Artif Intell Law (2007) 15:429–447
DOI 10.1007/s10506-007-9050-4

ORIGINAL PAPER

# Formal models of coherence and legal epistemology

**Amalia Amaya**

Received: 28 October 2006 / Accepted: 29 March 2007 / Published online: 17 May 2007


**Abstract** This paper argues that formal models of coherence are useful for constructing a legal epistemology. Two main formal approaches to coherence are examined: coherence-based models of belief revision and the theory of coherence as constraint satisfaction. It is shown that these approaches shed light on central aspects of a coherentist legal epistemology, such as the concept of coherence, the dynamics of coherentist justification in law, and the mechanisms whereby coherence may be built in the course of legal decision-making.

**Keywords** Legal epistemology · Coherence · Belief revision · Constraint satisfaction

## 1 Introduction

‘‘‘Coherence’ is not something that we have an algorithm for, but something that we ultimately judge by ‘seat on the pants’ feel’’—wrote Putnam.[1] That coherence resists formalization is also a claim that some legal coherentists—like Wintgens[2]—are willing to endorse. Yet, given the relevance of coherence to legal justification, it is crucial to give a precise account of the notion of coherence and of the inner workings of coherentist justification.[3] In this paper, I would like to examine some

[1] Putnam (1981, pp. 132–133).

[2] Wintgens (2007, forthcoming).

[3] Coherence theories of legal justification have been very popular in the last decades. While few legal scholars would endorse an account of legal justification in terms of coherence, the view that coherence is at least an important ingredient of legal justification is widely shared in contemporary legal theory. For a discussion of the current state of the coherence theory in law, see Amaya (2006, chapters 1 and 2).

A. Amaya (✉)
Harvard Law School, Harvard University, 1557 Massachusetts Avenue, Cambridge, MA 02138, USA
e-mail: aamayana@law.harvard.edu

formal approaches to the notion of coherence. More specifically, I will examine formal approaches to coherence-based theories of belief revision and theory change. I shall argue that, despite claims to the contrary, these approaches do illuminate important aspects of legal coherentism, even if—to be sure—no formal account to date has the resources to fully capture the complexities of coherence-based legal reasoning.

The structure of this paper is as follows. In Sect. 2, I provide an overview of the main theory of belief revision, that is, the AGM model, and argue that a coherentist interpretation of this theory is inadequate for several reasons. Nonetheless, AGM provides the basic framework within which other models of belief change that have the resources to model coherentist justification have been developed. In Sect. 3, I introduce one such model, by Olsson, which is the most detailed proposal as to how belief revision theory may be put to work to formalize a coherentist epistemology. Section 4 discusses the potential of belief revision formalisms for modeling legal coherentism. A different formal approach to coherence, coming not from the realm of philosophical logic but rather from the field of computational philosophy of science, may be found in the work of Paul Thagard, which is discussed in Sect. 5. I conclude this paper with some general remarks on the relevance of formal theories of coherence to non-formal analyses of coherence and justification in law.

## 2 The AGM theory: a coherentist interpretation

Formal approaches to the notion of coherence have been developed in the area of belief revision. Belief revision is an extensive and growing (both in quantity and sophistication) body of literature which aims at formalizing what is involved in rational belief change.[4] The most influential of the belief revision formalisms is the AGM model, so called after its three originators Carlos Alchourrón, Peter Gärdenfors, and David Makinson.[5] In this model, the beliefs held by an individual are represented by 'belief sets', where a belief set is a set of sentences that is closed under logical consequence. This model distinguishes between three types of belief revision operations, i.e., 'contraction', 'revision', and 'expansion'. Expansion consists in adding a sentence $p$ (together with its logical consequences) to the belief set $K$. The expansion of $K$ by $p$ is denoted $K + p$. In contraction, a specified sentence $p$ is retracted. The result of contracting a belief set $K$ with respect to $p$ is denoted $K \div p$. Last, in revisions, a new sentence that is inconsistent with a belief set $K$ is added. In this case, in order to maintain consistency in the resulting belief system, it is necessary to remove some of the old sentences in $K$. The new belief set that results from revising $K$ by $p$ is denoted $K * p$.

The main objective of AGM is to provide formal representations of these three types of changes. In order to do so, AGM follows two complementary strategies, namely, it formulates a number of rationality postulates (largely motivated by a

[4] For a clear and brief introduction to belief revision formalisms, see Gärdenfors (1992) and Gärdenfors and Rott (1995). A more in-depth introduction is provided by Hansson (1999a).

[5] Alchourrón et al. (1985).

principle of informational economy) that should be satisfied by the appropriate belief changes and it presents explicit constructions of these operations. One of the central results of AGM is a set of representation theorems which connect both approaches by showing that the rationality postulates indeed characterize the proposed operations of belief revision. Let us briefly see how AGM formalizes these three operations.

'Expansions' of belief systems are the easiest to handle. In AGM, the expansion of $K$ by $p$ is defined as the logical closure of $K$ together with $p$: $(K + p = \mathrm{Cn}(K \cup \{p\})$. A number of rationality postulates are proposed which uniquely characterize the expansion of $K$ by $p$ in terms of $K$ and $p$ only.[6]

'Revisions' can be seen as a composition of contractions and expansion. More precisely, in order to revise $K$ by $p$, one first contracts $K$ with respect to $p$, and then expands $K$ by $p$. This composition of operations gives rise to the following definition of a revision operator, which is called the 'Levi identity': $K * p = (K \div p) + p$. AGM also formulates a number of rationality postulates for revision.[7] These postulates, however, do not suffice to determine a revision function, but only constrain possible revision functions. The reason is that, from a logical point of view, there are several ways of specifying the revision of a belief set. Given the Levi identity, an explicit construction of a contraction function also yields a solution to the problem of constructing a revision function.

The basic function for 'contractions' in the AGM model is a 'partial meet contraction' function: $K \div p = \cap\gamma(K \perp p)$. The set $K \perp p$ is the set of maximal subsets of $K$ that do not imply $p$, whereas $\gamma$ is a selection function that selects the ''best'' elements of $K \perp p$. Thus, the outcome of the partial selection function is equal to the intersection of the set of selected elements of $K \perp p$. For contractions, the following rationality postulates are proposed:

$(K \div 1)$ For any sentence $p$ and any belief set $K$, $K \div p$ is a belief set (Closure)

$(K \div 2)$ $K \div p \subseteq K$ (Inclusion)

$(K \div 3)$ If $p \notin K$, then $K \div p = K$ (Vacuity)

$(K \div 4)$ If not $\vdash p$, then $p \notin K \div p$ (Success)

$(K \div 5)$ If $p \in K$, then $K \subseteq (K \div p) + p$ (Recovery)

$(K \div 6)$ If $\vdash p \leftrightarrow q$, then $K \div p = K \div q$ (Extensionality)

The first postulate requires that the output of a contraction function be a belief set. The second one requires that no new belief occur in $K \div p$. According to $(K \div 3)$, if $p$ is not in $K$, then the contraction of $K$ by $p$ should have no effect. The fourth postulate states that the contraction be successful, that is, that the sentence to be contracted must not be a logical consequence of the beliefs retained in $K \div p$ (unless $p$ is logically valid). $(K \div 5)$ postulates that all beliefs in $K$ are recovered after first contracting and then expanding by the same belief. The sixth postulate

[6] For the rationality postulates for expansion and representation theorems, see Gärdenfors (1988, pp. 48–52).

[7] For a statement of the postulates for revision, see Gärdenfors (1988, pp. 52–60).

ensures that the contraction of a set by logically equivalent sentences yields the same result.[8] AGM obtained a 'representation theorem' that shows that the basic postulates indeed characterize the class of partial meet contraction functions: ''For every belief set $K$, '$\div$' is a partial meet contraction function *iff* '$\div$' satisfies postulates $(K \div 1) - (K \div 6)$.''

The AGM model has been claimed by many to formalize a coherentist epistemology.[9] Three main features may seem to support this interpretation: its focus on logical structure rather than on inferential pedigree, the fact that its rationality postulates are mostly motivated by the objective of making minimal changes while maintaining the coherence of the belief set, and the lack of a formal distinction between a set of basic or foundational beliefs and a set of non-basic beliefs. However, despite appearances to the contrary, AGM does not provide an adequate framework for formalizing a coherentist epistemology for a number of reasons.

First, in the AGM model, a belief state is taken to be coherent provided that it is consistent and closed under logical consequence. This conception of coherence is highly problematic and does not correspond well with the notion of coherence that characterizes the coherentist tradition. To start with, it is more than doubtful that consistency should be taken to be a necessary condition of coherence.[10] In addition, as Olsson and Hansson have shown, the requirement of deductive closure is incompatible with basic criteria of coherence and incapable of capturing the gradational nature of this concept.[11] Second, in the AGM model, new information must always be accepted. This is guaranteed by the 'success postulate', according to which the sentence that encodes the input must be a member of the expanded or revised belief set. However, it is a common feature of coherentist theories of justification that input is not always accepted but rather evaluated in the light of the background system of beliefs. And last, AGM has a reduced scope of application in that it only formalizes changes of belief that occur when the agent receives new information that is inconsistent with her present epistemic state, that is, 'external' changes of belief. But coherence is a standard of rationality that is also relevant to 'internal' changes of belief, that is to say, to revisions that are not mediated by new input. Thus, AGM does not adequately formalize the main tenets of a coherentist epistemology. Nonetheless, it has provided the main framework for developing

[8] These six postulates are called the 'basic set' of postulates. Two further postulates for contractions with respect to conjunctions are added: 'intersection', i.e., $K \div p \cap K \div q \subseteq K \div p\&q$, and 'conjunction', i.e., If $p \notin K \div p\&q$, then $K \div p\&q \subseteq K \div p$. The former requires that the beliefs that are both in the contracted set $K \div p$ and $K \div q$, are also in the contraction of $K$ by $p$ and $q$. The last postulate expresses the idea that everything that is retained in $K \div p\&q$ is also retained in $K \div p$.

[9] The most detailed argument for a coherentist interpretation of AGM has been provided by Gärdenfors (1990)—reprinted in Gärdenfors (2005). In this work, Gärdenfors contrasts AGM with an alternative approach to belief revision, namely, Doyle's Truth Mantainance System (TMS), which, in his view, follows the foundations theory. Dolye has agreed with Gärdenfors concerning the epistemological interpretation of AGM and TMS, but argued that TMS provides the most practical means of mechanizing coherence approaches in Dolyle (1992). For a critique of the view that AGM may be plausibly interpreted as a version of coherentism, see Hansson and Olsson (1999) and Schaffer (2002).

[10] See Sect. 4 below.

[11] Hansson and Olsson (1999).

formal approaches to belief revision which are more in accordance with the coherentist view of justification. Olsson's work is the most articulated attempt to date to bring together belief revision formalisms and a coherentist epistemology.[12]

## 3 Olsson's coherentist approach to belief revision

Olsson has provided a coherentist interpretation of Hansson's theory of semi-revision. Hansson's theory is more in line with the coherentist ideas than the AGM framework.[13] First, Hansson's theory does not represent epistemic states as belief sets but rather as 'belief bases', that is, as sets of sentences that are not closed under logical consequence. Second, it is a non-prioritized model of belief revision. Non-prioritized models do not assign a special priority to new information due to its novelty, and therefore they are capable of modeling changes in which new information is not always accepted but rather weighted against old information.[14] And last, Hansson's theory has the resources to formalize not only changes of belief induced by epistemic input, but also internal to changes of belief. Hansson constructs the operation of semi-revision of a belief base $K$ by a sentence $p$ on the basis of two suboperations:

(i) Expand $K$ by $p$.
(ii) Consolidate $K$.

In the first step, the input sentence is added to the belief base. In the second step, the belief base is 'consolidated', which is the operation whereby a belief base is made consistent. In Olsson's generalized version of Hansson's theory, the requirement of consistency is replaced by a requirement of coherence. According to Olsson, coherence has two dimensions: consistency and stability. Olsson introduces a 'stability set' $S$ to serve as the formal counterpart of the concept of stability. A stability set $S$ is a set of sets of sentences, i.e., $S \subseteqq P(L)$. The idea is that the stability set contains all sets of beliefs that are epistemically stable. Hence, a given set $A$ is stable with respect to $S$ if and only if $A \in S$. More precisely, Olsson defines coherence as follows:

*Coherence*. Let $S \subseteq P(L)$. A set $A \subseteq L$ is coherent with respect to $S$ if and only if $(i) A \in S$, and (ii) $A$ is consistent.

With this concept of coherence in hand, Olsson then develops a coherentist interpretation of the main operations of Hansson's theory, i.e., consolidation and semi-revision. In Olsson's approach, unlike Hansson's, to consolidate is not to make a belief state coherent, in the narrow sense of making it consistent, but to change it

[12] See Olsson (1997a, b, 1998, 1999).

[13] Hansson (1994, 1997a).

[14] For a brief introduction to non-prioritized models of belief revision, see Hansson (1997b). Hansson provides a survey of recent research in this area in Hansson (1999b).

so that it is also stable. Perhaps, from an internal point of view, the most obvious reason to revise is the presence of some inconsistency. However, there are more subtle forms of epistemic conflict than inconsistency, for example, when one belief is very implausible given other beliefs, or when we cannot explain why some beliefs should be held true. In these cases, it would be irrational to settle with having merely consistent beliefs. Thus, argues Olsson, consolidation must be understood as the process of internally changing one's system of beliefs so as to achieve stability, in a way that cannot be reduced to consistency. Olsson's approach to consolidation differs from Hansson's in another respect. Whereas Hansson considers that a system can be consolidated only by subtracting beliefs, Olsson claims that this is only one type of consolidation. Klein and Warfield have argued that coherence can be achieved by subtracting beliefs as well as by adding beliefs.[15] Following Klein and Warfield, Olsson has distinguished between two kinds of consolidation: 'subtractive consolidation' and 'additive consolidation'.

The 'subtractive strategy' amounts to subtracting a belief (and perhaps with it many more) from an incoherent set, thereby rendering it coherent. Olsson defines the operation consolidation, which is also of a partial meet kind, as follows:

> An operation $>$ is an operation of subtractive partial meet consolidation if and only if $K> = \cap\gamma_K(K\perp^{\perp}{}_S)$

where $K\perp^{\perp}{}_S$ is the set of all maximally inclusive coherent subsets of $K$, and $\gamma$ is a function which for each $K$ selects a subset of $K\perp^{\perp}{}_S$. Thus, the subtractive consolidation of a set $K$ is the intersection of some (the best) maximally inclusive coherent subsets of $K$. The 'additive strategy' consists of adding one or more beliefs to a consistent set of beliefs in order to render it coherent. Olsson constructs a function of additive partial meet consolidation by analogy with the construction of its subtractive version except that instead of looking at maximal coherent subsets, he focuses on the minimal coherent supersets of $K$, denoted by $K \Uparrow^{\perp}{}_S$. The best supersets of $K$ are then selected by an additive selection function $\gamma_K$. Formally:

> An operation $<$ is an operation of additive partial meet consolidation if and only if $K< = \cap\gamma_K(K \Uparrow^{\perp}{}_S)$

Olsson presents a number of postulates for both subtractive and additive consolidation, and several representation theorems which show the conditions under which the proposed functions satisfy these postulates.[16]

Using the generalized partial meet consolidation, Olsson proposes a coherence interpretation of semi-revision.[17] He constructs an operation of partial meet semi-revision that parallels Hansson's definition of partial meet semi-revision in terms of partial meet consolidation except that both operations are coherence-oriented, as

[15] Klein and Warfield (1994).

[16] For a statement of the postulates and representation theorems for subtractive and additive consolidation, see Olsson (1998).

[17] Olsson (1997b).

opposed to consistency-oriented. Let $\div^{\perp}_{S}$ denote the operation of partial meet consolidation arising from using $S$ as the stability set. The generalized or 'stable' version of partial meet semi-revision can be defined as follows:

> An operation ? is an operation of partial meet semi-revision if and only if $A?\alpha = (A \cup \{\alpha\}) \div^{\perp}_{S}$.

Thus, the coherence version of an operation of partial meet semi-revision amounts to consolidating (in the sense of making consistent and stable) the set theoretical union of the original set and the new information.

Olsson presents the following postulates for generalized partial meet semi-revision:

(i) $K?\alpha$ is coherent (Coherence)
(ii) If $K?\alpha \subseteq K \cup \{\alpha\}$, then $K?\ \alpha$ is coherent (Coherence for Identity)
(iii) $K?\alpha \subseteq K \cup \{\alpha\}$ (Inclusion)
(iv) If $\beta \in K \backslash K?\alpha$, then there is some $K'$ with $K?\alpha \subseteq K' \subseteq K \cup \{\alpha\}$ such that (a) $K'$ is coherent, and (b) $K''$ is incoherent for all $K''$ such that $K' \cup \{\beta\} \subseteq K'' \subseteq K \cup \{\alpha\}$ (Strong Coherence Relevance)
(v) $(K \cup \{\alpha\})?\alpha = K?\alpha$ (Pre-Expansion)
(vi) if $\alpha$, $\beta \in K$, then $K?\alpha = K?\beta$ (Internal Exchange)

The first postulate requires that the result of semi-revision be, not merely consistent but also coherent. The second postulate says that if the semi-revision of a set is the set-theoretical union of the original set and the new information, then the semi-revised set must be coherent. The third postulate says that such result is included in the set theoretical addition of $\alpha$ to $K$. This postulate, notes Olsson, is valid only for semi-revision that uses subtractive consolidation, and not for the kind of semi-revision in which additive strategies are employed. Strong Coherence Relevance states that if a belief is given up in the semi-revision process, then it must have induced incoherence in a way that could not be remedied by considering more conservative subsets of $K \cup \{\alpha\}$. Pre-expansion says that when we semi-revise by $\alpha$, it is irrelevant whether such a sentence is already believed or not, as it should be if, as coherentism claims, the only factor that matters to justification is whether or not a belief fits together with the rest of the beliefs. Internal exchange says that the effect of considering the effect of a particular belief is the same as the effect on $K$ of considering any other belief. Olsson shows that this postulate, given the relation between consolidation and semi-revision, implies the holistic principle according to which the effect of considering one single belief is the same as the effect of considering all beliefs at once.

Olsson has shown that the operation of stable partial meet semi-revision satisfies all the foregoing postulates, except for the 'Coherence' postulate. The result after semi-revising is coherent only if the underlying stability set is intersective. A set $S$ is intersective if and only if $S$ is closed under logical intersection. For example, the stability set $S_{\text{cn}}$ is intersective: the intersection of a family of logically closed sets is also a logically closed set. Stability sets corresponding to negative coherence (i.e.,

coherence as absence of conflict) can be assumed to be intersective. For instance, if both A and B are probabilistically inconsistent, then so is A ∩ B. However, stability sets corresponding to the positive aspect of coherence (i.e., coherence as presence of support) cannot plausibly be assumed to be intersective. Thus, it cannot be guaranteed that performing the operation of generalized partial meet semi-revision will yield a coherent set as a result. As Olsson recognizes, this result suggests that generalized partial meet semi-revision can plausibly model coherentist belief revision only if coherence is interpreted as negative coherence.

An interesting feature of Olsson's theory is that it allows us to define two key concepts of a coherentist epistemology, to wit, the concepts of systemic and relational coherence. 'Systemic' models of coherence take coherence to be a property of a system of beliefs. In contrast, 'relational' models claim that coherence is a relation between a given belief and a system of beliefs, rather than a property of the whole system.[18] While these two concepts are very much used in the coherentist literature, the question of how they should be defined and how they relate to each other is in need of clarification. In Olsson's approach, a proposition coheres with a set if it belongs to the system that results from semi-revising that set. Formally,

$$\alpha?\text{-coheres with } K, \text{ if and only if } \alpha \in K?\alpha$$

Olsson argues that systemic coherence can be defined in terms of relational coherence: a system is coherent just in case each of its members coheres in its turn with the remaining members. Thus, the relation between relational and systemic coherence can be formally stated as follows:

$$K \text{ is coherent if and only if, for each } \alpha \in K, \alpha?\text{-coheres with } K\backslash\{\alpha\}.^{19}$$

In a later work, Olsson has taken relational coherence to be a relation between two sets of propositions, rather than a relation between a proposition and a set.[20] He has defined relational coherence as follows:

$$\text{A coheres with B if and only if } A \cup B \in \text{COH}$$

where COH is the class of all coherent sets of sentences. The relation between relational and systemic coherence is captured by the 'Residual Coherence' principle:

$$A \in \text{COH if and only if, for each } \alpha \in A, \alpha \text{ coheres with } A\backslash\{\alpha\}.$$

How do these notions of relational and systemic coherence enter into the process of justification? In Olsson's view, we do not statically compare whether new

[18] On the distinction between systemic and relational coherence as well as on the relations between them, see Bender (1989, pp. 2–3), Baltelborth (1999, 218f), and Williams (1996, p. 276).

[20] Olsson (1999).

[19] Olsson (1997b, p. 120).

information coheres with a system and then decide whether we are justified in accepting it or not. Instead, justification is a dynamic process in which new information is merged with old information in order to arrive at the best system given our total information. He has suggested that we use a 'merge operation' to model this dynamic approach to justification. Merge operations, as opposed to semi-revision, do not take the input to be a single sentence. Fuhrmann has defined a merge operation $\circ$ in which two belief bases are combined into a consistent one.[21] According to Fuhrmann, these operations are, like semi-revision, of a partial meet variety. Like semi-revision, these operations, which model externally motivated changes of belief, can also be defined in terms of internal changes, e.g., consolidation. Let ! be the consolidator operator:

$$A \circ B = A \cup B!$$

Olsson's definition of merge differs from Fuhrmann's in two important respects: the result of merge is required to be coherent, and not merely consistent, and it is not a partial meet function. Let $K\bigtriangledown$ be the set of largest coherent subsets of $K$, and $\gamma$ a selection function. The elements of $K\bigtriangledown$ are the candidate interpretations of the total information—old and new. Among those elements, the selection function will select the best candidate system. Olsson defines the merge of two sets A and B as follows:

$$A \circ B = \gamma((A \cup B)\nabla)$$

That is, merging A and B results in the best system among the maximally inclusive coherent subsystems of $A \cup B$. This system is the most justified one, given our information. The selection mechanism may be based on a notion of comparative systemic coherence, so that the candidate that exhibits a higher degree of systemic coherence is chosen. Then, A will (relationally) cohere with B if the merging of A and B is the union of these sets, i.e., A coheres with B if and only if $A \circ B = A \cup B$. Thus, in Olsson's approach, the question of whether we are justified in accepting a new piece of information, and change our beliefs accordingly, can be read off from the system: if it belongs to the system that results from merging, then we are justified in accepting it.

## 4 Belief revision formalisms and legal coherentism

Belief revision formalisms provide us with the tools for thinking more precisely about coherence and its role in legal justification. The most important contribution of these formalisms to a coherentist legal epistemology is that of working out in detail a number of coherence-enhancing mechanisms. Coherence theories of legal justification have remained vague about how legal decision-makers should reach coherence in the course of legal inquiry and deliberation. Belief revision theories help us clarify this issue by providing an account of the different operations by which we may bring a belief set to coherence. More specifically, the belief revision

[21] Fuhrmann (1997, pp. 80–85).

theorists' distinction between two kinds of change, namely, internal and external, and their corresponding operations, i.e., consolidation and merge (or semi-revision, if the input is a singleton) is extremely useful for explaining the different mechanisms whereby the alternative hypotheses that are being considered in the course of legal decision-making may be rendered coherent. These hypotheses come in two kinds: 'explanatory' hypotheses about the facts under dispute and 'interpretative' hypotheses about what the law requires. Thus, belief revision formalisms are valuable, from an epistemic point of view, insofar as they shed light on the issue of how legal decision-makers reach the most coherent hypothesis (which is, by the coherentist standards, the most justified one) when reasoning about both disputed questions of fact and disputed questions of law.

Faced with a pool of alternative explanatory or interpretative hypotheses, it is desirable that legal decision-makers make the best possible case for each of them, before selecting one of them as justified. From a coherentist perspective, this amounts to performing 'internal changes' on each of the decision alternatives so as to render them as coherent as they can be. The different kinds of consolidation clarify how the alternative explanatory and interpretative hypotheses may be modified in the direction of greater coherence. First, 'additive' consolidation is exceedingly useful for maximizing the coherence of the alternative hypotheses under consideration. For instance, a legal decision-maker may render an explanatory hypothesis coherent by adding a belief in a testimony that explains away an inconsistency between such a hypothesis and a piece of evidence at trial. Or one may enhance the coherence of an interpretative hypothesis by adding a belief in an overarching principle that irons out the discrepancies between the proposed hypothesis and a relevant body of precedent.

'Subtractive' consolidation is also helpful for making the competing hypotheses about the facts and the law coherent. One may determine that one should reject a particular piece of evidence on the grounds of its incoherence with an explanatory hypothesis which is strongly supported by the rest of the evidence at trial. Or one may increase the coherence of an interpretative hypothesis which explains a substantial body of precedent and statutory laws by ruling out as mistaken a precedent which conflicts with the principles upon which such an interpretation relies.

In addition to additive and subtractive strategies, a 're-interpretative' strategy, which employs both addition and subtraction, is also very useful for construing coherence in the course of legal decision-making.[22] For example, one may enhance the coherence of an explanatory hypothesis of the facts under dispute by re-interpreting a conflicting piece of evidence so that it supports the hypothesis. Similarly, the coherence of an interpretative hypothesis may be increased by re-interpreting a body of precedent, which apparently incoheres with such a hypothesis, in the light of an alternative principle as lending support for (rather than conflicting with) such an interpretative hypothesis.

Legal inquiry is a dynamic process in which sometimes new information that conflicts with one's accepted beliefs about either the facts under dispute or the law

[22] I borrow the term 're-interpretation' from Conte (1999, p. 88).

is received. In these cases, 'external' changes are called for to restore the coherence of one's views. 'Merge' (and 'semi-revision') helps us explain the process whereby legal decision-makers may integrate into a coherent system the total information that they have—old and new. The idea is that processing new information requires the revision of one's total information state. This leads to the generation of a number of alternative candidate interpretations from which the most coherent one (which may or may not contain the new piece of information) is selected. This process has two steps: the first step consists of adding the new information; and the second consists of consolidating the new total system. Here, we may also distinguish between three different strategies whereby the coherence of the explanatory or interpretative hypotheses under consideration may be maximized, in light of new information, depending on the kind of consolidation (subtractive, additive, or a combination of both) that is used.

Confronted with a new piece of information—for example, a new piece of evidence or a novel analogy—the legal decision-maker has to decide whether she is justified in accepting it or not. The suggestion is that the processing of this new information is made by first including it in one's set of accepted beliefs about either the facts or the law, and then by maximizing the coherence of the set that results from merging the old information with the new. For instance, in light of a new conflicting testimony, the legal decision-maker would reason by first, supposing that the testimony is reliable, and then by working out whatever modifications are necessary to make the whole decision alternative as coherent as it can be: this might be done by eliminating some elements from the set—for instance, some evidence that conflicts with this testimony—by adding new elements—e.g., accepting a corroborating testimony as evidence for one's hypothesis—or by re-interpreting some elements—e.g., altering the evaluation of other pieces of evidence so that they cohere with the new testimony. Of course, the outcome of the decision process might well be that the testimony ought to be rejected—and this would amount to rendering the system that includes both the old and new information coherent by subtractive consolidation. Likewise, a novel analogy that militates against a proposed interpretative hypothesis may lead one to revise one's views about the law (through additive, subtractive, and re-interpretative strategies).

These belief revision operations are effective strategies for rendering each of the explanatory and interpretative hypotheses under consideration as coherent as they can be. Not only are they appealing, from a normative standpoint, but they also characterize quite well the kind of operations performed by legal decision-makers when searching for the most coherent arrangement of the different elements that enter into their decisions. Simon has persuasively shown that legal decision-makers restructure the legal materials so as to build up a coherent mental model of the decision task by means of what he refers to as 'gate-keeping', 'bolstering', and 'rule-selecting'.[23] These operations correspond, he says, with most typologies offered by cognitive consistency theorists, to wit, changing elements, adding new elements, and decreasing the importance of dissonant elements. There is substantial psychological evidence that we use these kind of operations for enhancing the

[23] Simon (2004, p. 85ff).

coherence (or reducing the incoherence) among our beliefs. These types of operations closely resemble the kind of operations, which belief revision formalisms advocate. Thus, belief revision operations not only seem to provide a useful classification of the strategies that legal decision-makers have at their disposal for enhancing the coherence of each of the decision alternatives, but they also describe the processes whereby they seem to construe coherence in the course of legal decision-making.

Despite the relevance of these formalisms to a coherentist legal epistemology, they fall short of formalizing important aspects of it. First, these formalisms embody an unrestricted commitment to holism, in that belief change operations are performed over the *whole* system of beliefs. This wholesale holism is problematic for various reasons. To start with, a holistic view of justification is psychologically implausible in that no real legal agent has the cognitive resources and memory capacities needed to perform the kind of inferences that such a view of justification requires. In addition, a holistic theory of justification fails to be an accurate description of our practices of legal justification for, clearly enough, judges and other legal decision-makers do not engage in the kind of global computations of coherence that holism demands. Last, holism is normatively troubling in that it makes the justification of any particular decision defeasible on grounds of its incoherence with any part of the legal decision-maker's belief system, no matter how unrelated. In light of these reasons, 'local' models of coherence, which take justification to be a matter of coherence among a relevant subset of beliefs, may be viewed as more plausible than 'global' models of coherence. Thus, belief revision operations need to be local rather than global for them to provide a helpful formalization of a coherentist legal epistemology. Recently, some research has been done in the direction of developing operations of change (e.g., 'localized' consolidation) that are meant to be performed only over some parts of the belief system.[24] This development is a very interesting attempt to formalize belief revision for resource-bounded agents.

Second, as argued, current work on belief revision is extremely helpful in that it provides legal decision-makers with an array of coherence-making strategies for refining competing explanatory and interpretative hypotheses. However, these formalisms fail to address the issue of how these hypotheses are generated in the first place, and what role coherence plays in this process. Three stages of legal inquiry may be distinguished: the 'context of discovery', in which novel explanatory and interpretative hypotheses are initially generated, the 'context of pursuit', where promising hypotheses are further developed and preliminary assessed, and the 'context of justification', which leads to the acceptance (or rejection) of a hypothesis as justified (or unjustified).[25] While belief revision formalisms are extremely helpful in the context of pursuit, they do not formalize the processes whereby innovative hypotheses are formulated. However, a full-blown coherentist theory of the dynamics of legal justification should give an account not only of the role that coherence plays in the pursuit and final acceptance of hypotheses, but also of

[24] See Wassermann (1999, 2001, 2003) as well as Hansson and Wassermann (2002).

[25] Sintonen and Kiikeri (2004, pp. 214–218).

coherence-driven hypothesis generation. There is some interesting work that employs belief revision formalisms to articulate a model of scientific inquiry as a process of rational hypothesis revision in the face of data, starting from a background theory.[26] However, this approach is not well suited to account for discoveries which involve conceptual innovations, and thus fails to account for the generation of truly novel hypotheses which question the accepted background.

Third, belief revision formalisms, and more specifically, Olsson's theory, help us define more precisely important notions of a coherentist epistemology, such as the notions of relational and systemic coherence. However, the general concept of coherence employed is still too rudimentary. To start with, consistency is taken to be a requirement of coherence. This is very problematical. In some cases, a complex system of beliefs, even if it contains some trivial inconsistencies, may be preferable to a much less rich system of beliefs that is perfectly consistent. As some have argued, sometimes it is reasonable and rational to be inconsistent.[27] In addition, given human logical fallibility, making consistency a requirement of coherence would have the unpalatable consequence of depriving real agents of justification. Arguably, not all sorts of inconsistency—as BonJour has suggested—are equally inimical to coherence and justification. While any sort of inconsistency detracts to some degree from the coherence and thus justification of a system of beliefs, perhaps the presence of some of them do not do so enough to make them fail to satisfy the threshold of coherence required for justification.[28] Recently, some proposals have been developed that model belief changes which maintain coherence, while allowing people to come to believe in contradictions.[29] However, consistency maintenance is still the driving force of most formal work in the area.

Not only does the negative dimension of coherence fails to be satisfactorily addressed within the framework of belief revision formalisms, but the positive, connectivity, dimension of this notion is also poorly understood. Olsson's introduction of the concept of 'stability' to capture the positive aspects of coherence is a significant step towards the formalization of coherence. However, Olsson leaves open whether stability should be defined in terms of derivability, probability, explanation, or other concepts, and thus the lack of precision regarding the notion of coherence is transferred to the notion of stability. In addition, it is a main problem for Olsson's generalized version of semi-revision that the coherence of the result cannot be guaranteed for positive coherence. Further, stability is seen as a matter of all-or-nothing, and thus degrees of coherence cannot be represented in the model. This is a serious short-coming, for the kind of coherence that is relevant to justification and, more specifically, to legal justification, is of a positive and gradational kind.

Last, belief revision formalisms artificially isolate beliefs from other constituents of the mind, such as emotions and preferences.[30] This is clearly an idealization

[26] See Martin and Osherson (1997, 1998, 2002).

[27] Harman (1995, p. 178, and pp. 183–186) and Nozick (1993, p. 85 and pp. 89–93).

[28] BonJour (1989, p. 284, 1999, p. 124).

[29] Priest (2001), Mares (2002), and Tanaka (2005).

[30] Hansson (2004, p. 256).

(perhaps a necessary one), for beliefs and other constituents of the mind are intertwined in real instances of reasoning. The relevance of emotional components to coherence-based legal reasoning cannot be obliterated. Thus, these formalisms clarify only *some* aspects of coherence-oriented belief change.

Thus, there are important limits to what these formalisms may help us to achieve. Nonetheless, despite their limitations, they are very useful for the purposes of constructing a well-structured coherentist epistemology for law, particularly, as we have seen, for better understanding the mechanics of coherentist justification in law.[31]

## 5 A computational theory of coherence

Another interesting formal approach to coherence has been developed by philosopher of science Paul Thagard. Thagard (in collaboration with Verbeurgt) has provided a general characterization of coherence as 'constraint satisfaction'. On this view, coherence maximization is a matter of maximizing the satisfaction of a set of positive and negative constraints among the elements of a given set. The idea is the following one. We start with a set $E$ of elements, which may be propositions or other representations (goals, actions, concepts, etc.). The problem is how we can accept some elements and reject others in a way that maximizes the coherence of $E$. The claim is that we turn $E$ into as coherent a whole as possible by taking into account the coherence and incoherence relations that hold between pairs of elements of $E$. These relations put constraints on what can be accepted or rejected. To maximize coherence, we partition $E$ into two disjoint subsets $A$, which contains the accepted elements, and $R$, which contains the rejected elements, in a way that takes into account the local coherence and incoherence relations. For example, if a hypothesis $h_1$ explains $e_1$, we want to ensure that if $h_1$ is accepted, so is $e_1$. On the other hand, if $h_1$ is inconsistent with $h_2$, then we will make sure that if $h_1$ is accepted, then $h_2$ is rejected. The coherence problem is thus that of dividing up $E$ into $A$ and $R$ in a way that best satisfies the most constraints. Thagard and Verbegeurt define more formally the coherence problem as follows:

> Let $E$ be a finite set of elements $\{e_i\}$ and $C$ be a set of constraints on $E$ understood as a set $\{(e_i, e_j)\}$ of pairs of elements of $E$. $C$ divides into $C+$, the positive constraints on $E$, and $C$-, the negative constraints on $E$. Each constraint is associated with a number $w$, which is the weight (strength) of the constraint. The [coherence] problem is to partition $E$ into two sets $A$ and $R$, in a way that maximizes compliance with the following two *coherence conditions*:
> 1. if $(e_i, e_j)$ is in $C+$, then $e_i$ is in $A$ if and only if $e_j$ is in $A$.
> 2. if $(e_i, e_j)$ is in $C-$, then $e_i$ is in $A$ if and only if $e_j$ is in $R$.

[31] The relevance of belief revision formalisms to the analysis of epistemological problems has been argued by Rott (2001, pp. 46–65) and (Hansson, 2004). On the connections between belief revision and issues in informal philosophy, see Hansson (2003). For a strong criticism of the utility of these formalisms for developing an epistemological theory, see Pollock and Gillies (2000).

> Let $W$ be the weight of the partition, that is, the sum of the weights of the satisfied constraints. The coherence problem is then to partition $E$ into $A$ and $R$ in a way that maximizes $W$.[32]

The coherence of a partition of $E$ is $W$, the sum of the weights of the constraints that satisfy the above conditions. Coherence is maximized if there is no other partition of $E$ that has greater total weight.

Thagard claims that this abstract characterization of coherence applies to a wide variety of problems, among others, epistemic justification, moral and legal justification, and discourse comprehension. The application of the constraint satisfaction approach to coherence requires the specification of the elements and the constraints that are relevant in a particular domain. The solution of a particular coherence problem—claims Thagard—involves the interaction of different kinds of coherence. He distinguishes six kinds of coherence: explanatory, analogical, deductive, perceptual, conceptual, and deliberative. Each kind requires different sorts of elements and constraints. Thagard has proposed theories for all these six kinds of coherence, which supplement the general conception of coherence as constraint satisfaction so that it can be applied to the solution of a particular coherence problem.

Here, I will only present Thagard's theory of explanatory coherence (TEC).[33] This theory is the first theory of coherence that Thagard has proposed, and the one that he has elaborated in more detail.[34] In fact, the theories of all the other kinds of coherence have been generated by analogy with the theory of explanatory coherence.[35] In explanatory coherence, elements are propositions that describe hypotheses or pieces of evidence. Positive and negative constraints are established by means of the following principles, which state the theory of explanatory coherence:

> *E1: Symmetry.* Explanatory coherence is a symmetrical relation. That is, two propositions, $p$ and $q$, cohere with each other equally.
> *E2: Explanation*. (a) A hypothesis coheres with what it explains, which can be either evidence or another hypothesis; (b) hypotheses that together explain some other proposition cohere with each other; and (c) the more hypotheses it takes to explain something, the lower the degree of coherence.
> *E3: Analogy.* Similar hypotheses that explain similar pieces of evidence cohere with each other.
> *E4: Data priority.* Propositions that describe the results of observation have a degree of acceptability on their own.
> *E5: Contradiction*. Contradictory propositions are incoherent with each other.

[32] Thagard and Verbeurgt (1998, p. 3).

[33] See Thagard (1989, 1992, 2000).

[34] As a matter of fact, the theory of explanatory coherence was proposed *before* the general theory of coherence as constraint satisfaction was developed.

[35] See Thagard (2000), for a statement and discussion of the theories of deliberative, conceptual, analogical, perceptual, and deductive coherence.

*E6: Competition*. If *p* and *q* both explain a proposition and if *p* and *q* are not explanatorily connected, then *p* and *q* are incoherent with each other.
*E7: Acceptance*. The acceptability of a proposition in a system of propositions depends on its coherence with them.

The theory of explanatory coherence has been implemented computationally in a connectionist program called ECHO. In ECHO, each proposition is represented by a unit, a highly simplified artificial neuron, connected with other units by excitatory and inhibitory links, which represent relations of coherence and incoherence. The degree of acceptance of propositions is modeled by the activation of units, which can range from 1 (acceptance) to 0 (rejection). Thagard's theory of explanatory coherence, and its computational implementation, has been applied to many examples of scientific reasoning, everyday reasoning, and, more important for our purposes, legal reasoning. In the legal context, it has been used to explain jurors' reasoning in some criminal cases, legal inferences from testimony, and judgments as to when a fact-finder's belief in the guilt-hypothesis is beyond a reasonable doubt.[36]

Thagard's theory of coherence as constraint satisfaction has an enormous potential for modeling coherence-based inference in law. First, it provides a more interesting notion of coherence, in which consistency is not taken to be a necessary requirement of coherence, for there may be cases where two negatively constrained elements both end up being accepted. Second, this theory has a good claim to psychological plausibility insofar as it endorses a moderate kind of holism, in which the acceptability of a particular element depends only on its relations with a relevant subset of elements. Third, Thagard's theory does much by way of solving some of the problems which beset coherence theories of justification, including coherence theories of legal justification. The principle of symmetry allows us to distinguish between circular chains of inference and mutually supporting inferences, and thus to put worries about the circularity of coherence-based inference to rest. This theory exhibits only a modest conservatism, in that it allows that previously accepted elements be dislodged by new elements, if they sufficiently cohere with other elements. In addition, Thagard's theory, by giving priority to some elements in being accepted, provides a way of meeting the charge that coherence theories do not give new input the role it ought to have in justification and, therefore, that they cut off justification from the world. Hence, the theory is also appealing at a normative level. Last, this theory can be easily extended to account for the role that emotions play in legal reasoning, as Thagard's recent work on 'emotional coherence' shows.[37]

Hence, Thagard's theory does significantly contribute to the development of a solid coherentist epistemology for law. Nonetheless, there are important aspects of a coherence theory for law that remain to be addressed within this framework. For one, Thagard's theory is a theory of hypothesis evaluation, and thus it does not account for the processes whereby the initial set of competing hypotheses and relevant evidence is generated in the first place. In addition, it does not incorporate

[36] See Thagard (1989, 2003, 2004a, b, 2005, 2006a, b).

[37] Thagard (2006b).

some important and specific aspects of legal reasoning, such as the relevance of the standards of proof or the presumption of innocence, in the context of fact-reasoning.[38] Besides, Thagard has developed legal applications of his theory exclusively to reasoning about disputed questions of fact. Nonetheless, there are good possibilities for application of Thagard's theory of coherence to reasoning about disputed questions of law.[39] This is exceedingly interesting, for it would enable the development of a unified account of coherence-based inference in law.

## 6 Conclusions

In this paper I have discussed some formal approaches to the notion of coherence, to wit, belief revision formalisms and Thagard's theory of coherence as constraint satisfaction, and argued that while these approaches fall short of providing a formalization of a complete coherence theory of legal justification, they advance this theory in a number of different fronts. More specifically, the formal approaches reviewed are extremely useful for clarifying three central aspects of legal coherentism. First, they provide us with a precise characterization of the notion of coherence; this is an important contribution, for a main problem of coherence theories of legal justification is that they rely on a notion of coherence that is too vague for the purposes of legal justification. Second, these theories give an account of the dynamic aspects of coherentist justification, which are very much neglected in non-formalized versions of legal coherentism that focus on the static dimension of justification. Last, informal coherence theories of legal justification fail to provide concrete guidance to legal decision-makers as to how coherence may be built in the course of inquiry and deliberation; formal approaches shed some light on this question by providing a precise account of coherence-enhancing mechanisms.

Hence, formal approaches to coherence such as those reviewed above can help us make coherentism a solid theory of justification for law. There is thus a symbiotic relationship between formal and informal approaches to legal coherentism. On the one hand, new developments in formalizations of coherence will help illuminate central issues in a coherentist legal epistemology. On the other hand, informal approaches to coherence and justification in law are an extremely useful guide for doing formal research in the area.

**Acknowledgements** I am grateful to Jorge Cerdio, Rossella Rubino, and Frederick Schauer for comments on an earlier draft of this paper. I thank an anonymous reviewer for his comments.

[38] See Thagard (2003, p. 366) and Amaya (2006, pp. 890–894) for suggestions about how Thagard's theory of coherence may be amended so as to incorporate the institutional constraints under which the evaluation of explanatory hypotheses in law proceeds.

[39] Bench-Capon and Sartor (2001) have adapted Thagard's approach to coherence to their theory of case-based reasoning as a kind of reasoning that involves theory construction, use, and theory evaluation. In this approach, cases are taken to provide evidence for the competing theories and cases, rules, and preferences are viewed as units. See also Bench-Capon and Sartor (2003, pp. 135–136). For a proposal as to how a theory of normative coherence for law could be developed on the basis of Thagard's framework, see Amaya (2006, pp. 897–905).

## References


Alchourrón CE, Gärdenfors P, Makinson D (1985) On the logic of theory change: partial meet contraction and revision functions. J Symbol Logic 50:510–530

Amaya A (2006) An inquiry into the nature of coherence and its role in legal argument, PhD thesis, European University Institute

Balterborth T (1999) Coherence and explanations. Erkenntnis 50:209–224

Bench-Capon T, Sartor G (2001) Theory-based explanation of case law domains. In: Proceedings of the eighth international conference on AI and Law, 12–21 St Louis, MO ACM Press, New York

Bench-Capon T, Sartor G (2003) A model of legal reasoning with cases incorporating theories and values. Artif Intel Law 150:97–113

Bender J (1989) Coherence, justification, and knowledge: the current debate. In: Bender J (ed) The current state of the coherence theory: critical essays on the epistemic theories of Keith Lehrer and Lawrence BonJour, with replies, Kluwer, Dordrecht, pp 1–14

BonJour L (1989) Replies and clarifications. In: Bender J (ed) The current state of the coherence theory, Kluwer, Dordrecht, pp 276–292

BonJour L (1999) The dialectic of foundationalism and coherentism. In: Greco J, Sosa E (eds) The Blackwell guide to epistemology, Blackwell, Oxford, pp 117–142

Conte ME (1999) Condizioni di Coerenza: Ricerche di Linguistica Testuale. Edizioni dell'Orso, Firenze

Doyle J (1992) Reason maintenance and belief revision: foundations vs coherence theories. In: Gärdenfors P (ed) Belief revision, Cambridge University Press, Cambridge, pp 29–51

Furhmann A (1997) An essay on contraction. CSLI Publications, Stanford

Gärdenfors P (1988) Knowledge in flux: modeling the dynamics of epistemic states. MIT Press, Cambridge

Gärdenfors P (1990) The dynamics of belief systems: foundations vs. coherence theories. Revue Internationale de Philosophie 172:24–46

Gärdenfors P (1992) Belief revision: an introduction. In: Gärdenfors P (ed) Belief revision. Cambridge University Press, Cambridge, pp 1–27

Gärdenfors P (2005) The dynamics of thought. Springer, Dordrecht

Gärdenfors P, Rott H (1995) Belief revision. In: Gabbay DM, Hogger CJ, Robinson JA (eds) Handbook of logic in artificial intelligence and logic programming, vol 4, Oxford University Press, Oxford, pp 35–132

Hansson SO (1994) Taking belief bases seriously. In: Prawitz D, Westertøahl D (eds) Logic and philosophy of science in Uppsala. Kluwer, Dordrecht, pp 13–28

Hansson SO (1997a) Semi-revision. J Appl Non-Classical Logic 7:151–175

Hansson SO (1997b) What's new isn't always best. Theoria 63:1–13

Hansson SO (1999a) A textbook of belief dynamics. Kluwer, Dordrecht

Hansson SO (1999b) A survey of non-prioritized belief revision. Erkenntnis 51:413–427

Hansson SO (2003) Ten philosophical problems in belief revision. J Logic Comput 13:37–49

Hansson SO, Wassermann R (2002) Local change. Studia Logica 70:49–76

Hansson SO (2004) Belief revision from an epistemological point of view. In: Niiniluoto I, Sintonen M, Wolénski J (eds) Handbook of epistemology. Kluwer, Dordrecht, pp 255–279

Hansson SO, Olsson EJ (1999) Providing foundations for coherentism. Erkenntnis 51:243–265

Harman G (1995) Rationality. In: Smith EE, Osherson DN (eds) An invitation to cognitive science, vol 3. MIT Press, Cambridge, pp 175–212

Klein P, Warfield TA (1994) What price coherence? Analysis 54:129–132

Mares ED (2002) A paraconsistent theory of belief revision. Erkenntnis 56:229–246

Martin E, Osherson D (1997) Scientific discovery based on belief revision. J Symbol Logic 62:1352–1370

Martin E, Osherson D (1998) Elements of scientific inquiry. MIT Press, Cambridge

Martin E, Osherson D (2002) Scientific discovery from the perspective of hypothesis acceptance. Philos Sci 69:S331–S341

Nozick R (1993) The nature of rationality. Princeton University Press, Princeton

Olsson EJ (1997a) Coherence: studies in epistemology and belief revision, PhD thesis, Uppsala University

Olsson EJ (1997b) A coherence interpretation of semi-revision. Theoria 63:105–134

Olsson EJ (1998) Making beliefs coherent. J Logic Lang Inform 7:143–163

Olsson EJ (1999) Cohering with. Erkenntnis 50:273–291

Pollock JL, Gillies AS (2000) Belief revision and epistemology. Synthese 122:69–92
Priest G (2001) Paraconsistent belief revision. Theoria 67:214–228
Putnam H (1981) Reason, truth and history. Cambridge University Press, Cambridge
Rott H (2001) Change, choice and inference: a study of belief revision and nonmonotonic reasoning. Oxford University Press, Oxford
Shaffer MJ (2002) Coherence, justification, and the AGM theory of belief revision. In: Bouchard Y (ed) Perspectives on coherentism. Scribe, Aylmer, pp 139–160
Simon D (2004) A third view of the black box: cognitive coherence in legal decision-making. Univ Chicago Law Rev 65:511–586
Sintonen M, Kiikeri M (2004) Scientific discovery. In: Niiniluoto, I, Sintonen, M, Wolénski, J (eds) Handbook of epistemology, Kluwer Academic Publishers, Dordrecht, pp 205–253
Tanaka K (2005) The AGM theory and inconsistent belief change. Logique et Analyse 48:113–151
Thagard P (1989) Explanatory coherence. Behav Brain Sci 12:435–467
Thagard P (1992) Conceptual revolutions. Princeton University Press, Princeton
Thagard P (2000) Coherence in thought and action. MIT Press, Cambridge
Thagard P (2003) Why wasn't OJ convicted? Emotional coherence in legal inference. Cogn Emot 17:361–383
Thagard P (2004a) Causal inference in legal decision making: explanatory coherence vs Bayesian networks. Appl Artif Intel 18:231–249
Thagard P (2004b) What is doubt and when is it reasonable? In: Ezcurdia M, Stainton R, Viger C (eds) New essays in the philosophy of language and mind. Can J Philos 30:391S–406S
Thagard P (2005) Testimony, credibility, and explanatory coherence. Erkentnnis 63:295–316
Thagard P (2006a) Evaluating explanations in law, science, and everyday life. Curr Direct Psychol Sci 15:141–145
Thagard P (2006b) Hot thought: mechanisms and applications of emotional cognition. MIT Press, Cambridge
Thagard P, Verbeurgt K (1998) Coherence as constraint satisfaction cognitive. Science 22:1–24
Wassermann R (1999) Resource bounded belief revision. Erkenntnis 50:429–446
Wassermann R (2001) On structured belief bases. In: Williams MA, Rott H (eds) Frontiers in belief revision. Kluwer, Dordrecht, pp 349–367
Wassermann R (2003) Generalized change and the meaning of rationality postulates. Studia Logica 73:200–319
Williams M (1996) Unnatural doubts: epistemological realism and the basis of skepticism. Princenton University Press, Princeton
Wintgens L (2007, forthcoming) Making sense of coherence: the level theory of coherence. Artif Intel Law