

Research Article

Incorporating Contagion in Portfolio Credit Risk Models Using Network Theory

Ioannis Anagnostou ^{1,2}, **Sumit Sourabh**^{1,2} and **Drona Kandhai**^{1,2}

¹Computational Science Lab, University of Amsterdam, Science Park 904, 1098XH Amsterdam, Netherlands

²Quantitative Analytics, ING Bank, Foppingadreef 7, 1102BD Amsterdam, Netherlands

Correspondence should be addressed to Ioannis Anagnostou; i.anagnostou@uva.nl

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Portfolio credit risk models estimate the range of potential losses due to defaults or deteriorations in credit quality. Most of these models perceive default correlation as fully captured by the dependence on a set of common underlying risk factors. In light of empirical evidence, the ability of such a conditional independence framework to accommodate for the occasional default clustering has been questioned repeatedly. Thus, financial institutions have relied on stressed correlations or alternative copulas with more extreme tail dependence. In this paper, we propose a different remedy—augmenting systematic risk factors with a contagious default mechanism which affects the entire universe of credits. We construct credit stress propagation networks and calibrate contagion parameters for infectious defaults. The resulting framework is implemented on synthetic test portfolios wherein the contagion effect is shown to have a significant impact on the tails of the loss distributions.

1. Introduction

One of the main challenges in measuring the risk of a bank's portfolio is modelling the dependence between default events. Joint defaults of many issuers over a fixed period of time may lead to extreme losses; therefore, understanding the structure and the impact of default dependence is essential. To address this problem, one has to take into consideration the existence of two distinct sources of default dependence. On the one hand, performance of different issuers depends on certain common underlying factors, such as interest rates or economic growth. These factors drive the evolution of a company's financial success, which is measured in terms of its rating class or the probability of default. On the other hand, default of an issuer may, too, have a direct impact on the probability of default of a second dependent issuer, a phenomenon known as contagion. Through contagion, economic distress initially affecting only one issuer can spread to a significant part of the portfolio or even the entire system. A good example of such a transmission of pressure is the Russian crisis of 1998-1999 which saw the defaults of corporate and

subsovereign issuers heavily clustered following the sovereign default [1].

Most portfolio credit risk models used by financial institutions neglect contagion and rely on the conditional independence assumption according to which, conditional on a set of common underlying factors, defaults occur independently. Examples of this approach include the Asymptotic Single Risk Factor (ASRF) model [2], industry extensions of the model presented by Merton [3] such as the KMV [4, 5] and CreditMetrics [6] models, and the two-factor model proposed recently by Basel Committee on Banking Supervision for the calculation of Default Risk Charge (DRC) to capture the default risk of trading book exposures [7]. A considerable amount of literature has been published on the conditional independence framework in standard portfolio models; see, for example, [8, 9].

Although conditional independence is a statistically and computationally convenient property, its empirical validity has been questioned on a number of occasions, where researchers investigated whether dependence on common factors can sufficiently explain the default clustering which

occurs from time to time. Schönbucher and Schubert [10] suggest that the default correlations that can be achieved with this approach are typically too low in comparison with empirical default correlations, although this problem becomes less severe when dealing with large diversified portfolios. Das et al. [11] use data on US corporations from 1979 to 2004 and reject the hypothesis that factor correlations can sufficiently explain the empirically observed default correlations in the presence of contagion. Since a realistic credit risk model is required to put the appropriate weight on scenarios where many joint defaults occur, one may choose to use alternative copulas with tail dependence which have the tendency to generate large losses simultaneously [12]. In that case, however, the probability distribution of large losses is specified a priori by the chosen copula, which seems rather unintuitive [13].

One of the first models to consider contagion in credit portfolios was developed by Davis and Lo [14]. They suggest a way of modelling default dependence through infection in a static framework. The main idea is that any defaulting issuer may infect any other issuer in the portfolio. Giesecke and Weber [15] propose a reduced-form model for contagion phenomena, assuming that they are due to the local interaction of companies in a business partner network. The authors provide an explicit Gaussian approximation of the distribution of portfolio losses and find that, typically, contagion processes have a second-order effect on portfolio losses. Lando and Nielsen [16] use a dynamic model in continuous time based on the notion of mutually exciting point processes. Apart from reduced-form models for contagion, which aim to capture the influence of infectious defaults to the default intensities of other issuers, structural models were developed as well. Jarrow and Yu [17] generalize existing models to include issuer-specific counterparty risks and illustrate their effect on the pricing of defaultable bonds and credit derivatives. Egloff et al. [18] use network-like connections between issuers that allow for a variety of infections between firms. However, their structural approach requires a detailed microeconomic knowledge of debt structure, making the application of this model in practice more difficult than that of Davis and Lo's simple model. In general, since the interdependencies between borrowers and lenders are complicated, structural analysis has mostly been applied to a small number of individual risks only.

Network theory can provide us with tools and insights that enable us to make sense of the complex interconnected nature of financial systems. Hence, following the 2008 crisis, network-based models have been frequently used to measure systemic risk in finance. Among the first papers to study contagion using network models was [19], where Allen and Gale show that a fully connected and homogeneous financial network results in an increased system stability. Contagion effects using network models have also been investigated in a number of related articles; see, for example, [20–24]. The issue of too-central-to-fail was shown to be possibly more important than too-big-to-fail by Battiston et al. in [25], where DebtRank, a metric for the systemic impact of financial institutions, was introduced. DebtRank was further extended in a series of articles; see, for example, [26–28].

The need for development of complexity-based tools in order to complement existing financial modelling approaches was emphasized by Battiston et al. [29], who called for a more integrated approach among academics from multiple disciplines, regulators, and practitioners.

Despite substantial literature on portfolio credit risk models and contagion in finance, specifying models, which take into account both common factors and contagion while distinguishing between the two effects clearly, still proves challenging. Moreover, most of the studies on contagion using network models focus on systemic risk and the resilience of the financial system to shocks. The qualitative nature of this line of research can hardly provide quantitative risk metrics that can be applied to models for measuring the risk of individual portfolios. The aforementioned drawback is perceived as an opportunity for expanding the current body of research by contributing a model that would account for common factors and contagion in networks alike. Given the wide use of factor models for calculating regulatory and economic capital, as well as for rating and analyzing structured credit products, an extended model that can also accommodate for infectious default events seems crucial.

Our paper takes up this challenge by introducing a portfolio credit risk model that can account for two channels of default dependence: common underlying factors and financial distress propagated from sovereigns to corporates and subsovereigns. We augment systematic factors with a contagion mechanism affecting the entire universe of credits, where the default probabilities of issuers in the portfolio are immediately affected by the default of the country where they are registered and operating. Our model allows for extreme scenarios with realistic numbers of joint defaults, while ensuring that the portfolio risk characteristics and the average loss remain unchanged. To estimate the contagion effect, we construct a network using credit default swaps (CDS) time series. We then use CountryRank, a network-based metric, introduced in [30] to quantify the impact of a sovereign default event on the credit quality of corporate issuers in the portfolio. In order to investigate the impact of our model on credit losses, we use synthetic test portfolios for which we generate loss distributions and study the effect of contagion on the associated risk measures. Finally, we analyze the sensitivity of the contagion impact to rating levels and CountryRank. Our analysis shows that credit losses increase significantly in the presence of contagion. Our contributions in this paper are thus threefold: First, we introduce a portfolio credit risk model which incorporates both common factors and contagion. Second, we use a credit stress propagation network constructed from real data to quantify the impact of deterioration of credit quality of the sovereigns on corporates. Third, we present the impact of accounting for contagion which can be useful for banks and regulators to quantify credit, model, or concentration risk in their portfolios.

The rest of the paper is organized as follows. Section 2 provides an overview of the general modelling framework. Section 3 presents the portfolio model with default contagion and illustrates the network model for the estimation of contagion effects. In Section 4 we present empirical analysis of two

synthetic portfolios. Finally, in Section 5, we summarize our findings and draw conclusions.

2. Merton-Type Models for Portfolio Credit Risk

Most financial institutions use models that are based on some form of the conditional independence assumption, according to which issuers depend on a set of common underlying factors. Factor models based on the Merton model are particularly popular for portfolio credit risk. Our model extends the multifactor Merton model to allow for credit contagion. In this section, we present the basic portfolio modelling setup, outline the model of Merton, and explain how it can be specified as a factor model. A more detailed presentation of the multivariate Merton model is provided by [9].

2.1. Basic Setup and Notations. This subsection introduces the basic notation and terminology that will be used throughout this paper. In addition, we define the main risk characteristics for portfolio credit risk.

The uncertainty of whether an issuer will fail to meet its financial obligations or not is measured by its *probability of default*. For comparison reasons, this is usually specified with respect to a fixed time interval, most commonly one year. The probability of default then describes the probability of a default occurring in the particular time interval. The *exposure at default* is a measure of the extent to which one is exposed to an issuer in the event of, and at the time of, that issuer's default. The default of an issuer does not necessarily imply that the creditor receives nothing from the issuer. The percentage of loss incurred over the overall exposure in the event of default is given by the *loss given default*. Typical values lie between 45% and 80%.

Consider a portfolio of m issuers, indexed by $i = 1, \dots, m$, and a fixed time horizon of $T = 1$ year. Denote by e_i the exposure at default of issuer i and by p_i its probability of default. Let q_i be the loss given default of issuer i . Denote by Y_i the default indicator, in the time period $[0, T]$. All issuers are assumed to be in a nondefault state at time $t = 0$. The default indicator Y_i is then a random variable defined by

$$Y_i = \begin{cases} 1 & \text{if issuer } i \text{ defaults} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

which clearly satisfies $\mathbb{P}(Y_i = 1) = p_i$. The overall portfolio loss is defined as the random variable

$$L := \sum_{i=1}^m q_i e_i Y_i. \quad (2)$$

With credit risk in mind, it is useful to distinguish potential losses in *expected losses*, which are relatively predictable and thus can easily be managed, and *unexpected losses*, which are more complicated to measure. Risk managers are more concerned with unexpected losses and focus on risk measures relating to the tail of the distribution of L .

2.2. The Model of Merton. Credit risk models are typically distinguished in structural and reduced-form models, according to their methodology. Structural models try to explain the mechanism by which default takes place, using variables such as asset and debt values. The model presented by Merton in [3] serves as the foundation for all these models. Consider an issuer whose asset value follows a stochastic process $(V_t)_{t \geq 0}$. The issuer finances itself with equity and debt. No dividends are paid and no new debt can be issued. In Merton's model the issuer's debt consists of a single zero-coupon bond with face value B and maturity T . The values at time t of equity and debt are denoted by S_t and B_t and the issuer's asset value is simply the sum of these; that is,

$$V_t = S_t + B_t, \quad t \in [0, T]. \quad (3)$$

Default occurs if the issuer misses a payment to its debtholders, which can happen only at the bond's maturity T . At time T , there are only two possible scenarios:

- (i) $V_T > B$: the value of the issuer's assets is higher than its debt. In this scenario the debtholders receive $B_T = B$, the shareholders receive the remainder $S_T = V_T - B$, and there is no default.
- (ii) $V_T \leq B$: the value of the issuer's assets is less than its debt. Hence, the issuer cannot meet its financial obligations and defaults. In that case, shareholders hand over control to the bondholders, who liquidate the assets and receive the liquidation value in lieu of the debt. Shareholders pay nothing and receive nothing; therefore we obtain $B_T = V_T$, $S_T = 0$.

For these simple observations, we obtain the below relations:

$$S_T = \max(V_T - B, 0) = (V_T - B)^+, \quad (4)$$

$$B_T = \min(V_T, B) = B - (B - V_T)^+. \quad (5)$$

Equation (4) implies that the issuer's equity at maturity T can be determined as the price of a European call option on the asset value V_t with strike price B and maturity T , while (5) implies that the value of debt at T is the sum of a default-free bond that guarantees payment of B plus a short European put option on the issuer's assets with strike price B .

It is assumed that under the physical probability measure \mathbb{P} the process $(V_t)_{t \geq 0}$ follows a geometric Brownian motion of the form

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t, \quad t \in [0, T], \quad (6)$$

where $\mu_V \in \mathbb{R}$ is the mean rate of return on the assets, $\sigma_V > 0$ is the asset volatility, and $(W_t)_{t \geq 0}$ is a Wiener process. The unique solution at time T of the stochastic differential equation (6) with initial value V_0 is given by

$$V_T = V_0 \exp\left(\left(\mu_V - \frac{\sigma_V^2}{2}\right)T + \sigma_V W_T\right) \quad (7)$$

which implies that

$$\ln V_T \sim \mathcal{N}\left(\ln V_0 + \left(\mu_V - \frac{\sigma_V^2}{2}\right)T, \sigma_V^2 T\right). \quad (8)$$

Hence, the real-world probability of default at time T , measured at time $t = 0$, is given by

$$\begin{aligned} \mathbb{P}(V_T \leq B) &= \mathbb{P}(\ln V_T \leq \ln B) \\ &= \Phi\left(\frac{\ln(B/V_0) - (\mu_V - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}\right). \end{aligned} \quad (9)$$

A core assumption of Merton's model is that asset returns are lognormally distributed, as can be seen in (8). It is widely acknowledged, however, that empirical distributions of asset returns tend to have heavier tails; thus, (9) may not be an accurate description of empirically observed default rates.

2.3. The Multivariate Merton Model. The model presented in Section 2.2 is concerned with the default of a single issuer. In order to estimate credit risk at a portfolio level, a multivariate version of the model is necessary. A multivariate geometric Brownian motion with drift vector $\boldsymbol{\mu}_V = (\mu_1, \dots, \mu_m)'$, vector of volatilities $\boldsymbol{\sigma}_V = (\sigma_1, \dots, \sigma_m)$, and correlation matrix Σ , is assumed for the dynamics of the multivariate asset value process $(\mathbf{V}_t)_{t \geq 0}$ with $\mathbf{V}_t = (V_{t,1}, \dots, V_{t,m})'$, so that for all i

$$V_{T,i} = V_{0,i} \exp\left(\left(\mu_i - \frac{1}{2}\sigma_i^2\right)T + \sigma_i W_{T,i}\right), \quad (10)$$

where the multivariate random vector \mathbf{W}_T with $\mathbf{W}_T = (W_{T,1}, \dots, W_{T,m})'$ is satisfying $\mathbf{W}_T \sim N_m(\mathbf{0}, T\Sigma)$. Default takes place if $V_{T,i} \leq B_i$, where B_i is the debt of company i . It is clear that the default probability in the model remains unchanged under simultaneous strictly increasing transformations of $V_{T,i}$ and B_i . Thus, one may define

$$\begin{aligned} X_i &:= \frac{\ln V_{T,i} - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T}{\sigma_i \sqrt{T}}, \\ d_i &:= \frac{\ln B_i - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T}{\sigma_i \sqrt{T}} \end{aligned} \quad (11)$$

and then default equivalently occurs if and only if $X_i \leq d_i$. Notice that X_i is the standardized asset value log-return $\ln V_{T,i} - \ln V_{0,i}$. It can be easily shown that the transformed variables satisfy $(X_1, \dots, X_m)' \sim N_m(\mathbf{0}, \Sigma)$ and their copula is the Gaussian copula. Thus, the probability of default for issuer i is satisfying $p_i = \Phi(d_i)$, where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. A graphical representation of Merton's model is shown in Figure 1. In most practical implementations of the model, portfolio losses are modelled by directly considering an m -dimensional random vector $\mathbf{X} = (X_1, \dots, X_m)'$ with $\mathbf{X} \sim N_m(\mathbf{0}, \Sigma)$ containing the standardized asset returns and a deterministic vector $\mathbf{d} = (d_1, \dots, d_m)$ containing the critical thresholds with $d_i = \Phi^{-1}(p_i)$ for given default probabilities p_i , $i = 1, \dots, m$. The default probabilities are usually estimated by historical default experience using external ratings by agencies or model-based approaches.

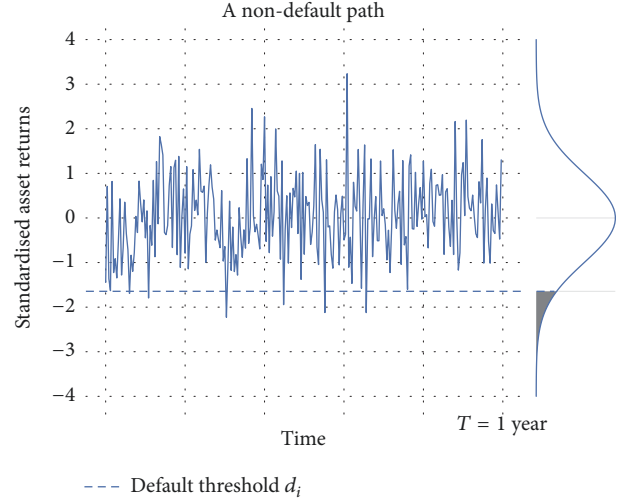


FIGURE 1: In Merton's model, default of issuer i occurs if at time T asset value $V_{T,i}$ falls below debt value B_i , or equivalently if $X_i := (\ln V_{T,i} - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T)/\sigma_i \sqrt{T}$ falls below the critical threshold $d_i := (\ln B_{T,i} - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T)/\sigma_i \sqrt{T}$. Since $X_i \sim \mathcal{N}(0, 1)$, i 's default probability, represented by the shaded area in the distribution plot, is satisfying $p_i = \Phi(d_i)$. Note that default can only take place at time T does not depend on the path of the asset value process.

2.4. Merton Model as a Factor Model. The number of parameters contained in the correlation matrix Σ grows polynomially in m , and thus, for large portfolios it is essential to have a more parsimonious parametrization which is accomplished using a factor model. Additionally, factor models are particularly attractive due to the fact that they offer an intuitive interpretation of credit risk in relation to the performance of industry, region, global economy, or any other relevant indexes that may affect issuers in a systematic way. In the following we show how Merton's model can be understood as a factor model. In the factor model approach, asset returns are linearly dependent on a vector \mathbf{F} of $p < m$ common underlying factors satisfying $\mathbf{F} \sim N_p(0, \Omega)$. Issuer i 's standardized asset return is assumed to be driven by an issuer-specific combination $\tilde{F}_i = \boldsymbol{\alpha}_i' \mathbf{F}$ of the systematic factors

$$X_i = \sqrt{\beta_i} \tilde{F}_i + \sqrt{1 - \beta_i} \epsilon_i, \quad (12)$$

where \tilde{F}_i and $\epsilon_1, \dots, \epsilon_m$ are independent standard normal variables and ϵ_i represents the idiosyncratic risk. Consequently, β_i can be seen as a measure of sensitivity of X_i to systematic risk, as it represents the proportion of the X_i variation that is explained by the systematic factors. The correlations between asset returns are given by

$$\begin{aligned} \rho(X_i, X_j) &= \text{cov}(X_i, X_j) = \sqrt{\beta_i \beta_j} \text{cov}(\tilde{F}_i, \tilde{F}_j) \\ &= \sqrt{\beta_i \beta_j} \boldsymbol{\alpha}_i' \boldsymbol{\Omega} \boldsymbol{\alpha}_j \end{aligned} \quad (13)$$

since \tilde{F}_i and $\epsilon_1, \dots, \epsilon_m$ are independent and standard normal and $\text{var}(X_i) = 1$.

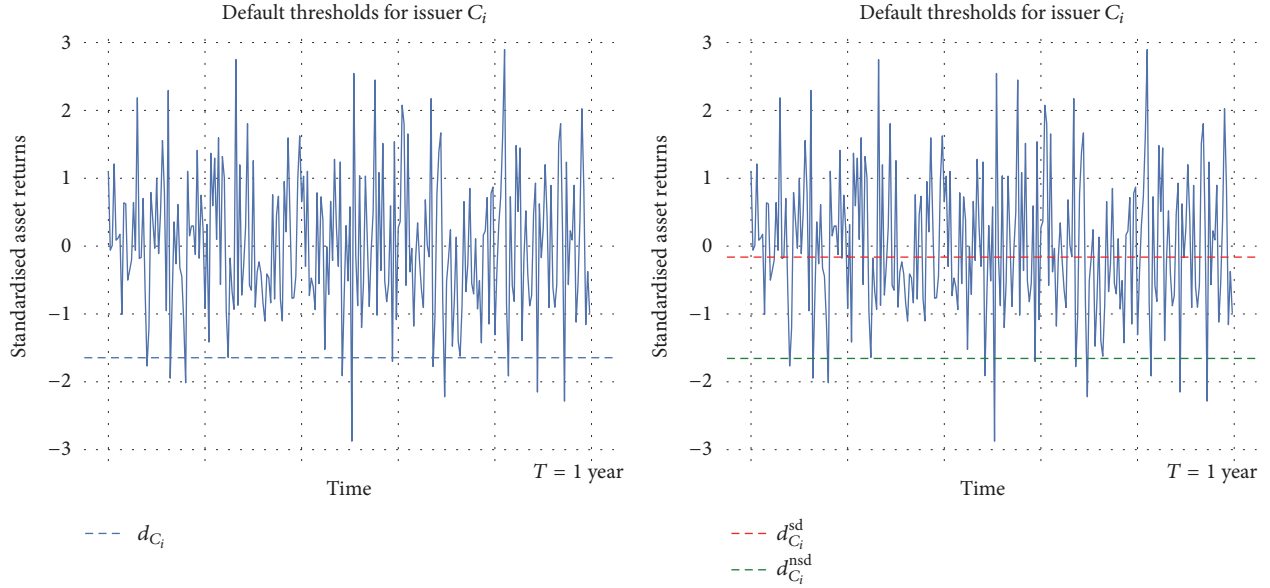


FIGURE 2: Under the standard Merton model, the default threshold d_{C_i} for corporate issuer C_i is set to be equal to $\Phi^{-1}(p_{C_i})$. Under the proposed model, the threshold increases in the event of sovereign default, making C_i 's default more likely as the contagion effect suggests.

3. A Model for Credit Contagion

In the multifactor Merton model specified in Section 2.4, the standardized asset returns X_i , $i = 1, \dots, m$, are assumed to be driven by a set of common underlying systematic factors, and the critical thresholds d_i , $i = 1, \dots, m$, are satisfying $d_i = \Phi^{-1}(p_i)$ for all i . The only source of default dependence in such a framework is the dependence on the systematic factors. In the model we propose, we assume that, in the event of a sovereign default, contagion will spread to the corporate issuers in the portfolio that are registered and operating in that country, causing default probability to be equal to their CountryRank. In Section 3.1, we demonstrate how to calibrate the critical thresholds so that each corporate's probability of default conditional on the default of the corresponding sovereign equals its CountryRank, while its unconditional default probability remains unchanged. In Section 3.2, we show how to construct a credit stress propagation network and estimate the CountryRank parameter.

3.1. Incorporating Contagion in Factor Models. Consider a corporate issuer C_i and its country of operation S . Denote by p_{C_i} the probability of default of C_i . Under the standard Merton model, default occurs if C_i 's standardized asset return X_{C_i} falls below its default threshold d_{C_i} . The critical threshold d_{C_i} is assumed to be equal to $\Phi^{-1}(p_{C_i})$ and is independent of the state of the country of operation S . In the proposed model, a corporate is subject to shocks from its country of operation; its corresponding state is described by a binary state variable. The state is considered to be stressed in the event of sovereign default. In this case, the issuer's default threshold increases, causing it more likely to default, as the contagion effect suggests. In case the corresponding

sovereign does not default, the corporates liquidity state is considered stable. We replace the default threshold d_{C_i} with $d_{C_i}^*$, where

$$d_{C_i}^* = \begin{cases} d_{C_i}^{sd} & \text{if the corresponding sovereign defaults} \\ d_{C_i}^{nsd} & \text{otherwise} \end{cases} \quad (14)$$

or equivalently

$$d_{C_i}^* = \mathbb{1}_{\{Y_S=1\}} d_{C_i}^{sd} + \mathbb{1}_{\{Y_S=0\}} d_{C_i}^{nsd}. \quad (15)$$

We denote by p_S the probability of default of the country of operation and by γ_{C_i} the CountryRank parameter which indicates the increased probability of default of C_i given the default of S . An example of the new default thresholds is shown in Figure 2. Our objective is to calibrate $d_{C_i}^{sd}$ and $d_{C_i}^{nsd}$ in such way that the overall default rate remains unchanged and $\mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1) = \gamma_{C_i}$. Denote by

$$\phi_2(x, y; \rho) := \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 + y^2 - 2\rho xy}{2(1-\rho^2)}\right), \quad (16)$$

$$\Phi_2(h, k; \rho) := \int_{-\infty}^h \int_{-\infty}^k \phi_2(x, y; \rho) dy dx$$

the density and distribution function of the bivariate standard normal distribution with correlation parameter $\rho \in (-1, 1)$.

Note that $d_{C_i}^*(\omega) = d_{C_i}^{\text{sd}}$ for $\omega \in \{Y_{C_i} = 1, Y_S = 1\} \subset \{Y_S = 1\}$, and $d_{C_i}^*(\omega) = d_{C_i}^{\text{nsd}}$ for $\omega \in \{Y_{C_i} = 1, Y_S = 0\} \subset \{Y_S = 0\}$. We rewrite $\mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1)$ in the following way:

$$\begin{aligned} & \mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1) \\ &= \frac{1}{\mathbb{P}(Y_S = 1)} \mathbb{P}(Y_{C_i} = 1, Y_S = 1) \\ &= \frac{1}{p_S} \mathbb{P}[X_{C_i} < d_{C_i}^{\text{sd}}, X_S < d_S] \\ &= \frac{1}{p_S} \Phi_2(d_{C_i}^{\text{sd}}, d_S; \rho_{SC_i}). \end{aligned} \quad (17)$$

Using the above representation and given $d_S = \Phi^{-1}(p_S)$ and ρ_{SC_i} , one can solve the equation

$$\mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1) = \gamma_{C_i} \quad (18)$$

over $d_{C_i}^{\text{sd}}$. We proceed to the derivation of $d_{C_i}^{\text{nsd}}$ in such way that the overall default probability remains equal to p_{C_i} . This constraint is important, since contagion is assumed to have no impact on the average loss. Clearly,

$$\begin{aligned} p_{C_i} &= \mathbb{P}(Y_{C_i} = 1) \\ &= \mathbb{P}(Y_{C_i} = 1, Y_S = 1) + \mathbb{P}(Y_{C_i} = 1, Y_S = 0) \\ &= \mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1) \mathbb{P}(Y_S = 1) \\ &\quad + \mathbb{P}(Y_{C_i} = 1, Y_S = 0) \end{aligned} \quad (19)$$

and thus

$$\mathbb{P}(Y_{C_i} = 1, Y_S = 0) = p_{C_i} - \gamma_{C_i} \cdot p_S. \quad (20)$$

The left-hand side of the above equation can be represented as follows:

$$\begin{aligned} \mathbb{P}(\text{corp.def} \cap \text{nosov.def}) &= \mathbb{P}[X_{C_i} < d_{C_i}^{\text{nsd}}, X_S > d_S] \\ &= \mathbb{P}[X_{C_i} < d_{C_i}^{\text{nsd}}] - \mathbb{P}[X_{C_i} < d_{C_i}^{\text{nsd}}, X_S < d_S] \\ &= \Phi(d_{C_i}^{\text{nsd}}) - \Phi_2(d_{C_i}^{\text{nsd}}, d_S; \rho_{SC_i}). \end{aligned} \quad (21)$$

By use of the above and given $d_S = \Phi^{-1}(p_S)$ and ρ_{SC_i} , one can solve the previous equation over $d_{C_i}^{\text{nsd}}$.

3.2. Estimation of CountryRank. In this section, we elaborate on the estimation of the CountryRank parameter [30], which serves as the probability of default of the corporate conditional on the default of the sovereign. In addition, we provide details on the construction of the credit stress propagation network.

3.2.1. CountryRank. In order to estimate contagion effects in a network of issuers, an algorithm such as DebtRank

[31] is necessary. In the DebtRank calculation process, stress propagates even in the absence of defaults and each node can propagate stress only once before becoming inactive. The level of distress for a previously undistressed node is given by the sum of incoming stress from its neighbors with a maximum value of 1. Summing up the incoming stress from neighboring nodes seems reasonable when trying to estimate the impact of one node or a set of nodes to a network of interconnected balance sheets where links represent lending relationships. However, when trying to quantify the probability of default of a corporate node given the infectious default of a sovereign node, one has to consider that there is significant overlap in terms of common stress, and thus, by summing we may be accounting for the same effect more than once. This effect is amplified in dense networks constructed from CDS data. Therefore, we introduce CountryRank as an alternative measure which is suited for our contagion model.

We assume that we have a hypothetical credit stress propagation network, where the nodes correspond to the issuers, including the sovereign, and the edges correspond to the impact of credit quality of one issuer on the other. The details of the network construction will be presented in Section 3.2.2. Given such a network, the CountryRank of the nodes can be defined recursively as follows:

- (i) First, we stress the sovereign node and as a result its CountryRank is 1.
- (ii) Let γ_S be the CountryRank of the sovereign and let $e_{(j,k)}$ denote the edge weight between nodes j and k . Given a node C_i , let $p = SC_1C_2 \cdots C_{i-1}C_i$ be a path without cycles from the sovereign node S , to the node C_i . The *weight* of the path p is defined as

$$w(p) = \gamma_S e_{(1,2)} \cdots e_{(i-1,i)}, \quad (22)$$

where $e_{(j,k)}$ are the respective edge weights between nodes j and k for $j \in \{1, \dots, i-1\}$ and $k \in \{2, \dots, i\}$. Let p_1, \dots, p_m be the set of all acyclic paths from the sovereign node to the corporate node C_i and let $w(p_1), \dots, w(p_m)$ be the corresponding weights. Then the CountryRank of node C_i is defined as

$$\gamma_{C_i} = \max_{1 \leq j \leq m} w(p_j). \quad (23)$$

In order to compute the conditional probability of default of a corporate given the sovereign default analytically, we would need the joint distribution of probabilities of default of the nodes, which has an exponential computational complexity, and it is therefore intractable. Thus, we approximate the conditional probability by choosing the path with the maximum weight in the above definition for CountryRank.

The example in Figure 3 illustrates calculation of CountryRank for a hypothetical network. The network consists of a sovereign node S and corporate nodes C_1, C_2, C_3, C_4 . The edge labels indicate weights in network between two nodes. We initially stress the sovereign node which results in a CountryRank of 1 for node S . In the next step, the stress propagates to node C_1 and as a result its CountryRank is 0.9.

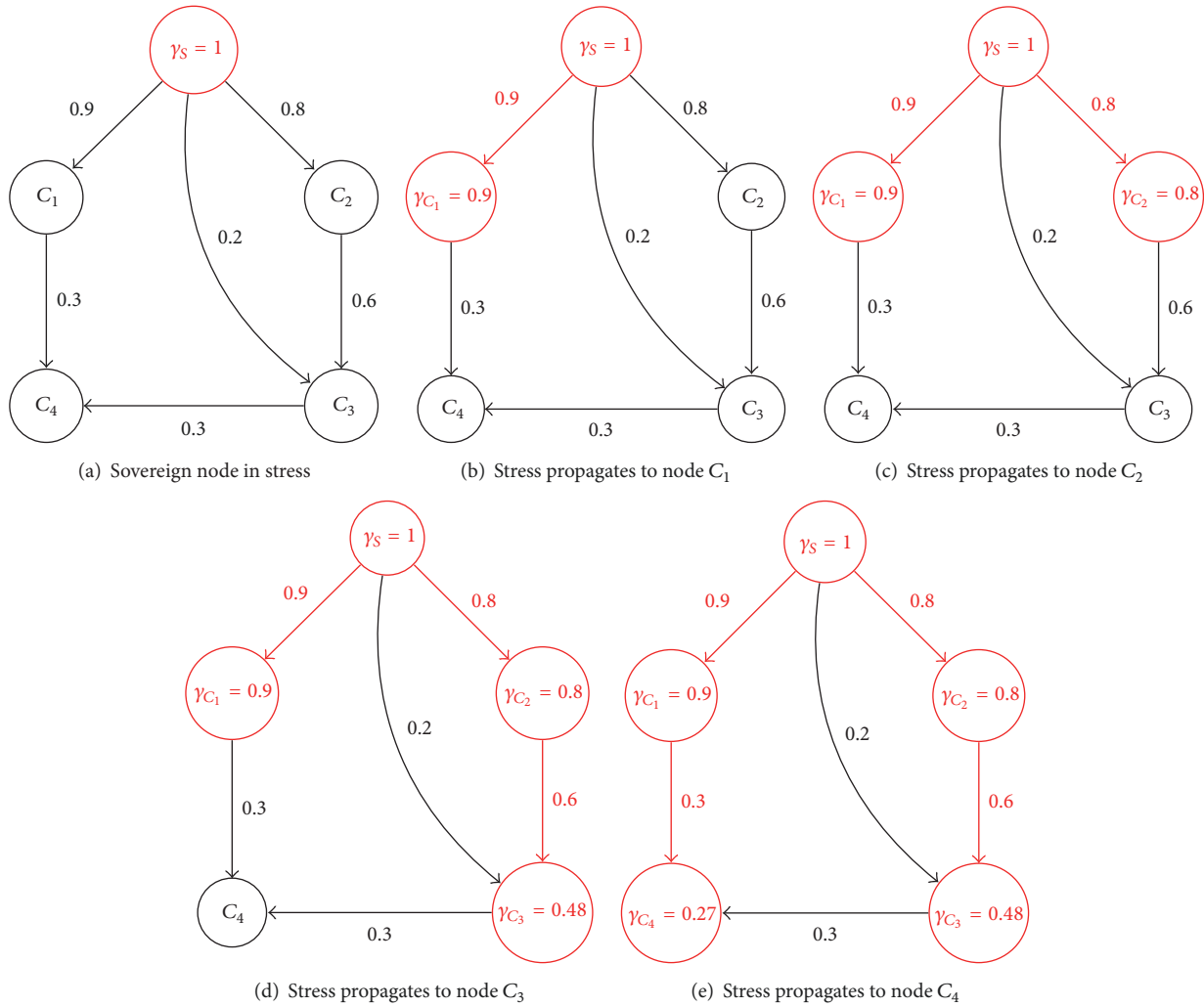


FIGURE 3: Illustration of the CountryRank parameter using a hypothetical network. The subfigures (a)–(e) show the propagation of stress in the network starting from the sovereign node to corporate nodes. At each step, the stress spreads to a node using the path with the maximum weight from the sovereign node.

Then, node C_2 gets stressed giving it a CountryRank value 0.8. For node C_3 , there are two paths from node S , so we pick the path through node C_2 having a higher weight of 0.48. Finally, there are three paths from node S to node C_4 , and the path with maximum weight is 0.27.

3.2.2. Network Construction. Credit default swap spreads are market-implied indicators of probability of default of an entity. A credit default swap is a financial contract in which a protection seller A insures a protection buyer B against the default of a third party C . More precisely, regular coupon payments with respect to a contractual notional N and a fixed rate s , the CDS spread, are swapped with a payment of $N(1 - RR)$ in the case of the default of C , where RR , the so-called recovery rate, is a contract parameter which represents the fraction of investment which is assumed be recovered in the case of default of C .

Modified ϵ -Draw-Up. We would like to measure to what extent changes in CDS spreads of different issuers occur simultaneously. For this, we use the notion of a *modified ϵ -draw-up* to quantify the impact of deterioration of credit quality of one issuer on the other. Modified ϵ -draw-up is an alteration of the ϵ -draw-ups notion which is introduced in [32]. In that article, the authors use the notion of ϵ -draw-ups to construct a network which models the conditional probabilities of spike-like comovements among pairs of CDS spreads. A modified ϵ -draw-up is defined as an upward movement in the time series in which the amplitude of the movement, that is, the difference between the subsequent local maxima and current local minima, is greater than a threshold ϵ . We record such local minima as the modified ϵ -draw-ups. The ϵ parameter for a local minima at time t is set to be the standard deviation in the time series between days $t - n$ and t , where n is chosen to be 10 days. Figure 5 shows

the time series of Russian Federation CDS with the calibrated modified ϵ -draw-ups using a history of 10 days for calibration.

Filtering Market Impact. Since we would like to measure the comovement of the time series i and j , we exclude the effect of the external market on these nodes as follows. We calibrate the ϵ -draw-ups for the CDS time series of an index that does not represent the region in question; for instance, for Russian issuers we choose the iTraxx index which is the composite CDS index of 125 CDS referencing European investment grade credit. Then, we filter out those ϵ -draw-ups of node i which are the same as the ϵ -draw-ups of the iTraxx index including a time lag τ . That is, if iTraxx has a modified ϵ -draw-up on day t , then we remove the modified ϵ -draw-ups of node i on days $t, t + 1, \dots, t + \tau$. We choose a time lag of 3 days for our calibration based on the input data which is consistent with the choice in [32].

Edges. After identifying the ϵ -draw-ups for all the issuers and filtering out the market impact, the edges in our network are constructed as follows. The weight of an edge in the credit stress propagation network from node i to node j is the conditional probability that if node i has an epsilon draw-up on day t , then node j also has an epsilon draw-up on days $t, t + 1, \dots, t + \tau$, where τ is the time lag. More precisely, let N_i be the number of ϵ -draw-ups of node i after filtering using iTraxx index and N_{ij} epsilon draw-ups of node i which are also epsilon draw-ups for node j with the time lag τ . Then, the edge weight w_{ij} between nodes i and j is defined as $w_{ij} = N_i/N_{ij}$. Figure 6 shows the minimum spanning tree of the credit stress propagation network constructed using the CDS spread time series data of Russian issuers.

Uncertainty in CountryRank. We test the robustness of our CountryRank calibration by varying the number of days used for ϵ -parameter. The figure in Appendix B shows that the ϵ -parameter for Russian Federation CDS time series remains stable when we vary the number of days. We initially obtain time series of ϵ -parameters by calculating standard deviation in the last $n = 10, 15,$ and 20 days on all local minima indices of Russian Federation CDS. Subsequently, we calculate the mean of the absolute differences between the epsilon time series calculated and express this in units of the mean of Russian Federation CDS time series. The percentage difference is 1.38% between the 10-day ϵ -parameter and 15-day ϵ -parameter and 2.22% between the 10-day and 20-day ϵ -parameters.

Further, we quantify the uncertainty in CountryRank parameter as follows. For an corporate node, we calculate the absolute difference in CountryRank calculated using $n = 15$ and 20 days with CountryRank using $n = 10$ days for the ϵ -parameter. We then calculate this difference as a percentage of the CountryRank calculated using 10 days for ϵ -parameter for all corporates and then compute their mean. The mean difference between CountryRank calibrated using $n = 15$ days and $n = 10$ days is 6.84% and $n = 20$ days and $n = 10$ days is 9.73% for the Russian CDS data set.

TABLE 1: Systematic factor: index mapping.

Factor	Index
Europe	MSCI EUROPE
Asia	MSCI AC ASIA
North America	MSCI NORTH AMERICA
Latin America	MSCI EM LATIN AMERICA
Middle East and Africa	MSCI FM AFRICA
Pacific	MSCI PACIFIC
Materials	MSCI WRLD/MATERIALS
Consumer products	MSCI WRLD/CONSUMER DISCR
Services	MSCI WRLD/CONSUMER SVC
Financial	MSCI WRLD/FINANCIALS
Industrial	MSCI WRLD/INDUSTRIALS
Government	ITRAXX SOVX GLOBAL LIQUID INVESTMENT GRADE

4. Numerical Experiments

We implement the framework presented in Section 3 to synthetic test portfolios and discuss the corresponding risk metrics. Further, we perform a set of sensitivity studies and explore the results.

4.1. Factor Model. We first set up a multifactor Merton model, as it was described in Section 2. We define a set of systematic factors that will represent region and sector effects. We choose 6 region and 6 sector factors, for which we select appropriate indexes, as shown in Table 1. We then use 10 years of index time series to derive the region and sector returns $F_{R(j)}$, $j = 1, \dots, 6$ and $F_{S(k)}$, $k = 1, \dots, 6$, respectively, and obtain an estimate of the correlation matrix Ω , shown in Figure 7. Subsequently, we map all issuers to one region and one sector factor, $F_{R(i)}$ and $F_{S(i)}$, respectively. For instance, a Dutch bank will be associated with Europe and financial factors. As a proxy of individual asset returns, we use 10 years of equity or CDS time series, depending on the data availability for each issuer. Finally, we standardize the individual returns time series ($X_{i,t}$) and perform the following Ordinary Least Squares regression against the systematic factor returns

$$X_{i,t} = \alpha_{R(i)}F_{R(i),t} + \alpha_{S(k)}F_{S(k),t} + \epsilon_{i,t} \quad (24)$$

to obtain $\hat{\alpha}_{R(i)}$, $\hat{\alpha}_{S(i)}$, and $\hat{\beta}_i = R^2$, where R^2 is the coefficient of determination, and it is higher for issuers whose returns are largely affected by the performance of the systematic factors.

4.2. Synthetic Test Portfolios. To investigate the properties of the contagion model, we set up 2 test portfolios. For these portfolios, the resulting risk measures are compared to those of the standard latent variable model with no contagion. Portfolio A consists of 1 Russian government bond and 17 bonds issued by corporations registered and operating in the Russian Federation. As it is illustrated in Table 2, the issuers are of medium and low credit quality. Portfolio B represents a similar but more diversified setup with 4 sovereign bonds

TABLE 2: Rating classification for the test portfolios.

Rating	Portfolio A		Portfolio B	
	Issuers	%	Issuers	%
AAA	-	0.00%	3	3.75%
AA	-	0.00%	3	3.75%
A	-	0.00%	22	27.50%
BBB	1	5.56%	39	48.75%
BB	15	83.33%	9	11.25%
B	2	11.11%	3	3.75%
CCC/C	-	0.00%	1	1.25%

TABLE 3: Sector classification for the test portfolios.

Sector	Portfolio A		Portfolio B	
	Issuers	%	Issuers	%
Materials	5	27.78%	12	15.00%
Consumer products	-	0.00%	12	15.00%
Services	3	16.67%	19	23.75%
Financial	7	38.89%	25	31.25%
Industrial	-	0.00%	6	7.50%
Government	3	16.67%	6	7.50%

issued by Germany, Italy, Netherlands, and Spain and 76 corporate bonds by issuers from the aforementioned countries. The sectors represented in Portfolios A and B are shown in Table 3. Both portfolios are assumed to be equally weighted with a total notional of €10 million.

4.3. Credit Stress Propagation Network. We use credit default swap data to construct the stress propagation network. The CDS raw data set consists of daily CDS liquid spreads for different maturities from 1 May 2014 to 31 March 2015 for Portfolio A and 1 July 2014 to 31 December 2015 for Portfolio B. These are averaged quotes from contributors rather than exercisable quotes. In addition, the data set also provides information on the names of the underlying reference entities, recovery rates, number of quote contributors, region, sector, average of the ratings from Standard & Poor's, Moody's, and Fitch Group of each entity, and currency of the quote. We use the normalized CDS spreads of entities for the 5-year tenor for our analysis. The CDS spreads time series of Russian issuers are illustrated in Figure 4.

4.4. Simulation Study. In order to generate portfolio loss distributions and derive the associated risk measures, we perform Monte Carlo simulations. This process entails generating joint realizations of the systematic and idiosyncratic risk factors and comparing the resulting critical variables with the corresponding default thresholds. By this comparison, we obtain the default indicator Y_i for each issuer and this enables us to calculate the overall portfolio loss for this trial. The only difference between the standard and the contagion model is that in the contagion model we first obtain the default indicators for the sovereigns, and their values determine which default thresholds are going to be

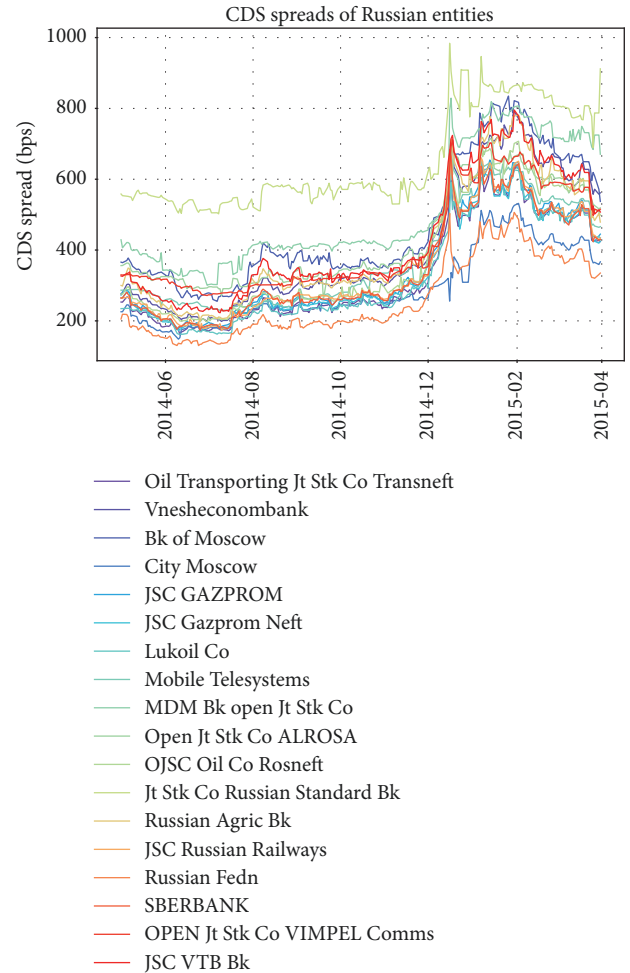
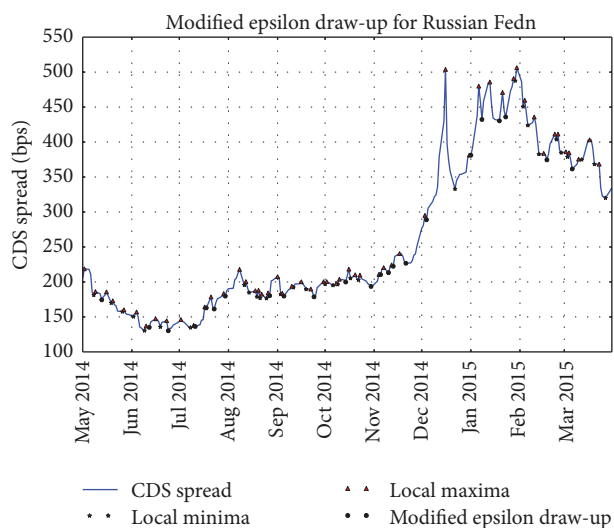


FIGURE 4: Time series of CDS spreads of Russian issuers.

TABLE 4: Portfolio losses for the test portfolios and additional risk due to contagion.

(a) Panel 1: Portfolio A				
Quantile	Loss standard model	Loss contagion model	Contagion impact	
99%	1,115,153	1,162,329	47,176	4%
99.50%	1,443,579	3,003,949	1,560,370	108%
99.90%	2,258,857	4,968,393	2,709,536	120%
99.99%	3,543,441	5,713,486	2,170,045	61%
Average loss	71,807	71,691		

(b) Panel 2: Portfolio B				
Quantile	Loss standard model	Loss contagion model	Contagion impact	
99%	373,013	379,929	6,915	2%
99.50%	471,497	520,467	48,971	10%
99.90%	775,773	1,009,426	233,653	30%
99.99%	1,350,279	1,847,795	497,516	37%
Average loss	44,850	44,872		

FIGURE 5: Time series for Russian Federation with local minima, local maxima, and modified ϵ -draw-ups.

used for the corporate issuers. The quantiles of the generated loss distributions as well as the percentage increase due to contagion are illustrated in Table 4. A liquidity horizon of 1 year is assumed throughout and the figures are based on a simulation with 10^6 samples.

For Portfolio A, the 99.90% quantile of the loss distribution under the standard factor model is €2,258,857, which corresponds to approximately 23% of the total notional. This figure jumps to €4,968,393 (almost 50% of the notional) under the model with contagion. As shown in Panel 1, contagion has a minimal effect on the 99% quantile, while at 99.5%, 99.90%, and 99.99% it results in an increase of 108%, 120%, and 61%, respectively. This is to be expected

as the probability of default for Russian Federation is less than 1% and thus, in more than 99.9% of our trials, default will not take place and contagion will not be triggered. For Portfolio B, the 99.90% quantile is considerably lower under both the standard and the contagion model, at €775,773 or 8% of the total notional and €1,009,426 or 10% of the total notional, respectively, reflecting lower default risk. One can observe that the model with contagion yields low additional losses at 99% and 99.5% quantiles, with a more significant impact at 99.90% and 99.99% (30% and 37%, respectively). An illustration of the additional losses due to contagion is given by Figure 8.

4.5. Sensitivity Analysis. In the following, we present a series of sensitivity studies and discuss the results. To achieve a candid comparison, we choose to perform this analysis on the single-sovereign Portfolio A. We vary the ratings of sovereign and corporates, as well as the CountryRank parameter, to draw conclusions about their impact on the loss distribution and verify the model properties.

4.5.1. Sovereign Rating. We start by exploring the impact of the credit quality of the sovereign. Table 5 shows the quantiles of the generated loss distributions under the standard latent variable model and the contagion model when the rating of the Russian Federation is 1 and 2 notches higher than the original rating (BB). It can be seen that the contagion effect appears less strong when the sovereign rating is higher. At the 99.9% quantile, the contagion impact drops from 120% to 62% for an upgraded sovereign rating of BBB. The drop is even higher, when upgrading the sovereign rating to A, with only 11% additional losses due to contagion. Apart from having a less significant impact at the 99.9% quantile, it is clear that, with a sovereign rating of A, the contagion impact is zero at the 99% and 99.5% levels, where the results of the

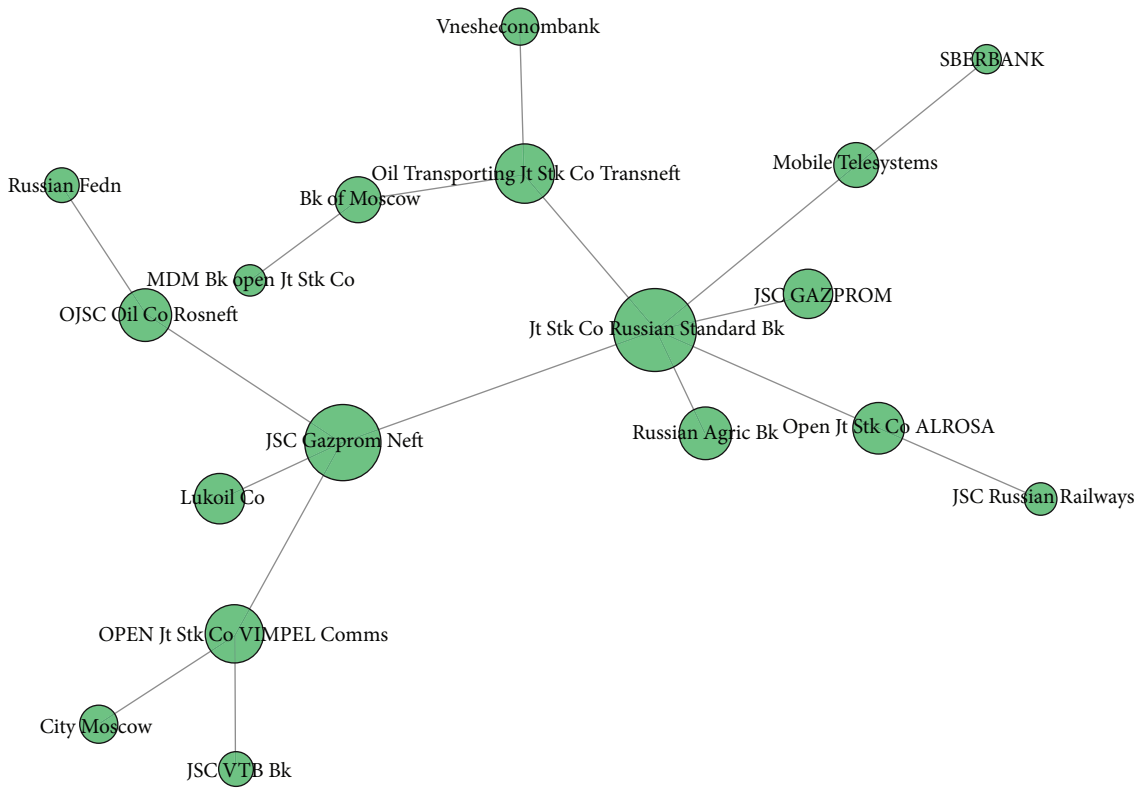


FIGURE 6: Minimum spanning tree for Russian issuers.

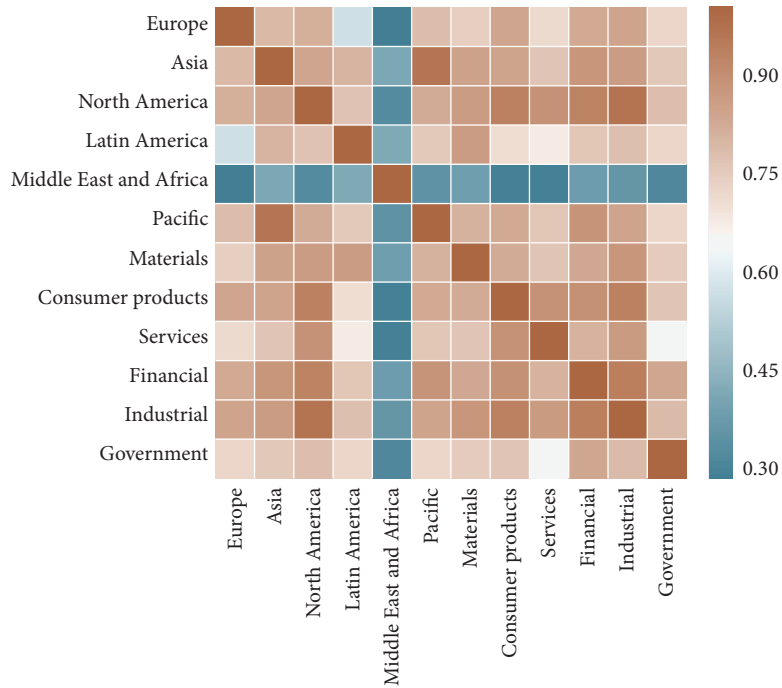


FIGURE 7: Estimated systematic factor correlation matrix $\hat{\Omega}$.

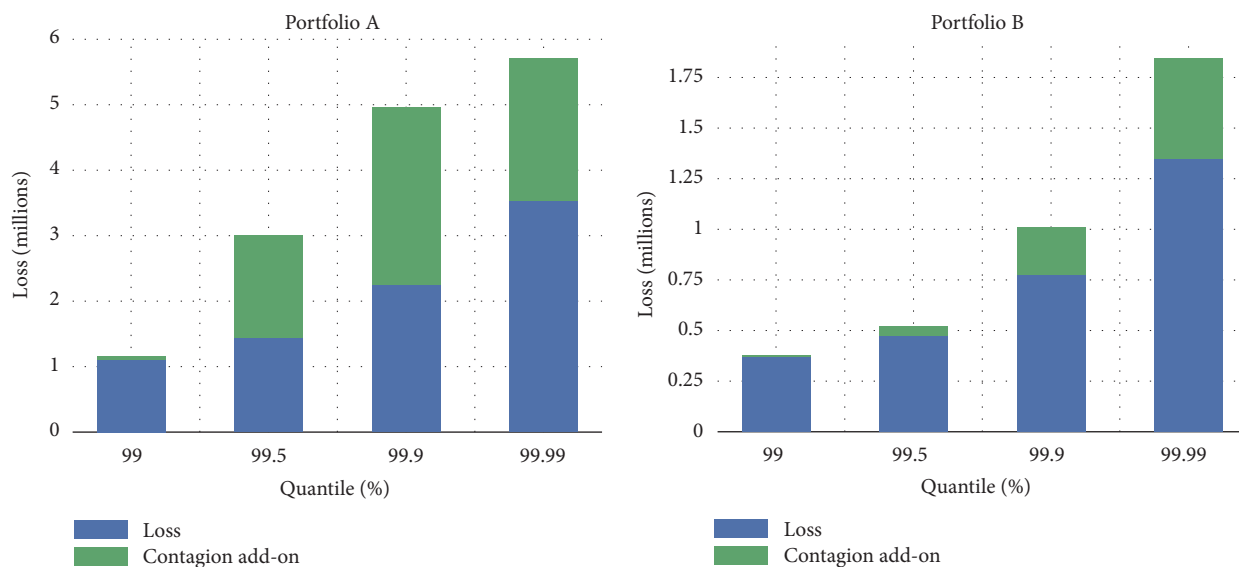


FIGURE 8: Additional losses due to contagion for Portfolios A and B.

TABLE 5: Varying the sovereign rating.

Sovereign rating	Quantile	Loss standard model	Loss contagion model	Contagion impact	
BB	99%	1,115,153	1,162,329	47,176	4%
	99.50%	1,443,579	3,003,949	1,560,370	108%
	99.90%	2,258,857	4,968,393	2,709,536	120%
	99.99%	3,543,441	5,713,486	2,170,045	61%
BBB	99%	1,115,153	1,115,153	-	0%
	99.50%	1,443,009	1,490,755	47,746	3%
	99.90%	2,229,742	3,613,625	1,383,883	62%
	99.99%	3,496,264	5,432,236	1,935,972	55%
A	99%	1,114,583	1,114,583	-	0%
	99.50%	1,443,009	1,443,009	-	0%
	99.90%	2,229,737	2,469,922	240,185	11%
	99.99%	3,455,199	5,056,639	1,601,439	46%

contagion model match those of the standard model. This is to be expected since a rating of A corresponds to a probability of default less than 0.01%, and as explained in Section 4.4, when sovereign default occurs seldom, the contagion effect can hardly be observed.

4.5.2. Corporate Default Probabilities. In the next test, the impact of corporate credit quality is investigated. As Table 6 illustrates, contagion has smaller impact when the corporate default probabilities are increased by 5%, which is in line with intuition since the autonomous (not sovereign induced) default probabilities are quite high, meaning that they are likely to default whether the corresponding sovereign defaults or not. For the same reason, the impact is even less significant when the corporate default probabilities are stressed by 10%.

4.5.3. CountryRank. In the last test, the sensitivity of the contagion impact to changes in the CountryRank is investigated. In Table 7, we test the contagion impact when CountryRank is stressed by 15% and 10%, respectively. The results are in line with intuition, with a milder contagion effect for lower CountryRank values and a stronger effect in case the parameter is increased.

5. Conclusions

In this paper, we present an extended factor model for portfolio credit risk which offers a breadth of possible applications to regulatory and economic capital calculations, as well as to the analysis of structured credit products. In the proposed framework, systematic risk factors are augmented

TABLE 6: Varying corporate default probabilities.

Corporate default probabilities	Quantile	Loss standard model	Loss contagion model	Contagion impact	
Unstressed	99%	1,115,153	1,162,329	47,176	4%
	99.50%	1,443,579	3,003,949	1,560,370	108%
	99.90%	2,258,857	4,968,393	2,709,536	120%
	99.99%	3,543,441	5,713,486	2,170,045	61%
Stressed by 5%	99%	1,115,153	1,250,570	135,417	12%
	99.50%	1,443,579	3,003,949	1,560,370	108%
	99.90%	2,333,935	4,968,393	2,634,458	113%
	99.99%	3,584,506	5,713,486	2,128,979	59%
Stressed by 10%	99%	1,162,329	1,260,422	98,093	8%
	99.50%	1,503,348	3,003,949	1,500,602	100%
	99.90%	2,375,570	4,968,393	2,592,823	109%
	99.99%	3,642,099	5,713,486	2,071,387	57%

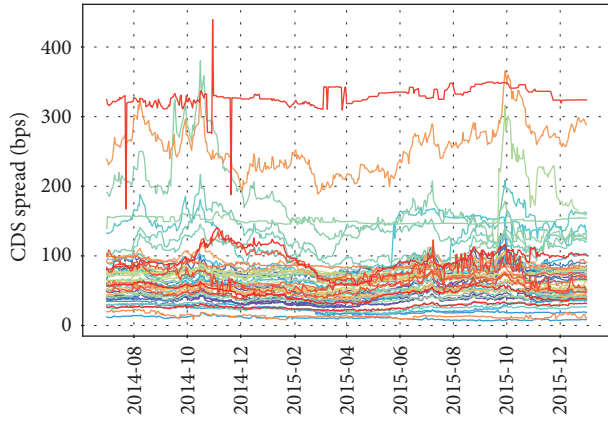
TABLE 7: Varying CountryRank.

CountryRank	Quantile	Loss standard model	Loss contagion model	Contagion impact	
Unstressed	99%	1,115,153	1,162,329	47,176	4%
	99.50%	1,443,579	3,003,949	1,560,370	108%
	99.90%	2,258,857	4,968,393	2,709,536	120%
	99.99%	3,543,441	5,713,486	2,170,045	61%
Stressed by 5%	99%	1,115,153	1,162,329	47,176	4%
	99.50%	1,443,579	3,196,958	1,753,379	121%
	99.90%	2,258,857	5,056,634	2,797,777	124%
	99.99%	3,543,441	5,713,495	2,170,054	61%
Stressed by 10%	99%	1,115,153	1,162,329	47,176	4%
	99.50%	1,443,579	3,389,398	1,945,818	135%
	99.90%	2,258,857	5,296,249	3,037,392	134%
	99.99%	3,543,441	5,801,727	2,258,286	64%

with an infectious default mechanism which affects the entire portfolio. Unlike models based on copulas with more extreme tail behavior, where the dependence structure of defaults is specified in advance, our model provides an intuitive approach, by first specifying the way sovereign defaults may affect the default probabilities of corporate issuers and then deriving the joint default distribution. The impact of sovereign defaults is quantified using a credit stress propagation network constructed from real data. Under this framework, we generate loss distributions for synthetic test portfolios and show that the contagion effect may have a profound impact on the upper tails.

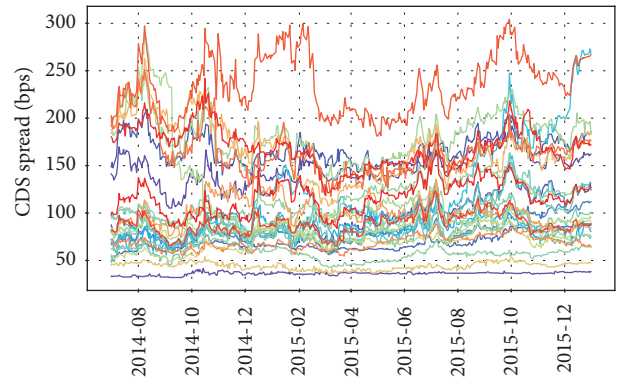
Our model provides a first step towards incorporating network effects in portfolio credit risk models. The model can

be extended in a number of ways such as accounting for stress propagation from a sovereign to corporates even without sovereign default or taking into consideration contagion between sovereigns. Another interesting topic for future research is characterizing the joint default distribution of issuers in credit stress propagation networks using Bayesian network methodologies, which may facilitate an improved approximation of the conditional default probabilities in comparison to the maximum weight path in the current definition of CountryRank. Finally, a conjecture worthy of further investigation is that a more connected structure for the credit stress propagation network leads to increased values for the CountryRank parameter, and, as a result, to higher additional losses due to contagion.



- | | |
|--------------------------------------|---------------------------|
| — Allianz SE | — Robert Bosch GmBH |
| — EnBW Energie Baden Wuerttemberg AG | — Rheinmetall AG |
| — BASF SE | — RWE AG |
| — Bertelsmann SE Co KGaA | — Siemens AG |
| — Bay Motoren Werke AG | — UniCredit Bk AG |
| — Bayer AG | — Volkswagen AG |
| — Bay Landbk Giroz | — Aegon N.V. |
| — Commerzbank AG | — Koninklijke Ahold N V |
| — Contl AG | — Akzo Nobel NV |
| — Daimler AG | — Eneco Hldg N V |
| — Deutsche Bk AG | — Essent N V |
| — Deutsche Bahn AG | — ING Groep NV |
| — Fed Rep Germany | — ING Bk N V |
| — Deutsche Post AG | — Koninklijke DSM NV |
| — Deutsche Telekom AG | — Koninklijke Philips NV |
| — EON SE | — Koninklijke KPN N V |
| — Fresenius SE & Co KGaA | — UPC Hldg BV |
| — Grohe Hldg Gmbh | — Kdom Neth |
| — Hannover Rueck SE | — Nielsen Co |
| — HeidelbergCement AG | — NN Group NV |
| — Henkel AG & Co KGaA | — PostNL NV |
| — Linde AG | — Rabobank Nederland |
| — Deutsche Lufthansa AG | — Royal Bk of Scotland NV |
| — METRO AG | — SNS Bk NV |
| — Munich Re | — Stmicroelectronics N V |
| — TUI AG | — Unilever N V |
| | — Wolters Kluwer N V |

FIGURE 9: Time series of CDS spreads of Dutch and German entities.



- | |
|---------------------------------------|
| — ALTADIS SA |
| — Bco de Sabadell S A |
| — Bco Bilbao Vizcaya Argentaria S A |
| — Bco Pop Espanol |
| — Endesa S A |
| — Gas Nat SDG SA |
| — Iberdrola SA |
| — Instituto de Cred Oficial |
| — REPSOL SA |
| — Bco SANTANDER SA |
| — Kdom Spain |
| — Telefonica S A |
| — Assicurazioni Generali S p A |
| — ATLANTIA SPA |
| — Mediobanca SpA |
| — Bco Pop S C |
| — CIR SpA CIE Industriali Riunite |
| — ENEL S p A |
| — ENI S.p.A. |
| — Edison S p A |
| — Finmeccanica S p A |
| — Rep Italy |
| — Bca Naz del Lavoro S p A |
| — Bca Pop di Milano Soc Coop a r l |
| — Bca Monte dei Paschi di Siena S p A |
| — Intesa Sanpaolo SpA |
| — Telecom Italia SpA |
| — UniCredit SpA |

FIGURE 10: Time series of CDS spreads of Spanish and Italian entities.

Appendix

A. CDS Spread Data

The data used to calibrate the credit stress propagation network for European issuers is the CDS spread data of Dutch, German, Italian, and Spanish issuers as shown in Figures 9 and 10.

B. Stability of ϵ -Parameter

The plot in Figure 11 shows the time series of the epsilon parameter for different number days used for ϵ -draw-up calibration.

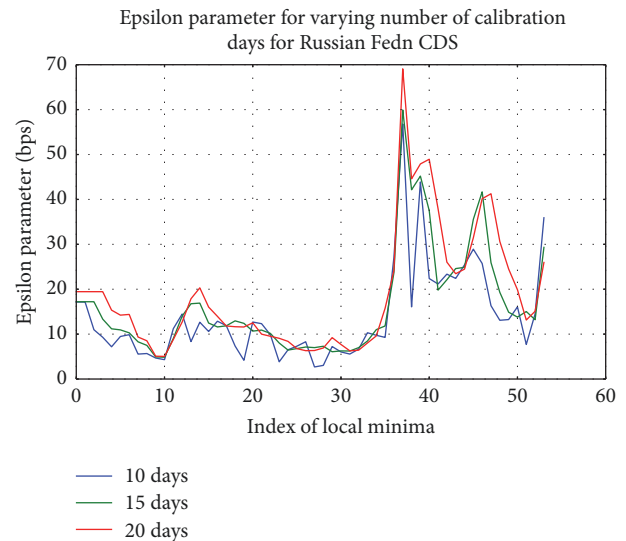


FIGURE 11: Stability of ϵ parameter for Russian Federation CDS.

Disclosure

The opinions expressed in this work are solely those of the authors and do not represent in any way those of their current and past employers.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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