Wave-Functionalism[1]

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Abstract

In this paper I present a new perspective for interpreting the wavefunction as a non-material, non-epistemic, non-representational entity. I endorse a functional view according to which the wavefunction is defined by its roles in the theory. I argue that this approach shares some similarities with the nomological account of the wave function as well as with the pragmatist and epistemic approaches to quantum theory, while avoiding the major objections of these alternatives.

Keywords: quantum ontology; wavefunction realism; primitive ontology; epistemic approaches; pragmatist quantum realism; epistemic/ontic distinction; constructive vs. principle explanations; functionalism.

1 Introduction

The wavefunction seems to play a crucial role in quantum mechanics: the Schrödinger equation is an equation for the temporal evolution of wavefunction. Moreover, the probabilities of measurement results are provided by the Born rule, which is usually expressed in terms of the wavefunction. In the context of realist approaches to quantum mechanics, these reasons provide an excellent rationale for endorsing wavefunction realism, the view that the wavefunction should be seen as representing a quantum state, i.e. a material field or physical entity. However, for a variety of reasons, other realist approaches reject the idea that the wavefunction represents matter in this way. One such perspective is given by the primitive ontology approach, according to which the wavefunction should be understood as a law of nature. Other realist attitudes following this lead, such as for instance the information-theoretic interpretation, have instead proposed that the wavefunction is epistemic. Furthermore, there are pragmatist approaches, in which the wavefunction is taken to be prescriptive about the experimenter's expectation of a given phenomenon. In this paper I start from the assumption that the wavefunction does not represent something physical, following the ideas of the approaches mentioned above, and *contra* wavefunction realism. All these

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views face major challenges. *My goal is to explore whether it is possible to construct an account of the wavefunction (understood as not representational), which is less problematical than the alternatives*. The result is a view, which I dubbed *wave-functionalism*, that is functionalist: following the pragmatists, the wavefunction is not defined in terms of what it represents but rather in terms of the functions it plays, so that *the wavefunction is as the wavefunction does*.

This is the structure of the paper. In Section 2 I overview the motivations and the most serious problems of the various views about the nature of the wavefunction. I start in the first subsection reviewing wavefunction realism, then I discuss the primitive ontology program, and finally I move to epistemic and pragmatic interpretations. Next, in Section 3 I present my view, *wave-functionalism.* I begin by reviewing the main ideas of functionalist approaches, and then I outline the different roles the wavefunction plays, comparing and contrasting wave-functionalism with the other approaches. Then I discuss the advantages of my view over the alternatives, and I anticipate possible objections to wave-functionalism and proposing some replies. The last section summarizes the main ingredients of my proposal.

2 What is the Wavefunction?

Scientific realists ask: *what is the quantum world like?* Opening physics books, a mathematical entity and an equation stand out: the wavefunction and its evolution equation, namely the Schrödinger equation. So, one natural thought is to take this object as describing the fundamental ontology. In this section I review the major realist approaches to the status and the nature of the wavefunction as well as their most challenging problems.

2.1 Wavefunction Realism: The Wavefunction Represents a Material Field

Given that the wavefunction appears in the theory's most fundamental equation and in the rules used to make predictions, a reasonable attitude seems to think of the wavefunction as representing matter. However, as emphasized by Schrödinger himself (1935), this is problematical for at least two reasons. For one, the cat problem: if the wavefunction is a physical field, then there are macroscopic superpositions that we actually do not observe. Realist solutions of the cat problem include the pilot-wave theory,[2] the spontaneous localization theory,[3] and the many-worlds theory.[4] Nonetheless, there seems to be a further problem, sometimes dubbed the configuration

[2] de Broglie (1928), Bohm (1952).
[3] Girardi, Rimini and Weber (1986).
[4] Everett (1957).

space problem. The wavefunction of the universe is defined over a $3n$ dimensional configuration space, where n is the apparent total number of particles. So, how can we interpret the wavefunction as something physically vibrating? Unlike the cat problem, the configuration space problem is more controversial.[5] Some deny there is a deep problem and embrace *wavefunction realism,* [6] according to which there is a new material field, sometimes called the quantum state, which the wavefunction mathematically represents. Then, wavefunction realists propose ways in which three-dimensional particles, as well as macroscopic objects, are suitably emergent from the field in the high-dimensional configuration space.[7]

Wavefunction realism, which was also motivated by the desire of keeping the theory local (even if only in configuration space),[8] has been criticized on grounds that the existence of a wave or field in a high-dimensional space is more problematical than the wavefunction realists are willing to admit. Some have argued that configuration space is an artificial construct, since there are no particles at the fundamental level and thus the introduction of such a space lacks justification.[9] Also, some have argued that the view is too revisionary: since configuration space is high-dimensional, the explanation of the macroscopic world has to be heavily revised when compared to the classical understanding.[10] Others have maintained that wavefunction realism makes the role of spacetime in the theory obscure, and it is difficult to extend to quantum field theories.[11] While wavefunction realists are actively working on solving these challenges, [12] some have instead proposed alternative views, which have in common that the idea that the wavefunction is still a material field, but not in configuration space.[13] However, even if promising, arguably these approaches still face some difficulties.[14]

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[5] See Albert and Ney (2013) for a review of the different approaches to it.

[6] See most notably Albert (1996, 2015), Ney (forthcoming), North (2013), and references therein.

[7] See Albert (2015) and Ney (forthcoming) for two different approaches on this.

[8] Ney (forthcoming) and references therein.

[9] "It seems a little paradoxical to construct a configuration space with the coordinates of points which do not exist." de Broglie (1928) translated in Bacciagaluppi and Valentini (2009) p. 380.

[10] Monton (2013), Allori (2013).

[11] See Wallace and Timpson (2010), Myrvold (2015).

[12] See Chen (2019) and references therein for more objections.

[13] One view considers the wavefunction as a three-dimensional poly-wave or multi-field which assigns values to regions of points (Forrest 1988, Belot 2012, Chen 2017, Hubert and Romano 2018). Another possibility is primitivism: the wavefunction represents an unanalyzable, nonlocal, material field (Maudlin 2019). Others have tried to reformulate quantum theory only in terms of local 'beables' (Norsen 2010), while spacetime state realism associates to each system a determinate property represented by a reduced density matrix (Wallace and Timpson 2010).

[14] Allori (2018) has objected that the multi-field view is unable to account for symmetries, while primitivism looks unnecessarily mysterious and dismissive (Allori 2020). Theories of local beables face

Be that as it may, presumably driven by these challenges, others have taken the radical view of eliminating, one way or the other, the wavefunction from the material ontology of the theory. In other words, the wavefunction does not describe matter: the wavefunction does not represent some new and peculiar material field which needs to be added to 'the furniture of the world.' Different approaches spell out what the wavefunction is in different ways, but they all agree that there is no material, physical quantum state the wavefunction mathematically represents. I wish to focus on this type of approach, and on the following question: *what is the best way of thinking of the wavefunction, if it is not representing a material field?* I articulate an answer to this question in Section 3, after having overviewed the main non-material theories of the wavefunction, their motivations, and their challenges in the remainder of this section.

2.2 Primitive Ontology: The Wavefunction Represents a Nomological Fact

Among those who find the configuration space problem serious enough to conclude that the wavefunction does not represent a material field, one finds the primitive ontology approach.[15] This view constitutes a realist approach to quantum theory, just like wavefunction realism. In contrast with it, however, it assumes that the fundamental ontology of matter is given by microscopic entities in three-dimensional space, or four-dimensional space-time: particles, fields, strings, spatio-temporal events (flashes). These entities constitute the primitive ontology of the theory, as opposed to the other variables present in the theory which do not represent matter, including the wavefunction. One popular way of thinking about the wavefunction in this framework is that it represents a law of nature. In other words, the quantum state is not material, but it is nomological. The main idea behind this nomological approach is captured by the popular slogan: 'the primitive ontology specifies what matter is, the wavefunction tells matter how to move.'[16] The central motivation for this nomological view of the wavefunction lies in the fact that the wavefunction, having a nomological role, is able to *compositionally and dynamically* explain why phenomena happen. This type of explanation is the one that kinetic theory provides of the laws of thermodynamics: gases are composed of Newtonian particles, and one explains, say, why gases expand when heated given the dynamics of their fundamental particles. Analogously, in the primitive ontology approach the observed macroscopic properties and the behavior of the macroscopic objects of our experience are explained in terms of these macroscopic objects being

serious technical challenges (Norsen 2010), and Maudlin (2019) has objected that the reduced density matrix proposed by spacetime state realists inverts the usual relation between parts and wholes because it is not separable.

[15] Allori, Goldstein, Tumulka, and Zanghì (2008), Allori (2013).

[16] Dürr, Goldstein, and Zanghì (2013), Goldstein and Zanghì (2013).

composed by the microscopic primitive ontology, whose evolution is dynamically governed by a law defined in terms of the wavefunction.[17]

What is considered by many the most serious problem of this view is that the wavefunction does not look like a law of nature. First, laws are time-independent entities, while the wavefunction evolves in time.[18] Primitive ontologists have replied that even if the wavefunction is time-dependent in the current theory, it may turn out not to be so in a future theory of quantum gravity.[19] However, some may still find this type of reply not very satisfactory, as one would want to know what the wavefunction is in the current theory without having to wait for a future theory.[20] Alternatively, one could reverse the argument and maintain that if our intuitions about what a law of nature, for instance time-independence, are at odds with the theory, then we should reject our intuitions.[21]

Another problem for the nomological approach is that, unlike laws, the wavefunction is not unique, as two wavefunctions may give rise to the same empirical predictions. To this challenge, one could reply that lack of uniqueness does not speak against the wavefunction being a law, as one could have different formulations of the same law, with different mathematical objects. However, this is controversial, as others have questioned that they are truly the same law.[22]

Be that as it may, to overcome these problems, people have proposed alternative ways of spelling out the idea that the wavefunction mathematically represent a nomological fact.[23] However, while these are interesting proposals, there is still work to be done as it is not straightforward how to implement the details, or uncontroversially respond to objections.[24]

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[17] Allori (2013).

[18] Brown and Wallace (2005).

[19] Dürr, Goldstein, and Zanghì (1997), Goldstein and Teufel (2001), Goldstein and Zanghì (2013).

[20] Wallace, p.c.; Albert, p.c..

[21] Callender (2015)

[22] Maudlin, p.c. See also subsection 3.5, objection 3. For more objections, see Belot (2012), Callender (2015), Esfeld *et al.* (2014), Suárez (2015).

[23] These proposals have been developed in a Humean framework (see Callender 2015, Esfeld 2014, Miller 2014, Bhogal and Perry 2017). A connected nomological approach is to think of the wavefunction as a dispositional property, or more generally as a property of the primitive ontology (Monton 2013; see also Esfeld *et al.* 2014, and Suàrez 2015 for the development of this approach in the framework of Bohmian mechanics). Other approaches that seems to belong to the same camp are structuralist accounts (North 2013).

[24] Many find the Humean accounts wanting, partly because they find Humeanism with respect to laws misguided for other reasons. Moreover, some others consider find the property approach to the wavefunction objectionable, given that the wavefunction does not behave like a traditional property

So, the question is the following: assuming one does not want to think of the wavefunction as representing material objects, *can we do better than the nomological approach in accounting for the nature of a non-material wavefunction*? In the next section I discuss the epistemic interpretations. The main idea there is to reject the ontic representational status of the wavefunction (proper of wavefunction realism) in favor of an epistemic one: the wavefunction does not represent something objective in the system under investigation; rather it represents our best knowledge of it. This framework is interesting as it provides an example of a realist approach which explains the phenomena in a distinctive way, which differs from the one proposed by the primitive ontology program.

2.3 Epistemic Interpretations: The Wavefunction Represents Our Knowledge

As in the primitive ontology approach, in these interpretations the wavefunction does not represent any material quantum state. However, while in the primitive ontology approach the wavefunction expresses a nomological fact, here it is epistemic: it represents our incomplete knowledge of the system and, by appearing in the Born rule, it determines our expectations about measurement outcomes.[25]

In virtue of being epistemic, these approaches have received many criticisms, most prominently that they cannot account for interference phenomena, and that they violate an important no-go theorem. This theorem, called PBR theorem from the initials of the authors,[26] is taken to show that, in line of the strategy used by the other no-go theorems, epistemic interpretations are in conflict with the statistical predictions of quantum mechanics. This is done by showing that if the wavefunction were epistemic, then it would require the existence of certain relations which are mathematically impossible. Therefore, it is argued, if quantum mechanics is empirically adequate, then the wavefunction must be ontic: it must objectively describe either a fact about matter, or a

(Suàrez 2015). Finally, structuralist accounts seem at the moment to be underdeveloped or too vague, as it is unclear what structure is in this context.

[25] Epistemic approaches can be distinguished into neo-Copenhagen accounts, and hidden variables ones. Einstein's original statistical interpretation (1949) is an example of a hidden variable epistemic approach. In fact, it states that quantum theory is fundamentally incomplete, as it does not specify hidden variables describing the reality under the phenomena, whose behavior is statistically well described by the wavefunction. For a more modern approach, see Spekkens (2007). In contrast, the neo-Copenhagen approaches reject that the above-mentioned hidden variables exist or are needed. Traces of this can be found for example in Heisenberg (1958). Neo-Copenhagen approaches include the information-theoretic approach (Bub and Pitowsky 2010, Bub 2015), Bayesian approaches (Fuchs 2010), and relational ones (Rovelli 1996). See Dunlap (2015) for a comparison between the primitive ontology approach and the information-theoretic interpretation.

[26] Pusey, Barrett, and Rudolph (2012).

nomological fact. These objections have been subject to an extensive examination, and proponents of epistemic approaches have provided several replies, which however in the eyes of many are still tentative.[27]

Usually, these approaches are regarded as instrumentalist, given that they give a central role to measurements outcomes. So, it may seem pointless that I even consider them here. However, some of these interpretations, most notably the information-theoretic interpretation of quantum theory, explicitly claim to provide a fully realist account of the quantum world.[28] The idea is that quantum mechanics is fundamentally about measurement outcomes, whose statistics are recovered by the constraints that Hilbert space structure imposes on the quantum dynamics.

Notice that this type of approach is explanatory in a distinctive way: it explains by systematizing the phenomena using general *principles,* in a *kinematic way*. An example of such a principle explanation is the explanation of heat phenomena by thermodynamics: thermodynamics uses principles such as "energy cannot be created or destroyed" and "entropy always increases" to determine what can or cannot happen. This way of explaining is in contrast with the type of explanation favored by the proponents of the primitive ontology approach, for instance. As discussed in the previous subsection, the primitive ontology explanation is compositional and dynamical: macroscopic objects are made of the microscopic entities the primitive ontology describes, and their behavior is recovered dynamically, in terms of a some sort of reductive or constructive explanation, like the one provided by kinetic theory for thermodynamics. In other words, realist epistemic interpretations do not provide a dynamical (or a causal) mechanism to explain. Rather, they claim to be explanatory by being able to account and predict which phenomena can occur in terms of principles: just as a melted piece of ice cannot re-freeze because it is forbidden by the second principle of thermodynamics according to which entropy always increases, in quantum mechanics we have interference under a given set of circumstances because this is what the quantum rules prescribe. To use a terminology introduced by Einstein (1919), there is a distinction between *constructive and principle theories*: constructive theories involve a dynamical reduction of macroscopic objects in terms of their microscopic, fundamental, constituents, while principle theories are formulated in terms of principles used to constrain physically possible processes. While the primitive ontology approach is built to provide constructive explanations, epistemic interpretations are principle theories.

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[27] See Leifer (2014), Gao (2017), Maudlin (2019), and references therein, for a discussion of objections and replies.

[28] Bub and Pitowsky (2010). See also Fuchs (2010) for a discussion of how quantum Bayesianism is a realist approach.

Because of this, the latter offer reasons why one *should expect* a phenomenon to happen, while the former provides the *reasons wh*y the phenomenon happens.[29]
This leads us straightforwardly to the next interesting view, namely pragmatist quantum realism, which shares the same attitude on explanation as the epistemic approaches.

2.4 Pragmatist Quantum Realism: The Wavefunction Prescribes Expectations

According to pragmatist quantum realism the wavefunction is prescriptive rather than descriptive: physical systems are *not* described by the wavefunction, which instead tells us what we should believe about a given physical situation.[30] This view is pragmatist in the sense that the wavefunction is not some entity to 'reify' or materialize.
Instead, in contrast with epistemic interpretations and with the primitive ontology program, in this approach the wavefunction is to be understood in terms of the roles, or functions, it has from our perspectives. The wavefunction provides objective 'information bridges' which concisely summarize what happens and enable us to make predictions about what to expect will happen next. Thus, the wavefunction has the role of explaining the phenomena in the specific sense that it explains why experimenters are justified in believing something will or will not happen. In the terminology introduced in the previous subsection, pragmatist quantum realism is a *principle theory*: the Born rule assigns phenomena probabilities of happening, and these probabilities express our degrees of belief that a given phenomenon will happen. In this respect, pragmatist quantum realism is similar to the epistemic interpretations. However, epistemic approaches may be thought of as pragmatic, rather than pragmatist, in that they prioritize recovering the empirical data, and assign a fundamental role to measurement results, while pragmatist quantum realism does not do that.[31] Moreover, by defining the wavefunction in terms of its explanatory role, the pragmatist provides a characterization of the wavefunction which is not epistemic. Rather, it is objective in such a way that, arguably, pragmatist quantum realism does not fall prey of the PBR theorem, in contrast with the epistemic interpretations.[32]

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[29] For more on the constructive/principle distinction in this context, see Bub and Pitowski (2010). Also, for a more general discussion of this distinction, see Flores (1999), Balashov and Janssen (2003), Brown and Pooley (2004), Brown (2005), Brown and Timpson (2006), Felline (2011), van Camp (2011).

[30] Healey (2015, 2017a, 2017b, 2020) and references therein. See also Friederich (2015).

[31] A pragmatist view is one in which, as held by the pragmatist school of philosophy, to understand a concept, such as the wavefunction, is to understand its functions, rather than what it represents. Instead a view can be called pragmatic when it focuses on the macroscopic domain, aiming at reproducing our experiences, rather than providing the microscopic description giving rise to the macroscopic data. I thank an anonymous referee for pointing this out to me.

[32] Notice the PBR theorem proves that the wavefunction has to be non-epistemic. Traditionally this has been taken to mean that the wavefunction has to be ontic, in the sense that the wavefunction represents

This view provides a non-material, non-epistemic, non-representational account of the wavefunction based on a particular understanding of explanation. It is therefore similar to the primitive ontology program in that it focuses on explanation and non-materiality of the wavefunction. However, there are at least two differences. First, in the primitive ontology approach the wavefunction is still representational, of a nomological rather than a material fact, while in pragmatist quantum realism the wavefunction completely ceases to be defined in these terms. Second, the two views differ in what they consider a satisfactory explanation. As anticipated, while the primitive ontologists favor a dynamical, compositional approach based on a constructive understanding of theories, pragmatists instead agree with epistemicists that principle theories are enough to provide satisfactory explanations: no constructive (dynamical and compositional) explanation is needed. These accounts explain by providing principles which limit the possible phenomena, just like thermodynamics explain heat phenomena. By stating that "gases expand when heated" one explains why this gas expands in the sense that, if I perform an experiment heating this gas, I expect it to expand. Still, people have pushed back against this attitude, maintaining that only constructive theories, which provide the reasons why a phenomenon obtains, are truly explanatory: I need to explain what heating is, and what in this gas makes it expand when heated. Otherwise, I merely have a description, and not an understanding of the phenomenon.[33]

Therefore, one may wonder whether it is possible to reconcile the desire of having a non-material, non-representational, ontic wavefunction as pragmatist quantum realism maintains, with the constructive explanation provided by the nomological approach. In the next section I propose a view which is aimed at combining these desiderata, with the aid of another philosophical position, namely functionalism.

3 Wave-Functionalism

something independently of any observer. Nonetheless, the epistemic-ontic distinction so understood is a false dichotomy, as it is not necessarily the case that if the wavefunction is not epistemic has to be ontic in this way. In fact, the following are all possible state of affairs: 1) the wavefunction represents a physical object (as wavefunction realism proposes); 2) the wavefunction represents a nomological fact (as the primitive ontologists claim); but also 3) the wave function is defined in terms of the functions it plays in the theory. While in the first two cases the wavefunction is ontic strictly speaking (being representational), in the third option the wavefunction is non-representational, but nonetheless it is non-epistemic. This fact alone allows to bypass the PBR theorem.

[33] See e.g. Jansson (2020), Lewis (2020) and references therein for criticisms, and Healey (2020) for a defense. See also Brown and Timpson (2006) for a criticism of the information-theoretic interpretation as not explanatory.

As I have discussed above, the issue is to find an effective way of thinking about the wavefunction as *non-representational, non-epistemic, but also constructively explanatory*. The nomological view helps with the explanatory part, but it does not provide a non-representational wavefunction, while pragmatist quantum realism helps with that, but it has the wrong explanatory framework. So, my proposal is a blended view, which suitably combines some ingredients of the approaches discussed in the previous section as to provide several advantages over each of them, while preserving their core motivations. I call my approach *wave-functionalism* because it defines the wavefunction in terms of its roles in the theory: the wavefunction is whatever functions it plays.
In this section I articulate this view. In the first subsection I briefly review functionalism in general, and sketch how one can use a similar strategy in the case of the wavefunction. Then, in Section 3.2 I discuss the roles the wavefunction in quantum theory. In the last two subsections I show how, by focusing on these roles rather than trying to define what the wavefunction represents, I can avoid the main obstacles of the approaches discussed above.

3.1 Functionalism: The Wavefunction is as the Wavefunction Does

Functionalism, broadly speaking, is the view that certain entities can be defined in terms of their functional roles. To use a powerful slogan, 'the table is as the table does.'[34] Strategies with a functionalist core have been used in the philosophy of mind for a long time.[35] In this context, by defining mental states in term of their functional roles, they can be realized by different physical systems. Mental states are reduced to their functions or may be thought of as non-fundamental but suitably emergent from physical states.

Recently functionalist approaches have been used in philosophy of physics as well. For instance, Knox (2013, 2014) argues that spacetime can be functionalized in the classical theory of gravity. That is, spacetime can be thought of as non-fundamental (emergent) in virtue of the fact that spacetime plays the role of defining inertial frames. Moreover Albert (2015) defends wavefunction realism using his own brand of functionalism. He argues that ordinary three-dimensional objects emerge by functional reduction: they are functionalized in terms of their causal roles, and the wavefunction dynamically uses these relations to give rise to the empirical evidence.[36] Lam and Wüthrich (2018) use functionalist strategies in quantum gravity. A common feature of the different approaches to quantum gravity is that spacetime is not fundamental but it is

[34] Or, as Forrest Gump's mother used to say: "Stupid is as stupid does."
[35] Starting from Putnam (1960).
[36] Ney (forthcoming) provides a criticism and discusses her own alternative.

functionally emergent from non-spatiotemporal structures.[37] Lam and Wüthrich argue that, in order for spacetime to emerge, it is sufficient to recover only those features which are functionally relevant in producing observations. Several objections have been raised against this view,[38] one of which is common with wavefunction realism, namely the problem of empirical incoherence. A theory is said to be empirically incoherent in case its truth undermines our empirical justification for believing it to be true. Arguably, any theory gets confirmation by spatiotemporal observations. However, a theory which rejects spacetime as fundamental entails that its fundamental entities are not spatiotemporal. Because of this, our observations are not observations at all, and thus provide no evidence for the theory in the first place. [39]

My idea, on which wave-functionalism is based, is to use functionalist strategies not applied to space, time, or spacetime, but to the wavefunction. In the rest of this section I am going to argue that my view is compatible with the non-material approaches to the wavefunction, such as the primitive ontology program, the epistemic interpretations, and pragmatist quantum realism. In particular, pragmatist quantum realism can be seen as having functionalist components: the wavefunction is understood in terms of the function it plays in the theory, namely prescribing the experimenter's expectations in a given experimental situation. Wave-functionalism is more general than that, as it is compatible with a more 'robust' type of explanation: as I elaborate below, I identify other roles for the wavefunction which interconnect with one another as to providing a constructive (dynamical and compositional) explanation able to giving us the reasons why a given phenomenon happens.

To sum up, along the lines of pragmatist quantum realism, I start from the idea the wavefunction is not to be understood in terms of what it represents. We should look neither for material facts (like wavefunction realists propose), nor for nomological facts (*contra* the primitive ontology approach). Nor we should look at the wavefunction as describing incomplete knowledge (in contrast with epistemic interpretations). Rather, the wavefunction is an objective ingredient of the theory, which is functionally defined in terms of the roles it plays. I show that by functionalizing the wavefunction in this way one can capture the intuitions that the wavefunction is law-like (as in nomological approaches), avoiding the problems of time-dependence and non-uniqueness. Also, one

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[37] For a review of spacetime emergence in quantum gravity see e.g., Huggett and Wüthrich (2013, forthcoming), Crowther, (2016), Le Bihan and Linnemann (2019), and references therein.
[38] For a discussion on the role of functionalism in the emergence of spacetime, see Le Bihan (forthcoming). See also Lam and Esfeld (2013), and Yates (forthcoming).
[39] Barrett (1999), Maudlin (2007) and Ney (2015) discuss the issue of empirical incoherence in the framework of quantum mechanics, while Huggett and Wüthrich (2013) address it in quantum gravity. For related considerations, see also Healey (2002).

can understand how the wavefunction can be non-material (as in epistemic approaches) but ontic, therefore bypassing no-go theorems. Then, one can make the wavefunction non-representational (like the pragmatists) without being committed to thinking of it as merely predicting expectations.

3.2 The Functions of the Wavefunction

So, what are the functions of the wavefunction which one may use to functionalize it? I think that one can identify the following different but interconnected roles which however all contribute to the same goal: *to effectively reproduce the empirical data in an explanatory way.*

First and foremost, there is an *empirical role*: the wavefunction contributes to providing whatever is needed to adequately recover the measurement results. Notice that this does not mean necessarily that one has to give a special status to measurement processes, experimental outcomes, or observers, like epistemic interpretations do. In fact, this empirical role merely refers to the fact that the wavefunction is one of the ingredients which allow to make predictions and to provide explanations. If one thinks of a theory as a black box which produces empirical predictions, the wavefunction is part of the machinery inside of the box that produces these predictions. This is what the wavefunction does also in the primitive ontology approach and in pragmatist quantum realism, neither of which is, strictly speaking, empiricist in this way. In executing this role, the wavefunction performs *a practical role in a pragmatic way*. In fact, the wavefunction generates the probability outcomes by being part of the Born rule, but one could choose another object for this role. For instance, one could generate the same probability distributions using another wavefunction which differs from the first by a phase.[40] Or, one could express the Born rule in terms of the density matrix. Or, more exotically, consider theories like the pilot-wave theory and the spontaneous collapse theory, which are originally defined respectively in terms of a Schrödinger-evolving wavefunction and a wavefunction which spontaneously and randomly collapses. One can show that it is possible to write two theories, respectively empirically equivalent to the pilot-wave theory and the spontaneous collapse theory, in terms of a 'collapsing' wavefunction and a Schrödinger evolving one.[41] For all practical and explanatory purposes, they are equivalent reformulations of the same theory, as they produce the very same empirical predictions, and explain the phenomena in the same way. The fact that quantum mechanics is formulated in terms of a Schrödinger-evolving wavefunction is purely pragmatic, as it is contingent to other super-empirical

[40] Indeed, the wavefunction is as a ray in Hilbert space, namely an equivalence class of objects which differ from a phase, because each element in the class generates the same empirical results.
[41] Allori, Goldstein, Tumulka, and Zanghì (2008).

considerations like for example simplicity and explanatory power: it is the simplest and most explanatory choice one could have made.

This empirical role alone is enough to guarantee that the wavefunction is able to explain why experiments are justified in believing in a set of experimental outcomes. That is, this empirical role is the only one needed in epistemic and pragmatist approaches. However, if one wishes to have an explanation of why these results obtain in terms of a constructive theory, as the primitive ontologist would (see subsection 2.2), one needs to identify other roles for the wavefunction. For one, the wavefunction has a *nomological role*. This is straightforward for the primitive ontology approach, in which the wavefunction governs the motion of the primitive ontology.[42] In the case of the epistemic approaches the dynamics does not plays much of a role, but the wavefunction can be seen as nomological in the sense that it takes part in the definition of the principles which characterize which phenomena actually happen. Similarly, according to the pragmatists, the nomological role is to provide 'information bridges' which describe the phenomena by concisely summarizing what happens.[43]

As the discussion above shows, by playing this nomological role, the wavefunction also executes its *explanatory role*: it is a necessary ingredient to account for the experimental results. In what way the wavefunction explains, whether constructively or in terms of principles depends on the type framework (see again subsection 2.2). In the primitive ontology approach the wavefunction is part of the ingredients necessary to define the dynamics for the primitive ontology, and in this way accounts for the empirical data compositionally and dynamically, as macroscopically produced by the 'trajectories' of the microscopic primitive ontology. That is, the explanation is constructive. In the epistemic and pragmatist interpretations the wavefunction provides the statistics of the measurement outcomes *via* the Born rule, therefore explaining the probabilistic data by imposing kinematical constraints on the possible phenomena, and by therefore predicting what an experimenter should expect. Accordingly, they are principle theories.

These roles combine together to define the wavefunction *functionally*: the wavefunction is whatever functions it plays in the theory. This is similar to the functionalist approaches discussed at the beginning of subsection 3.1: in order to account for observations, it is sufficient to recover only those features which are functionally relevant in producing the empirical evidence in an explanatory manner. As anticipated,

[42] In the context of the pilot-wave theory, the wavefunction can be taken as a force or a potential, but one does not think of it as material either (even if Belousek 2003 argues otherwise).
[43] Healey (2017a).

this can be done either constructively, in terms of the dynamics of the primitive ontology, or using Hilbert space principles or quantum rules to systematize the phenomena.

3.3 Further Characterizations

As we just saw, wave-functionalism captures many of the motivations of the interpretations in which the wavefunction is considered non-material. In this subsection I further explore the view, and in the next subsection I show how wave-functionalism does not fall prey of the major objections of the alternative views. Before doing that, let me make a couple of remarks.

First, in this view the wavefunction is an *ingredient in the law,* rather than a law itself. Depending on the approach considered, laws of nature systematize the phenomena, prescribe matter how to move, and/or tell experimenters what to expect. When we write down laws in physics books, we use different symbols and mathematical entities. Some of these symbols describe constants of nature. Some others are variables, some of which represent masses and charges, for instance, and some others represent potentials. Among these variables there is also the wavefunction: as such, it is one of the ingredients a law must possess in order to 'carry out' the tasks mentioned above. Just like constants and potentials, the wavefunction is a part of the law: it is one of the pieces, one of the ingredients present in the law. To use a not entirely adequate analogy, one could say that a natural law is similar to a birthday cake. The function of a birthday cake is to make the birthday boy or girl happy, and to fulfil its role the cake needs its ingredients, each of which has its own function. Sugar is to sweeten, while flour, butter and egg serve to provide structure to the compound and to amalgamate it. Similarly, a law has the role of governing the motion of physical bodies and/or systematizing the phenomena. To do that, it needs ingredients. Each ingredient has its function, and they all combine to allow the law to fulfil its role. The wavefunction looks more similar to potentials than to any other ingredients in the law, in the fact that they both conveniently express the interaction and the motion of matter.[44] This is the sense in which the wavefunction is part of the law: it encodes part of what is needed to fulfil its nomological role. In a language that is more common in functionalist approaches, one may say that the wavefunction *partially realizes* the law.

Second, the wavefunction is *only one of the possible realizers* of the functions mentioned in the previous section. So, there is a sense in which I should say not that the wavefunction is functionalized, but rather the ingredient of the law the wavefunction realizes is

[44] Indeed, at least initially, the pilot-wave theory was described in terms of the quantum potential, containing the wavefunction, and acting on matter, made of particles, as another interaction.

functionalized. To understand, continue with the cake analogy. Some ingredients in the cake may not be indispensable, in the sense that their function may be multiply realized: white sugar can be substituted by cane sugar; flour with starch, butter with margarine, eggs with just yolk. Likewise, the different ingredients in the law, among which there is the wavefunction, may be multiply realized: one could use other mathematical objects and still realize the same functions. In classical physics one can write Newton's equation in terms of the gravitational field, or in terms of the gravitational potential: they fulfil the same role. Similarly, in quantum theory one could write the Born rule using this wavefunction, or the same wavefunction shifted by a phase, or a density matrix, or whatnot, since all these objects realize the same functions.

Finally, *wave-functionalism is not reductionist* in the sense that the realizers (the wavefunction and the other ingredients in the law) needs to be specified over and above the distribution of matter in the world. This is equivalent to say that God, at the beginning of time, created the world, including its laws. The laws contain the necessary ingredients that allows them to play their nomological role, whatever one may think it is. One way of conveniently realize the various functions is in terms of the wavefunction, but that is not the only way, as we have seen.

3.4 Advantages over the Alternatives

Now let me consider how my view solves the problems of the alternative approaches about the nature of the wavefunction, starting from the nomological approach. In wave-functionalism, as we have just seen, the wavefunction is an ingredient of the law. Because of that, unlike in the case of the nomological approach which considers the wavefunction itself as a law of nature, here the wavefunction does not have to be unique, like potentials are not unique: as a potential is an equivalence class of objects which differ by a constant, the wavefunction is an equivalence class of objects which differ by a phase. Since they both give rise to the same phenomena (macroscopic and microscopic), they are equivalent, for all practical and explanatory purposes. Moreover, in wave-functionalism it does not matter whether the wavefunction evolves in time or not: this is a problem only if one thinks the wavefunction in itself is a law, while in this case it is merely part of the mechanism that generates what we observe and which we want to explain. As in classical physics sometimes we need time-dependent potentials, in quantum theory we need a wavefunction which evolves in time. Thus, to sum up, the wavefunction plays a nomological role but it is not itself a law, therefore avoiding the objections of the nomological view.

Wave-functionalism shares with the epistemic view the idea that the wavefunction does not represent matter. However, in my view the wavefunction is not epistemic: it does not represent the incomplete knowledge of a system. Rather, the wavefunction is functionally defined in terms of the roles it plays in the theory, mainly to recover the experimental data in an efficient and explanatory manner. As such, the wavefunction is objective, as it exists independently of the existence of observers and their minds. As a consequence, the wavefunction is ontic, and wave-functionalism does not fall prey of the PBR theorem.

Then, wave-functionalism overcomes the challenge, faced by pragmatist quantum realism, of not being sufficiently explanatory. The problem was that both pragmatism and epistemic views are explanatory only in a weak sense, as they account for why experimenters are justified in expecting some phenomenon without spelling out why the phenomenon actually takes place. Conversely, if someone wishes to know what caused the phenomenon, or what grounds it, or what is the mechanics that produces it, they will be left unsatisfied by these approaches. Instead wave-functionalism is more general because it is not committed to this conception of explanation. In fact, one could understand the explanatory role of the wavefunction dynamically and compositionally, in terms of a constructive explanation, as the primitive ontologists do: the wavefunction is part of the dynamical law which governs the microscopic entities and reproduces the macroscopic observed behavior. In fact, consider for example the interference-diffraction pattern produced in a two-slit experiment. One can explain it in two ways. First, one can say that the dots are distributed the way they are because they follow the pattern dictated by the square module of the wavefunction. This is compatible with wave-functionalism and it is a principle theory explanation: the wavefunction plays the role of constraining the observed phenomena and tells us where to expect the dots. This is also what the pragmatist would say. Moreover, within wave-functionalism, one can also say that the reason why the dots are where they are is because the wavefunction is part of the law governing the motion of matter, as the primitive ontologists would say. In other words, wave-functionalism is more flexible than pragmatist quantum realism because while pragmatist quantum realism is compatible with only the principle type of explanation, wave-function realism is compatible with both, as it is also able to accommodate constructive explanations.[45]

As a side remark, let me notice that wave-functionalism also avoids the main problems of wavefunction realism, namely the configuration space problem and the problem of empirical incoherence. In fact, both problems arise when there is a mismatch between the space of our observations and the fundamental space postulated by the theory, like in wavefunction realism and quantum gravity. However, since in wave-functionalism the wavefunction is not representational, there is never a reason to introduce a

[45] See also subsection 3.5, objection 4.

fundamental space which is different from ordinary three-dimensional space. Hence, these problems never arise.

3.5 Possible Objections and Replies

In this section I anticipate some objections to wave-functionalism. Some of them stem from misunderstandings of my position, so the replies I offer may also serve to clarify the view.

1. *Is the wavefunction functionally reduced? What are its functional realizers?*

Elaboration. Functionalist approaches are typically reductionist about the object with gets functionalized. For instance, mental states do not exist over and above physical states. In Albert's functionalism, three-dimensional objects are not postulated to exist over and above the wavefunction. In wave-functionalism the wavefunction is reduced to its roles. Does that mean that the wavefunction does not exist over and above the distribution of matter? Or, in the case of epistemic or pragmatist interpretations, over and above the distribution of the empirical outcomes or the experimenter's expectations? That seems wrong: one needs to postulate the wavefunction over and above them.
Moreover, by focusing on the functions and not the representation, functionalism in the philosophy of mind allows mental states may be multiply realized. For instance, they are realized in humans by brain states, while they could be realized by positronic circuits in Commander Data. In the case of wavefunction, what are the functional realizers?

Reply:
Functionalism does not need to entail reduction, as it is consistent also with elimination, and dualism about the realizers. Wave-functionalism is not reductionist, as the wavefunction (or any other realizer) has to be specified in addition to the material ontology, and this usually done specifying the laws.
As discussed in subsection 3.3, what is functionalized is the ingredient in the law which the wavefunction happens to realize. That is, the wavefunction is one of the possible realizers, another realizer could be a wavefunction with a different phase, or a density matrix.

2. *Why do we need this view in addition to the primitive ontology approach, pragmatism and epistemicism?*

Elaboration. Wave-functionalism is seen as an alternative to these various proposals. But why do we even need this alternative? In all likelihood, primitive ontologists will see no need for it, as they believe that the problems to the nomological view are not serious.

Similarly, the proponents of the epistemic and pragmatist interpretations have replies to their respective challenges: they have put forward ways out from the no-go theorems and advocate that principle theories provide satisfactory explanations. So, who needs this view?

Reply:
It is indeed very possible that purists of these approaches may not be moved by wave-functionalism, as they may insist that the problems of their views are less severe than I have made them. Likewise, consider an empiricist, or someone who cares about accurately reproducing the data without feeling the necessity of specifying some microscopic mechanisms to constructively explain these data (like the epistemicists), but who is bothered by the epistemic character of the wavefunction typical of these approaches. They can become pragmatists, and, since they are not preoccupied by the type of explanation proposed by principle theories, they will find little interest in wave-functionalism. However, my purpose is to explore whether one could 'have it all,' so to speak. For example, if someone is moved by the objections to the nomological view but they do not want to move to wavefunction realism (because they find the configuration space problem to be serious), or to epistemicism (because they also find the no-go theorem to be challenging), or to pragmatism (because they would like to have the constructive explanations), what options have they got left? Wave-functionalism is proposed for people like this. Wave-functionalism relaxes the original ontic nomological view, which is still representational, to make it functionally non-representational, preserving its constructive explanation. Also, consider someone who thinks that both primitive ontologists and empiricists are wrong in thinking of the wavefunction as representing something (as the pragmatists), but they are bothered by the type of explanation principle theories provide. Wave-functionalism gives them an option: the wavefunction is functionally defined, and therefore it is non-representational, but being part of the law and of the constructive, dynamic explanatory schema there is no worry about explanation.

3. *What does it mean that the wavefunction is a part of the law?*
Elaboration. Laws do not have parts or ingredients, laws are laws: they either govern the phenomena or they describe them in a Humean fashion or are used to constrain the phenomena. The notion of parthood as applied to laws is mysterious. Therefore, the approach trades one mystery (the wavefunction is a time-dependent, non-unique law) for another (the wavefunction is part of the law), hiding all the problems into a unacceptably vague terminology: 'part of the law.'

Reply:

Laws of nature are what they are, but when we write them down in physics books, we use different symbols and mathematical entities, including constants of nature, masses, charges, the wavefunction, and the potentials. All these ingredients are necessary for the law to play its role, which is understood differently by the various proposals. As emphasized above, the ingredient realized by the wavefunction could be realized by other mathematical objects. Therefore, in a nutshell, to say that the wavefunction is part of the law means that it is one of the possible realizers of certain functions which appear in the law.

Follow up:
This seems to imply that whenever one writes the law in terms of different realizers or different ingredients, one has a new law, and thus a new theory. For instance, the pilot-wave theory written in terms of the guidance equation and the pilot-wave theory written in terms of the quantum potential would come out as different theories because they have different ingredients in the law. This seems wrong.

Reply:
They are not different theories because they do not have different laws. The two examples express two different ways of writing the same law in terms of different realizers. Even if one writes the law in terms of different realizers, the law is still the same, if its content remains untouched. That is, the law is still the same as long as it generates the same behavior for matter, systematizes the phenomena in the same way, or prescribes agents to have certain beliefs. As already mentioned, the law is what it is, but one can write it differently: one can write Newton's equation with a given potential, and also with the same potential with an additional constant. The (partial) nomological content of the wavefunction can be encoded in the quantum potential, or in something else. What is essential is what the wavefunction encodes or realizes, rather than the fact that it is encoded or realized in the wavefunction, as opposed in something else. One does not need the wavefunction in itself: it is just one of the possible realizers of the various functions. This should not sound particularly strange from a Humean perspective, because in this view laws are merely convenient summaries which God would write on a T-shirt for our benefit. Some Humean approaches do not seem committed to reifying the wavefunction or any other symbol or variable in the laws, and this is exactly what I am proposing here.[46]

[46] I have in mind Esfeld's super-Humeanism (2014), or Bhogal and Perry's version of Humeanism (2017). They relax the condition, proper of the more traditional Lewisian Humeans, that fundamental laws should mention only perfectly natural properties in the Humean mosaic. Lewisians instead may resist this view. In fact, their best system would include the Schrödinger equation as an axiom, and this would mention the wavefunction, which therefore should be part of the Humean mosaic.
Thank you for an anonymous reviewer for pressing me on this point.

4. *How can wave-functionalism be explanatory if the wavefunction does not materially exist?*
Elaboration. In the history of science, people have postulated the existence of new entities to explain new data: neutrinos, electromagnetic fields, dark matter, and so on. Arguably, the wavefunction was introduced in quantum theory for the same reason: to explain, say, the double-slit experiment. However, if wave-functionalism claims that the wavefunction does not represent anything in the world, how is it possible that it is able to explain the phenomena?

Reply:
Pragmatist quantum realists face the same challenge, and their defenders respond that they have a different idea about what explanation requires: this interpretation does not explain by providing a description of the physical world but rather by providing reasons to expect certain experimental data. In any case, I do not see the reason to think that all the ingredients in an explanation needs to be physically real. Historically, electromagnetic fields were initially considered to be useful bookkeeping devices, without taking them as material fields. One could have raised the very same objection in that context, but instead no one complained that classical electrodynamics was not explanatory. I believe they thought that theory was nonetheless explanatory because, even if the electromagnetic fields were understood as mere fictions, they were still carrying a piece of the relevant nomological information needed to the law to generate the empirically adequate behavior. In other words, people were realist about laws, or nomological facts, rather than about their individual ingredients. This is what wave-functionalism proposes: it is in virtue of the fact that the law exists, rather than the wavefunction in particular, that the view is explanatory.

5. *How can wave-functionalism solve the configuration space problem if it postulates nomological facts in configuration space?*
Elaboration. Think about the double-slit experiment. The wavefunction realist has a ready explanation: there is a physical wave crossing both slits which therefore creates two waves which generate the interference and diffraction pattern on the screen. The epistemicist has more problems: if the wavefunction represent incomplete knowledge of the system, how is it that it generates a physical phenomenon? Wave-functionalism could explain two-slit interference in terms of the wavefunction though as a potential, but it would have to be defined over configuration space. If so, it looks like one has to privilege configuration space as the appropriate arena for that law to operate in. If part of the motivation for wave-functionalism is to avoid appeals to configuration space, then the view seems to fail.

Reply:

I think it is the main innovation of quantum mechanics that configuration space has to enter one way or another, and this is because quantum nonlocality seems to be a fact of nature. So, everyone will have to deal with this. Wave-functionalism says that there are non-local nomological facts encoded in, or realized by, the wavefunction. This is not ideal, but it does not seem to be as problematical as saying that there is a physical ontology in configuration space. In fact, I perceive material bodies to be in three-dimensional space more directly that the way I 'perceive' laws: I only see their indirect effect through the motion of the material bodies. So, I seem more justified in saying that the material bodies are three-dimensional than in saying that the law is. Therefore, it seems more problematical to say that these bodies are in configuration space, rather than to say that the law is. Moreover, especially from a Humean perspective, it is easier to make sense of the law being in configuration space, rather than physical objects. In fact, a law in configuration space is one in which the interaction is more complex and interconnected than originally thought. Instead, as the various approaches developed by wavefunction realists have shown, it is much more convoluted to spell out what it means for a physical body to 'vibrate' in configuration space. Because of this I think that, since everyone has to deal with configuration space one way or the other, wave-functionalism, by inserting it in the law rather than in the material ontology, choses the best route compatible with the nonlocality of quantum theory.

4 Conclusion

Wave-functionalism, the view proposed in this paper, claims that the wavefunction is defined functionally: the wavefunction is whatever functions it plays in the theory. These are the several distinctive but connected roles the wavefunction may play in the theory:

1) An *empirical* role, in that the wavefunction is an essential ingredient in reproducing the experimental results;
2) A *nomological* role, in that the wavefunction is one of the ingredients to be specified in the law;
3) An *explanatory* role, in that the wavefunction plays a crucial part in the compositional, dynamical explanatory mechanism of the theory.

I have argued that in my account the wavefunction, by being *functional*, may be thought as:

1) *Non-epistemic* (as the epistemic interpretations) *without being material* (in contrast with wavefunction realism) or *nomological* (in contrast with the primitive ontology approach);
2) *Non-material without being representational* (as pragmatists quantum realism);
3) *Nomological without being a law* (in contrast with the primitive ontology approach);

4) *Constructively explanatory* (as the primitive ontology approach).

By ensuring all these features, wave-functionalism is able to avoid the major objections of the competing approaches. In fact:

1) By having a *nomological role without being a law* (that is, given that the wavefunction is only one possible realizer), it does not matter whether the wavefunction is time-evolving, or unique;
2) By having an *explanatory role tied to laws of nature*, the wavefunction does not have to be wedded to the explanation provided by principle theories;
3) By being *non-representational but also non-epistemic*, no-go theorems are bypassed.

For these reasons, wave-functionalism provides a general understanding of the wavefunction that could be adopted by those who have reasons to think that the wavefunction does not represent material objects, but also care about an objective, constructive, dynamical and compositional explanation of the phenomena.

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