Some remarks on relational nature of gauge symmetry

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Abstract

We review the gauge theories on the relational point of view. With this new insight we approach the Yang Mills theories and the problem of confinement. We point out that we can intuit the relational gauge nature from the Gribov mechanism in the study of confinement in strong coupled gauge theories.

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1 Introduction

In contemporary physics the understanding of physical systems in terms of relations between their fundamental entities has been addressed in different contexts. An important example comes from the foundations of quantum mechanics, the relational quantum mechanics [1] that defends the thesis of the rejection of absolute state of a system in favor of a weaker notion of state and values of physical quantities related to something else. Another is to try to understand or interpret the concept of gauge in relational terms [2] (and see [3] for some discussions), by reviewing the usual interpretation of gauge as mathematical redundancy. This discussion appears in the case of electromagnetism as a view of thinking the Maxwell potential, a variable non-gauge-invariant, as more fundamental and not its derivative. See [4] to an approach of electromagnetism in these terms.

The quantum and the gauge, two important foundations of our most successful theories. It is relevant that they point in the same direction, an even greater foundation at the core of nature, the relations itself should be more important than the individuals themselves. We will concentrate on the gauge that has electromagnetism as the great paradigm, and above references give the tone of the approach we want to address the relational term here. Gauge symmetries are omnipresent and fundamental in contemporary theoretical physics because all four fundamental interactions, electromagnetism, the strong and weak nuclear forces, gravitation through general relativity, and several models of unification, such as string theories are described by theories of this type. Successful examples of gauge theories are quantum electrodynamics (QED) and quantum chromodynamics (QCD) [5] that are part of the Standard Model of particle physics. In light of this we ask ourselves about the true nature of what the gauge is, and we seek to exploit fully its consequences. We also emphasize that interesting philosophical discussions have recently appeared in the literature, see [2] and references therein.

In this paper we begin by reviewing an example of dynamic of a classical gauge system shown in Rovelli's paper "Why Gauge?" (RWG) [2], recalling their interpretation and getting new insight into the nature of the gauge, and we use these ideas to address the QED and Yang Mills theories (YM) [5], and in particular the QCD. Then we will analyze some aspects of quantization of such gauge theories in this point of view. Finally we discuss relational nature of gauge symmetry in confined phase of QCD. The problem of confinement of quarks and gluons [6] is a challenging issue, which is relevant in the context of general investigations of strongly coupled gauge theories. From the an analytical perspective, a possible approach to the problem of confinement in YM theories, comes from the analysis of the Gribov copies [7]. This is known as Gribov problem, and the model of Gribov–Zwanziger (GZ) [8, 9] has an important role. We will interpret the Gribov problem (or more generally, Gribov mechanism) in these relational terms, ie, we will see that gauge in the strongly coupled gauge theories (which naturally suggests more interaction, coupling and relation) reveals a very particular structure of our world, one relational structure between physical quantities in the sense that each individual or system can be thought of or described in terms of their relations with other individuals or systems and that this type of relation is more fundamental. Indicate this relational structure and pointing out that it can also be obtained and generalized from the Gribov mechanism, we consider the step beyond the considerations of the RWG.

2 Notes on relational classical gauge

Let's review a simple prototype of a gauge system, used in the RWG, to help to visualize the relational character involving gauge theories. We will see that this dynamic system already shows the relational feature that later we will argue that is present in the successful gauge theories QED and QCD, ie, general property that has to do with gauge itself.

Consider then a system S_1 formed by a set of N variables $x_n(t)$ (with $n = 1...N$) whose dynamics is governed by the Lagrangian:

$$\mathcal{L}_1 = \frac{1}{2}\sum_{n=1}^{N-1}(\dot{x}_{n+1} - \dot{x}_n)^2 \tag{1}$$

and the equations of motion are invariant under the gauge transformations

$$x_n \to x'_n = x_n + \lambda \tag{2}$$

for an arbitrary function $\lambda(t)$. The evolution of $x_n(t)$ is undetermined by the equations of motion. A complete set of gauge invariant quantities is given by

$$a_n = x_{n+1} - x_n \tag{3}$$

with $n = 1...N-1$.

And similarly consider a system S_2 consisting by a set of M variables $y_n(t)$ (com $n = 1...M$) whose dynamics is governed by the Lagrangian:

$$\mathcal{L}_2 = \frac{1}{2}\sum_{n=1}^{M-1}(\dot{y}_{n+1} - \dot{y}_n)^2 \tag{4}$$

and the equations of motion are invariant under the gauge transformations

$$y_n \to y'_n = y_n + \lambda' \tag{5}$$

for an arbitrary function $\lambda'(t)$. As before, the evolution of $y_n(t)$ is undetermined by the equations of motion. A complete set of gauge invariant quantities is given by

$$b_n = y_{n+1} - y_n \tag{6}$$

with $n = 1...N-1$. Now, consider the coupled system defined by the variables $x_n(t)$, $y_n(t)$, and the Lagrangian

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{12} \tag{7}$$

where

$$\mathcal{L}_{12} = \frac{1}{2}(\dot{y}_1 - \dot{x}_N)^2 \tag{8}$$

The coupled system is invariant under the gauge transformations (2) and (5) if $\lambda'(t) = \lambda(t)$. Its observable gauge invariant are more than those of the individual systems: they are given by the gauge invariant observables of the first system, the observables of the second, plus a new one:

$$c = y_1 - x_N \tag{9}$$

which depends on gauge variables of both systems. An important conclusion here (of RWG) is that gauge non invariant variables store potential of relation with other systems, like a measurement apparatus for example. In this sense it is a "partial observable" [21].

In addition to the original considerations of the paper in question we can observe and intuit from this example that the essential information of a system of gauge, S_1 for example, can be taken from some kind of sum of interrelations of the system itself (a_n) more the information coming from the coupling with one or more other systems, in this case, the gauge invariant observable c that captures the information of the two coupling systems. Thus we can anticipate a conclusion that can also be obtained from the main gauge theories, which we will see below, that these results may be interpreted heuristically for a gauge system, S_1, as:

$$S_1 = \sum(relations(internal + whole)) \,. \tag{10}$$

That is, for a system S_i, we can represent in the following way:

$$S_i = \sum_j S_i \mathcal{O}_{ij} S_j \,, \tag{11}$$

where $\mathcal{O}_{ij}$ is an object that relates the S_i, S_j.

Let's now do the same study for QED, the great paradigm of a gauge theory. Our system S_1 now involves the gauge field A_μ (photon) with

$$\mathcal{L}_1 = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \,, \tag{12}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{13}$$

$F_{\mu\nu}$ and then $\mathcal{L}_1$ are invariant under the gauge transformations (symmetries):

$$A_\mu \to A_\mu^{'} = A_\mu + \partial_\mu \alpha \tag{14}$$

The system S_2 in QED involves the Dirac field for the electron or positron $(\psi, \bar{\psi})$ and:

$$\mathcal{L}_2 = i\bar{\psi}\gamma^\mu \partial_\mu \psi. \tag{15}$$

with gauge invariance:

$$\psi \to \psi' = e^{i\alpha'} \psi. \tag{16}$$

The coupled system is:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{12} \tag{17}$$

where

$$\mathcal{L}_{12} = -e\bar{\psi}\gamma_\mu A^\mu \psi \tag{18}$$

The coupled system is invariant under the gauge transformations (14) and (16) if $\alpha'(x) = -e\alpha(x)$. This is an abelian gauge symmetry with the gauge group U(1), therefore the QED is a U(1) gauge theory. Note that the interaction term $\mathcal{L}_{12}$ is essential for the full description of the system S_1 and in the framework of quantum field theories it is calculated by the well known tools of perturbation theory [5]. That is, in a gauge system of the type S_1, the relational information with the whole, in the case the system S_2 with which it is coupled, coming through perturbative corrections where the gauge non invariant variable plays important role in according to interpretation (10). We will address the issue of perturbative expansion in quantum version in the next section.

We now generalize the analogy to nonabelian gauge theories [5]. We have then SU(N) gauge theories. In this case we have richer theory with more subsystems that previous cases. Now considering only the bosonic term of QED we have more than one system involved, the gauge field is now A^a_μ (in the adjoint representation of SU(N)), with $a = 1 \dots N^2 - 1$. Thus the analogue to (12) is the set of systems A^a_μ and their interactions, described by the Yang Mills Lagrangian:

$$\mathcal{L}_{YM} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} \,, \tag{19}$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \,, \tag{20}$$

and

$$[T^a, T^b] = i f^{abc} T^c \,, \tag{21}$$

where the matrices T^a are called generators of the group defined in the appropriate representation, the constants of proportionality f^{abc} are the structure constants and g is the coupling constant. The nonabelian gauge transformations is more complex and we can write it in infinitesimal form as follows:

$$A^a_\mu \to A^{a'}_\mu = A^a_\mu - D^{ab}_\mu \alpha^b \,, \tag{22}$$

with D^{ab}_μ being the covariant derivative in the adjoint representation

$$D^{ab}_\mu = \delta^{ab} \partial_\mu - g f^{abc} A^c_\mu \,. \tag{23}$$

Note that $\mathcal{L}_{YM}$ can also be written as:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \dots + \mathcal{L}_{N^2-1} + \mathcal{L}_{int} \tag{24}$$

with

$$\mathcal{L}_1 = (\partial^\mu A^{1\nu} - \partial^\nu A^{1\mu})(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu) \tag{25}$$

and the other $\mathcal{L}_2, \dots, \mathcal{L}_{N^2-1}$ analogously. And:

$$\mathcal{L}_{int} = \partial^\mu A^{a\nu} g f^{abc} A^b_\mu A^c_\nu + \dots \tag{26}$$

Thus here the system S_1 involves the field A^1_μ and $\mathcal{L}_1$, and a complete description of this system should also take into consideration its relations with other bosonic fields computed in the term $\mathcal{L}_{int}$ (perturbatively). In accordance with (10).

To conclude this classical analysis, consider the example of QCD which consists of also consider the term of matter (fermions):

$$\mathcal{L}_m = \bar{\psi}_i i (\gamma^\mu D_\mu)_{ij} \psi_j \,, \tag{27}$$

where color indices and possible mass terms are suppressed, and which contains the fermions ψ_i e $\bar{\psi}_i$ belonging to the fundamental representation of SU(N), that is infinitesimally:

$$\delta \bar{\psi}_i = -i g \alpha^a T^a_{ij} \bar{\psi}_j \,, \tag{28}$$

and D^{ij}_μ the covariant derivative in the fundamental representation

$$D_\mu^{ij} = \delta^{ij}\partial_\mu - igA_\mu^a(T^a)^{ij} \,. \tag{29}$$

The indices i, j run to 1, ..., N. The γ^μ are the Dirac gamma matrices.
Note that $\mathcal{L}_m$ can also be written as:

$$\mathcal{L}_m = \mathcal{L}_{m1} + \mathcal{L}_{m2} + ... + \mathcal{L}_{m(N)} + \mathcal{L}_{m(int)} \,, \tag{30}$$

with

$$\mathcal{L}_{m1} = \bar{\psi}_1 i(\gamma^\mu\partial_\mu)\psi_1 \,, \tag{31}$$

and the others $\mathcal{L}_{m2}, ..., \mathcal{L}_{m(N)}$ analogously. And:

$$\mathcal{L}_{m(int)} = \bar{\psi}_i i(\gamma^\mu igA_\mu^a(T^a)^{ij})\psi_j \,. \tag{32}$$

Thus in the same way of the previous examples here the systemS_1 involves the field $\bar{\psi}_1, \psi_1$ and $\mathcal{L}_{m1}$ and a complete description of this system should also take into consideration its relations with other fermionic and bosonic fields computed in the term $\mathcal{L}_{m(int)}$. In this way we can interpret the S_1 system in terms of the relational structure (11).

3 Notes on relational quantum gauge

Let us consider a theory involving a set of fields $\Phi(x)$ in a dimension D of space-time. The (classical) dynamics is defined by the action [5, 10]:

$$S(\Phi) = \int dx\mathcal{L}(x) = S_0(\Phi) + S_{int}(\Phi) \,. \tag{33}$$

The Lagrangian has the general form

$$\mathcal{L}(x) = \frac{1}{2}\Phi(x)K^{ij}(\partial)\Phi(x) + \mathcal{L}_{int} = \mathcal{L}_0 + \mathcal{L}_{int} \,. \tag{34}$$

K^{ij} is some invertible differential operator, usually a second-order polynomial in ∂ for the bosonic field and first order for the fermionic, the quadratic part $\mathcal{L}_0$ of the Lagrangian corresponds to the free theory, while $\mathcal{L}_{\text{int}}$ describes the interactions. For example for QED, as we saw in the previous section, $\mathcal{L}_0$ would be formed by the equations (12) plus (15) and $\mathcal{L}_{\text{int}}$ equation (18).

The objects of the corresponding quantum theory one wants to compute, are the Green functions, ie the vacuum expectation values of time-ordered product of field operators:

$$G_{i_1 \dots i_N}(x_1, \cdots, x_N) = < T\Phi_{i_1}(x_1)\cdots\Phi_{i_N}(x_N) > \,. \tag{35}$$

These Green functions can be collected together in generating functional $Z(J)$, a formal power series in "classical sources" $J^i(x)$ and formally given by Feynman path integrals

$$Z(J) = \mathcal{N}\int \mathcal{D}\Phi e^{-\frac{1}{g}(S(\Phi)+\int dx J^i(x)\Phi_i(x)} \,, \tag{36}$$

where $\mathcal{N}$ is a numerical factor. The solution of the free theory ($\mathcal{L}_{\text{int}} = 0$ in (34)) is given by

$$Z_{free}(J) = e^{\frac{1}{2g^2}\int dx_1 dx_2 J^{i_1}(x_1)J^{i_2}(x_2)\triangle_{i_1 i_2}(x_1,x_2)} \,, \tag{37}$$

where $\triangle_{i_1 i_2}(x_1, x_2)$ is the free propagator obtained by inverting the waveform operator $K^{ij}(\partial)$ from (34).

In the case of full theory with interactions, a formal solution is given by [5]

$$Z(J) = \mathcal{N} e^{-\frac{1}{g} S_{int}(-g \frac{\delta}{\delta J})} Z_{free}(J) \,. \tag{38}$$

This expression leads to the well known perturbative expansion of Green functions in terms of Feynman graphs.

Thus we have that the quantization (tools of quantum field theory) of the theory allows computing the interaction terms, that we are interpreting as structures of relational type, (18), (26) and (32) from perturbative form.

In the case of gauge theories there are another important ingredient to consider in the quantization process. For example, in the case of QCD and QED that we are using, $\mathcal{L}_0$ in (34) would be respectively (12) or parte of (19) consisting of (25) and similar and the corresponding functional generator $Z(J)$ given by (36). But this integral has no meaning, which can already be seen in calculating the free part, since the operator $K^{ij}(\partial)$ of QED or QCD can not be inverted in the calculation of the propagators. This is usually seen as a consequence of the vector potential, the gauge field, can be chosen arbitrary due to gauge freedom. The gauge field theories are examples of field theories with degenerate functional action. Ie, the gauge variable has a redundancy, but in the interpretation that we are studying this has to do with the character or potential to relation of the part gauge non invariant. Thus an effective procedure in the treatment of this redundancy should be something that handles well this "relational potential".

In the usual procedure, redundant variables should be removed from the gauge theory by considering conditions for gauge fixing. This procedure was due to Ludvig Faddeev and Victor Popov with the introduction of the ghost fields [11, 5]. When this is done, the gauge invariance is lost and it is not clear how to deal with physics. The contemporary approach to address these problems involves the construction of BRST [12]. The central idea of the construction of BRST is to replace the original gauge symmetry by the BRST symmetry, which must be present even after the gauge fixing. An important observation is that in the BRST approach fields and fermionic ghosts are considered together [13]). An extended action involving all these variables can be constructed in a way that is BRST invariant. The additional terms are:

$$\mathcal{L}_{gf} = -\frac{1}{2\xi} (\partial \cdot A)^2 \,, \tag{39}$$

for the gauge fixing and

$$\mathcal{L}_{gh} = -(\bar{c}^a \partial_\mu \partial^\mu c^a + g \bar{c}^a f^{abc} \partial_\mu A^{b\mu} c^c) \,, \tag{40}$$

for the ghosts. here we use c^a for the ghost fields and $\bar{c}^a$ for the anti-ghosts. With the inclusion of these terms the functional generator $Z(J)$ given by (36) is the expression usually used to derive the Feynman rules.

Another way to see this or "interpretation" is to say that the gauge parameter gained life or dynamic, ie, the ghost is a way to encapsulate the "potential" or "handles" relational of gauge non invariant part after fixation, so they are, with gauge fixing, the new "partial observables". In QED it decouples but in more strongly relational theories like QCD they will engage with the other fields involved, ie, diagrams involving ghosts must also be considered in perturbation theory.

Note that in a general quantum fields theory, the interaction term can have generally any form and do not necessarily support (11) so that the perturbative calculations (38) can compute other things. But in practice the interaction terms are strongly constrained by arguments of simplicity, by proper exploration of symmetries (where the gauge symmetry plays a central role) and other attributes of the theory so that we expect that in a gauge theory necessarily the interaction term will bring relational influences of the type (11) by the analysis we're doing here, and it is remarkable that the QED and QCD work in this way.

4 Gribov mechanism and relational gauge

As noted above much of the relations between systems of gauge are computed perturbatively. Now, what if a given theory is in a regime where you can not do perturbation theory, ie, a regime where the coupling constant is not small enough for that? Then we have the theory in a regime known as nonperturbative and so other techniques must be developed to study relational effects in this regime. That's what happens with QCD and we will study this here. Two important phenomena in the nonperturbative region of QCD are confinement (the absence of color charged particles in the QCD spectrum) and the dynamic chiral symmetry breaking. Although the confinement and chiral symmetry breaking are well accepted, its treatment still inspires an active research, see for example [14, 15]. The study of this regime of QCD and approaches that explore the gauge symmetry in this context is of course of great value for understanding the relational character involving the gauge due to perturbation theory to fail precisely where the system is extremely relational. And the insight that we have of this discussion is that we need tools to capture relational influences of the type (11) in the theory. It is what we see from equations (25) and (26). If (26) can not be computed perturbatively we are left with the following information from the system S_1: only A^1_μ with himself (equation 25) missing $A^1_\mu \leftrightarrow A^2_\mu,\ A^3_\mu ...$.

We'll focus on the problem of confinement [6] where there are several approaches, even complementary, as lattice QCD, Dyson-Schwinger equations, Sum rules among other, refer [6] and references therein. As we point out in the introduction, in the analytical point of view, one possible approach (important by the use in a refined way of the gauge symmetry) to the problem of confinement comes from the analysis of Gribov copies. We consider this approach important because it captures relational information between $A^1_\mu \leftrightarrow A^2_\mu,\ A^3_\mu ...$ in a non perturbative way and still own approach itself makes explicit the relational character of the gauge in the sense proposed here.

In order to clarify the understanding of the Gribov problem and its relation to our analysis we present below a brief description of the problem in Landau gauge [16]. The $SU(N)$ Yang-Mills Euclidean action in the Landau gauge is built with the terms (19, 39 and 40) and we can re-write here in the following way:

$$S_{\rm YM} = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i b^a\, \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) , \tag{41}$$

here, A^a_μ is the gluons gauge field , b^a is the Lagrange multiplier field enforcing the Landau gauge condition, $\partial_\mu A^a_\mu = 0$, $(\bar{c}^a, c^a)$ are the anti-commuting scalar fields, the Faddeev-Popov ghosts, and g is the coupling constant of the theory. The indices $(a, b, c, \dots)$ run to 1 at (N^2-1) and f^{abc} are the totally antisymmetric structure constants of the Lie algebra of generators of $SU(N)$. Also, this action is invariant under the following nilpotent BRST transformations:

$$sA^a_\mu = -D^{ab}_\mu c^b, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c, \quad s\bar{c}^a = ib^a, \quad sb^a = 0. \tag{42}$$

Even with the gauge fixing by the Faddeev-Popov method, Gribov showed in [7][1] that there are still configurations of fields obeying the Landau gauge linked for gauge transformations, that is, there is still equivalent configurations, or copies being taken into account in the Feynman path integral. In other words there is a remaining ambiguity due to the existence of normalized zeros modes of the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu D^{ab}_\mu . \tag{43}$$

Gribov also showed that to eliminate these copies to the domain of integration the functional integral should be restricted to a certain region Ω, which is defined as the set of field configurations performing

[1] see [17] for a pedagogical review.

Landau gauge condition, in which the Faddeev-Popov operator is strictly positive, namely

$$\Omega := \{ A^a_\mu \,|\, \partial_\mu A^a_\mu = 0,\, \mathcal{M}^{ab}(A) > 0 \} \,. \tag{44}$$

Its boundary, $\partial\Omega$, where the first zero eigenvalue of Faddeev-Popov operator appears, is known as Gribov horizon.

The second step in this approach to the Gribov problem is the GZ theory, which consists in a renormalizable and local way to implement the restriction to the first Gribov region. Indeed, Zwanziger notes that the restriction can be implemented by adding the following term in the action (41):

$$S_{\rm GZ} = S_{\rm YM} + \gamma^4 H(A) \,, \tag{45}$$

where, $H(A)$ is called horizon function (a non-local term which is localized in GZ theory),

$$H(A) = g^2 \int d^4x\, d^4y\, A^b_\mu(x) f^{abc} [\mathcal{M}^{-1}]^{ad}(x,y) f^{dec} A^e_\mu(y) \,. \tag{46}$$

Zwanziger conjecture ([18, 19, 20]) that an exact nonperturbative quantization of continuous gauge theories, is provided by the formula of Faddeev-Popov in the Landau gauge restricted to the Gribov region, which can be implemented by the horizon function above. We will not go into more details of this approach and its nuances, and we suggest in addition to the references above the review [16].

Thus, what we reviewed here is that with the action of YM in hand when we quantize it we need to fix the gauge. If we do it in the Faddeev-Popov approach, gauge parameters acquires dynamic in the theory, are the ghosts of Faddeev-Popov, and enables a good understanding on perturbative region. But when we go to the region of strong coupling (infrared) we note that the gauge fixing was still not enough, as if part of the gauge invariance migrate to the variance of gauge (relational nature) and we need to insert in the action a mixing term of the type (46).

As in usual gauge theories the gauge symmetry help in selecting a new term of interaction that makes sense in the non-perturbative region, the horizon function, which implements the interactions, or we can say, captures the relations. The inclusion of this term explicit information about this region giving indications of change of theory spectrum and the propagators in the infrared, being an important approach to the problem of confinement in these theories. In accordance with the discussion of the end of the last section, this interaction term brings relational influences of the type (11), ie a sum of interactions or more precisely relationships between each field A^a_μ with itself and the other fields. We can even go further in the interpretation, it is as if in a certain regime of the system interactions or internal relations in the system itself, or with the exterior, became more important than their individual parts and that the measure has to do with the gauge in these theories. In the case of QCD on the ultraviolet or QED, relational character is smaller, and the perturbative approach works well to capture such relations. The more towards the infrared QCD, the relational character is more important with the gauge parameter acquiring new importance and new treatment being necessary as in the Gribov mechanism. This suggests that even physical quantities can have contributions coming from these sectors. Occur mixed propagators, propagators that can be interpreted as confining and condensates (composite particles) emerge as fundamental entities.

Thus in this discussion within the strongly coupled gauge theories also we got this idea that seems to be more general, that the gauge has to do with the relational structure of a system, a structure of type (11).

To complete this analysis we put in other similar words, structures that appear in (46) ($\sum_b A^1_\mu \mathcal{O}^{1b} A^b_\mu$) can be conceived as follows: the individual A^1_μ lost significance in the regime of theory in question and thus emerges a relational view:

$$A^1_\mu \to \sum_b A^1_\mu \mathcal{O}^{1b} A^b_\mu \,, \tag{47}$$

where $\mathcal{O}^{ab}$ may be a generalization of $[\mathcal{M}^{-1}]^{ab}$.

The simple gauge symmetry itself captures this behavior:

$$\delta A_{\mu}^{a} = -D_{\mu}^{ab}\theta^{b}\,, \tag{48}$$

that becomes clearer with BRST:

$$sA_{\mu}^{a} = -D_{\mu}^{ab}c^{b}\,, \tag{49}$$

where the gauge parameter gets dynamic. On this point of view the gauge field maps internal relations of the system with the surroundings through the relational mix (47), or a parameter coupled (48), or another field doing this role (49).

Finally we can see that it is possible the inclusion of fermions in this approach. When we consider the full QCD, the system should become more relational. We must include in action (45) the terms for free fermions, of the type (31) and terms of interaction fermionic and bosonic (32). Being a more relational phenomenon ghosts should play an important role here (see section VI of [22] and references therein), but we will not go into here. Let's address the issue that in the region nonperturbative we could address the fermions the same way as bosons, so that a new interaction term in the molds of the horizon function acquired importance. This term keeps the structure of the type (11). Thus we have for fermions, a horizon function in the mold of (46), that insert already at the level of free terms, the mix between the different flavors:

$$H(\lambda) = \int d^4x\, \bar{\lambda}_{\alpha}^{i} f_{ik}^{a} [\mathcal{M}^{-1}]^{kl} f_{jl}^{a} \lambda_{\alpha}^{j}\,. \tag{50}$$

whose simplest construction is to test

$$H(\lambda) = \int d^4x\, \bar{\lambda}_{\alpha}^{i} f_{ik}^{a} \frac{\gamma^3 \delta^{\alpha\beta}}{\partial^2} f_{jk}^{a} \lambda_{\beta}^{j}\,. \tag{51}$$

This type of term is inserted and studied in [15, 23] providing important results. An important detail is that the usual term of interaction in the ultraviolet always involves the gauge field, but this horizon function has interactions just between fermionic flavors what is new.

Thus in nonabelian theories a way to recover relational information lost in nonperturbative region can be done with inclusion of terms of the type of the horizon function H above, it maintains the structure (11) that has the interaction terms in gauge theories, and is a consequence of a improved treatment of the "partial observables" of the theory.

5 Conclusions

Our physical world certainly has many structures and should change according to the way we scrutinize it. We point out here that the gauge, which is important in building other tools or theories that has great success in particle physics, being essential in the construction of the standard model of particle physics, reveals a very interesting structure at the core of the matter, that each object or system can be defined or understood as the sum of its relations, $S_i = \sum_j S_i \mathcal{O}_{ij} S_j$, and even more, when selecting such structures it allows to reconstruct information of certain physical quantities of interest by perturbation theory. The gauge is also essential in nonperturbative regimes as we saw with the Gribov mechanism where such structures can be reinsert in the theory consistently and we see that they play an important role. Note that the old thread to take gauge pontencial as essential is contemplated by what we saw of the Gribov mechanism and relational gauge symmetry.

It is interesting that as in other contexts, the gauge in the Gribov mechanism is usually viewed as redundance, but in all these situations we can interpret this redundancy as nature or relational reserve in terms of "partial observables" (RWG handles). We saw that in this context the old gauge parameters become ghosts when the gauge is fixed, they shall carry that relational reserves of gauge non invariant variable and the new terms of coupling that emerges has a very suggestive form called horizon function. The "partial observables" play important role in this context, they will compose to form the real physical observables.

Finally we wish to emphasize the thesis defended in this work, that the relational structure that emerges with the gauge is necessarily of the type (10), so we can see the concept of relation as an irreducible basic datum, and we think that this may have implications in other areas such theories of unification, dark energy or be brought to the level of more general principle as suggestively elaborated in [24].

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