

Working Backwards with Copi's Inference Rules*

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Copi and Cohen suggest that students construct a formal proof by "working backwards" from the conclusion by looking for some statement or statements from which it can be deduced and then trying to deduce those intermediate statements from the premises.ⁱ What follows is an elaboration of this suggestion. I describe an almost mechanical procedure for determining from which statement(s) the conclusion can be deduced and the rules by which the required inferences can be made. This method is designed to forestall the quandary in which many beginners find themselves: not knowing how to get started.

We begin by dividing the nine inference rules into two categories: the detachment rules—Modus Ponens (MP), Disjunctive Syllogism (DS) and Simplification (Simp)ⁱⁱ—and the "construction rules," all the rest. Following this division, we specify consequents, right disjuncts, and left conjuncts as the "detachable part," that is, the elements of the truth-functionally compound statements of which our arguments are composed that may, under specified circumstances, be written as new lines in a proof.

The construction rules are employed to "construct" statements of various sorts: Constructive Dilemma (CD) and Addition (Add) to form disjunctions, Hypothetical Syllogism (HS) and Absorption (Abs) to form conditionals, Modus Tollens (MT) to form negations, and Conjunction (Conj) to form conjunctions. One may further distinguish between different ways of forming a conditional: Abs is used when one wants to construct out of *one* conditional, a conditional whose consequent is a conjunction (of the antecedent and consequent of the original conditional); HS, on the other hand, is used to construct a conditional when one has *two* conditionals, one of which has the antecedent of the required conditional as its antecedent, and the other of which has the required conditional's consequent as its consequent, the two conditionals being "linked" in virtue of the conditional supplying the antecedent having as its consequent the same formula as the conditional supplying the consequent has as its antecedent. We may also distinguish between the ways in which the two construction rules for forming disjunctions are employed: Add is used in those situations in which the left disjunct of the disjunction one needs to derive is already on a line by itself, that is, is a previous line in the proof (i.e., is one of the premises or a line derived therefrom); CD, on the other hand, is used to derive a disjunction if the disjunction is made up of the consequents of conjoined (or conjoinable) conditionals whose antecedents are disjoined on a line (or are disjoinable on a line).

Having described the situations in which each rule is applicable, an instructor can proceed to explain in terms of these situations how to work backwards in constructing a formal proof. The basic idea is to determine by *process of elimination* how one is to derive the conclusion and, if necessary, the other formulas one "needs" in order to apply the rule that will yield the conclusion. One will have worked backwards far enough just in case the formula that one needs is either given as a premise or obtainable either by applying Simp, Abs. or Add (the rules applicable to one line).

Here is an example of how to develop a proof "strategy." Consider the following argument:

1. $(Z \cdot A) \supset (B \cdot C)$
2. $Z \supset A / \therefore Z \supset (B \cdot C)$ ⁱⁱⁱ

We begin by asking of the conclusion (the formula that one ultimately needs on a line) whether or not it is a detachable part.^{iv} It is not. (The two detachable parts of the premises are the two consequents, neither one of which is the conclusion.) We can rule out, then, as ways of deriving the conclusion MP, DS, and Simp, i.e., all of detachment rules. We thus

proceed to the construction rules. There are two ways to construct a conditional, HS and Abs. Determining which one ought to be used in the present circumstances requires noticing that there is no way to derive a conditional of the conclusion's form — $p \supset (q \cdot r)$ — by using Abs. Thus, one is left knowing that the last move in the construction of the proof will be HS. One should then note that one needs two conditionals to apply HS in such a way as to derive the conclusion: $Z \supset q, q \supset (B \cdot C)$

Trial and error may be required here to determine that it is $(Z \cdot A) \supset (B \cdot C)$ that should play the role of $q \supset r$ in this context. Having determined this, however, one should note that one needs $Z \supset (Z \cdot A)$ in order to do the required HS. If one doesn't see straight off that this conditional is obtainable by applying Abs to 2, one can make this determination by repeating the process of asking whether or not it's a detachable part and, if not, which construction rule(s) (in being applied) yield conditionals (and then noticing that HS can't be applied to the given conditionals), leaving Abs as the only candidate. If one still doesn't see that it is 2 to which Abs ought to be applied, one should just go ahead and apply it to both lines and then consider which "production" can be used to do the required HS. In condensed form, the strategy for the construction of the argument's proof would look like this:

Need: $Z \supset (B \cdot C)$

Use: HS on $Z \supset q + q \supset (B \cdot C)$

Need: $Z \supset q$ (assuming that $(Z \cdot A) \supset (B \cdot C)$ plays the role of $q \supset (B \cdot C)$)

Use: Abs on 2

Here is another example of how to apply this method:

1. $(H \supset I) \cdot (J \supset K)$

2. $K \vee H$

3. $\sim K / \therefore I \vee$

Need: I

Use: MP on $H \supset I$ (Here the formula I need *isn't* a detachable part of a line. Rather it is a detachable part of a detachable part. Thus, I need to acquire not only the conditional's antecedent as in applications of MP in which the conditional is by itself on a line. I need to also derive the conditional itself.)

Need: $H \supset I, H$

Use: (for $H \supset I$) Simp on 1

Need: H

Use: DS on 2

Need: $\sim K$ (given as a premise so one may now begin the proof)

4. H 2, 3, DS

5. $H \supset I$ 1, Simp

6. I 4, 5, MP

Though the proofs are longer in parts V and VI, there is no problem on which this method won't work. One is simply to determine the rule one is to apply to derive the conclusion and the line(s) one needs in order to make that move. This determination is made by the process of elimination, by asking a series of questions until an answer gives the appropriate rule. Things do not proceed as smoothly once Copi's system is expanded to include the Rules of Replacement. Nevertheless, the method is still helpful, since one can consider the new rules as additional construction rules and transform the conclusion into a

logically equivalent formula in the event that one cannot see how to derive it in the form in which it is presented in the argument.

In practice, the procedure has the effect of helping students associate each rule with the situations in which it is applicable, the inability to make this connection being a major impediment to learning how to construct a formal proof.

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ⁱ Irving Copi and Carl Cohen, *Introduction to Logic*, 9th edition (New York: Macmillan Publishing), p. 391. My suggestions are intended to be helpful to students who have completed the exercises designed to familiarize one with instances of each one of the Rules of Inference.

ⁱⁱ Here I employ Copi's abbreviations for the names of the rules, given on pp. 378-379.

ⁱⁱⁱ Given on p. 382 of Copi.

^{iv} Or: Is it a detachable part of a detachable part?

^v Copi, p. 383