FINITE ALTERNATING-MOVE ARBITRATION SCHEMES AND THE EQUAL AREA SOLUTION

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Abstract: We start by considering the Alternate Strike (AS) scheme, a real-life arbitration scheme where two parties select an arbitrator by alternately crossing off at each round one name from a given panel of arbitrators. We find out that the AS scheme is not invariant to "bad" alternatives. We then consider another alternating-move scheme, the Voting by Alternating Offers and Vetoes (VAOV) scheme, which is invariant to bad alternatives. We fully characterize the subgame perfect equilibrium outcome sets of these above two schemes in terms of the rankings of the parties over the alternatives only. We also identify some of the typical equilibria of these above two schemes. We then analyze two additional alternating-move schemes in which players' current proposals have to either honor or enhance their previous proposals. We show that the first scheme's equilibrium outcome set coincides with that of the AS scheme, and the equilibrium outcome set of the second scheme coincides with that of the VAOV scheme. Finally, it turns out that all schemes' equilibrium outcome sets converge to the Equal Area solution's outcome of cooperative bargaining problem, *if* the alternatives are distributed uniformly over the comprehensive utility possibility set and *as* the number of alternatives tends to infinity. *Journal of Economic Literature* Classification Number: C72.

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1. INTRODUCTION

One of the main agencies involved in the appointment of arbitrators in labor-management disputes is the Federal Mediation and Conciliation Service (FMCS), which nominates arbitrators in over thirty thousand cases per year. The FMCS provides the disputing parties with a panel of seven names and allows them to select an arbitrator by alternately crossing off one name from that list at each round. This scheme is termed the Alternate Strike (AS) scheme. We fully characterize the subgame perfect equilibrium outcome set of this scheme in terms of the rankings of the parties over the arbitrators only. We also identify a typical equilibrium.[1]

It turns out that the AS scheme is not invariant to "bad" alternatives, i.e., to alternatives that are not regarded highly by any player (to be made precise later).[2] We then consider another alternating-move scheme, namely the Voting by Alternating Offers and Vetoes (VAOV) scheme, which is invariant to bad alternatives. In the VAOV scheme, the two players take turns making offers until an alternative is accepted; any offer rejected by a player is taken out of consideration, and if no offer is accepted, the last remaining alternative is the outcome. We first characterize the subgame perfect equilibrium outcome set of this scheme in terms of the rankings of the parties over the alternatives only. We also identify a typical equilibrium.

In fact, our setup can also be considered as a finite bargaining setup. Bargaining theory has traditionally assumed a continuum of feasible outcomes. However, many real-life bargaining situations involve a finite number of alternatives, such as two managers choosing from among a few job candidates, or a husband and her wife choosing from among a few homes or automobiles.[3] Another characteristic of many real-life bargaining situations is that past concessions have significant relevance. Either past concessions have to be honored or new offers have to improve upon the past offers.[4] We will show

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[1] Except for a few studies in the mid-1980s, the economics literature has entirely ignored the part of the arbitration process which precedes the appointment of the arbitrator (i.e., the part that involves the selection of arbitrators). This is surprising since empirical evidence suggests that arbitrators do not have identical preference functions (Ashenfelter and Bloom (1984)) and that the parties do not have consensus on the characteristics of "good arbitrators" (Bloom and Cavanagh (1986)). The AS scheme is the only scheme that is used by the FMCS in over thirty thousand cases a year, and thus it possibly affects the lives of about a million employees every year. Therefore, it would be relevant and useful to analyze it.

[2] Since a scheme such as the AS scheme is one of the alternating-move schemes in this paper, we will use the terms "arbitrator" and "alternative" interchangeably when we refer to that scheme.

[3] Another motivation for studying the finite case is that two finite bargaining situations that involve different alternatives can end up being represented by the same utility possibility set of Nash's bargaining problem (1950). That is, that utility possibility set can suppress many relevant differences among substantially different finite bargaining setups. This happens because that set utility possibility set is generated by randomization among payoffs of the finite alternatives.

[4] One can imagine a union-management bargaining case where there are a few hourly-wage possibilities (typically following some prominence levels - see Albers and Albers (1983)). There can also be a few fringe benefit packages - each

that such alternating-move schemes can provide additional insights concerning the above two schemes, as the next two paragraphs will indicate.

In the *Enhancing Past Concessions* scheme, two players take turns making offers until an alternative is accepted; any rejected alternative is taken out of consideration, and at any stage each Player i's offer must be preferred by his opponent Player j to Player i's previous offers. If only one unrejected alternative remains, it becomes the outcome. It turns out that the subgame perfect equilibrium outcome set of this scheme coincides with that of the AS scheme.

In the *Honoring Past Concessions* scheme, two players take turns making offers and all offers are on the table until one is accepted; at any stage, a Player i can either (1) accept the last offer of Player j or (2) accept any of the previous offers made by Player j (which Player i had not accepted when they were offered) or (3) choose not to accept the current offer as well as any of the past offers. If only one rejected alternative remains, the player who is offered that alternative has to either accept it or accept one of the previous offers made by his opponent. It turns out that the subgame perfect equilibrium outcome set of this scheme coincides with that of the VAOV scheme.

Finally, we show that all of the above schemes' equilibrium outcome sets converge to the Equal Area solution's outcome *if* the alternatives are distributed uniformly over the comprehensive utility possibility set and *as* the number of alternatives tends to infinity. (The outcome of the Equal Area solution is the intersection of the Pareto frontier and the straight line that goes through the disagreement point and cuts S into two equal areas.)

Section 2, analyzes the AS scheme. Section 3 analyzes the VAOV scheme. In Section 4, the Enhancing Past Concessions and the Honoring Past Concessions schemes are analyzed. In Section 5, we show under what circumstances all schemes' equilibrium outcomes converge to the Equal Area solution's outcome. Section 6 concludes. The proofs of our results are in the Appendix.

2. STRATEGIC ANALYSIS OF THE 'ALTERNATE STRIKE' SCHEME

Consider a *finite set of alternatives*, A. In the Alternate Strike scheme, the cardinality of A, #A, is seven. But our results will hold for any #A. We denote the two parties by L and M ("Labor" and "Management"). Players' preferences $\succeq_L$ and $\succeq_M$ over the alternatives in A are assumed to be complete, transitive and *anti-symmetric* (i.e., a player is indifferent between two alternatives **a** and **b** iff **a** = **b**). Given a *set of alternatives* A, and a *profile* $(\succeq_L,\succeq_M)$, let $(A,\succeq_L,\succeq_M)$ denote a

involving a various health insurance possibilities (e.g., PPO and HMO options of a few insurance providers) -, and a few pension plan possibilities (e.g., TIAA-CREF or some state-funded plan), as well as, say, some dental plan possibilities. In many on-going relationships, it is conceivable that each party's preferences on such finitely many packages can be guessed rather accurately by the other party.

problem. (We will abbreviate $(A,\succeq_L,\succeq_M)$ by using $(\succeq_L, \succeq_M)$ or A only when no confusion will arise from doing so.)

In the *Alternate Strike* scheme, the sequence of the moves is as follows where i,j = L,M, i ≠ j: Player i first vetoes some **a** ∈ A. Then Player j vetoes some **b** ∈ A\\{**a**}. Player i vetoes some **a'** ∈ A\\{**a,b**}. Player j vetoes some **b'** ∈ A\\{**a,b,a'**}. Player i vetoes some **a"** ∈ A\\{**a,b,a',b'**}. Player j vetoes some **b"** ∈ A\\{**a,b,a',b',a"**}. Then the only remaining arbitrator **c** in A\\{**a,b,a',b',a",b"**} is selected to arbitrate the dispute between L and M.

The FMCS provides the two parties with the resumés of the arbitrators (these resumés include information such as each arbitrator's training and prior decisions). It is reasonable to assume that, in many enduring relationships, 'each party knows its opponent's rankings over the arbitrators, but not necessarily the intensity of its opponent's preferences over any two arbitrators' and that 'these rankings are common knowledge between the parties but are not known to outsiders'. This also makes our framework tractable.[5]

DEFINITION 2.1: Given any $(A,\succeq_L, \succeq_M)$, let $P_i^A(\mathbf{a}) = \{\mathbf{a'} \in A \mid \mathbf{a'} \succ_i \mathbf{a}\}$ be the *set of alternatives that a Player i (strictly) prefers to some* ***a*** *in* A, i = L,M.

Observe that given any two **a, b** ∈ A, $\#P_i^A(\mathbf{a}) < \#P_i^A(\mathbf{b})$ means that Player i (strictly) prefers **a** to **b**. $\#P_i^A(\mathbf{a})$ will also represent a *Player i's ranking of an alternative* **a** in some A. Thus, if $\#P_i^A(\mathbf{a}) = 0$, then **a** is Player i's most preferred alternative in A; similarly, if $\#P_i^A(\mathbf{a}) = \#A-1$, then **a** is Player i's least preferred alternative in A. Let $\mathbf{a} \succ \mathbf{b}$ denote the situation where both players (strictly) prefer **a** to **b**. Let $A^* = \{\mathbf{a} \in A \mid \mathbf{a'} \succ \mathbf{a} \Rightarrow \mathbf{a'} \notin A\}$ be the *set of efficient alternatives* in A.

EXAMPLE 2.1: Consider A = {**a, b, c, d, e, f, g,**} with the profile $\mathbf{a} \succ_L \mathbf{b} \succ_L \mathbf{c} \succ_L \mathbf{d} \succ_L \mathbf{e} \succ_L \mathbf{f} \succ_L \mathbf{g}$ and $\mathbf{g} \succ_M \mathbf{e} \succ_M \mathbf{c} \succ_M \mathbf{a} \succ_M \mathbf{b} \succ_M \mathbf{d} \succ_M \mathbf{f}$. Alternatively, each **a** in any problem $(A,\succeq_L,\succeq_M)$ can be expressed in terms of players' rankings of **a**. The alternatives in this example's problem $(A,\succeq_L,\succeq_M)$ can be expressed as **a** = (0,3), **b** = (1,4), **c** = (2,2), **d** = (3,5), **e** = (4,1), **f** = (5,6), **g** = (6,0). The efficient alternatives are **a**, **c**, **e**, and **g**.

[5] To be sure, one of the most important aspects present in reality is the existence of uncertainty about the other player's rankings (especially when the two parties have not dealt with each other sufficiently many times in the past). Dealing with such uncertainty, however, makes this framework very complicated. To illustrate the extent of the complications, consider a problem with four alternatives. Suppose L knows his own rankings. Concerning M's rankings there can be twelve possibilities. Given that L assigns a probability distribution over these possibilities, not only M's ordinal preferences but also M's cardinal preferences will be relevant. But then L will also need to form a probability distribution over M's cardinal preferences over these twelve possibilities. As the number of alternatives grows, this complexity grows much faster.

Insert Figure 1 about here

The equilibrium outcome(s) of the AS scheme can be identified using an algorithm, the "Simultaneous Naive Elimination" algorithm.

DEFINITION 2.2: *The Simultaneous Naive Elimination algorithm*: Given any $(A, \succeq_L, \succeq_M)$, let $A_1 = A$. Let w_L^{At} be L's least preferred arbitrator in some A_t, and w_M^{At} be M's least preferred arbitrator in some A_t (observe that $w_L^{At} = w_M^{At}$ is possible). For any integer $t \geq 2$, let $A_t = A_{t-1} \backslash \{w_L^{At-1}, w_M^{At-1}\}$. Let t* be the smallest integer such that A_{t*} is empty. We shall denote by $\boldsymbol{\sigma}_L^A$ L's most preferred alternative in A_{t*-1} and by $\boldsymbol{\sigma}_M^A$ M's most preferred alternative in A_{t*-1}.

In Example 2.1, observe that $\boldsymbol{\sigma}_L^A = \boldsymbol{\sigma}_M^A = \boldsymbol{\sigma}^A = \mathbf{a}$. To see that note the following: given $A_1 = A = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}\}$, we have $w_L^{A1} = \mathbf{g}$, $w_M^{A1} = \mathbf{f}$, and thus, $A_2 = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}\}$; then, given A_2, we have $w_L^{A2} = \mathbf{e}$, $w_M^{A2} = \mathbf{d}$, and thus $A_3 = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$; consequently, given A_3, we have $w_L^{A3} = \mathbf{c}$, $w_M^{A3} = \mathbf{b}$, and thus $A_4 = \{\mathbf{a}\}$. Hence, $\boldsymbol{\sigma}_L^A = \boldsymbol{\sigma}_M^A = \boldsymbol{\sigma}^A = \mathbf{a}$. Thus, t* = 5, since $w_L^{A4} = w_M^{A4} = \mathbf{a}$. $A_{t*-1} = A_4 = \{\mathbf{a}\}$. Observe that M prefers **g**, **e**, and **c** to $\boldsymbol{\sigma}_L^A = \boldsymbol{\sigma}_M^A = \mathbf{a}$ while L does not prefer any alternative to $\boldsymbol{\sigma}_L^A = \boldsymbol{\sigma}_M^A = \boldsymbol{\sigma}^A = \mathbf{a}$.

DEFINITION 2.3: Given any $(A, \succeq_L, \succeq_M)$, suppose **a** and **a'** are alternatives in A such that $\mathbf{a} \succ_i \mathbf{a'}$ and $\mathbf{a'} \succ_j \mathbf{a}$, i,j =1,2, $i \neq j$. Then **a** and **a'** are said to be *adjacent* alternatives iff there is no $\underline{\mathbf{a}} \in A^*$ such that $\mathbf{a} \succ_i \underline{\mathbf{a}} \succ_i \mathbf{a'}$ and $\mathbf{a'} \succ_j \underline{\mathbf{a}} \succ_j \mathbf{a}$.

LEMMA 1. *Given any* $(A, \succeq_L, \succeq_M)$,

(1) $\boldsymbol{\sigma}_L^A$ *and* $\boldsymbol{\sigma}_M^A$ *are well-defined;*

(2) $\boldsymbol{\sigma}_L^A$ *and* $\boldsymbol{\sigma}_M^A$ *are efficient;*

(3) $\boldsymbol{\sigma}_L^A$ *and* $\boldsymbol{\sigma}_M^A$ *are adjacent alternatives when* $\boldsymbol{\sigma}_L^A \neq \boldsymbol{\sigma}_M^A$.

The AS scheme is a sequential game of perfect information. Therefore, here the notion of equilibrium is *subgame perfection*; a strategy profile **s*** is a subgame perfect equilibrium if and only if the strategy of each player is optimal at each decision node given the strategies of the other player. The first part of the next result fully characterizes the circumstances under which each $\boldsymbol{\sigma}_i^A$ becomes the subgame perfect equilibrium outcome. The second part provides a typical equilibrium.

THEOREM 1: *Given any* $(A, \succsim_L, \succsim_M)$,

(1) If $\boldsymbol{\sigma}_i^A \neq \boldsymbol{\sigma}_j^A$ *and Player i has the first move, then* $\boldsymbol{\sigma}_i^A$ *is the outcome of any subgame perfect equilibrium of the AS scheme. If* $\boldsymbol{\sigma}_L^A = \boldsymbol{\sigma}_M^A = \boldsymbol{\sigma}^A$*, then it is the outcome of any subgame perfect equilibrium of the AS scheme regardless of who has the first move.*

(2) A subgame perfect equilibrium strategy for Player i = L is as follows: At any stage B, given $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$*, veto* $\boldsymbol{\sigma}_M^B$*. Given* $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$*, veto any alternative that Player M prefers to* $\boldsymbol{\sigma}^B$ *if there is any such alternative; veto any alternative* $\boldsymbol{a} \neq \boldsymbol{\sigma}^B$ *otherwise.*

Consider Example 2.1. Suppose L moves first. L will first veto any of **g**, **e** or **c** (all of which M prefers to $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B = \mathbf{a}$). After L vetoes any of them, it is M's turn to move. Since L does not prefer any alternative to $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B = \mathbf{a}$, M will veto any of **f**, **d** or **b**. Suppose L vetoes **g** and then M vetoes **f**. Then at his second move, L will veto any of **e** or **c**. Following that, at his second move, M will veto any of **d** or **b**. Suppose L vetoes **e** and then M vetoes **d** at their second moves. Ensuing that, at their final moves, L will veto **c** and then M will veto **b**. The outcome will be **a**. If M moves first instead of L, observe that the outcome will still be **a**.

In the remarks below, we will focus on more specific features of this result.

REMARK 2.1: *The two possible equilibrium outcomes of the AS scheme are efficient adjacent arbitrators*: This follows from Theorem 1 which has established that the subgame perfect equilibrium outcome of the AS scheme is either $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ when $\boldsymbol{\sigma}_L^A \neq \boldsymbol{\sigma}_M^A$, and from Part 3 of Lemma 1 which established that they are adjacent efficient arbitrators.

REMARK 2.2: *Multiple outcomes and first-mover advantage*: Consider $(A, \succsim_L, \succsim_M)$ = {**a** = (0,2), **b** = (1,3), **c** = (2,0), **d** = (3,1)}). We will first illustrate that the equilibrium outcomes can differ depeding on who moves first. By Part 2 of Theorem 1, L will start by vetoing **c**. To see why M will not veto **a**, observe that he knows that in that case L would veto **d** next round and consequently make **b** the outcome, which from M's perspective is worse than **a**. Thus, M will veto **b**. Then L will veto **d**, and **a** will become the outcome. Similarly, **c** will be the outcome if M has the first move. *Clearly, once the first mover is known, the equilibrium outcome is unique.* This example also illustrates the first-mover advantage in the AS scheme.

REMARK 2.3: *Multiple equilibria with a unique equilibrium outcome*: Consider $(A, \succsim_L, \succsim_M)$ = {**a** = (0,4), **b** = (1,3), **c** = (2,2), **d** = (3,1), **e** = (4,0)}. Suppose L starts; then L moves at rounds one and three, and M moves at rounds two and four. Observe that it does not matter which of **d** and **e** Player L will veto first and second; likewise, it does not matter which of **a** and **b** Player M will veto first and second. The equilibrium outcome will still be **c**. Since A is a symmetric set, **c** is the equilibrium outcome regardless of who moves first. This illustrates the presence of multiple equilibria even with a unique equilibrium outcome.

REMARK 2.4: A *reciprocal elimination of parties' preferred arbitrators need not occur in the AS scheme*: In some sets, M's least preferred arbitrator may coincide with L's most preferred arbitrator while L's least preferred arbitrator may coincide with one of M's lowest ranked arbitrators. In Example 2.1, L's least preferred arbitrator is **g** while M's least preferred arbitrator is **f**. Such an unreciprocal elimination of some of M's preferred arbitrators might take place several times; as a result (such as in Example 2.1), the most preferred arbitrator of one party can be the outcome regardless of who moves first.

REMARK 2.5: *The equilibrium outcome set is not invariant with respect to alternatives that are dominated by both equilibrium outcomes (i.e., "bad" alternatives)*: Consider the following two problems $(A,\succeq_L, \succeq_M) = \{\mathbf{a} = (0,3), \mathbf{b} = (1,2), \mathbf{c} = (2,1), \mathbf{d} = (3,0)\}$ and $(A',\succeq'_L,\succeq'_M) = \{\mathbf{a} = (0,3), \mathbf{b} = (1,2), \mathbf{c} = (2,1), \mathbf{e} = (3,4), \mathbf{d} = (4,0)\}$. By Theorem 1, $\boldsymbol{\sigma}_L(A,\succeq_L,\succeq_M) = \mathbf{b}$, $\boldsymbol{\sigma}_M(A,\succeq_L,\succeq_M) = \mathbf{c}$. Clearly, the only difference between $(A,\succeq_L,\succeq_M)$ and $(A',\succeq'_L,\succeq'_M)$ is **e** which is dominated by both **b** and **c**. Notice, however, that $\boldsymbol{\sigma}_L(A',\succeq'_L,\succeq'_M) = \boldsymbol{\sigma}_M(A,'\succeq'_L,\succeq'_M) = \mathbf{b}$.[6] Remark 2.4 provides an explanation as to what causes such an unsatisfactory result for the AS scheme.

3. THE 'VAOV' SCHEME

Now, we will analyze a scheme that, unlike the AS scheme, is invariant with respect to bad alternatives, namely the VAOV scheme (see Anbarci (1993)). In the *Voting by Alternating Offers and Vetoes* (VAOV) scheme, first, Player i offers some $\mathbf{a} \in A$. Player j can either accept **a** or veto it and offer some $\mathbf{b} \in A\backslash\{\mathbf{a}\}$. If Player j vetoes **a** and offers **b**, Player i can either accept **b** or veto it and offer some $\mathbf{c} \in A\backslash\{\mathbf{a},\mathbf{b}\}$, and so on. This procedure continues either until some alternative in A is offered and accepted or until only one unvetoed alternative remains in A, which is the outcome. Consider:

$$\boldsymbol{\gamma}_L^A = \underset{\mathbf{a}\in A^*}{\text{argmin}} \{\#P_M^A(\mathbf{a}) - \#P_L^A(\mathbf{a}) \mid \#P_M^A(\mathbf{a}) \geq \#P_L^A(\mathbf{a})\},\ \boldsymbol{\gamma}_M^A = \underset{\mathbf{a}\in A^*}{\text{argmin}} \{\#P_L^A(\mathbf{a}) - \#P_M^A(\mathbf{a}) \mid \#P_L^A(\mathbf{a}) \geq \#P_M^A(\mathbf{a})\}.^7$$

Let $A^*_i = \{\mathbf{a} \in A^* \mid \#P_j^A(\mathbf{a}) \geq \#P_i^A(\mathbf{a})\}$, $i,j = L,M$, $i \neq j$. That is, any alternative **a** in A^*_i is efficient and Player i prefers fewer number of alternatives to it than Player j does. Note that $\boldsymbol{\gamma}_i^A$ is Player j's most preferred alternative in A^*_i. Observe that, in Example 2.1, $A^*_L = \{\mathbf{a},\mathbf{c}\}$ and $A^*_M = \{\mathbf{c}, \mathbf{e}, \mathbf{g}\}$; thus, $\boldsymbol{\gamma}_L^A = \boldsymbol{\gamma}_M^A = \mathbf{c}$ (recall that, in the same example,

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6 Here is how $\boldsymbol{\sigma}_L(A,\succeq_L,\succeq_M) = \mathbf{b}$, $\boldsymbol{\sigma}_M(A,\succeq_L,\succeq_M) = \mathbf{c}$, and $\boldsymbol{\sigma}_L(A'\succeq'_L,\succeq'_M) = \boldsymbol{\sigma}_M(A'\succeq'_L,\succeq'_M) = \mathbf{b}$ are obtained. Observe that, with $(A,\succeq_L,\succeq_M)$, first **d** and **a** (which are symmetric) are eliminated since **d** is L's and **a** is M's least preferred arbitrator among the remaining ones. This leaves **b** and **c** as the possible outcomes. With $(A',\succeq'_L,\succeq'_M)$, first **d** and **e** are eliminated since **d** is L's and **e** is M's least preferred arbitrator. Then **c** and **a** are eliminated since **c** is L's and **a** is M's least preferred arbitrator among the remaining ones. This leaves **b** as the only possible outcome (i.e., regardless of which party moves first).

7 Anbarci (1993) shows that $\boldsymbol{\gamma}_L^A$ or $\boldsymbol{\gamma}_M^A$ are well-defined and that $\boldsymbol{\gamma}_L^A$ or $\boldsymbol{\gamma}_M^A$ are adjacent efficient alternatives when $\boldsymbol{\gamma}_L^A \neq \boldsymbol{\gamma}_M^A$.

$\sigma_L^A = \sigma_M^A = a$).[8]

Theorem 1 in Anbarci (1993) showed that the subgame perfect equilibrium outcome set of the VAOV scheme contains either γ_L^A or γ_M^A or both. Clearly, this result is not precise.[9] When $\gamma_L^A \neq \gamma_M^A$, it does not specify which of these two alternatives will become the equilibrium outcome under what circumstances. Thus, our next result is a significantly sharper version of Theorem 1 in Anbarci (1993). A problem $(A,\succeq_L,\succeq_M)$ is *balanced* iff $\#P_L^A(\gamma_M^A) = \#P_M^A(\gamma_L^A)$ and *unbalanced* iff $\#P_i^A(\gamma_j^A) > \#P_j^A(\gamma_i^A)$ i,j = L,M, i ≠ j. Let $D^A = \{a \in A \mid \gamma_i^A \succ a \ \forall i = L,M\}$.

THEOREM 2: *Given any* $(A,\succeq_L,\succeq_M)$,

(1) Consider the balanced problem where $\gamma_L^A = \gamma_M^A = \gamma^A$. *Then* γ^A *is the subgame perfect equilibrium outcome regardless of who has the first move. Consider the remaining balanced problem where* $\gamma_L^A \neq \gamma_M^A$ *and suppose Player i has the first move. Then the subgame perfect equilibrium outcome is* γ_i^A *if* $\#D^A$ *is odd, and it is* γ_j^A *if* $\#D^A$ *is even or zero. Consider the unbalanced problem and suppose* $\#P_i^A(\gamma_j^A) > \#P_j^A(\gamma_i^A)$. *Then the subgame perfect equilibrium outcome is* γ_i^A *regardless of who has the first move, i,j = L,M, i ≠ j.*

(2) A subgame perfect equilibrium strategy for Player i = L is as follows: Offer some $a \in D^B$ *if* $D^B \neq \varnothing$. *Otherwise, offer some* $a \in P_L^B(\gamma_M^B)\backslash\gamma_L^B$ *if* $P_L^B(\gamma_M^B)\backslash\gamma_L^B \neq \varnothing$. *Otherwise, offer* γ_L^B. *If* $[\#P_L^B(\gamma_M^B) \geq \#P_M^B(\gamma_L^B)]$ *or* $[\#P_M^B(\gamma_L^B) - \#P_L^B(\gamma_M^B) = 1$ *and* $D^B \neq \varnothing]$, *accept any* $a \in P_L^B(\gamma_L^B) \cup \gamma_L^B$ *and reject any other* a. *If* $[\#P_M^B(\gamma_L^B) - \#P_L^B(\gamma_M^B) = 1$ *and* $D^B = \varnothing]$ *and* $[\#P_M^B(\gamma_L^B) - \#P_L^B(\gamma_M^B) > 1]$, *accept any* $a \in P_L^B(\gamma_M^B) \cup \gamma_M^B$ *and reject any other* a.[10]

In the remarks below, we will focus on more specific features of this result.

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[8] Suppose that $\#A^*_L \geq 2$ and $\#A^*_M \geq 2$. That is, for each player there are two or more efficient alternatives that he prefers at least as much as his opponent does. Then observe that the AS scheme is not very conducive to a compromise in that one of the player's most preferred alternative can be the outcome (see Example 2.1 for instance). Given the above condition, the VAOV scheme outcome, however, can never be any player's most preferred alternative (it simply follows from the definitions of γ_M^A and γ_L^A, since γ_M^A is L's most preferred alternative in $\#A^*_M$ and γ_L^A is M's most preferred alternative in $\#A^*_L$).

[9] Anbarci (1993)'s main concern is establishing the link between these outcomes and the Equal Area solution outcome as the number of alternatives tends to infinity (by using some structure to generate the equilibrium alternatives). Thus, for that paper's intent and purpose a result more precise than that is not needed.

[10] There are additional problems when more than two players are considered in all of the schemes analyzed in this paper. The first one is that the number of subgame perfect equilibrium outcomes will grow with the number of players, n; i.e., there can be n possible equilibrium outcomes depending on who starts first. In addition, especially in the case of the VAOV scheme, there can several different rules concerning the vetoing of an alternative. That is, when an alternative is offered, it can be eliminated after the next player rejects it or it can eliminated after a certain number of players reject it (possibly all of the other players).

REMARK 3.1: *The two possible equilibrium outcomes of the VAOV scheme are efficient adjacent arbitrators*: This follows from Theorem 3 above and from Part 3 of Lemma 1 in Anbarci (1993) which established that they are adjacent efficient alternatives.

REMARK 3.2: *Multiple outcomes depending on who moves first*: This states the obvious. Consider $(A, \succeq_L, \succeq_M) = \{$**a** $= (0,2)$, **b** $= (2,0)\}$ and suppose L starts by offering **a**. M will reject **a** and the outcome will be **b**. Likewise, suppose M starts by offering **b**. L will reject **b** and the outcome will be **a**.

REMARK 3.3: *Multiple equilibria with a unique equilibrium outcome*: Consider $(A, \succeq_L, \succeq_M) = \{$**a** $= (0,2)$, **b** $= (1,1)$, **c** $= (2,0)\}$ and suppose L starts by offering **a**. Then M will reject **a**. Note that whether M then offers **b** or **c** does not matter: if M offers **c**, L will reject it and the outcome will be **b**, and if M offers **b**, L will accept it, which will thus become the outcome.

REMARK 3.4: *No generic first- or second-mover advantage*: Consider the situation where the numbers of alternatives that L and M prefer to $\boldsymbol{\gamma}_M^A$ and $\boldsymbol{\gamma}_L^A$ respectively are equal. When #A is even, there is a last-mover advantage: Consider $(A, \succeq_L, \succeq_M) = \{$**a** $= (0,2)$, **b** $= (1,3)$, **c** $= (2,0)$, **d** $= (3,1)\}$; observe that the outcome is **c** if L makes the first offer, and **a** if M makes the first offer. When #A is odd, there is a first-mover advantage: consider A $= \{$**a** $= (0,1)$, **b** $= (1,0)$, **c** $= (2,2)\}$ and observe that the outcome is **a** when moves first, and **b** when M moves first.

REMARK 3.5: *The VAOV scheme's outcome balances the numbers of alternatives that the parties find preferable to it "as much as possible."* Unlike the AS scheme (as alluded to in Remark 2.4), in the VAOV scheme, reciprocal elimination of preferred alternatives must take place. When $\boldsymbol{\gamma}_i^A$ is the outcome regardless of who has the first move, however, one of the players has more alternatives that he prefers to $\boldsymbol{\gamma}_i^A$ than the other player does. Then the VAOV scheme's outcome balances the numbers of alternatives that the parties find preferable to it only "as much as possible."

REMARK 3.6: *The first offer can be accepted in equilibrium*: Consider $(A, \succeq_L, \succeq_M) = \{$**a** $= (0,2)$, **b** $= (1,1)$, **c** $= (2,0)\}$ and suppose L starts by offering **b**. Note that, if M rejects **b**, the outcome will be **a**. Thus, M will accept **b**.

4. THE 'ENHANCING PAST CONCESSIONS' AND 'HONORING PAST CONCESSIONS' SCHEMES

First consider the *Enhancing Past Concessions* scheme. First Player i offers some **a** $\in$ A. Player j can either accept **a** or veto it and offer some **b** $\in$ A\{**a**}. If j vetoes **a** and offers **b**, Player i can either accept **b** or veto it and offer some **c** $\in$ A\{**a**,**b**}, and so on. During subsequent rounds, both players are further restricted in that they may not make an offer which, from the other player's point of view, is worse than an offer he has already vetoed. In other words, suppose that at some round Player i has offered some **a'** which consequently got vetoed by Player j; then at any later round Player i has the move, he is only allowed to offer an alternative **a"** such that **a"** $\succ_j$ **a'**. This procedure stops either when an alternative is accepted by one of the players or there is only one remaining alternative **a***, in which case **a*** becomes the outcome.

THEOREM 3: *The subgame-perfect equilibrium outcome set of the Enhancing Past Concessions scheme coincides with that of the AS scheme.*

Next, consider the *Honoring Past Concessions* scheme. First Player i offers some **a** ∈ A. Player j can either accept **a** or reject it and offer some **b** ∈ A\{**a**}. If Player j rejects **a** and offers **b**, Player i can either accept **b** or reject it and offer some **c** ∈ A\{**a**,**b**}. If Player i rejects **b** and offers **c**, Player j can either accept an alternative from {**a**,**c**} or reject them and offer some **d** ∈ A\{**a**,**b**,**c**} (i.e., a player has to offer a yet unoffered alternative; also a player can accept an alternative only from the ones that his opponent has offered up to that point). Then Player i can either accept an alternative from {**b**,**d**} or reject them, and so on. This procedure continues until either some alternative is offered and accepted or only one unoffered alternative **a*** remains. Suppose that Player i gets to offer **a***, then Player j has to accept either **a*** or any alternative that Player i has offered before; the accepted alternative becomes the outcome.

THEOREM 4: *The subgame-perfect equilibrium outcome set of the Honoring Past Concessions scheme coincides with that of the VAOV scheme.*

5. THE EQUILIBRIUM OUTCOMES OF THE ARBITRATION SCHEMES AND THE EQUAL AREA SOLUTION OF NASH'S BARGAINING SETUP

Nash's Bargaining Problem (which is the traditional cooperative bargaining theory)[11] is a pair (S,d) where the *utility possibility set*, S, is a subset of $\mathbb{R}^2_+$, and d ∈ S is the *disagreement point* such that there is some x ∈ S with x > d.[12] The agents receive d unless they unanimously agree on a compromise x in S. Let B^2_0 be the class of pairs (S,d) where d = 0, and $S \subset \mathbb{R}^2_+$ is convex, compact and comprehensive (*comprehensiveness of S in* B^2_0: If x ∈ S, then any $0 \leq y \leq x$ is also in S). With B^2_0, the notation of Nash's bargaining problem reduces from the pair (S,d) to S where any x in S is such that $x \geqq 0$. Given B^2_0, a *solution* is a function F associating with every S in B^2_0 a point F(S) in S. Let ∂S denote the Pareto frontier of S (i.e., the set

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[11] Nash (1950, 1953) considered bargaining problems between two players and introduced two approaches to modeling such problems: the strategic (non-cooperative) and axiomatic (cooperative) approaches. The attempt to combine them is known as the Nash program, which this paper will try to follow (at the finitely-many-alternatives setup as well as its relationship with Nash's cooperative bargaining setup).

[12] Vector inequalities: Given x, x' ∈ $\mathbb{R}^2_+$, $x \geqq x'$ means $x_i \geqq x_i'$ for all i = 1,2; $x \geq x'$ means $x \geqq x'$ and $x \neq x'$; $x > x'$ means $x_i > x_i'$ for all i = 1,2.

of weakly Pareto optimal points in S); $\partial S = \{x \in \mathbf{S} \mid \nexists\, x' \in S \text{ with } x' > x\}$.

By Theorem 1, $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ are the unique subgame-perfect equilibrium outcomes of the AS scheme, and by Anbarci (1993), $\boldsymbol{\gamma}_L^A$ and $\boldsymbol{\gamma}_M^A$ are the unique subgame-perfect equilibrium outcomes of the VAOV scheme. But note that $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ (and $\boldsymbol{\gamma}_L^A$ and $\boldsymbol{\gamma}_M^A$) can be quite apart from each other for some finite A. This seems problematic since, depending on the situation, each player would like to be the first- or second-mover. Anbarci (1993) showed that if the alternatives in A are selected from S such that they are distributed uniformly over S in a particular way, then they converge as #A goes to infinity. Furthermore, they converge to the Equal Area solution $\boldsymbol{\alpha}$'s outcome $\boldsymbol{\alpha}(S)$, which is the intersection point of ∂S and the straight line that goes through 0 and cuts S into two equal areas (see Anbarci (1993) and Anbarci and Bigelow (1994)).

To generate some A from S, Anbarci (1993) first approximated S with a finite number of cells, and then selected one point from each cell. This approximation is created using a grid of rectangular cells that covers S completely. Let $b_i = \max\{x_i \mid x \in S\}$. Let n = 1,2,3,... denote the grid parameter such that the length of a cell is a_1/n and its height is a_2/n. Call any cell that contains some part of ∂S a *boundary cell*; observe that only boundary cells can contain some points outside S (it can easily be seen that Anbarci (1993)'s result is robust to making the cell length and height equal).

One alternative will be selected from each cell. To make use of Kuhn's backward induction theorem here, no player should be indifferent between two or more alternatives unless the other player is also indifferent between them. One can find many procedures to select alternatives to A from S such that Kuhn's theorem can be utilized. Anbarci (1993) selected alternatives by creating a slight bias against the less efficient cells; this is essentially identical to assuming that each Player i has lexicographic preferences such that he chooses a over a' if he prefers a over a' or if he is indifferent between a and a' *and* Player j prefers a over a' (one can see that Anbarci (1993)'s result would still hold even if the slight bias is created against more efficient cells, where players have somewhat negatively interdependent lexicographic preferences).

Specifically, there are n rows, $R_1,\ldots,R_n$, and n columns, $C_1,\ldots,C_n$. R_1 contains the origin as well as a_1, and C_1 contains the origin and a_2. Let r_k denote the boundary cell of R_k, and let c_k denote the boundary cell of C_k, k = 1,2,...,n. Let ∂r_k denote Pareto frontier of r_k, and let ∂c_k denote the Pareto frontier of c_k. In each R_k, find the point $m(\partial r_k)$, which bisects the arc-length of ∂r_k, and connect $m(\partial r_k)$ to the southwest corner of R_k through a straight line. Likewise, in each C_k, find the point $m(\partial c_k)$, which bisects the arc-length of ∂c_k, and connect it to the southwest corner of C_k through a straight line. The intersections of these row lines and column lines provide the set of alternatives A. Let P* denote this procedure (for more details of P*, see Anbarci (1993) and the figures therein).

Observe that any a in A selected through P* is either in ∂S or is strictly dominated by another alternative which is in ∂S. Also observe that, for each n and a given utility profile, #A will be invariant with respect to affine transformations of players' utility functions. As n goes to infinity, the distance between any two adjacent alternatives becomes insignificant. Anbarci (1993) showed that, as n tends to infinity, γ_L^A and γ_M^A (which are adjacent efficient alternatives) tend to $\alpha(S)$.

Let $\Sigma(A) = \{\sigma_L^A, \sigma_M^A\}$ and $\Gamma(A) = \{\gamma_L^A, \gamma_M^A\}$ in any problem $(A, \succeq_L, \succeq_M)$. It will turn that σ_L^A and σ_M^A too tend to $\alpha(S)$, as n tends to infinity. This seems surprising since for any arbitrary A (i.e., for any A that was not generated through P*), $\Sigma(A)$ and $\Gamma(A)$ could be significantly different. The key to the understanding of why $\Sigma(A)$ and $\Gamma(A)$ tend to $\alpha(S)$ is that, in some equilibrium in either scheme, players take turns in removal of alternatives from opposite sides of the line that connects the origin to $\alpha(S)$. This becomes identical to removing equal areas from opposite sides of that line. Since a finer grid (i.e., a grid with a higher n) approximates S better, the equilibrium outcomes of these schemes approximate $\alpha(S)$ better as n tends to infinity.

THEOREM 5: *Suppose P* constructs A from S. As $n \to \infty$, σ_L^A and σ_M^A converge to $\alpha(S)$.*

REMARK 5.1: Here we extend the domain of non-cooperative foundations for the Equal Area solution towards a particular direction: when there is an alternating-move scheme with finitely many alternatives, which are distributed uniformly over the utility possibility set S, then the subgame-perfect outcome of that scheme converges to the Equal Area solution outcome. However, when the alternatives are *not* uniformly distributed over S, the solution outcomes of these schemes do not converge to the outcome of the Equal Area solution.[13]

REMARK 5.2: Is the above setup the only setup under which the AS and VAOV schemes have the same outcome sets? The answer is negative. The proof of Theorem 5 provides clues about the answer of this question. If the two players end up eliminating almost equal numbers of alternatives in $D^A = \{\mathbf{a} \in A \mid f_i(A) \succ \mathbf{a}\ \forall i = L,M\}$, then the two schemes will have the same outcome. Note that this is possible if either (i) for any w_L^{At}, dominated by both σ_L^A and σ_M^A, w_M^{At} is also dominated by both σ_L^A and σ_M^A, or (ii) for any w_L^{At}, dominated by both σ_L^A and σ_M^A, if w_M^{At} is not dominated by both σ_L^A and σ_M^A,.there is another $w_M^{At'}$ that is dominated by both σ_L^A and σ_M^A *and* $w_L^{At'}$ is not dominated by both σ_L^A and σ_M^A.

6. CONCLUDING REMARKS

To summarize, this paper studies alternating-move bargaining situations and arbitration schemes in which only a finite set of feasible alternatives is present. Many real-life bargaining situations have this feature, but the literature has mainly

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[13] Uniform distribution of alternatives over S is used to highlight the importance of the Equal Area solution. In addition, comprehensiveness of S turns out to be important because, without comprehensiveness of S, the Equal Area solution outcome need not be efficient whereas the SPE outcomes of the AS and VAOV schemes are efficient.

dealt with the case of infinite choice sets. The two main contributions of this paper are, first, to study the of finite alternating-move bargaining situations (arbitration schemes); and, second, to provide a general link between the finite-alternatives bargaining situations and the infinite-alternatives bargaining situations and consequently to provide robust non-cooperative foundations for the Equal Area solution.

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APPENDIX

PROOF OF LEMMA 1: (1) Observe that at each round of the Simultaneous Naive Elimination algorithm, at least one and at most two alternatives are eliminated before the last stage t*- 1 is reached. Consequently, at least one and at most two efficient alternatives reach t*-1 (efficiency of the alternatives in the outcome set is established in Part 2 of this lemma).

(2) Without loss of generality, suppose that $\boldsymbol{\sigma}_L^A$ is not efficient. Then there must be some $\mathbf{a} \in A$ such that $\mathbf{a} \succ \boldsymbol{\sigma}_L^A$. But since $\mathbf{a} \succ_L \boldsymbol{\sigma}_L^A$, then $\boldsymbol{\sigma}_L^A$ would be eliminated before **a** gets eliminated or $\mathbf{a} = \boldsymbol{\sigma}_M^A$; i.e., in either case, $\boldsymbol{\sigma}_L^A$ would not be an outcome of the Simultaneous Naive Elimination algorithm, a contradiction.

(3) Suppose $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ are not adjacent. Then there is some $\mathbf{a} \in A$ such that $\boldsymbol{\sigma}_L^A \succ_L \mathbf{a} \succ_L \boldsymbol{\sigma}_M^A$ and $\boldsymbol{\sigma}_M^A \succ_M \mathbf{a} \succ_M \boldsymbol{\sigma}_L^A$. But then either $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ or both would be eliminated before **a** gets eliminated, a contradiction. ▌

PROOF OF THEOREM 1: We will need the following lemma (the proofs of Parts (i) and (ii) are straightforward and, thus, will be omitted):

LEMMA A1.1. *Consider the Simultaneous Naive Elimination algorithm on two problems* $(B, \succsim_L, \succsim_M)$ *and* $(B\backslash a, \succsim_L, \succsim_M)$.

(i) Suppose that $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$. *Suppose* $B\backslash a$ *is such that* $\boldsymbol{\sigma}_L^B$ *is removed from B. Then* $\boldsymbol{\sigma}_M^B$ *becomes* $\boldsymbol{\sigma}^{B\backslash a}$.

(ii) Suppose that $\#B > 1$ *and* $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$. *Suppose* $B\backslash a$ *is such that* $\boldsymbol{\sigma}^B$ *is removed from B. Then* $w_L^{Bt^*-2}$ *becomes* $\boldsymbol{\sigma}_M^{B\backslash a}$ *and* $w_M^{Bt^*-2}$ *becomes* $\boldsymbol{\sigma}_L^{B\backslash a}$.

(iii) Regardless of whether $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$ *or* $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$, *suppose that* $B\backslash a$ *is such that some* w_L^{Bt} *with* $w_L^{Bt} = w_M^{Bt}$ *for any t,* $t < t^*-1$, *is removed from B. Then each* $\boldsymbol{\sigma}_i^B$ *becomes* $\boldsymbol{\sigma}_i^{B\backslash a}$.

(iv) Suppose that $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$. *Suppose that* $B\backslash a$ *is such that some* w_L^{Bt} *with* $w_L^{Bt} \neq w_M^{Bt}$ *for any t,* $t < t^*-1$, *is removed from B. Then either [each* $\boldsymbol{\sigma}_i^B$ *becomes* $\boldsymbol{\sigma}_i^{B\backslash a}$*] or [*$\boldsymbol{\sigma}_L^B$ *becomes* $\boldsymbol{\sigma}^{B\backslash a}$ *and* $\boldsymbol{\sigma}_M^B$ *becomes* $w_L^{(B\backslash a)t^*-2}$*].*

(v) Suppose that $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$. *Suppose that* $B\backslash a$ *is such that some* w_L^{Bt} *with* $w_L^{Bt} \neq w_M^{Bt}$ *for any t,* $t < t^*-1$, *is removed from B. Then either [*$\boldsymbol{\sigma}^B$ *becomes* $\boldsymbol{\sigma}^{B\backslash a}$*] or [*$\boldsymbol{\sigma}^B$ *becomes* $\boldsymbol{\sigma}_M^{B\backslash a}$ *and* $w_M^{Bt^*-2}$ *becomes* $\boldsymbol{\sigma}_L^{B\backslash a}$*].*

PROOF OF LEMMA A1.1: (iii) Note that, if some w_L^{Bt} such that $w_L^{Bt} = w_M^{Bt}$ for any t, $t < t^*-1$, is removed from B, then each $w_i^{Bt'}$ for any t', $t < t'$, becomes $w_i^{(B\backslash a)t'+1}$ at $B\backslash a$. Then, clearly, each $\boldsymbol{\sigma}_i^B$ becomes $\boldsymbol{\sigma}_i^{B\backslash a}$.

(iv) and (v) Note that when some w_L^{Bt} such that $w_L^{Bt} \neq w_M^{Bt}$ for any t, $t < t^*-1$, is removed from B, then either w_M^{Bt} will become $w_L^{(B\backslash a)t} = w_M^{(B\backslash a)t}$ or w_L^{Bt+1} will become $w_L^{(B\backslash a)t} \neq w_M^{(B\backslash a)t} = w_M^{Bt}$. If the former case holds, trivially [each $\boldsymbol{\sigma}_i^B$ becomes $\boldsymbol{\sigma}_i^{B\backslash a}$] in (iv) and [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$] in (v). If the latter case holds, then we are back to our starting point with the difference that t is replaced with t+1. Thus, we only have to focus on the case $t = t^*-2$.

Consider (iv). Suppose $w_L^{Bt^*-2} \neq w_M^{Bt^*-2}$ and $\boldsymbol{\sigma}_M^B \succ w_M^{Bt^*-2}$ at B, then $w_M^{Bt^*-2}$ becomes $w_L^{(B\backslash a)t^*-2} = w_M^{(B\backslash a)t^*-2}$ in which case we have [each $\boldsymbol{\sigma}_i^B$ becomes $\boldsymbol{\sigma}_i^{B\backslash a}$]. Now consider the remaining situation $w_L^{Bt^*-2} \neq w_M^{Bt^*-2}$ and $\boldsymbol{\sigma}_M^B \nsucc w_M^{Bt^*-2}$ at B. Then by definition of any $\boldsymbol{\sigma}_i^B$ and $w_M^{Bt^*-2}$, we have $w_M^{Bt^*-2} \succ_L \boldsymbol{\sigma}_M^B$ (and, of course, $\boldsymbol{\sigma}_M^B \succ_M w_M^{Bt^*-2}$). Then we will have [$\boldsymbol{\sigma}_L^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$ and $\boldsymbol{\sigma}_M^B$ becomes $w_L^{(B\backslash a)t^*-2}$].

Consider (v). Suppose $w_L^{Bt^*-2} \neq w_M^{Bt^*-2}$ and $\boldsymbol{\sigma}^B \succ w_M^{Bt^*-2}$ at B, then $w_M^{Bt^*-2}$ becomes $w_L^{(B\backslash a)t^*-2} = w_M^{(B\backslash a)t^*-2}$ in which case we have [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$]. Now consider the remaining situation $w_L^{Bt^*-2} \neq w_M^{Bt^*-2}$ and $\boldsymbol{\sigma}^B \nsucc w_M^{Bt^*-2}$ at B. Then by definition of $\boldsymbol{\sigma}^B$ and $w_M^{Bt^*-2}$, we have $w_M^{Bt^*-2} \succ_L \boldsymbol{\sigma}^B$. Then we will have [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}_M^{B\backslash a}$ and $w_M^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_L^{B\backslash a}$]. ▌

(1) We will use an inductive argument. Clearly, given any $(B,\succeq_L,\succeq_M)$, the result is correct if B contains only two efficient alternatives, and there is no arbitrator selection problem when B is a singleton. We suppose that our claim has been established for B\a and we want to establish it for B. Suppose that L uses the following strategy: if $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$, veto $w_L^{Bt^*-2}$; if $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$, veto $\boldsymbol{\sigma}_M^B$.

We will first show that when, without loss of generality, L moves at B, he will be able to make $\boldsymbol{\sigma}_L^B$ the outcome against an opponent who follows a SPE strategy.

Suppose that $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$.

Suppose that L vetoes $\boldsymbol{\sigma}_M^B$. Then by Lemma A1.1(i), $\boldsymbol{\sigma}_L^B$ will become $\boldsymbol{\sigma}^{B\backslash a}$ at B\a.

Suppose that $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$.

Suppose that L vetoes $w_L^{Bt^*-2}$. Then by Lemma A1.1(v), either [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}_M^{B\backslash a}$ and $w_M^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_L^{B\backslash a}$] or [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$].

Now, we will show that, when a SPE-following M moves at B, $\boldsymbol{\sigma}_L^{B\backslash a}$ (including the possibility $\boldsymbol{\sigma}_L^{B\backslash a} = \boldsymbol{\sigma}_M^{B\backslash a} = \boldsymbol{\sigma}^{B\backslash a}$) will become the outcome.

Suppose that $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$.

Suppose that M vetoes $\boldsymbol{\sigma}_L^B$ at B. Then, by Lemma A1.1(i), $\boldsymbol{\sigma}_M^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$; thus by our induction hypothesis, $\boldsymbol{\sigma}_M^B$ (and hence $\boldsymbol{\sigma}^{B\backslash a} = \boldsymbol{\sigma}_L^B$) will become the outcome.

Suppose that M vetoes some w_L^{Bt} such that $w_L^{Bt} = w_M^{Bt}$. Then, by Lemma A1.1(iii), $\boldsymbol{\sigma}_L^B$ and $\boldsymbol{\sigma}_M^B$ will become $\boldsymbol{\sigma}_L^{B\backslash a}$ and $\boldsymbol{\sigma}_M^{B\backslash a}$ respectively; thus, by our induction hypothesis, $\boldsymbol{\sigma}_L^B$ (and hence $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome.

Suppose that M vetoes some w_M^{Bt} such that $w_M^{Bt} \neq w_L^{Bt}$. Then, by Lemma A1.1(iv), either [each $\boldsymbol{\sigma}_i^B$ becomes $\boldsymbol{\sigma}_i^{B\backslash a}$] or [$\boldsymbol{\sigma}_L^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$ and $\boldsymbol{\sigma}_M^B$ becomes $w_L^{(B\backslash a)t^*-2}$]. Thus, by our induction hypothesis, $\boldsymbol{\sigma}_L^B$ (and hence $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome.

Suppose that M vetoes some w_L^{Bt} such that $w_L^{Bt} \neq w_M^{Bt}$. Then, by Lemma A1.1(iv), either [each $\boldsymbol{\sigma}_i^B$ becomes $\boldsymbol{\sigma}_i^{B\backslash a}$] or [$\boldsymbol{\sigma}_M^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$ and $\boldsymbol{\sigma}_L^B$ becomes $w_M^{(B\backslash a)t^*-2}$]. Thus, by our induction hypothesis, either $\boldsymbol{\sigma}_L^B$ or $\boldsymbol{\sigma}_M^B$ (but in any case, $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome.

Thus, overall, when $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$, a rational M will veto $\boldsymbol{\sigma}_L^B$ since it unambiguously makes $\boldsymbol{\sigma}_M^B$ (and hence $\boldsymbol{\sigma}^{B\backslash a} = \boldsymbol{\sigma}_L^B$) the outcome by our induction hypothesis. Vetoing anything else either can make or will make $\boldsymbol{\sigma}_L^B$ the outcome.

Suppose that $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$.

Suppose that M vetoes $\boldsymbol{\sigma}^B$. Then, by Lemma A1.1(ii), $w_L^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_M^{B\backslash a}$ and $w_M^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_L^{B\backslash a}$; thus, by our induction hypothesis, $w_M^{Bt^*-2}$ (and hence $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome. (By definition, $\boldsymbol{\sigma}^B \succ_M w_M^{Bt^*-2}$.)

Suppose that M vetoes some w_L^{Bt} such that $w_L^{Bt} = w_M^{Bt}$. Then, by Lemma A1.1(iii), $\boldsymbol{\sigma}^B$ will become $\boldsymbol{\sigma}^{B\backslash a}$; thus, by our induction hypothesis, $\boldsymbol{\sigma}^B$ (and hence $\boldsymbol{\sigma}^{B\backslash a}$) will become the outcome.

Suppose that M vetoes some w_M^{Bt} such that $w_M^{Bt} \neq w_L^{Bt}$. Then, by Lemma A1.1(v), either [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$] or [$w_L^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_M^{B\backslash a}$ and $\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}_L^{B\backslash a}$]. Thus, by our induction hypothesis, $\boldsymbol{\sigma}^B$ (and hence $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome.

Suppose that M vetoes some w_L^{Bt} such that $w_L^{Bt} \neq w_M^{Bt}$. Then, by Lemma A1.1(v), either [$\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}^{B\backslash a}$] or [$w_M^{Bt^*-2}$ becomes $\boldsymbol{\sigma}_L^{B\backslash a}$ and $\boldsymbol{\sigma}^B$ becomes $\boldsymbol{\sigma}_M^{B\backslash a}$]. Thus, by our induction hypothesis, $\boldsymbol{\sigma}^B$ or $w_M^{Bt^*-2}$ (but in any case, $\boldsymbol{\sigma}_L^{B\backslash a}$) will become the outcome.

Thus, overall, when $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$, M will veto some w_M^{Bt} (regardless of $w_M^{Bt} \neq w_L^{Bt}$ or $w_M^{Bt} = w_L^{Bt}$) since this move unambiguously makes $\boldsymbol{\sigma}^B$ the outcome by our induction hypothesis. Vetoing $\boldsymbol{\sigma}^B$ will make $w_M^{Bt^*-2}$ the outcome. Vetoing some w_L^{Bt} such that $w_L^{Bt} \neq w_M^{Bt}$ can make $w_M^{Bt^*-2}$ or $\boldsymbol{\sigma}^B$ the outcome.

(2) When $\boldsymbol{\sigma}_L^B \neq \boldsymbol{\sigma}_M^B$, this strategy is the same as L's strategy in the proof of Part 1 of this theorem.

When $\boldsymbol{\sigma}_L^B = \boldsymbol{\sigma}_M^B = \boldsymbol{\sigma}^B$, by the proof of Part 1 of this theorem, against a SPE-following opponent M, L cannot make any $\mathbf{a} \succ_L \boldsymbol{\sigma}^B$ the outcome. On the other hand, $w_L^{Bt^*-2}$ is not the only possible alternative that he must veto. He can prevent any alternative $\mathbf{b} \succ_M \boldsymbol{\sigma}^B$ from becoming the outcome by vetoing such alternatives in no particular order. This is because, by definition of $\boldsymbol{\sigma}^B$, L will have sufficient number of moves to veto all w_L^{Bt} such that $w_L^{Bt} \neq w_M^{Bt}$ even if $w_L^{Bt} \succ_M \boldsymbol{\sigma}^B$ for all such w_L^{Bt} and even if there are no w_L^{Bt} such that $w_L^{Bt} = w_M^{Bt}$ (any w_L^{Bt} such that $w_L^{Bt} = w_M^{Bt}$ is dominated by $\boldsymbol{\sigma}^B$ by the Simultaneous Naive Elimination algorithm). Clearly, once there are no such alternatives, L can veto any alternative but $\boldsymbol{\sigma}^B$.

▌

PROOF OF THEOREM 2: In this proof (and Theorem 3's proof), $\boldsymbol{\gamma}^A$ will abbreviate $\boldsymbol{\gamma}_L^A = \boldsymbol{\gamma}_M^A$. First we need the following definitions and Lemma A2.1. Recall $D^A = \{\mathbf{a} \in A \mid \boldsymbol{\gamma}_i^A \succ \mathbf{a}\ \forall i = L,M\}$. Let $Q_i^A = \{\mathbf{a} \in A \mid \mathbf{a} \succ_i \boldsymbol{\gamma}_j^A \text{ and } \boldsymbol{\gamma}_i^A \succ \mathbf{a},\ i,j = L,M,\ i \neq j\}$.

LEMMA A2.1: *(i) Suppose that* $\boldsymbol{\gamma}_L^A = \boldsymbol{\gamma}_M^A$. *Then* $Q_L^A = Q_M^A = \varnothing$ *and* $\#P_L^A(\boldsymbol{\gamma}^A) = \#P_M^A(\boldsymbol{\gamma}^A)$.

(ii) Suppose that $\boldsymbol{\gamma}_L^A \neq \boldsymbol{\gamma}_M^A$. *Then* $P_M^A(\boldsymbol{\gamma}_L^A)$, $P_L^A(\boldsymbol{\gamma}_M^A)$ *and* D^A *partition A, and* $\boldsymbol{\gamma}_i^A \in P_i^A(\boldsymbol{\gamma}_j^A)$. *Suppose that* $\boldsymbol{\gamma}_L^A = \boldsymbol{\gamma}_M^A$. *Then* $P_M^A(\boldsymbol{\gamma}^A)$, $P_L^A(\boldsymbol{\gamma}^A)$, $\boldsymbol{\gamma}^A$ *and* D^A *partition A.*

(iii) Suppose that $\boldsymbol{\gamma}_L^A \neq \boldsymbol{\gamma}_M^A$. *Then each* $P_i^A(\boldsymbol{\gamma}_j^A)$ *is partitioned by* $P_i^A(\boldsymbol{\gamma}_i^A)$, $\boldsymbol{\gamma}_i^A$ *and* Q_i^A.

PROOF OF LEMMA A2.1: (i) By definition, each $Q_i^A = \varnothing$ is when $\boldsymbol{\Gamma}(A) = \{\boldsymbol{\gamma}^A\}$. $\#P_L^A(\boldsymbol{\gamma}^A) = \#P_M^A(\boldsymbol{\gamma}^A)$ follows from the definition of $\boldsymbol{\Gamma}(A)$ when $\boldsymbol{\Gamma}(A) = \{\boldsymbol{\gamma}^A\}$.

(ii) This is Lemma A1(i) in the Appendix of Anbarci (1993).

(iii) It follows from the definitions of $\boldsymbol{\gamma}_i^A$, $P_i(\boldsymbol{\gamma}_i^A)$, and Q_i.

We will also need the next lemma which follows from Lemma A2.1 and definitions (thus, its lengthy but straightforward proof will be omitted).

LEMMA A2.2. *Consider two problems* $(B, \succeq_L, \succeq_M)$ *and* $(B\backslash a, \succeq_L, \succeq_M)$.

(i) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) = \#P_j^B(\boldsymbol{\gamma}_i^B)$, $i,j = L,M$, $i \neq j$. *Suppose* $B\backslash a$ *is such that* $\boldsymbol{\gamma}_i^B$ *is removed from B. Then some* $\mathbf{a}_i \in P_i^B(\boldsymbol{\gamma}_j^B) \backslash \boldsymbol{\gamma}_i^B$ *becomes* $\boldsymbol{\gamma}_i^{B\backslash a}$ *and* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) < \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(ii) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) > \#P_j^B(\boldsymbol{\gamma}_i^B)$, $i,j = L,M$, $i \neq j$. *Suppose* $B\backslash a$ *is such that* $\boldsymbol{\gamma}_i^B$ *is removed from B. Then some* $\mathbf{a}_i \in P_i^B(\boldsymbol{\gamma}_j^B) \backslash \boldsymbol{\gamma}_i^B$ *becomes* $\boldsymbol{\gamma}_i^{B\backslash a}$ *and* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) \geq \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(iii) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) > \#P_j^B(\boldsymbol{\gamma}_i^B)$*, i,j = L,M, i ≠ j. Suppose B\a is such that* $\boldsymbol{\gamma}_j^B$ *is removed from B. Then some* $\mathbf{a}_j \in P_j^B(\boldsymbol{\gamma}_i^B)\backslash\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *and* $\boldsymbol{\gamma}_i^B$ *becomes* $\boldsymbol{\gamma}_i^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) > \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(iv) Suppose B\a is such that some $\mathbf{a} \in D^B$ *is removed from B. Then each* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that still* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) = \#P_i^B(\boldsymbol{\gamma}_j^B)$*, i,j = L,M.*

(v) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) = \#P_j^B(\boldsymbol{\gamma}_i^B)$*, i,j = L,M, i ≠ j. Suppose B\a is such that* $\mathbf{a}_i \in P_i^B(\boldsymbol{\gamma}_j^B)\backslash\boldsymbol{\gamma}_j^B$ *is removed from B. Then each* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) < \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(vi) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) > \#P_j^B(\boldsymbol{\gamma}_i^B)$*, i,j = L,M, i ≠ j. Suppose B\a is such that* $\mathbf{a}_i \in P_i^B(\boldsymbol{\gamma}_j^B)\backslash\boldsymbol{\gamma}_j^B$ *is removed from B. Then each* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) \geq \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(vii) Suppose that $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_i^B(\boldsymbol{\gamma}_j^B) > \#P_j^B(\boldsymbol{\gamma}_i^B)$*, i,j = L,M, i ≠ j. Suppose B\a is such that* $\mathbf{a}_j \in P_j^B(\boldsymbol{\gamma}_i^B)\boldsymbol{\gamma}_j^B$ *is removed from B. Then each* $\boldsymbol{\gamma}_j^B$ *becomes* $\boldsymbol{\gamma}_j^{B\backslash a}$ *such that* $\#P_i^{B\backslash a}(\boldsymbol{\gamma}_j^{B\backslash a}) < \#P_j^{B\backslash a}(\boldsymbol{\gamma}_i^{B\backslash a})$.

(1) We will use an inductive argument. Clearly, given any $(B,\succeq_L,\succeq_M)$, the result is correct if B contains only two efficient alternatives, and there is no arbitrator selection problem when B is a singleton. We suppose that our claim has been established for B\a and we want to establish it for B. Without loss of generality, let i = L and j = M. Suppose that L uses the following strategy: Offer some $\mathbf{a} \in D^B$ if $D^B \neq \varnothing$. Otherwise, offer some $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$ if $P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B \neq \varnothing$. Otherwise, offer $\boldsymbol{\gamma}_L^B$. If $[\#P_L^B(\boldsymbol{\gamma}_M^B) \geq \#P_M^B(\boldsymbol{\gamma}_L^B)]$ or $[\#P_M^B(\boldsymbol{\gamma}_L^B) - \#P_L^B(\boldsymbol{\gamma}_M^B) = 1$ and $D^B \neq \varnothing]$, accept any $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_L^B)\cup\boldsymbol{\gamma}_L^B$ and reject any other $\mathbf{a}$. If $[\#P_M^B(\boldsymbol{\gamma}_L^B) - \#P_L^B(\boldsymbol{\gamma}_M^B) = 1$ and $D^B = \varnothing]$ and $[\#P_M^B(\boldsymbol{\gamma}_L^B) - \#P_L^B(\boldsymbol{\gamma}_M^B) > 1]$, accept any $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\cup\boldsymbol{\gamma}_M^B$ and reject any other $\mathbf{a}$.

We will first show that, against an M who follows a SPE strategy, when M offers at B, L will be able to make $\boldsymbol{\gamma}_L^B$ the outcome if $[\#P_L^B(\boldsymbol{\gamma}_M^B) > \#P_M^B(\boldsymbol{\gamma}_L^B)]$ and $[\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$ and $\#D^B$ is even], and L will be able to make $\boldsymbol{\gamma}_M^B$ the outcome otherwise. Then we will show that, against an M who follows a SPE strategy, when L offers at B, L will be able to make $\boldsymbol{\gamma}_L^B$ the outcome if $[\#P_L^B(\boldsymbol{\gamma}_M^B) > \#P_M^B(\boldsymbol{\gamma}_L^B)]$ and $[\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$ and $\#D^B$ is odd], and L will be able to make $\boldsymbol{\gamma}_M^B$ the outcome otherwise.

We will not consider the case $\boldsymbol{\gamma}_L^B = \boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}^B$ which has been considered in the proof of Theorem 1 in Anbarci (1993). We will consider the remaining two cases: (1) $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ and $\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$, and $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ and $\#P_L^B(\boldsymbol{\gamma}_M^B) > \#P_M^B(\boldsymbol{\gamma}_L^B)$.

Case 1: $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$.

Suppose M offers some $\mathbf{a} \in D^B$. If L vetoes it, by Lemma A2.2(iv), each $\boldsymbol{\gamma}_j^B$ becomes $\boldsymbol{\gamma}_j^{B\backslash a}$, i,j = L,M. Since, by definition, L prefers any $\boldsymbol{\gamma}_j^B$ to any $\mathbf{a} \in D^B$, he will veto it. If $\#D^B$ is even, then $\#D^{B\backslash a}$ will be odd. Thus, by our induction hypothesis, $\boldsymbol{\gamma}_L^B = \boldsymbol{\gamma}_L^{B\backslash a}$ will become the outcome. If $\#D^B$ is odd, then $\#D^{B\backslash a}$ will be even or zero. Thus, by our induction hypothesis, $\boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}_M^{B\backslash a}$ will become the outcome.

Suppose M offers some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$. Then L will veto it since by Lemma A2.2(v), $\boldsymbol{\gamma}_M^B$ will become $\boldsymbol{\gamma}_M^{B\backslash a}$ and $\boldsymbol{\gamma}_L^B$ will become $\boldsymbol{\gamma}_L^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) > \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$. Thus, by our induction hypothesis $\boldsymbol{\gamma}_L^B = \boldsymbol{\gamma}_L^{B\backslash a}$ will become the outcome. Hence, a rational M would not offer some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$ if $\#D^B$ is odd (recall that "if $\#D^B$ is odd, then $\#D^{B\backslash a}$ will be even or zero; by our induction hypothesis, $\boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}_M^{B\backslash a}$ becomes the outcome.")

Suppose M offers $\boldsymbol{\gamma}_M^B$. Then L will veto it since, by Lemma A2.2(i), some $\mathbf{a}_i \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$ becomes $\boldsymbol{\gamma}_M^{B\backslash a}$ and $\boldsymbol{\gamma}_j^B$

becomes $\boldsymbol{\gamma}_j^{B\backslash a}$ such that $\#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a}) < \#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a})$. Thus, by our induction hypothesis, $\boldsymbol{\gamma}_L^B$ will become the outcome. Hence, again a rational M would not offer some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$ if $\#D^B$ is odd.

Suppose M offers $\boldsymbol{\gamma}_L^B$. Then L will accept it since, by Lemma A2.2(i), otherwise $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) < \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$ will hold, and, by our induction hypothesis, $\boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}_M^{B\backslash a}$ will become the outcome. By definition, L prefers $\boldsymbol{\gamma}_L^B$ to $\boldsymbol{\gamma}_M^B$ and M prefers $\boldsymbol{\gamma}_M^B$ to $\boldsymbol{\gamma}_L^B$. Hence, again a rational M would not offer some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$ if $\#D^B$ is odd.

Suppose M offers some $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$. Then L will accept it, since, by Lemma A2.2(v), otherwise $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) < \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$ would hold, and, by our induction hypothesis, $\boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}_M^{B\backslash a}$ would become the outcome. By definition, L prefers such an alternative $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)$ to $\boldsymbol{\gamma}_M^B$ and M prefers $\boldsymbol{\gamma}_L^B$ to any $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$. Thus, a rational M would not offer $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$.

Thus, when $\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$, against a rational M who makes the offer at some B, L can make $\boldsymbol{\gamma}_L^B$ the outcome if $\#D^B$ is even or zero, and $\boldsymbol{\gamma}_M^B$ otherwise.

Suppose $D^B \neq \varnothing$. Suppose L offers some $\mathbf{a} \in D^B$. If M vetoes such an $\mathbf{a}$, by Lemma A2.2(iv), each $\boldsymbol{\gamma}_i^B$ becomes $\boldsymbol{\gamma}_i^{B\backslash a}$ at B\a when some $\mathbf{a} \in D^B$ is removed from B, i,j = L,M. Since M prefers any $\boldsymbol{\gamma}_i^B$ to such an $\mathbf{a}$, M will veto it. Thus, if $\#D^B$ is even, $\boldsymbol{\gamma}_M^B$ will become the outcome by our induction hypothesis. Otherwise, $\boldsymbol{\gamma}_L^B$ will become the outcome.

Suppose $D^B = \varnothing$. Suppose $P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$ is non-empty. Suppose L offers some $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$. If M vetoes such an $\mathbf{a}$, by Lemma A2.2(v) $\boldsymbol{\gamma}_M^B$ will become $\boldsymbol{\gamma}_M^{B\backslash a}$ and $\boldsymbol{\gamma}_L^B$ will become $\boldsymbol{\gamma}_L^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) < \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$. Since M prefers any $\boldsymbol{\gamma}_i^B$ to such an $\mathbf{a}$, a rational M will veto it. Thus, by our induction, hypothesis $\boldsymbol{\gamma}_M^B = \boldsymbol{\gamma}_M^{B\backslash a}$ will become the outcome.

Suppose $D^B = \varnothing$. Suppose $P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B = \varnothing$; then clearly $P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B = \varnothing$ too since our case presumes $P_L^B(\boldsymbol{\gamma}_M^B) = P_M^B(\boldsymbol{\gamma}_L^B)$. Hence, $\boldsymbol{\gamma}_L^B$ and $\boldsymbol{\gamma}_M^B$ will be the only alternatives left. Then, by our induction, $\boldsymbol{\gamma}_M^B$ will become the outcome.

Thus, when $\#P_L^B(\boldsymbol{\gamma}_M^B) = \#P_M^B(\boldsymbol{\gamma}_L^B)$, against a rational M, L can make $\boldsymbol{\gamma}_L^B$ the outcome when he has the offer at B unless $\#D^B$ is even or zero.

Case 2: $\boldsymbol{\gamma}_L^B \neq \boldsymbol{\gamma}_M^B$ *and* $\#P_L^B(\boldsymbol{\gamma}_M^B) > \#P_M^B(\boldsymbol{\gamma}_L^B)$.

Suppose M offers some $\mathbf{a} \in D^B$. By definition, L prefers any $\boldsymbol{\gamma}_j^B$ to any $\mathbf{a} \in D^B$. Then L will veto it since, by Lemma A2.2(iv), each $\boldsymbol{\gamma}_j^B$ becomes $\boldsymbol{\gamma}_j^{B\backslash a}$, i,j = L,M and by our induction hypothesis, $\boldsymbol{\gamma}_L^B$ will become the outcome.

Suppose M offers some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$. By definition, L prefers any $\boldsymbol{\gamma}_L^B$ to any $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)$. Then L will veto it since, by Lemma A2.2(vii), each $\boldsymbol{\gamma}_j^B$ becomes $\boldsymbol{\gamma}_j^{B\backslash a}$, i,j = L,M such that $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) > \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$. Thus by our induction hypothesis $\boldsymbol{\gamma}_L^B$ will become the outcome.

Suppose M offers $\boldsymbol{\gamma}_M^B$. By definition, L prefers any $\boldsymbol{\gamma}_L^B$ to any $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)$. Then L will veto $\boldsymbol{\gamma}_M^B$, since, by Lemma A2.2(iii), some $\mathbf{a} \in P_M^B(\boldsymbol{\gamma}_L^B)\backslash\boldsymbol{\gamma}_M^B$ becomes $\boldsymbol{\gamma}_M^{B\backslash a}$ and $\boldsymbol{\gamma}_L^B$ becomes $\boldsymbol{\gamma}_L^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) > \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$, and by our induction hypothesis, $\boldsymbol{\gamma}_L^B$ will become the outcome.

Suppose M offers $\boldsymbol{\gamma}_L^B$. By definition, L prefers $\boldsymbol{\gamma}_L^B$ to $\boldsymbol{\gamma}_M^B$. Suppose L vetoes it. Then, by Lemma A2.2(ii), some $\mathbf{a} \in P_L^B(\boldsymbol{\gamma}_M^B)\backslash\boldsymbol{\gamma}_L^B$ would become $\boldsymbol{\gamma}_L^{B\backslash a}$ and $\boldsymbol{\gamma}_M^B$ would become $\boldsymbol{\gamma}_M^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) \geq \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$. Suppose $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) = \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$ would hold. In that case, by our induction hypothesis, if $\#D^B$ is even or zero, $\boldsymbol{\gamma}_M^B$ would become the outcome; if $\#D^B$ is odd, $\boldsymbol{\gamma}_L^{B\backslash a}$ would become the outcome. Suppose $\#P_L^{B\backslash a}(\boldsymbol{\gamma}_M^{B\backslash a}) > \#P_M^{B\backslash a}(\boldsymbol{\gamma}_L^{B\backslash a})$ would hold. Then by our induction hypothesis, $\boldsymbol{\gamma}_L^B$ would become the outcome. However, if L accepts $\boldsymbol{\gamma}_L^B$, it will become the outcome. Thus, L will accept γ_L^B except in the case where some $\gamma_L^{B\backslash a}$ which L prefers to γ_L^B will become the outcome.

Suppose M offers some $\mathbf{a} \in P_L^B(\gamma_M^B)\backslash\gamma_L^B$. By Lemma A2.2(vi), if such an **a** is vetoed, then each γ_j^B would become $\gamma_j^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\gamma_M^{B\backslash a}) \geq \#P_M^{B\backslash a}(\gamma_L^{B\backslash a})$. If L prefers such an **a** to γ_L^B, then L will accept it. But clearly by definition M prefers γ_L^B to any such **a**, and, say, by offering some $\mathbf{a} \in P_M^B(\gamma_L^B)\backslash\gamma_M^B$ or γ_L^B, M could make γ_L^B the outcome; thus, a rational M would not offer any **a** that L prefers to γ_L^B. Thus, this only leaves the possibility that L prefers γ_L^B to such an **a**. Then L will accept it if $\#P_L^B(\gamma_M^B) - \#P_M^B(\gamma_L^B) = 1$ and $\#D^B$ is even or zero, since otherwise by our induction hypothesis γ_M^B would become the outcome. L will reject such an **a** if [$\#P_L^B(\gamma_M^B) - \#P_M^B(\gamma_L^B) = 1$ and $\#D^B$ is even or zero] does not hold (i.e., if either [$\#P_L^B(\gamma_M^B) - \#P_M^B(\gamma_L^B) > 1$] or [$\#P_L^B(\gamma_M^B) - \#P_M^B(\gamma_L^B) = 1$ and $\#D^B$ is odd] holds), since in that case by our induction hypothesis γ_L^B will become the outcome.

Thus, when $\#P_L^B(\gamma_M^B) > \#P_M^B(\gamma_L^B)$, against a rational M who makes the offer at some B, L can make γ_L^B the outcome.

Suppose $D^B \neq \emptyset$. Suppose L offers some $\mathbf{a} \in D^B$. If M vetoes such an **a**, by Lemma A2.2(iv), each γ_i^B becomes $\gamma_i^{B\backslash a}$ at B\a when some $\mathbf{a} \in D^B$ is removed from B, i,j = L,M. Since M prefers any γ_i^B to such an **a**, M will veto it. Thus, γ_L^B will become the outcome by our induction hypothesis.

Suppose $D^B = \emptyset$. Suppose $P_L^B(\gamma_M^B)\backslash\gamma_L^B$ is non-empty. Suppose L offers some $\mathbf{a} \in P_L^B(\gamma_M^B)\backslash\gamma_L^B$. If M vetoes such an **a**, by Lemma A2.2(vi), γ_M^B will become $\gamma_M^{B\backslash a}$ and γ_L^B will become $\gamma_L^{B\backslash a}$ such that $\#P_L^{B\backslash a}(\gamma_M^{B\backslash a}) \geq \#P_M^{B\backslash a}(\gamma_L^{B\backslash a})$ with $D^B = \emptyset$. Thus, by our induction, hypothesis γ_L^B will become the outcome.

Thus, when $\#P_L^B(\gamma_M^B) > \#P_M^B(\gamma_L^B)$, against a rational M, L can make γ_L^B the outcome when he has the offer at B.

(2) It directly follows from the proof of Part 1 of this Theorem. ▌

PROOF OF THEOREM 3: Consider the following strategy *s*, which is a non-equilibrium strategy for Player i: if offered, accept either σ_j^A or any $\mathbf{a} \in P_i^A(\sigma_j^A)$ and veto any other alternative; if with the move, offer Player j's least preferred alternative.[14] It is straightforward to verify that, against a player who follows a subgame-perfect equilibrium strategy, L can guarantee σ_M^A and M can guarantee σ_L^A by using *s*. Thus, at any B a Player i can secure σ_j^A at worst and σ_i^A at best against a Player j who follows a subgame-perfect equilibrium strategy. This, will suffice to prove our theorem since by Lemma 1(iii) there is no efficient $\mathbf{a} \in B$ such that $\sigma_i^A \succ_i \mathbf{a} \succ \sigma_j^A$, i,j = L,M, i ≠ j. ▌

PROOF OF THEOREM 4: A minor modification of the proof of Theorem 2 above will suffice; it will be omitted here. ▌

PROOF OF THEOREM 5: As n tends to infinity, σ_L^A and σ_M^A, will converge when $\sigma_L^A \neq \sigma_M^A$ since (i) by Lemma 1(3), σ_L^A and σ_M^A are adjacent alternatives when $\sigma_L^A \neq \sigma_M^A$, and (ii) any two adjacent alternatives will converge as n tends

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14 To see that *s* is not an equilibrium strategy, consider A = {**a**,**b**} and suppose that $\mathbf{a} \succ_L \mathbf{b}$ and $\mathbf{b} \succ_M \mathbf{a}$, and suppose that L has the first move and offers **a**. Observe that *s* dictates that M accepts **a**.

to infinity. Now we need to show that they will converge to α. Suppose L is the first-mover and his utility in S is measured on the horizontal axis.

By the Simultaneous Naive Elimination algorithm (which identifies $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$), L will start eliminating the alternatives (and thus cells) in C_1, beginning with the cell closest to the horizontal axis and continuing with the adjacent cell, and so on; then he will eliminate cells in C_2, beginning with the cell closest to the horizontal axis and continuing with the adjacent cell, and so on. M will start eliminating the alternatives (and thus cells) in R_1 first, beginning with the cell closest to the vertical axis and continuing with the adjacent cell, and so on; then he will eliminate cells in R_2 beginning with the cell closest to the vertical axis and continuing with the adjacent cell, and so on. (At some stages, observe that they may eliminate the same alternative by the Simultaneous Naive Elimination algorithm.)

By the Simultaneous Naive Elimination algorithm, either $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ will remain at the end. Note the following facts: (1) Due to the pattern of elimination of alternatives (and thus cells) by the players given by the Simultaneous Naive Elimination algorithm, until either $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ remains, trivially both players will have removed equal numbers of alternatives less preferred than $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ by one or both players. (2) In particular, "the number of alternatives that are less preferred than $\boldsymbol{\sigma}_L^A$ by both players" and "the number of alternatives that are less preferred than $\boldsymbol{\sigma}_M^A$ by both players will converge" since (i) $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ converge as n tends to infinity, (ii) the alternatives that are jointly dominated by $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ are uniformly distributed in S (by P*), and (iii) players follow the pattern of elimination of alternatives described above.

Thus, as n tends to infinity, (1) and (2) above imply that "the number of alternatives that L prefers to $\boldsymbol{\sigma}_M^A$ *and* M does not prefer to $\boldsymbol{\sigma}_M^A$" and "the number of alternatives that M prefers to $\boldsymbol{\sigma}_L^A$ *and* L does not prefer to $\boldsymbol{\sigma}_L^A$" must converge too since, until either $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ remains, trivially both players will have removed equal numbers of alternatives, which are less preferred than $\boldsymbol{\sigma}_L^A$ or $\boldsymbol{\sigma}_M^A$ by one or both players (also, since $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$ converge as n tends to infinity, "the number of alternatives that L prefers to $\boldsymbol{\sigma}_L^A$ *and* M does not prefer to $\boldsymbol{\sigma}_L^A$" and "the number of alternatives that M prefers to $\boldsymbol{\sigma}_M^A$ *and* L does not prefer to $\boldsymbol{\sigma}_M^A$" must converge too). But, by Anbarci (1993), we know that, as n tends to infinity, "the number of alternatives that L prefers to $\boldsymbol{\gamma}_M^A$ *and* M does not prefer to $\boldsymbol{\gamma}_M^A$" and "the number of alternatives that M prefers to $\boldsymbol{\gamma}_L^A$ *and* L does not prefer to $\boldsymbol{\gamma}_L^A$" converge too. Since, by Anbarci (1993) $\boldsymbol{\gamma}_L^A$ and $\boldsymbol{\gamma}_M^A$ converge to α, so must $\boldsymbol{\sigma}_L^A$ and $\boldsymbol{\sigma}_M^A$. ▌

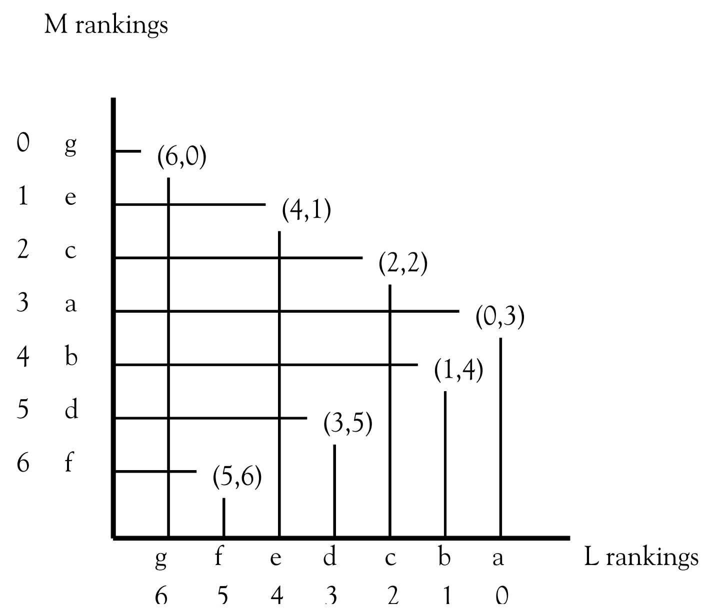
M rankings
0 g
1 e
2 c
3 a
4 b
5 d
6 f
(6,0)
(4,1)
(2,2)
(0,3)
(1,4)
(3,5)
(5,6)
g f e d c b a
6 5 4 3 2 1 0
L rankings

FIGURE 1

REFERENCES

Albers, W. and Albers, H: (1983), On the prominence structure of the decimal system, in R. Scholz, ed., *Decision Making Under Uncertainty*, North Holland: 271-287.

Anbarci, N.: (1993), Noncooperative foundations of the area monotonic solution, *Quarterly Journal of Economics* 108: 245-258.

Anbarci, N. and Bigelow, J.: (1994), The area monotonic solution to the cooperative bargaining problem, *Mathematical Social Sciences* 28: 133-142.

Ashenfelter, O. and Bloom, D.: (1984), Models of arbitrator behavior: theory and evidence, *American Economic Review* 74: 111-24.

Bloom, D. and Cavanagh, C.: (1986), An analysis of the selection of arbitrators, *American Economic Review*, 76: 408-22.

Nash, J. :(1950), The Bargaining Problem, *Econometrica* 18: 155-162.

Nash, J. :(1953), Two-Person Cooperative Games, *Econometrica* 21: 128-140.