stage, this preliminary extension is no longer hypothetical or *ideal* (in the sense of a useful fiction), but instead is transposed onto a *transcendentally real* field on which its ensemble of possibilities rest (and thus, also, a fortiori, as the condition of whatever is the case, its actualization). "The Ideal extension becomes the very basis or the condition of possibility for the real domain."[42] The 'intrinsic complement' of the first step of ontological extension—viz. the 'formal or logical moment' of the transcendental, which "subsume[s] both the real and the ideal"—is the 'superior abstraction' which authorizes the isomorphism between logical and phases spaces, because it is now the whole virtual expression of all its possible states: "in the case of Hertz and Boltzmann, this moment is accomplished by the idea of 'phase space' by which all possible states of a system are represented. In the *Tractatus* we find 'logical space'."[43] Logical space—given that its scope goes beyond mechanics—is therefore the metaphysical generalization of the coordination of phase space.

Despite the isomorphism of coordination being such that any conception of logical space is mistaken without due consideration of phase space as a conceptual form, there is a point of contrast between them, which we must now use to elucidate some issues arising in the solipsism of the TLP. A crucial explicit (exogenous) presupposition of phase spaces in statistical mechanics is that the phenomenon in question must be considered an individual or closed system. This presupposition allows the states of a phase space to be implemented in behavioral predictions. A more or less arbitrary spatial constraint, delimiting a certain volume as a frame for its elements, provides the basis of the measure or comparison of micro-states, and the evaluative map for the coarse graining of the theory into macro-states, on the basis of like distributions of elements in the possible the micro-states of a given volume.[44] The relative justification for this is the differential-geometrical derivation of the conservation of volume under possible changes of states in the distribution of the particles in that volume.[45] In this way, for physical phase spaces, the *content* of the system is ignored in relation to the *limits* of that system.[46] No such equivalence can be found in relation to logical space, and no such exogenous presupposition can be given in relation to its limits. If this *were* the

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40 This is how the atomic theory of point-masses was used in the origins of statistical mechanics, since it had not yet been verified experimentally.

41 Pârvu, "'*Meine Grundgedanke ist*…'", 264.

42 Ibid., 264.

43 Ibid., 265. There is much more to be said for the intricacies of Pârvu's delimitation of the Tractarian version of a structural metaphysics, especially as it is entangled with transcendental philosophy, which we, unfortunately, have no space for here.

44 This presupposition is also necessary for Boltzmann's probabilistic understanding of entropy.

45 See, Nolte, "The Tangled Tale", 35: "In the derivation of dynamical probability distributions, Boltzmann required the use of what we now know as conservation of volume in phase space." It nonetheless still requires the number of particles (and thus the number of possible states) to remain fixed a priori.

46 Pears, "Solipsism in the *Tractatus*", 72-73, indicates that this is not the case with logic, that the contents of logical space themselves "fix the limits of the receptacle." This he uses to argue why showing the limits is possible, but not saying them.

case for logical space then Wittgenstein should be able to step outside his own theoretical frame and give us the totality of elementary propositions; of course he cannot give us this totality.

Let us now return to some of the consequences of the inherence of logic, which was alone capable of assuring the relation between language and the world. If it is true that logical space determines the possibility of the world—that is, the absolute limit—it follows that whatever those limits are *could not be otherwise*, and, since they represent the boundary of the possible itself, it thus becomes impossible to say how it is that they are the limits, at least from the point of view of a closed system: "[f]eeling the world as a limited whole—it is this that is mystical."[47] Since logical space determines both the conditions of sense *and* possible states of affairs, how is it that we come to view logic as intelligibly able to fix rules for the implementation of signs, of saying, since in a sense, we could not err, i.e., logical form is already at work in everything and all possible propositions must occur within the bounds of the space they articulate?[48] This is the signal conundrum—what I will call the Tractarian paradox of immanence—which occupies the remarks of the TLP from 5.4 until the end of the treatise.[49] It directly precipitates the discussion of solipsism, since it seems to be an ineliminable case-in-point of the paradox.

4 Solipsism, the metaphysical subject, and singularity

Because logical space is the form of the world, or determines its possibility necessarily, and language is coextensive with that space, it is obvious that the limits of language mean the limits of the world. And since—again by dint of logical necessity—it is "the only language I understand" ("*der Sprache, die allein ich verstehe*")[50] it follows easily that "the limits of my language mean the limits of my world."[51] But we cannot countenance the empirical subject here, so we are obliged to think the metaphysical subject in a non-trivial way:[52]

> The I of solipsism shrinks to a point without extension, and there remains the reality coordinated with it.
> ...[T]he metaphysical subject, the limit of the world—not part of it.[53]

Since we have established that logical space articulates the form of the world, the limits of logic, then we must assume that solipsism maintains a relation with logical space. Because the metaphysical subject is also associated with the limit, we know too that it is not a content of logical space in any ordinary sense— that is, the subject is not an empirical content among others. Based on the assertion in the quote above we also know that the subject must be a subject-point. But solipsism cannot be attested to because it cannot

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47 TLP 6.45.

48 This is also what is behind Wittgenstein's invocation in the *Notebooks*: "Logic must take care of itself.", 2e. See also, TLP 5.473.

49 There is a related issue pertaining to idea of a boundary which it seems it must have in order for depiction to work, in order for there to be anything like "logico-pictorial form". If there is a boundary, then, one would assume logical space has a shape, which Wittgenstein seems to gesture at in at least one place: "I also always find myself at a particular point in my visual space, so my visual space has as it were a shape." *Notebooks*, 86; cf. TLP 5.6331. If logical space were formalized, it may be possible to specify the shape of its space immanently using relations within the space (by analogy to the way in which topologies can be determined from geometrical facts in the space), which may shed light on other logical relations that obtain. This is, however, beyond both my prowess and the scope of the issues here.

50 Ibid. 5.62.

51 Ibid. 5.6. It is not insignificant that Wittgenstein choses the verb 'to mean' here instead of 'to be'. This puts the limit in a referential relationship of language with the world, since Wittgenstein has in mind the Fregean sense of *Beteutung* as 'reference'. "A name means an object" (3.203) has the same indicative or indexical sense.

52 Many commentators have given trivial accounts of Wittgenstein's solipsism for different reasons: the realist wants to make it trivial in order to save realism from well-known classical problems of solipsistic skepticism (cf. Hintikka), the idealist wants to turn it into a trivial (read: non-synthetic, non-contentful, or 'thin') form of Kant's "I think", the "transcendental unity of apperception" (cf. Tang, Hacker). 'New' Wittgensteinians want to make it theoretically trivial, but ethically or practically strategic, and they do so in various ways (cf. Floyd). All of these are unsatisfying accounts because they cannot provide any positive formulation of Wittgenstein's concept—this difficulty is in part, no doubt, because he says it can only be shown.

53 TLP 6.4.

have a sense which lies inside the world—this is to say no more than has already been said regarding the limits of language and its relation to immanence.

The ordinary points of logical space—as is hopefully clear from context by now—are the objects of states of affairs. As we saw with Hamilton, these objects are never independent of (or isolated from) the possible states of affairs (complexes) which they constitute. The relationality of objects determines that sense only occurs at the level of logical relations in form spaces, or put to the side of language, at the syntactical level of the proposition. Names in-and-of themselves have no sense; they are merely indicators or 'deputizing symbols'. Objects are (in themselves) simple. Because they are simple, they are points in the space. However, since objects can in no conceptually or functionally significant way be considered the limit of logical space, and since these points (as internal to the sense conditions or truth functionality of the possibilities with which the employment of logical space concerns itself) are constitutive of its functions, we call these points regular or ordinary (merely possible or actually real). By contrast, the metaphysical subject, as a point, does not behave as the ordinary points do, but in a logically degenerate way. In some sense this point, by virtue of the limit, virtually contains all the coordinations and possibilities of logical space, but not in such a way as to be amenable to truth functionality (that is, amenable to empirical articulation). The metaphysical subject cannot be an empirical feature of the world, but it is nonetheless not hypothetical or ideal.[54] The metaphysical subject is a transcendentally real point: I call such a limit-point a singularity.

A singularity in mathematics is a point in some system—often a coordinate system—whose function is undefined, or a problematic point in which an ordinary function behaves in a 'degenerate' way. "In general, a singularity is a point at which an equation, surface, etc., blows up or becomes degenerate."[55] For example, consider the function f(x), $a/x = b$, where a and b are positive, such that as x approaches 0, f(x) is 'undefined' or blows up to infinity at the limit: $a/0 = \infty$. For this function, we would call the value $x = 0$ a singularity. Such a point is 'degenerate' insofar as it poses problems for the rules governing the ordinary functions of arithmetic.

More geometrically, consider the geographic coordinate system of longitude and latitude on the Earth. Lines of longitude define the distribution of time-zones on the Earth, allowing us to determine, given the velocity of an object on a given path, how many time-zones will be traversed in a time t. However, at the poles of the Earth, these are undefined, leading to an output where we could traverse all time zones an infinite number of times, in an infinitesimal duration, for arbitrarily small velocities. Here the singularities of the poles pose problems for the metrical evaluation of distances over time, it is degenerate with respect to the functional specification of the time zone. These are however, trivial examples—so-called 'removable' or 'non-essential' singularities.

The metaphysical subject, so it seems, is ineliminable or 'essential.' Nevertheless, the examples serve a didactic purpose for us. We can see that in some sense, the values or coordinates of the system as a whole are virtually contained in these singularities: all the real numbers are 'contained' in an infinite set; all the time zones are 'contained' at the poles, etc. If Tractarian objects in logical space are its ordinary points, then the limit-point of the subject is a singularity in the sense in which all the ordinary results of truth functionality are virtually 'contained,' though are not themselves truth-functional in the singularity, neither are they trivially *senseless* (*sinnlos*) in the way in which tautology and contradiction are. The metaphysical subject is a degenerate point in the coordination of logical space. But from this perspective, solipsism "coincides with pure realism", "...there remains the reality *co-ordinated* with it"[56] since all the truth-functional mapping of propositions to states of affairs holds equally well for all ordinary points in logical space. Solipsism cannot be said, not because it is senseless, but because it is *nonsensical* (*unsinnig*) to try and determine the singularity of the metaphysical subject in the world (from which its determinations degenerate), which alone would allow it to be truth-functional. It is in this precise sense the limit of the world, and not part of it—it is nonsensical because singular and yet ineliminable. Thus if the limit of the world and the boundary of logical space are equivalent we could present it as in Figure 1:[57]

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54 TLP 5.631-5.633.
55 Weisstein, "Singularity".
56 TLP 5.64, my italics.
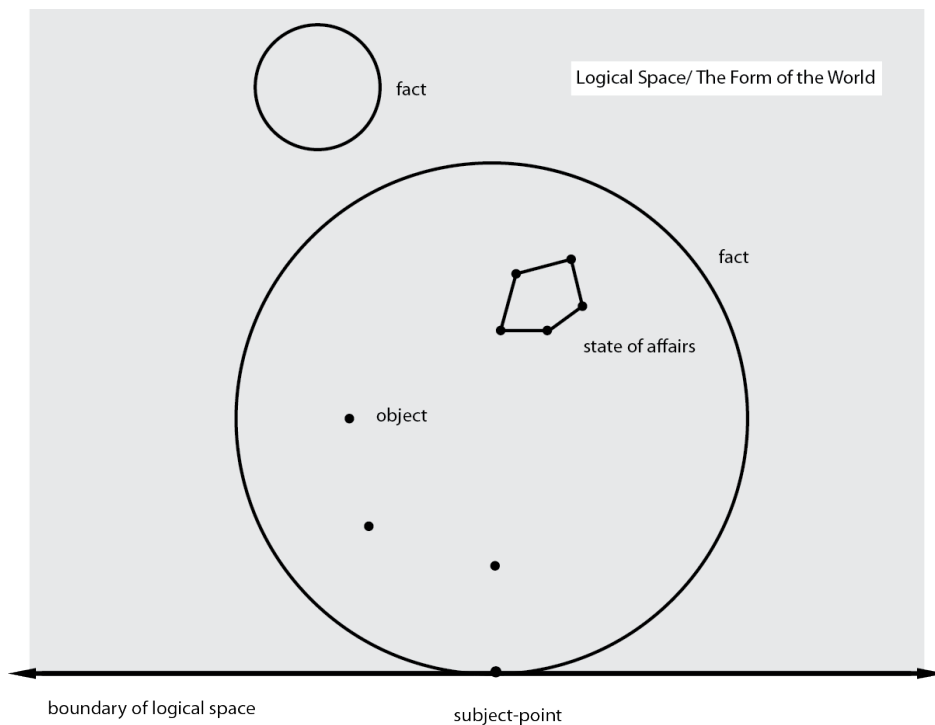57 Of course, Figure 1 is meant only as a heuristic cartoon.

Figure 1

Let us assume that logical space is bounded but infinite as in a hyperbolic space. [58] Then the subject-point as the limit of the world, although bounded, appears—from its own perspective—unbounded, without limit. And since in some sense, the whole of logical space is 'contained' at this limit, credence can be given to its characterization as solipsistic, even if it is also, in another sense, not genuinely 'restrictive,' for reasons of logical necessity. This would explain why I never run up against the limit of the world (empirically), and why I am not even be able to say what the limit is (because of course there is 'nothing', as it were, on the other side of the boundary). Neither could I *say* that the subject is the limit of the world, I could never attest to the truth of solipsism.

Finally then, we can say that as a singularity in logical space, undefined in relation to truth-value, the metaphysical subject obtains concretely the paradox of immanence consequent to the inherence of logic necessary for the Tractarian theory of language—the very thing that precipitates the implosion of the theoretical apparatus of the *Tractatus* from the inside.

5 Postscript on transcendental singularities: What kind of 'subject' is metaphysical?

Just as there are attractors in meteorological, economic, and population dynamics; centers of gravity where no body is present; points of resonance—positive and negative feedback—in sonorous and physical systems; and critical thresholds of temperature and other parameters in the phase transitions of the material properties of objects, there is also the involvement of singularities in the genesis of experience. As we have already stated, our concern here has been to make sense of solipsism in the TLP, but solipsism (classically defined) relates a locus of experience (its partiality) to the conditions of its possibility (e.g., the objects of its 'knowledge'), while excluding those conditions as accessible to its perspective. If experience had no locus, if it weren't partial, there would be no grounds for skepticism about what

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58 Since logical multiplicity is determined by the number of objects, but this number is not given, there should be no objection to the assumption of its magnitude.

appears in it (either about objects in-themselves or about other minds) since experience would amount to a simple adequation with what is real *in toto*, but if any given experience *can* be acknowledged as partial, then the apprehension of its own perspectivism must be possible from the outset, and thus, it must then have, internal to its own manifold, access to its conditions. Kant hatched upon this solution, but required of it the unfortunate form of the 'I' in the transcendental unity of apperception. This ensured the unity of the self wherein experience as consciousness is said to be 'possible' and assumed that *only* the form of its possibility suffices to vouchsafe for all experience. My contention here is that Wittgenstein is the first—obliquely, obscurely, negatively, and unconsciously—to show in what way the collapse of solipsism with pure realism signifies nothing other than the genesis of sense, and gives realism back to experience, without recourse to the form of consciousness. In a sense, the relation between experience and realism in the TLP was already dimly viewed by the Vienna Circle, which then tried to imprison a full-blooded notion of experience within the courthouse of philosophy, as it attempted, too, to adjudicate the validity of scientific enterprise through an ultimate appeal to logically pure notions of observation and verification.[59] Science, in any case, had no need of the judgment of philosophy to verify its successes. In this interpretation, Carnap tragically misapprehended Wittgenstein's silence, the silence of the TLP, as a denigration of what cannot be 'said' in experience (metaphysics, ethics, aesthetics, etc.), that is, what cannot be given the overt and explicit empirical form through which natural scientific knowledge has flourished. Such denigration, of course, was anything but Wittgenstein's aim. My claim is that in the relation between experience and the real, Wittgenstein's notion of the metaphysical subject entertains a subterranean alliance with Deleuze's dramatic ramification of the concept of experience, its entanglement with an equally ramified version of the transcendental, and his conception of singularities as they pertain to each of the former.

As we agreed, Hintikka is right to object to the interpretation of Wittgenstein's solipsism as a private language, or a form of linguistic idealism, in the personal or subjectivist sense of the term. But, as we have shown, the subject is not given by a complex of propositions constituting the limit of the world. This would make the subject both empirical and composite; it is instead transcendental and singular. This is why we have insisted on its characterization as a singularity: only a singularity constitutes both the limit and the point, and effectively renders the transcendental concrete rather than abstract. But what does this mean for experience? What kind of 'subject' is metaphysical?

If the solipsism of the TLP is heterodox, it is because it is not a personal solipsism (as for example one can imagine both in 17th century rationalism and in the sense-impression-based skepticism of the 18th century). Rather the metaphysical subject is impersonal. If it retains a nominal form of the 'I' in the sense in which "the world is my world" or in which "I am the microcosm," is this artifact not due to Wittgenstein's perspective being colored by the language he had inherited—namely, the language of the idealist who could not think outside the forms of consciousness and agency (Kant's 'I think', Schopenhauer's 'will')? After all, "there is no subject which thinks or entertains ideas"[60] and, "[i]t is impossible to speak about the will insofar as it is the subject of ethical attributes."[61]

Given what we have already brought to bear against Hintikka's view of the complex and its empirical character, the subject-point (the singularity) is not only transcendental and impersonal, but also pre-individual. There is no 'empirical subject' just in the sense in which empirically constituted individuals are already composed of facts described at a certain level of propositional complexity (and which can be decomposed or analyzed). According to Wittgenstein, this is a problem for empirical psychology, but not for philosophy. Again, this relates to what we took issue with in Hintikka's view, its reliance on the notation of propositional attitudes, which Wittgenstein rejects in Russell. It presupposes a thinking subject, which appears at once empirical yet hopelessly transcendent (rather than transcendental), because it hovers over the proposition, that which articulates states of affairs—the facts of its own being.

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59 See, for example, Carnap, *The Logical Syntax of Language*.
60 TLP 5.631.
61 Ibid., 6.423.

The characteristics of the metaphysical subject, on the other hand, as impersonal and pre-individual, are precisely what constitute Deleuze's definition of singularities in a properly transcendental manner. Deleuze accounts for the genesis of experience within real experience itself, without for all that eliminating the genetic difference between the transcendental and the empirical, but also without making of their difference a dualism:

> We seek to determine an impersonal and pre-individual transcendental field, which does not resemble the corresponding empirical fields, and which nevertheless is not confused with an undifferentiated depth. This field cannot be determined as that of a consciousness... A consciousness is nothing without a synthesis of unification, but there is no synthesis of unification of consciousness without the form of the I, or the point of view of the Self. What is neither individual nor personal are, on the contrary, emissions of singularities...Singularities are the true transcendental events... Far from being individual or personal, singularities preside over the genesis of individuals and persons; they are distributed in a 'potential' which admits neither Self nor I, but which produces them by actualizing or realizing itself, although the figures of this actualization do not at all resemble the realized potential.[62]

It is certainly not our contention that Wittgenstein had an account of the productivity of the singularity as such or of the ways in which the transcendental and the empirical need not be categorically opposed to one another but, on the contrary, must be insisted upon as immanent and interrelated. But, whatever the shortcomings of the TLP relative to Deleuze's program, did Wittgenstein not assemble an obscure and confused forecast of it? Aren't the metaphysical subject and logical space the coordinates by which the genesis of experience could be distributed in a potential? This seems to be hampered only by the fact that, for Wittgenstein, there is only a single singularity: he had not yet understood that singularities—and the paradoxical characteristic of their being at once limit and point, structure and genesis, neutrality and productivity—only occur in the plural. And that this plurality of singularities is distributed and constituted by its very empirical instantiation—just as the Tractarian objects must be, since they represent the genesis of *logical structure* by virtue of the ineliminable concomitance of objects that is itself their *individual form*.

For Deleuze, by contrast, singularities only exist in an open multiplicity of indefinitely co-constituted ontological expressions, not by virtue of an a priori set of objective possibilities—that is, not in the determinate totality of the 'limited whole'. Nonetheless, through the singular subject—which is, in the final analysis, his peculiar loneliness, his solipsism—Wittgenstein attempted to embrace the world entire.

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62 Deleuze, *Logic of Sense,* 102-3. We cannot stress highly enough the importance of Deleuze's notion of the transcendental and its relation to experience and the notion of the singularity, for—viewed carefully, perspicuously, uncaptively enough—it promises the greatest contributions to a univocal account of mind and matter, which suffers neither from spiritualism, nor from the myopia with which reductionism draws its opaque conclusions about what is (and is not) possible in nature. It offers a metaphysical point of view whereby differences in kind are *genuinely* possible, and a position from which it is possible, also, to imagine the genesis of experience from without, rather than by stifling its originary externality from within (e.g. in the form of consciousness). It can be shown how many of the most trenchant problems of analytic philosophy of mind (whether of the phenomenalist or of the reductionist) stem from poorly posed problems, and the blunt and clichéd instruments of conceptual complacency. Unfortunately, it cannot be our task to convince the reader here.

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