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ON THE REALITY OF SPACETIME GEOMETRY AND THE WAVEFUNCTION

by

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Abstract

The action-reaction principle (AR) is examined in three contexts: 1) the inertial-gravitational interaction between a particle and space-time geometry, 2) protective observation of an extended wave function of a single particle, and 3) the causal-stochastic or Bohm interpretation of quantum mechanics. A new criterion of reality is formulated using the AR principle. This criterion implies that the wave function of a single particle is real and justifies in the Bohm interpretation the dual ontology of the particle and its associated wave function. But it is concluded that the Bohm theory is not dynamically complete because the particle and its associated wave function do not satisfy the AR principle.

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0. Introduction

It is well known that the dynamics of particles and fields, in classical and quantum physics, may be described by the action principle. The space-time translational invariance of the action of the system under consideration implies that the energy- momentum of the system is conserved. This means that the different components of the system satisfy the action-reaction (AR) principle. But if the action does not have translational invariance, then we would say, rather than give up energy- momentum conservation, that there is some *external* influence on the system, so that the internal components of the system do satisfy the AR principle. This suggests that the AR principle is more fundamental than any other law of physics, as if it is a condition for the reality and being of the entities in a physical theory.

We shall re-examine, in section I, the inertial guidance of the motion of a particle by space-time geometry, and the particle's reaction back on the space-time geometry which is called gravitation. It is argued again, in section II, that this interaction must be quantum mechanical in order to satisy a stronger version of the AR principle. In section III, we apply the AR principle to the wave function interacting with a current. A new criterion for the reality of an entity, namely that it must satisfy the AR principle with another entity, is formulated. This reality criterion is applied, in section IV, to the Bohm or the causal-stochastic interpretation of quantum mechanics to justify the dual ontology given to the particle and its associated wave function in some versions of the Bohm theory.

I. Space-time Geometry, Action-Reaction Principle, and Dynamical Completeness

The general theory of relativity, according to Einstein, removed a defect common to both Newtonian mechanics and special relativity. In both of these theories, Einstein attributed to the 'ether', the term he used in 1924 to refer to the system of inertial frames, an 'absolute' status. Such an ether is of course required to account for the existence of inertial effects such as the centrifugal and coriolis forces in non-inertial frames, as in the celebrated rotating bucket experiment of Newton. Yet according to Einstein (1),

"Each physical object influences and in general is influenced in turn by others. The latter, however, is not true of the ether in Newtonian mechanics. The inertia-producing property of this ether, in accordance with classical mechanics, is precisely *not* to be influenced, either by configuration of matter, or by anything else. For this reason, one may call it 'absolute'. That something real has to be conceived as the cause for the preference of an inertial system over a non-inertial system is a fact that physicists have only come to understand in recent years. ... Also, following the special theory of relativity, the ether was absolute, because its influence on inertia and light propagation was thought to be independent of physical influences of any kind. ... The ether of the general theory of relativity differs from that of classical mechanics or the special theory of relativity respectively, in so far as it is not 'absolute', but is determined in its locally variable properties by ponderable matter. This determination is complete if the universe is closed and spatially finite."

On another occasion, he wrote(2) that in the general theory:

"Space and time were thereby divested not of their reality but of their causal absoluteness - i.e., affecting but not affected - which Newton had been compelled to ascribe to them in order to formulate the laws then known."

Einstein clearly saw the causal absoluteness of the inertial structure (what we today would call the projective structure(3)) of both the Newtonian and Minkowskian spacetimes as not just a feature lost in the transition to general relativity but one that was incompatible with any reasonable theory of motion. As he wrote in 1922 (4):

"... it is contrary to the mode of thinking in science to conceive of a thing (the space-time continuum) which acts itself, but which cannot be acted upon. This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics."

We shall say that two physical entities satisfy the *Action- Reaction (AR) principle*, if they interact in such a manner that each entity both acts on and is acted on by the other entity. We shall encapsulate Einstein's intuition in the statement that a physical theory is ***dynamically complete*** if all the entities postulated in the theory pairwise satisfy the AR principle. Spacetime structure in general relativity is affected, if not wholly determined, by the distribution of matter, as well as itself determining the privileged motion of free bodies. General relativity is thus dynamically complete.

It was the connection in Einstein's thinking between the AR principle in its spacetime manifestation and Mach's principle, that led him to suggest in the first passage above that the complete implementation of the AR principle only holds in general relativity for a closed finite universe. This was because in an infinite universe, asymptotic spacetime structure could not be fixed by the distribution of matter. However, it seems that Einstein's eventual doubts about the validity of Mach's principle, which in 1924 he dismissed(1) as involving action-at-a-distance, did not extend to an advantage the spacetime continuum in general relativity enjoyed in relation to its predecessors, viz. the satisfaction of the AR principle. The difference between the two lies in the locality principle being satisfied in relativity theories, as opposed to Newtonian theories which involve action at a distance. By the locality principle we mean that each object can only influence an object immediately neighboring it in space-time. A distant object may be influenced only by this influence propagating in space-time from the first object to the second. In the statement of the AR principle above, for the special case of relativity theories, the interaction should be restricted to occur between neighboring objects in this sense. To be precise, in general relativity the AR principle is satisfied because

$$G^{\mu\nu} = 8\pi\kappa T^{\mu\nu}, T^{\mu\nu}{}_{;\nu} = 0, \tag{1}$$

where $G^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu}$ is the energy-momentum tensor. (1) ensures that the AR principle is satisfied for the material objects that contribute to $T^{\mu\nu}$ and the space-time geometry which incorporate the gravitational field in their *local* interactions with one another.

Thus even though the AR principle is Machian, when it is combined with the locality principle it acquires an anti-Machian flavor in the following sense: The space-time geometry immediately surrounding a particle needs to be regarded as an object or a medium which influences the motion of the particle (the origin of inertia), but in turn is influenced by the energy- momentum of the particle (the origin of gravity). So, we cannot regard the local inertial properties to be a consequence of the distant stars, as Mach contended in his famous critique of Newton's rotating bucket experiment, in any immediate sense. This anti-Machian nature of the locality principle is at the heart of the difficulties in formulating Mach's principle in relativity theories. On the other hand, general relativity is Machian in the sense that space-time is dynamical unlike in Newtonian physics in which it is absolute.

These considerations suggest that the AR principle above is more fundamental than Mach's principle. In Newtonian physics, the AR principle applied to material bodies is consistent with Mach's principle if we require that the inertia of local matter is due to distant stars. In relativistic theories, the inertial properties are explained as being due to the interaction between matter and the space-time region in its immediate vicinity, both of which satisfy the AR principle. The Newtonian and Einsteinian theories may be expected to satisfy the AR principle, whether they satisfy Mach's principle or not.

II. Quantum Theory and the Action Reaction Principle

Another possible violation of dynamical completeness may exist in the determination of the geometric phase by the geometry of the projective Hilbert space in quantum mechanics(5). This geometric phase factor is due to parallel transport around a closed curve (holonomy transformation) in the projective Hilbert space with respect to a non-flat connection, which arises from the inner product in the Hilbert space. But the quantum system undergoing the cyclic evolution does not react back on the geometry of this space, according to present theory. Such a geometrical structure may therefore be 'absolute' in Einstein's sense. If the connection in the projective Hilbert space is said to 'act' on the phase of the wavefunction, then the theory is not dynamically complete. What this means is that such geometric structure plays no real independent dynamical role in quantum mechanics. Just as the projective structure in Newtonian and Minkowski spacetime acts merely as a codification, and not a dynamical cause, of free body motion, so the geometric structure of the projective Hilbert space is essentially a codification of a feature of Schrdinger dynamics that is independent of the Hamiltonian which drives the state around a given closed curve. This suggests that we should perhaps let the geometry of the projective Hilbert space become dynamical so that the system can react back on it.

However, quantum mechanics may itself play an important role in clarifying the nature of the dynamical completeness in general relativity. It seems natural for the purposes of empirically determining the gravitational field to make recourse to quantum, and not classical 'probes', or test particles. In one such approach using quantum probes(8), the gravitational field is defined in terms of a path dependent phase factor, an element of the Poincare group, in a fashion analogous to the determination of the electromagnetic field by means of the Dirac phase factor, an element of the $U(1)$ group, in quantum mechanics. The action of this gravitational phase factor on the wave function of a spinless particle with fairly well defined energy-momentum, gives rise to a phase factor $exp(i\phi)$, where the phase

$$\phi = -\frac{mc}{\hbar}s, \tag{2}$$

with m being the mass of this particle and s is the space-time distance determined by the metric along the unperturbed classical trajectory(6,7,8). That is the phase is the space-time distance measured in units of Compton wave-length.

A special case, which illustrates (2), is the neutron interferometry experiment(9) in the presence of the earth's gravitational field(10). Neglecting curvature effects, we can choose a coordinate system, with the $z-$axis vertical, in which the only component of the metric tensor $g_{\mu\nu}$ which differs from its Minkowskian value is $g_{00} = 1 + \frac{2gz}{c^2}$, where g is the acceleration due to gravity. In this experiment, the two horizontal parallel neutron beams are at different heights, say $z = 0$ and $z = H$. Hence, the phase difference between the interfering beams, from (2), is $\Delta\phi = -\frac{mc}{\hbar}(\sqrt{g_{00}(H)}ct - \sqrt{g_{00}(0)}ct) = -\frac{mgHt}{\hbar}$, where t is the time of flight of the unperturbed horizontal beam. Then $t = \frac{L}{v} = \frac{mL}{\hbar k}$, where v and k are the velocity and wave number of the unperturbed horizontal beam. Thus the phase shift is $\Delta\phi = -\frac{m^2 gA}{\hbar^2 k}$, where A is the oriented area enclosed by the interfering beams, which was first obtained by Overhauser and Colella$^{(10)}$ by means of a purely non relativistic derivation.

An interesting aspect of the above derivation is that this phase shift may be regarded as a manifestation of time dilation in a gravitational field that is responsible for the variation of g_{00} with height. Equivalently, this phase shift may be regarded as being due to the gravitational red shift undergone by the neutron frequency as it ascends the gravitational field, which has been observed for light by Pound and Rebka, although in the present case a purely Newtonian derivation of the observed result is also possible owing to the fact that the neutron is massive, which enables a Newtonian approximation.

The mass-dependence of the phase shift (2) implies that the effect of the space-time metric on test particles is dynamical in the sense that it is different for test particles with different masses. This is unlike in classical general relativity, in which the spacetime structure acts on a test particle in a mass-independent manner. This mass-dependence of the phase, may be construed as accounting in turn for the reaction of mass on the spacetime structure, as opposed to some other property of matter, which was pointed out by Anandan$^{(8)}$. In contrast, in classical general relativity, the prediction that the reaction of matter on spacetime involves the mass introduces a certain degree of arbitrariness. This arbitrariness is removed by quantum theory. Thus just as much as general relativity dynamically completes special relativity, as observed by Einstein, quantum theory dynamically completes general relativity in the sense of removing this arbitrariness$^{(8)}$.

This suggests that we should modify the AR principle to a stronger AR principle by requiring that two entities satisfy this principle if each entity acts and is acted upon by the other entity, and the action and reaction between them depend on the same quantities or variables. In the above example, these are the mass of the particle and the metric of the space-time geometry it is interacting with. They are both involved in the quantum mechanical action of the geometry on the particle as well as the reaction of the particle on the geometry. Similarly, a stronger version of dynamical completeness of a theory is that all entities in the theory satisfy, pairwise, the stronger version of the AR principle.

Notice that the quantum probe is incapable of giving any direct operational meaning to the geodesics of the affine connection, for the obvious reason that in most interpretations of QM quantum systems fail to have definite trajectories, whether in free fall or otherwise. (The most important counterexample is provided by the Bohm interpretation, to which we return later. But even in this interpretation, free-fall particle trajectories do not follow geodesics of the connection.) However, the affine connection may be operationally determined by parallel transporting spin systems around closed curves, which then are acted upon by the corresponding holonomy transformation that is an element of the Lorentz group.

Classically, there is nothing dynamical about this because all spin systems undergo the *same* Lorentz transformation when parallel transported around a given closed curve. But quantum mechanically we can take the *superposition* of the original state with the Lorentz transformed state. This superposition is different for different spins because the Lorentz transformation acts differently on different spin states, by definition of spin states. Such a superposition occurs when two coherent beams are interfered in a curved space-time: The two interfering states are related partly by the $SL(2, C)$ holonomy transformation associated with a closed curve through the interfering beams(6,8). Therefore, this superposition is in principle observable.

As an example, consider a closed path in spacetime for which the holonomy transformation of the connection is the rotation by an angle θ about an axis, which we shall take to be the $z-$axis. Consider a spin state $|j>$ that is an eigenstate of the spin angular momentum operator J_z with eigenvalue j, which is an integer or half-integer. The effect of the holonomy transformations is to multiply this state by the phase factor $exp(ij\theta)$. A general spin state of a particle is a superposition of these states $\{|j>\}$ with a given total angular momentum. (The state of the particle is of course characterized also by other quantum numbers such as momentum.) It is clear therefore that the interference pattern is affected in a manner that depends on the spin state.

Hence, the coupling of spin to the space-time connection is dynamical in the sense that a given connection influences quantum states with different spins differently. So, the above stronger version of the AR principle implies that spin must react back on the connection. The connection may then be expected to be independent of the metric, i. e. it may contain torsion(11).

The modification of the motion of an arbitrary wave function due to gravity, according to Huygen's principle, is determined by the phase shift in the interference of secondary wavelets. Therefore the above mentioned phase shifts that depend on mass and spin, which correspond to the Casimir operators of the Poincare group, imply that gravity affects the motion of an arbitrary quantum state in a manner which depends on these two dynamical quantities. Then, in accordance with the strong AR principle, these quantites react back on gravity. Hence, quantum mechanics completes the picture of dynamical completeness provided in classical general relativity.

III. The Action-Reaction Principle and the Reality of the Wavefunction

Let us start by supposing quantum mechanics is complete, in the sense that pure ensembles are von-Neumann homogeneous(12), or equivalently, that there are no hidden variables. The quantum mechanical indeterminacies (as specified by the Heisenberg-Robertson indeterminacy relations) associated with a given system such as an electron are then considered as intrinsic limits on the definability of the relevant observables associated with the single, given system. Since this objective fuzziness of the properties of a single system is determined by the wavefunction associated with it, it is not unreasonable to conclude that the wavefunction is an entity associated with the *single* system, and not in the first instance an ensemble of similarly prepared systems.

To see that the wavefunction satisfies the AR principle, it is enough to consider the following two quantum mechanical effects. The first is the magnetic Aharonov-Bohm effect(13): the current in the solenoid acts on the wavefunction of the electron so as to produce a shift in the interference pattern on a screen. Since, even if the intensity is low so that the there is only one electron at a time, the accumulated electrons registered on the screen gives the interference pattern, we may suppose that the current acts on each electron. The second is a recent thought experiment(14), in which an electron moves between two separated, identical boxes containing a second electron, the latter being in a superposition of the two stationary states corresponding to confinement in each box. If the second electron wave function is 'protected'(14,15) by means of tunneling between the two boxes, then the moving electron is predicted to accelerate in a straight, undeflected line between the two boxes as if the stationary electron is simultaneously in both boxes. That is the extended wavefunction is acting on the current, preventing it from being deflected towards one of the boxes as would occur in the usual measurement (when the electron wave function inside the boxes is not protected). The change of speed of the moving electron implies that its propagation is also different from it would be if the boxes were empty.

Thus while in the Aharonov-Bohm effect, the current in the solenoid acts on the extended wave function (because the phase shift is proportional to the integral of the vector potential due to the current around the wave function) of a single electron, in the double box protective experiment mentioned above, the *extended* wave function of a *single* electron acts on the current. Thus the wave function satisfies the AR principle. (Although the AR principle is demonstrated here by means of two separate experiments, this is because either the action or reaction is negligible in each experiment under the conditions in which it is performed.) This suggests a new criterion of *reality*: an object may be regarded as real if it satisfies the AR principle with another, (in principle) observable object.

The usual criterion for the reality of an object used by physicists is that the object should be observable at least in principle. Because it is possible to protectively observe the wave function of a single particle, although this is done under special conditions, it was argued that the wave function of a single particle is real(14,15). But now we have introduced a new criterion of reality, using the AR principle, which reinforces the view that the wave function of a single particle is real.

IV. The Action Reaction Principle and the Bohm theory

The principal problem in the foundations of quantum theory is the difficulty in accounting for the existence of definite (von Neumann measurement) events in the world, which confronts any interpretation postulating that the theory is complete, in the sense given in the beginning of section III. As is well known, radically different attempts have been made to solve the problem, including introducing spontaneous collapse mechanisms as corrections to the time-dependent Schrdinger equation, and extending the quantum ontology to a many-worlds scenario. Yet another approach involves introducing hidden variables, and hence (following the work of Bell and others) the necessity of nonlocality.

That such radical steps have been taken to solve the problem (involving theories that suffer from one or more of a series of defects, such as ad hocness, lack of definiteness and violation of Lorentz covariance) testify to the gravity of the problem and the failure of any real consensus as to the broad nature of its likely solution. However, in the stakes of definiteness and clarity, the Bohm hidden variable interpretation, also called the De-Broglie- Bohm, or the causal, or the causal-stochastic interpretation, scores high marks. In the Bohm theory of non-relativistic quantum mechanics, each particle has a definite spatial position at each instant of time (the spatial coordinate being the hidden variable), the instantaneous velocity of the particle is proportional to the gradient of the phase of the wavefunction evaluated at its location. For a survey of the causal interpretation, see refs. 16-18 and references therein.

It may appear at first sight that the Bohm interpretation suffers from the defect that Einstein(19) attributed in 1905 to the ether-based version of electrodynamics: it "leads to asymmetries which do not appear to be inherent in the phenomena". In the thought-experiment involving two boxes, mentioned in section III, the trajectory of the moving, undeflected particle defines an axis of (mirror) symmetry relative to the wavefunction of the confined particle. The latter wave function that is symmetrical with respect to this axis is all that is needed to predict the motion. Yet the Bohm particle corresponding to the confined electron may be at all times at rest in one of the boxes, which appears to violate this symmetry.

But, unlike its classical counterpart, the Bohm particle is *not* the localized seat of the associated dynamical properties, such as mass, charge and magnetic moment. This was shown(20) for the 'passive' properties such as the passive gravitational mass, and the passive electric charge, i. e. they must be associated with the extended wavefunction, and not with the Bohm particle. The strong AR principle implies that the 'active' counterparts of these properties suffer a similar fate. The above example of the wave function in the two boxes, which may interact with the moving particle by gravitational or electromagnetic interaction, now indeed shows that the 'active' properties such as the active gravitational mass and the active electric charge may also be associated with the entire wave function at least during protective observations. Also, if the Bohm particle, which is at rest in one of the two boxes, were influencing the moving particle, then this would violate the above mentioned symmetry, in this thought experiment. Therefore, these active properties cannot be associated with the Bohm particle, and must be associated with the wave function.

In the 'dualist' version of the Bohm theory, reality is attributed both to the particle trajectories and to the wavefunction, the latter being treated as a real 'ψ-field' which causally acts on the particle, 'informing' it as to where to go. But the particle does not react back on the wave function. So, they do not satisfy the AR principle between them. Therefore, neither the particle nor the wave function could be regarded as real on the basis of this interaction alone, according to the criterion of reality we introduced above. However, by means of the protective observation and the criterion of reality given in section

III, we showed that the wave function is real.

Can we demonstrate similarly the reality of the particle using the AR principle? Experimentally, when the wave function interacts with a macroscopic screen a spot appears on a portion of the screen. This shows that the AR principle is satisfied by this portion and an entity it interacts with. If we do not collapse the wave function to the region of this spot or adopt the Everett many worlds view, then we are compelled to treat this entity as different from the wave function, and it is called the 'particle'. Under these assumptions, one may be justified is assigning reality to the particle as well. But this particle may not have the Bohm trajectory, which has not been observed, and may not have a trajectory at all. But everything we have said so far is consistent with it having the Bohm trajectory. Therefore, we may conclude that both the particle and the wave are real, under the above assumptions, according to the new criterion of reality.

Nevertheless, in our view, the Bohm theory suffers from two defects. First, as already mentioned, it violates the AR principle, which prevents this theory from being dynamically complete. We have seen, in section III, that the ψ-field of an electron, say, influences and is influenced by a separate current, so it satisfies the AR principle in this context. But the Bohm particle associated with the ψ- field fails to satisfy the principle in relation to the ψ-field itself. Of course, it is affected by the ψ-field in the manner mentioned earlier. But the evolution of the ψ-field is in no way affected by the presence or nature of the particle trajectory: it is governed by the time-dependent Schrodinger equation willy- nilly. Holland [ref. 17, p. 79] has made the same observation and suggested that Newton's third law is violated in this theory.

The other defect is that without introducing by fiat the standard probability distribution equal to $|\psi|^2$ for an ensemble of Bohm particles at some point in its evolution, it is impossible to reproduce the statistics predicted in quantum mechanics for that ensemble and the (happy) inability of entangled quantum systems to be used, despite the action-at-a-distance occurring between them in the Bohm theory, to send superluminal signals. For this reason, it was argued that the Bohm interpretation describes the evolution of a mixed state, as opposed to a pure state, of the particle$^{(21)}$. This is a generic feature of all hidden variable theories.

The ψ-field thus exercises a mysterious double act: it 'informs' a given particle how to move, and also determines the density of its 'sisters' in space. This explains the terminology 'causal-stochastic' for this interpretation. So, the wave function plays an epistemological role as far as predicting the position of the particle is concerned, and an ontological role as far as protective observations are concerned. It is important to give an explanation for this dual role. This question has been addressed by Durr et al$^{(}$22).

As for the dynamical incompleteness in the Bohm theory mentioned earlier, recognition of the seriousness of the problem has recently led Squires$^{(23)}$ to take steps in an attempt to modify Bohm dynamics and thereby remove this defect. In our view, unless it can be remedied, the Bohm theory fails to provide a satisfactory account of the nature of particle trajectories, and hence of the process of obtaining definite outcomes in measurement which is its true raison d'tre.

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