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ORIGINAL RESEARCH

On the Ramsey Test Analysis of 'Because'

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Abstract The well-known formal semantics of conditionals due to Stalnaker (in: Rescher (ed) Studies in logical theory, Blackwell, Oxford, 1968), Lewis (Counterfactuals, Blackwell, Oxford, 1973a), and Gärdenfors (in: Niiniluoto, Tuomela (eds) The logic and 1140 epistemology of scientific change, North-Holland, Amsterdam, 1978, Knowledge in flux, MIT Press, Cambridge, 1988) all fail to distinguish between trivially and nontrivially true indicative conditionals. This problem has been addressed by Rott (Erkenntnis 25(3):345–370, 1986) in terms of a strengthened Ramsey Test. In this paper, we refine Rott's strengthened Ramsey Test and the corresponding analysis of explanatory relations. We show that our final analysis captures the presumed asymmetry between explanans and explanandum much better than Rott's original analysis.

1 Introduction

The assertion of a conditional 'if α, then γ' commonly implies that the antecedent α is in some way relevant for the consequent γ. The semantics of variably strict conditionals by Lewis (1973a) and Stalnaker (1968) spells out the relation of relevance between antecedent and consequent via a system of spheres of possible worlds. This semantics yields plausible results for counterfactual conditionals. However, it fails to account for the relevance between antecedent and consequent in

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the indicative case. That is, if α and γ are true in the actual world, then $\alpha > \gamma$ is true in that world, independently of whether there is any connection between α and γ. For example, 'If Munich is a town in Germany, then Lund is a town in Sweden' is true, provided that Munich is a town in Germany and Lund a town in Sweden. This seems to be an absurd consequence.

The Ramsey Test approach to conditionals by Gärdenfors (1978, 1988) faces an analogous problem: if α and γ are believed to be true, then $\alpha > \gamma$ must be accepted in that approach. Hence, the formal semantics of conditionals by Stalnaker (1968), Lewis (1973a), and Gärdenfors (1978, 1988) fail to distinguish between trivially true and non-trivially true indicative conditionals.

In what follows, we attempt to analyse the presumed relevance between antecedent and consequent by means of a strengthened Ramsey Test. More specifically, we suggest that a conditional be accepted iff it passes the following test:

> First, suspend judgement about the antecedent and the consequent. Second, add the antecedent (hypothetically) to your stock of explicit beliefs. Finally, consider whether or not the consequent is entailed by your explicit beliefs.

We believe that this variant of a strengthened Ramsey Test has interesting applications in different areas of philosophical logic:

(1) The analysis of indicative, subjunctive, and counterfactual conditionals in natural language.
(2) The analysis of the conjunction 'because' in natural language.
(3) The logical analysis of quantitative scientific explanations.
(4) The conditional analysis of causation.

There is work underway by the authors on all of these topics. In this paper, however, we focus on the semantic analysis of the word 'because' in natural language.

Let us briefly explain why the problem of relevance between antecedent and consequent is particularly pressing for an analysis of 'because' in everyday and scientific contexts. If a speaker asserts 'γ because of α', then he or she already believes, or knows, that α and γ. Hence, the standard Ramsey Test conditional $\alpha > \gamma$ is far too weak for a conditional analysis of 'because'. For, this conditional does not require the antecedent to be relevant for the consequent in case α and γ are believed to be true.

In addition to the relevance between explanans and explanandum, our analysis aims to account for the presumed asymmetry of explanatory relations. As is widely agreed upon, the presence of a tower may well explain the occurrence of a shadow, but not vice versa. That is, we endorse 'there is a shadow because of the tower', but not 'there is a tower because of the shadow'. The structure of this simple example captures a large class of asymmetric explanatory relations. We take it as a starting point to eventually work out a general account of scientific explanations.

The following analysis of 'because' will be shown to yield the intended results for the tower-shadow scenario. Let $\gg$ designate our strengthened Ramsey Test conditional. That is, $\alpha \gg \gamma$ iff, after suspending judgment about α and γ, an agent can infer γ from the supposition of α (in the context of further beliefs in the background). The schema of our definition is then:

$$\text{Because } \alpha, \gamma \text{ (relative to } K(S)) \quad \text{iff} \quad \alpha \gg \gamma \in K(S) \quad \text{and} \quad \alpha, \gamma \in K(S)$$

where $K(S)$ designates the belief set of the epistemic state S. We represent epistemic states by belief bases rather than belief sets. This proves crucial to account for the asymmetry between explanans and explanandum.

The next step is to generalise the tower-shadow scenario. We shall specify the inferential relations on the basis of which our analysis verifies statements of the form ‘γ because of α’, provided the underlying representation of epistemic states satisfies certain conventions. While this analysis captures the presumed asymmetry for the tower-shadow scenario as well as further classes of explanatory relations, there remain cases for which the analysis yields symmetric explanations. We therefore conclude with a proposal for a strictly asymmetric Ramsey Test conditional, which in turn yields a strictly asymmetric analysis of ‘because’.

The present investigation is very much inspired by the work of Hans Rott (1986) on strengthening the Ramsey Test and his corresponding analysis of ‘because’. We will show, however, that Rott’s analysis fails to account for the asymmetry of explanatory relations in the case of the tower-shadow scenario and a related class of explanatory relations. This is why we propose an alternative strengthening of the Ramsey Test.

Methodologically, we are working upward from the applications to the formal theory. This strategy seems preferable, for example, when it comes to choosing between different variants of a strengthened Ramsey Test. Moreover, it is worth noting that we take certain intuitions about the propriety of explanatory relations for granted, for instance, the intuition of asymmetry in the tower-shadow example. Our analysis thus aims to capture this and related intuitions about the propriety of explanatory directions.

2 Belief Revision Theory

2.1 Belief Revision: Basic Ideas

Belief revision theory provides us with a precise semantics of belief changes for the Ramsey Test. Let us therefore very briefly review the basic ideas of this theory. Let K be a set of formulas that represent the beliefs of an agent and α a formula that represents a single belief. In the AGM framework. In the AGM framework, as developed by Alchourrón et al. (1985), one distinguishes three types of belief change of a belief set K by a formula α:

(1) Expansions $K + \alpha$
(2) Revisions $K * \alpha$
(3) Contractions $K - \alpha$.

An expansion of K by α consists in the addition of a new belief α to the belief set K. This operation is not constrained by any considerations as to whether the new epistemic input α is consistent with the set K of present beliefs. Hence, none of the present beliefs is retracted by an expansion.

A revision of K by α, by contrast, can be described as the *consistent integration* of a new epistemic input α into a belief system K. If α is consistent with K, it holds that $K + A = K * \alpha$. If, however, α is not consistent with K, some of the present beliefs are to be retracted in order to believe α.

A contraction of K by α, finally, consists in retracting a certain formula α from the presently accepted system of beliefs. This operation will be used to define the *suspension of judgement about* α in our strengthened version of the Ramsey Test.

Belief changes can be defined in various ways. In what follows, we explain two approaches to the determination of belief revisions and contractions. First, entrenchment based revisions, which are part of the classical AGM theory. Second, partial meet base revisions, which use the AGM framework but have been developed at a later stage. We do not introduce the AGM postulates for belief revisions and contractions as they will mostly remain in the background. For an accessible exposition of the classical AGM theory, including the postulates, the reader is referred to Gärdenfors (1988).

2.2 Entrenchment Based Revisions

If we revise a belief set K by a new belief α that is not consistent with K, some beliefs of K need to be retracted so as to consistently integrate α into the belief system. In other words, we need to retract $\neg\alpha$ first in order to be able to accept α. This idea has been expressed by the Levi identity:

$$K * \alpha = (K - \neg\alpha) + \alpha. \tag{Levi identity}$$

The challenge arising here is to find a sensible way of retracting $\neg\alpha$ from K. From a logical point of view, there is no unique solution to this problem, set aside trivial belief revision problems. For, there are several subsets K' of K such that $\neg\alpha \notin K'$. How shall we choose among those subsets?

Belief revision theory tells us that an operation of contraction (as well as that of a revision) should be guided by two principles. First, the *conservativity principle*: when forced to change our beliefs, we should retain as many as possible of the present beliefs. Second, certain beliefs are more firmly established than others. When revising our beliefs, we should maintain the former and be prepared to give up the latter, at least if this is logically possible. The two principles have been formalised by the theory of entrenchment based revisions.

Let us begin with the formal characterisation of the epistemic entrenchment relation. $\alpha \leq \beta$ means that α is at most as entrenched as β. The following postulates formally characterise this relation (Gärdenfors 1988, pp. 89–91):

$$\text{If } \alpha \leq \beta \quad \text{and} \quad \beta \leq \chi, \text{ then } \alpha \leq \chi \tag{EE1}$$

$$\text{If } \alpha \vdash \beta, \text{ then } \alpha \leq \beta \tag{EE2}$$

$$\alpha \leq \alpha \wedge \beta \ \text{ or } \ \beta \leq \alpha \wedge \beta \tag{EE3}$$

$$\text{When } K \neq K_{\perp},\ \alpha \notin K \ \text{ iff } \ \alpha \leq \beta \text{ for all } \beta \in K \tag{EE4}$$

$$\text{If } \ \beta \leq \alpha \ \text{ for all } \beta \in \mathcal{L}\text{, then } \alpha \in Cn(\emptyset). \tag{EE5}$$

where $\mathcal{L}$ is the set of all formulas of the formal language used to analyse belief changes, and $K_{\perp}$ the absurd belief set containing all elements of $\mathcal{L}$.

Epistemic entrenchment orderings and contractions are interdefinable by (G-) which has been introduced by Gärdenfors and Makinson (1988). In what follows, we shall use only the direction from epistemic entrenchment orderings to contractions:

$$\begin{aligned} &\beta \in K - \alpha \ \text{ iff } \ \beta \in K \\ &\qquad \text{and either } \ \alpha < (\alpha \vee \beta) \ \text{ or } \ \alpha \in Cn(\emptyset). \end{aligned} \tag{G$-$}$$

So, a belief β of K will remain in the belief set after a contraction with α iff either α is strictly less epistemically entrenched than $\alpha \vee \beta$ or α is a logical truth. As one would expect, the strict and equivalence relations of epistemic entrenchment are defined as $\alpha < \beta$ iff $\alpha \leq \beta$ but not $\beta \leq \alpha$, and $\alpha \sim \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$. Once contractions are defined, revisions can be determined using the Levi identity.

The classical AGM theory assumes that belief sets are logically closed. That is, $K = Cn(K)$, where Cn is a consequence operation that satisfies certain standard properties, such as monotonicity, compactness, and the deduction theorem (Hansson 1999, Ch. 1^{+}). In this paper, we assume Cn to be given by classical logic. Henceforth, Cn is always used to designate the consequence operation of classical propositional logic.

2.3 Belief Bases

The study of belief bases and revisions thereof is intended to achieve a more realistic representation of epistemic states and their dynamics. It can be seen as a cognitively more adequate refinement of classical belief revision theory, which only investigates changes of logically closed belief sets. Why are belief sets felt to be a deficient representation of epistemic states from a cognitive point of view? The problem is that even for languages of propositional logic, any belief set is infinite. This contrasts with the finiteness of human minds and computers. As human minds have only a finite capacity to memorise sentences that are accepted, so have computers only a finite storage.[1]

Unlike belief sets, belief bases are allowed to be finite and are usually assumed to be so. The idea is to have a set H of explicit beliefs that represents all further implicit beliefs in the sense that the latter beliefs are consequences of H. In formal terms:

[1] The study of belief base changes has been originated by Sven Ove Hansson. Much of what we are going to say about such revisions draws on his *Textbook of Belief Dynamics* (Hansson 1999).

$$K(H) = Inf(H).$$

K contains all beliefs of the agent with a belief base H, i. e. the explicit beliefs and those beliefs that the agent is committed to accept because they are inferable from the explicit beliefs. *Inf* is an inferential closure operation. We assume that this operation is given by classical logic. Thus, $K(H) = Cn(H)$.

2.4 Partial Meet Base Revision

A contraction of a belief base H by α can be defined using the notion of a *remainder set* $H \perp \alpha$ (Hansson 1999, p. 12):

Definition 1 $H \perp \alpha$

Let H be a set of formulas and α a formula. $H' \in H \perp \alpha$ iff

(1) $H' \subseteq H$
(2) $\alpha \notin Cn(H')$
(3) there is no H'' such that $H' \subset H'' \subseteq H$ and $\alpha \notin Cn(H'')$.

A simple means to define the contraction of H by α is to take the intersection of the members of the remainder set $H \perp \alpha$:

$$H - \alpha = \bigcap H \perp \alpha. \tag{FMBC}$$

This way of defining a contraction is also referred to as *full meet base contraction*.

We can refine this way of determining contractions by invoking the idea of an epistemic ordering among the members of $H - \alpha$. Suppose $\leq$ is a binary transitive relation. $A \leq A'$ means that A' is epistemically not inferior to A. To put it more simply, $A \leq A'$ means that A' is epistemically at least as good as A. Using such an epistemic ordering, we can define a selection function for the remainder set as follows:

$$\sigma(H \perp \alpha) = \{H' \in H \perp \alpha \mid H'' \leq H' \text{ for all } H'' \in H \perp \alpha\}. \tag{Def σ}$$

Then, we take the selected members of the remainder set to define the contraction of H by α:

$$H - \alpha = \bigcap \sigma(H \perp \alpha). \tag{PMBC}$$

It remains to explain the expansion of a belief base H by α, which is straightforward:

$$H + \alpha = H \cup \{\alpha\}. \tag{H+α}$$

Now we are in a position to put everything together, thus defining partial meet base revisions:

$$H * \alpha = \bigcap \sigma(H \perp \neg\alpha) + \alpha. \tag{PMBR}$$

2.5 Prioritised Belief Bases

While the idea of an epistemic ordering of beliefs is quite plausible, it is far from clear how to order the subsets of a set of beliefs. This does not matter for studying the formal properties of belief changes, but it does so for studying concrete examples. Hence, we finally show how an epistemic ordering among the members of a belief base can be translated into an ordering among the subsets of such a base.

Drawing on the work by Brewka (1991), we assume the epistemic ordering among the items of H to be a strict weak ordering. Such an ordering can be represented by a sequence of subsets of H:

$$\mathbf{H} = \langle H_1, \ldots, H_n \rangle.$$

where $H_1, \ldots, H_n$ is a partition of H. $\mathbf{H}$ is called a *a prioritised belief base*. $H_1, \ldots, H_n$ are sets of formulas that represent explicit beliefs, and the indices represent an epistemic ranking of the beliefs. H_1 is the set of the most firmly established beliefs, the beliefs in H_2 have secondary priority, etc.

This prioritisation of beliefs can be used to define an epistemic ordering among the subsets of H:[2]

Definition 2 $H'' \leq H'$

Let H be a set of formulas, and H'' and H' be subsets of H. $H'' \leq H'$ iff there is no i $(1 \leq i \leq n)$ such that

(1) $H' \cap H_i \subset H'' \cap H_i$
(2) for all $j < i$ $(j \geq 1), H'' \cap H_j = H' \cap H_j$.

In the following investigation, we assume that our belief base has exactly two levels of epistemic priority: the upper level, containing the generalisations, and the lower level, which contains our beliefs about atomic facts. These levels of epistemic priority affect the determination of belief changes: when we retract a belief α, we retract first beliefs about atomic facts before we retract generalisations. If necessary, we also retract generalisations, but only if the retraction of α cannot be achieved by retractions of beliefs about atomic facts. For the considerations to follow, it may be helpful to have a graphical representation of such a prioritised belief base in mind:

G
L

G stands for the set of generalisations, while L contains the beliefs about the atomic facts. L is a set of literals. A literal is an atomic formula or its negation.

If we need to distinguish between strict and *ceteris paribus* laws, we can do so by distinguishing between two corresponding levels of generalisations. Strict laws have

[2] This definition is inspired by Brewka (1991), but the resulting belief revision operation is not equivalent with the one defined there.

priority over *ceteris paribus* laws. The notion of a generalisation subsumes strict and non-strict laws.

The present convention about generalisations and literals allow us to define the revision of epistemic states (in addition to the revision of belief sets of epistemic states). That is,

$$(H, <) * \alpha$$

has a well defined meaning insofar as Definition 2, (Def σ), and (PMBR) together define the revised belief base H', while the revised epistemic ordering $<'$ is determined by the simple convention that generalisations have priority over literals. So there is an epistemic state $(H', <')$ such that $(H', <') = (H, <) * \alpha$. Iterated belief base revisions are thus well understood.

Such are the basic ideas and definitions about belief changes that will be used in the present analysis of 'because'. We study the properties of belief set revisions using an epistemic entrenchment ordering of beliefs. Belief base revisions are studied in terms of partial meet base revisions with an underlying selection function that is defined by a prioritised belief base.

Why do we not study belief set revisions in terms of partial meet belief set revisions? The simple reason for this choice is that the idea of an epistemic entrenchment ordering is easier applicable than the idea of an epistemic ordering of subsets of a logically closed (and so infinite) belief set. Despite the differences between partial meet belief set revisions and entrechment based belief set revisions, it has been shown that any entrenchment based belief set revision can be represented by a partial meet belief set revision, and vice versa (Gärdenfors 1988, Ch. 4). Hence, results about the former can be translated into results about the latter.

2.6 Why Belief Bases?

In the final analysis of 'because', we shall use belief base revisions rather than belief set revisions. For, the combination of belief bases with our novel variant of a strengthened Ramsey Test allows us to capture the asymmetry of explanatory relations for a large class of scenarios, including the famous tower-shadow scenario. Belief set revisions of the classical AGM theory, by contrast, turn out not to be suited for this purpose (cf. Sects. 5 and 6).

Admittedly, belief base revision theory is less well established than belief set revision theory. This is surprising in light of distinctive merits of belief bases if compared with belief sets. First, there is the above indicated finite-memory argument in favour of belief bases. A belief set is an infinite entity and so cannot be fully comprehended by a human mind, at least on a literal understanding of comprehension. Likewise, a computer cannot store a belief set for obvious reasons.

Second, relatedly, while the study of formal properties is not much impeded by the infinite character of belief sets, the study of concrete examples certainly is. Note that even the notation $K = Cn(\alpha, \beta, \gamma)$ is misleading if K is supposed to be a belief set. For, it suggests that the belief set K is generated by the belief base $\{\alpha, \beta, \gamma\}$. This is misleading because belief set revisions differ from belief base generated revisions as regards their formal properties (cf. Hansson 1999, Ch. 4). Moreover, it is just impossible to completely specify the epistemic priority ordering for belief set revisions unless we view this ordering to be generated by priorities among the members of a belief base. In the latter case, we study belief base generated revisions but not belief set revisions. It is therefore not surprising that, in the belief set revision literature, examples of concrete belief changes are hardly formalised. It is next to impossible to find a fully formalised application of belief set revision theory, even to toy examples. Using belief bases, by contrast, makes applying belief revision theory to particular examples much easier.

In sum, belief bases are cognitively more plausible and much easier to use when it comes to formalising belief systems that concern specific examples. However, there is also an influential objection to belief bases. This objection appeals to the *principle of the irrelevance of syntax* (Dalal 1988), which is sometimes violated in the belief base approach. Contrary to this principle, Brewka (1991) has pointed out that choosing the formulation $\{p \wedge q\}$ over $\{p, q\}$ may well be intended to make a difference. This choice is justified, for example, if p and q are only to be given up together.

We are not convinced that the principle of the irrelevance of syntax is justified from a cognitive perspective. Contrary to this principle, one can point out that the study of belief bases carries on what Benthem (2008) has termed the *cognitive turn* in philosophical logic, i. e. the development of logical systems that aim to represent and to theoretically explain human reasoning. For the above indicated reasons, a finitely bounded human mind has no alternative to working with belief bases. Note, finally, that there is a very simple way to respect the principle of the irrelevance of syntax within the belief base approach. It suffices to require that the members of a belief base conform to a specific logical form. For example, we can require to represent generalisations by disjunctions of literals, while beliefs about atomic facts be represented by literals. The latter requirement has already been made explicit.

3 The Ramsey Test

3.1 The Ramsey Test by Ramsey

Ramsey (1950, footnote 1) proposes the following evaluation procedure for conditionals that is known as the Ramsey Test (RT):

> If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense 'If p, q' and 'If p, $\overline{q}$' are contradictories. [...] If either party believes *not p* for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses.

The RT is an epistemic evaluation recipe for conditionals in the sense that the evaluation depends on the beliefs of the agent(s) involved in the hypothetical discussion. This evaluation recipe for an epistemic agent has been pointedly expressed by Stalnaker (1968, p. 102):

First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true.

3.2 The Ramsey Tests by Gärdenfors and Levi

The AGM theory allows Gärdenfors (1988, Ch. 7) to concisely formalise the epistemic recipe of the Ramsey Test:

$$\alpha > \gamma \in K \text{ iff } \gamma \in K * \alpha. \tag{RT$_G$}$$

Thus, a conditional $\alpha > \gamma$ is accepted in K iff γ is believed in the course of revising K by α.

Writing $K * \alpha$ and speaking of belief set revisions is misleading insofar as this suggests that it is the belief set itself that is revised. This is not quite correct because there is no sensible way of uniquely determining the revision of a belief set by a new epistemic input. It is rather the belief set of a particular epistemic state that is revised, according to the AGM theory. As indicated in the previous section, epistemic states can be represented in various ways. Syntactic representation schemes commonly have the form of a pair $(A, <)$, where A is a set of formulas and $<$ an epistemic ordering among formulas or sets of formulas. A is logically closed for belief set revision schemes, while it does not have to be so for belief base revision schemes. Most possible world approaches to belief revision work with epistemic states of the form $(W, <)$, where W is a set of possible worlds and $<$ a plausibility ordering among these worlds (cf. Grove 1988).

It is thus more appropriate to write $K(S) * \alpha$ or $K(A, <) * \alpha$ and to speak of the revision of the belief set of an epistemic state. In this notation, S stands for an epistemic state. As regards the Ramsey Test, it seems consequently more appropriate to write:

$$\alpha > \gamma \in K_{>}(S) \text{ iff } \gamma \in K(S) * \alpha. \tag{RT_L}$$

$K(S)$ stands for the beliefs in non-modal propositions, i.e. beliefs that can be expressed by formulas of classical logic without any conditional or modal operator. $K_{>}$, by contrast, stands for the conditionals accepted, or believed, by the agent on the basis of the Ramsey Test.

In spirit, the distinction between $K(S)$ and $K_{>}(S)$ goes back to Levi (1988). It was also Levi (1988) who emphasised another distinction, viz. between believing and merely accepting conditionals, in light of a famous triviality theorem proved by Gärdenfors (1986). If we merely accept conditionals without viewing them as truth-apt, we avoid the fatal consequences of the triviality theorem for the Ramsey Test.[3]

We shall discuss triviality briefly in Sect. 6.4. There, it will be shown that an important premise of the proof by Gärdenfors (1986) is violated for our variant of a strengthened Ramsey Test. This allows us to remain neutral as to whether

[3] The triviality theorem continues to provoke lively research in belief revision theory (see, e.g., Rott 2011; Leitgeb 2010; Bradley 2007).

conditionals are properly believed or merely accepted. As (RT_L) gives us a clearer instruction of how to carry out a particular Ramsey Test, we prefer (RT_L) over (RT_G) as formulation of the Ramsey Test. A similar strategy has been recommended by Hansson (1992) who shows that triviality can be avoided by taking belief bases to represent the epistemic states underlying the Ramsey Test. Following Hansson (1992), we favour (RT_L) without making a commitment as regards the belief-acceptance distinction for conditionals.

3.3 Absurdity: Relevance Issues of the Ramsey Test

(RT_G) leads to the absurdity that any two accepted formulas α, β bear a conditional relation between each other, as has been shown by Rott (1986). Suppose $\alpha, \beta \in K$. By the AGM belief revision postulates, we know that, if $\alpha \in K$, then $K * \alpha = K$.[4] Therefore, $\beta \in K * \alpha$. We conclude by (RT_G) that $\alpha > \beta \in K$. Hence,

$$\text{If } \alpha, \beta \in K, \text{ then } \alpha > \beta \in K. \tag{Absurdity}$$

(Absurdity) expresses that a (RT_G) agent accepts a conditional connection between any two formulas she accepts. If, for example, 'Munich is a town in Germany' and 'Lund is a town in Sweden' is accepted by an agent, then (RT_G) prescribes that 'If Munich is a town in Germany, then Lund is a town in Sweden' should also be accepted.

The just observed problem carries over to a Ramsey Test analysis of 'because' as proposed by Ramsey (1950, p. 156, our emphasis) himself; there he relates conditional sentences 'If α, then γ' and sentences 'Because α, γ' by stating:

> *because* is merely a variant on *if*, when [the antecedent] p is known to be true.

It is a consequence of this view that, if $\alpha \in K$, then 'if' and 'because' coincide. Thus, given Ramsey's view, (RT_G) prescribes that our agent accepts the sentences 'Because Munich is a town in Germany, Lund is a town in Sweden', and the converse 'Because Lund is a town in Sweden, Munich is a town in Germany'. In more general terms, (Absurdity) entails that a (RT_G) agent accepts any because sentence composed of any two accepted formulas, once Ramsey's analysis of 'because' is adopted. This is in any case not less troubling than the (Absurdity) of the merely conditional reading. Moreover, the absurdity violates the asymmetry of one usage of because in natural language, viz. the one according to which the acceptance of 'because α, β' precludes the acceptance of 'because β, α', at least for some α, β. We might, for instance, accept that we sometimes get injured because we often play football, but then we would not accept that we often play football because we get sometimes injured.[5]

[4] For the list of AGM belief revision postulates, see for example Gärdenfors (1988, pp. 54–55).

[5] Although the latter because sentence seems totally fine in a context, in which the agent performs a so-called *inference to the best explanation*: repeatedly playing football may be the best explanation for occasional injuries. This reasoning *towards* (as opposed to *from*) the putative explanatory 'causes' seems to justify the usage of 'because' in the other direction. So peculiar as natural language is, we do not want to ban this usage of 'because' from natural language. For now, we just want to focus on the one usage showing the asymmetry without being entirely sure that this usage is strictly asymmetric. For example, 'because p and q are true, $p \wedge q$ is true' does not seem to preclude 'because $p \wedge q$ is true, p and q are true'.

In sum, (RT_G) fails to capture the semantics of indicative conditionals. This failure leads to verifying absurd explanatory relations if we accept Ramsey's analysis of 'because' in terms the Ramsey Test. The underlying problem is that the conditional connective $>$ does not express a proper relation of relevance between the antecedent and the consequent. Rott (1986) proposes to invalidate (Absurdity) by modifying (RT_G) such that the mere acceptance of α, γ does not result in the acceptance of 'If α, γ', as we shall see in the next section.

4 Rott's Ramsey Test Analysis of 'Because'

Rott (1986) embeds a parallel analysis of 'if' and 'because' in a systematic theory of universal conditionals. Universal conditionals, Rott claims, are not instantiated in natural language. However, he proposes a semantics of the natural language conjunctions 'if', 'if ... might', 'because', 'though' and 'even if' by specifying constraints on the acceptance of 'antecedents' and 'consequents' of the respective universal conditionals.[6]

Rott aims to analyse the type of 'because' that points to a reason or an explanation. The basic idea of Rott's analysis is that 'Because α, γ' be synonymous to 'α is a reason or an explanation for γ'. In an explanatory sentence, for example, 'because' may be seen as a connective that relates explanans and explanandum. As such a pointer 'because α, γ' expresses a relation of positive relevance between (explanans) α and (explanandum) γ. However, we have seen in the last section that (RT_G) does not capture a proper conditional connection of (positive) relevance between antecedent and consequent.

Rott's analysis of 'because' is driven by considerations of how to establish a relation of positive relevance, and this means for a start to find ways to invalidate (Absurdity). One such way consists in (i) the modification of (RT_G) to his Strong Ramsey Test; another way in (ii) the contraction of the belief set by the consequent, before the set is revised by the respective antecedent. The implementation of (i) and (ii) in Rott's analysis gives rise to the scheme of universal pro-conditionals. This scheme allows him to derive a semantics of the indicative and subjunctive ifs and of a certain 'because' of natural language.[7]

[6] 'Antecedent' is here a generalisation of the antecedent of a conditional sentence. It stands for 'subordinate clause' of the respective sentence. This mirrors Rott's view that all of the mentioned conjunctions are derived from a framework of universal conditionals. In detail, the indicative and subjunctive ifs and 'because' fall into the category of universal pro-conditionals, 'though' into the category of universal contra-conditionals, and 'even if' into the category of universal un-conditionals. See Rott (1986, pp. 355–363).

[7] Considerations of how to systematically categorise conditionals result in the schemes of universal contra- and un-conditionals as well.

4.1 The Strong Ramsey Test and the Contraction of the Belief Set by the Consequent

In Sect. 3.3, we have seen that (RT_G) together with Ramsey's analysis of 'because' fails since this analysis validates (Absurdity). This failure requires a modification of (RT_G) that invalidates (Absurdity). Rott's idea is to strengthen (RT_G), which results in his 'Strong Ramsey Test':

$$\alpha \ggg \gamma \in K \text{ iff } \gamma \in K * \alpha \text{ and } \gamma \notin K * \neg\alpha. \qquad (SRT_R)$$

We obtain in the situation where a (SRT_R) agent already accepts α, γ:

$$\text{If } \alpha, \gamma \in K, \text{ then } [\alpha \ggg \gamma \in K \text{ iff } \gamma \notin K * \neg\alpha]. \qquad (1)$$

Implication (1) shows how (SRT_R) invalidates the (Absurdity) that the mere acceptance of α and γ is sufficient for the acceptance of 'Because α, γ'. The reason is that the second conjunct of the right-hand-side of (SRT_R) still needs to be satisfied. This modification makes (SRT_R) "more adequate for natural language conditionals than" Gärdenfors's Ramsey Test, so Rott argues, since "it explicitly requires the antecedent to be positively relevant for the consequent" (Rott 1986, p. 352).[8]

The situation represented by implication (1) requires the (SRT_R) agent to perform a contrary-to-fact supposition. The counterfactual supposition of $\neg\alpha$ needs to retract the accepted γ from the belief set. In this sense the contrary-to-fact supposition 'makes a difference' as to whether γ is accepted.[9]

An alternative method to invalidate (Absurdity) consists in the contraction of the belief set by the consequent before (RT_G) is applied.

$$\alpha > \gamma \in K \text{ iff } \gamma \in (K - \gamma) * \alpha. \qquad (RT_G^-)$$

In the situation where a (RT_G^-) agent already accepts α, γ, the consequent γ may not be in the belief set K after a contraction by γ and a subsequent revision by α, i. e. $\gamma \notin (K - \gamma) * \alpha$. The consequent γ is only accepted if it is a consequence of the contracted belief set $(K - \gamma)$ revised by the antecedent α. We may say that (the belief expressed by) α is an inferential epistemic reason for γ such that the supposition of α epistemically brings about the acceptance of γ.

4.2 Universal Pro-conditionals and 'Because'

If we amend the (SRT_R) and its dual by (RT_G^-), we obtain the scheme of universal pro-conditionals that is according to Rott (1986, p. 355) "perfect for the analysis of

[8] This idea has recently been exploited in an analysis of evidential support by Chandler (2013).

[9] (SRT_R) structurally resembles Lewis (1973b)'s notion of causal dependence in terms of counterfactual conditionals. Using $\Rightarrow$ for causal dependence, we can transcribe Lewis's idea into the notation of belief revision: $\alpha \Rightarrow \gamma \in K$ iff $\gamma \in K * \alpha$ and $\neg\gamma \in K * \neg\alpha$. Note that Lewis's causal dependence requires a stronger version of difference making than (SRT_R), viz. the adoption of $\neg\gamma$ in $K * \neg\alpha$ in contrast to the mere retraction of γ. Moreover, Lewis might say that 'γ because α' means γ is causally dependent on α, when α and γ are (believed to be) true. Given Lewis (1973a)'s semantics for counterfactuals, we obtain the following implication paralleling (1): If $\alpha, \gamma \in K$, then $[\alpha \Rightarrow \gamma \in K \text{ iff } \neg\gamma \in K * \neg\alpha]$.

what you can call a ‘conditional connection’”. We write $\Rightarrow$ for the universal pro-conditional. The scheme is then given by

$$\begin{aligned} \alpha \Rightarrow \gamma \in K \quad &\text{iff } [\alpha \ggg \gamma \in (K - \gamma)] \text{ or } [\neg\alpha \ggg \neg\gamma \in (K - \gamma)] \\ &\text{iff } [\gamma \in (K - \gamma) * \alpha \text{ and } \gamma \notin (K - \gamma) * \neg\alpha] \\ &\text{or } [\neg\gamma \notin (K - \gamma) * \alpha \text{ and } \neg\gamma \in (K - \gamma) * \neg\alpha]. \end{aligned} \tag{UPC}$$

(UPC) says that there is a conditional connection (of positive relevance) between antecedent α and consequent γ iff (i) α lets us infer γ in the context of $K - \gamma$, and the supposition of $\neg\alpha$ makes a difference as to whether γ is accepted, *or* (ii) $\neg\alpha$ lets us infer $\neg\gamma$ in the context of $K - \gamma$, and the supposition of α makes a difference as to whether $\neg\gamma$ is accepted.

Rott derives the natural language ifs and ‘because’ from the scheme (UPC) by specifying acceptance constraints on antecedent and consequent of the respective connective. For universal pro-conditionals the acceptance constraint is that the acceptance status of the antecedent and the consequent is the same. In accordance with Ramsey’s view, ‘because α, γ’ is only accepted if the antecedent α is accepted. Let ${}^a\!\Rightarrow$ be the connective of Rott’s ‘because’, where the superscript ^{a} indicates that the antecedent is accepted. Then Rott’s analysis of the natural language ‘because’ is given by

$$\begin{aligned} \alpha \, {}^a\!\Rightarrow \gamma \in K \quad &\text{iff } \alpha \Rightarrow \gamma \in K \text{ and } \alpha, \gamma \in K \\ &\text{iff } \gamma \in (K - \gamma) * \alpha \text{ or } \neg\gamma \in (K - \gamma) * \neg\alpha \\ &\text{and } \alpha, \gamma \in K. \end{aligned} \tag{Because$_R$}$$

(Because$_R$) can be derived from (UPC) using a proposition in Rott (1986, p. 350):

Proposition 1 *If $\gamma \in K * \alpha$ and $\gamma \in K * \neg\alpha$ then $\gamma \in K$.*

Proposition 2 *Let K be a non-absurd belief set and γ a non-tautology. Then* (UPC) *and $\alpha, \gamma \in K$ implies* (Because$_R$).

Proposition 2 and all subsequent propositions are proven in the Appendix.

5 Symmetry Problems of Rott’s ‘Because’

5.1 A General Symmetry Problem

We can show now that (Because$_R$) is symmetric for a large class of potentially explanatory relations. For this to be achieved, we distinguish between trivial and non-trivial implications in K. As the members of K are non-modal propositional formulas, laws and generalisations are to be represented by material implications. However, not all implications in a belief set K represent instances of generalisations. As is well known, if $\neg\alpha \in K$, then, for any γ, $\alpha \rightarrow \gamma \in K$. Likewise, if $\gamma \in K$, then, for any α, $\alpha \rightarrow \gamma \in K$. How can we distinguish, then, between trivial implications and non-trivial implications in K, on the understanding that only the latter represent instances of genuine generalisations? Arguably, a material implication is non-trivial in a belief set K iff it "survives" a contraction by the negation of the antecedent and a contraction by the consequent:

Definition 3 Non-trivial implication in K
K contains an implication $\alpha \rightarrow \gamma$ non-trivially iff

(1) $\alpha \rightarrow \gamma \in K$, and
(2) $\alpha \rightarrow \gamma \in K - \neg\alpha$, and
(3) $\alpha \rightarrow \gamma \in K - \gamma$.

Now, if non-trivial implications represent instances of generalisations, it is reasonable to assume that non-trivial implications are more entrenched than literals and conjunctions of literals.[10] For, this assumption guarantees that generalisations are available for counterfactual considerations. To see this consider the following: suppose $\neg\alpha$, $\alpha \rightarrow \gamma \in K$, where $\alpha \rightarrow \gamma$ is a non-trivial implication, such as 'if it snows on the street, the street gets white'. Further, suppose that $\neg\alpha < \alpha \rightarrow \gamma$, where $\alpha < \beta$ means that β is strictly more entrenched than α. By (G-) and the entrenchment postulate (EE2), it holds then that $\alpha \rightarrow \gamma \in K - \neg\alpha$. By the Levi identity, this implies that $\gamma \in K * \alpha$, as it should be. The street would get white if it were to snow on the street.

If, by contrast, $\alpha \rightarrow \gamma \leq \neg\alpha$, we would have (i) $\neg\alpha \vee \gamma \leq \neg\alpha$. By (EE2), however, we know that (ii) $\neg\alpha \leq \neg\alpha \vee \gamma$. Using $\vdash (\alpha \rightarrow \gamma) \leftrightarrow (\neg\alpha \vee \gamma)$, we can infer from (i), (ii), and (G-) that $\alpha \rightarrow \gamma \notin K - \neg\alpha$. Hence, our generalisation represented by $\alpha \rightarrow \gamma$ would not be available for counterfactual considerations on the hypothetical assumption of α. We could not infer that the street would get white if it were to snow on the street. It goes without saying that this result is highly counterintuitive.

We must wonder, finally, whether or not a non-trivial implication $\alpha \rightarrow \gamma$ can be retracted in the course of a revision by a proposition β that is neither related to the antecedent α nor to the consequent γ. Such a retraction does not seem reasonable at all, even though there may be an entrenchment ordering that requires it. For example, we should not retract 'if it snows on the street, the street gets white' if we get to know that Munich is a town in Germany, Anna goes to the party, etc. The case study in Sects. 5.2 and 6.3 will further support the claim that implications representing generalisations must be more entrenched than literals and conjunctions of literals.

Hence, it is reasonable to make the following assumption:

Assumption 1 Let $\alpha \rightarrow \gamma$ be a non-trivial implication in K. Let δ, β be literals or conjunctions of literals. $<_K$ denotes the entrenchment ordering associated with the beliefs of K and $<_{K-\delta}$ the entrenchment ordering of the beliefs of $K - \delta$. Then, $\beta <_K \alpha \rightarrow \gamma$ and $\beta <_{K-\delta} \alpha \rightarrow \gamma$.

[10] See Hansson (1999, p. 96) for a brief justification of why law-like statements should—in most cases—be epistemically more entrenched than factual statements.

Thus, the second conjunct of this assumption says that the implication $\alpha \rightarrow \gamma$ remains more entrenched than literals and conjunctions thereof, after a contraction of K by a literal or a conjunction of literals.

We can show that ${}^{a}\!\!\Rightarrow$ is symmetric, if $\alpha \rightarrow \gamma$ is an accepted non-trivial implication.

Proposition 3 *Let α and γ be literals or conjunctions of literals. Further, $\alpha, \gamma \in K$. Suppose that $\alpha \rightarrow \gamma$ is a non-trivial implication in K and Assumption 1 holds for $\alpha \rightarrow \gamma$. Then $\alpha\, {}^{a}\!\!\Rightarrow \gamma \in K$ and $\gamma\, {}^{a}\!\!\Rightarrow \alpha \in K$.*

Proposition 3 says that, under Assumption 1, if $\alpha \rightarrow \gamma$ is a non-trivial implication in K, then ${}^{a}\!\!\Rightarrow$ is symmetric since both $\alpha\, {}^{a}\!\!\Rightarrow \gamma \in K$ and $\gamma\, {}^{a}\!\!\Rightarrow \alpha \in K$. In particular, we have the following problem for Rott's ${}^{a}\!\!\Rightarrow$: if $\gamma \rightarrow \alpha$ is a non-trivial implication in K, then $\alpha\, {}^{a}\!\!\Rightarrow \gamma \in K$. This means, for instance, if an agent accepts the sensible generalisation 'if there is lightning, then there is thunder', the agent is also committed to accept 'because there is thunder, there is lightning'. Of course, from an information-theoretic point of view, it is sensible to say that we *believe* there is lightning, because we *believe* that there is thunder. However, we are interested in the asymmetric usage of 'because' that goes beyond a purely information-theoretic relation. To be more precise, we aim to define an epistemic relation of bringing about according to which we *accept*, for instance, that lightning brings about thunder, but we should not *accept* the converse.

5.2 Further Symmetry Problems

We have seen that the notion of because implemented in ${}^{a}\!\!\Rightarrow$ is symmetric on some reasonable assumptions. However, as we shall see shortly, the scope of Proposition 3 is limited. By means of a simple example scenario, we lift the limitation by showing that ${}^{a}\!\!\Rightarrow$ does not capture intuitively asymmetric relations of relevance. The scenario illustrates the symmetry of ${}^{a}\!\!\Rightarrow$ over and above Proposition 3. It may be understood as revealing further symmetry problems for (Because$_R$) by characterising another class of problematic applications. The characterisation of a class means here that once the reader understands the underlying structure of the example scenario, she may easily come up with her own examples of the problematic class.

Suppose there is a tower (t), the sun is shining (s), so that the sun casts a shadow (sh).[11] (t, s, and sh are propositional constants to be used for the below formalisation of the example.) Intuitively, the presence of the tower and the sunlight *explain* that there is a shadow, but not vice versa, i.e. there being a shadow does not explain that there is a tower. After all, there might be, for instance, another opaque object exposed to sunlight. However, it seems that the following common-sense generalisation is entailed by our background knowledge:

[11] The example is similar to the famous tower-shadow scenario, for which there is wide agreement that the height of the tower together with the altitude of the sun explain the length of the shadow, but not vice versa. However, see Van Fraassen (1980, pp. 132–34) for an interesting challenge of this agreement involving the notion of relevance. Note that we simplified the original tower-shadow scenario such that a wider class of examples succumbs to the asymmetry problem of Rott's (Because$_R$).

$$t \wedge s \rightarrow sh. \tag{2}$$

We assume in our scenario that (2) is epistemically more entrenched in the agent's background knowledge than the facts t, s, and sh. Let us assume, moreover, that $K = Cn(\{t, s, sh, t \wedge s \rightarrow sh\})$. Note that (2) plays the role of a non-trivial implication that remains more entrenched than any literals or conjunctions of literals after the contraction of the epistemic state by some literals or conjunctions thereof. Assumption 1 is thereby satisfied, and thus (2) constitutes a special case of Proposition 3 according to which a non-trivial implication may be of the form $\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n \rightarrow \gamma$ for a finite $n \in \mathbb{N}$, where the α_i are literals. If such a non-trivial implication with a conjunction of antecedent conditions is accepted, we obtain $\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n \overset{a}{\Rightarrow} \gamma$ as well as $\gamma \overset{a}{\Rightarrow} \alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n$. In particular, suppose the non-trivial implication (2) with $n = 2$. Then a (Because$_R$) agent is committed to both the plausible direction that 'There is a shadow, because there is a tower and the sun is shining' and the less plausible direction that 'There is a tower and the sun is shining, because there is a shadow'.

Following our cognitive habits, we do not list all of the antecedent conditions when using 'because', especially if there are many. Rather we only state the pertinent ones given a particular contextual knowledge. If this is true, the scope of Proposition 3 is limited. For, this proposition does not tell us whether $\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n \rightarrow \gamma$ being a non-trivial implication entails that $\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_{n-k} \overset{a}{\Rightarrow} \gamma$ and $\gamma \overset{a}{\Rightarrow} \alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_{n-l}$, where $1 \leq k, l < n$. In what follows we show that further symmetry problems arise for non-trivial implications of the logical form $\alpha_1 \wedge \alpha_2 \rightarrow \gamma$, and because sentences of the form $\alpha_1 \overset{a}{\Rightarrow} \gamma$ and $\gamma \overset{a}{\Rightarrow} \alpha_1$, where α_i and γ are literals, respectively. We illustrate this further class of symmetry problems by proving the following proposition about the tower-shadow scenario, in which an antecedent condition of a generalisation remains implicit in the background knowledge, viz. 'the sun is shining'. Thereby we show that $\overset{a}{\Rightarrow}$ may express a symmetric relation even if Proposition 3 is not applicable.

Proposition 4 *Assume a* (Because$_R$) *agent accepts all facts and the generalisation of the tower-shadow scenario*, i. e. $t, s, sh, t \wedge s \rightarrow sh \in K$, *where the order of epistemic entrenchment is* $t, s, sh < t \wedge s \rightarrow sh$. *Then,* $t \overset{a}{\Rightarrow} sh \in K$ *if* $t \leq sh$ *and* $sh \overset{a}{\Rightarrow} t \in K$ *if* $sh \leq t$.

The proposition shows that Rott's analysis only verifies the desired direction 'because of the tower there is a shadow', if 'there is a tower' is at most as entrenched as 'there is a shadow', and thus both beliefs are given up when the belief set is contracted by 'there is a shadow'. Moreover, a (Because$_R$) agent is committed to believe the undesired direction 'because of the shadow there is a tower', if 'there is a shadow' is at most as entrenched as 'there is a tower'. The question is, of course, why should some of those atomic beliefs be more or less entrenched? It seems to be an ad hoc strategy to assume the entrenchment ordering that verifies the intended results. Why should the belief that there is a tower be strictly less entrenched than the belief that there is a shadow?

The underlying structure of the tower-shadow scenario illustrates the tendency of (Because$_R$) to express a symmetric relation. In addition to the symmetry shown by

Proposition 3, the scenario illustrates why $^{a}\Rightarrow$ is prone to symmetry problems, even if certain antecedent conditions are not explicitly stated. The tower-shadow scenario thus lets us recognize symmetry problems for (Because$_R$) even beyond the assumptions of Proposition 3.

Rott seems to be aware of the symmetry problems. He writes in Rott (1986, p. 347):

> [If] someone insists that *because* is positively about an asymmetric causal relation in the world, I have to confess that I cannot give a satisfactory interpretation of this 'causal' *because*. I shall concentrate on the 'informative' *because* specifying just reasons. Yet I conjecture that this 'informative' *because* is the more common and the more general one, and that the 'causal' *because* can eventually be characterised as a special case by a few non-epistemic conditions.

The quote is interesting in at least three respects. (i) Rott distinguishes between several interpretations of the word 'because' that correspond to different usages of the word. He calls the usage that points to reasons 'informative', and the usage that expresses an 'asymmetric causal relation' 'causal'. (ii) Rott assumes that a concept of causation needs to satisfy 'a few non-epistemic conditions'. Hence, he writes that a causal relation be 'in the world', although he outlines a purely epistemic account. (iii) Rott conjectures that the ontological 'causal' usage is derivative as a special case from the epistemic 'informative' usage.

We agree that the word 'because', like many natural language connectives, has several interpretations, and thus is used in a variety of ways. As for the non-epistemic conditions that a concept of causation may have to satisfy, the requirement that a cause precedes its effect seems still promising. As is well known, this requirement is central to Hume's account of causation.[12] Moreover, the requirement of temporal precedence has been adopted by Spohn (2006) in his ranking-theoretic elaboration of the basic Humean idea about causation. We shall not further pursue this line here, but confine ourselves to finding an epistemic interpretation of 'because' that is asymmetric. We leave it open, however, whether or not this interpretation deserves to be called 'causal'. In doing so, we bracket the topic of causation for the time being and leave it to another occasion.[13]

Recall from Sect. 4 Rott's basic idea that the 'informative' usage of 'because' expresses a reason or an explanation. But then Rott's analysis runs into the following difficulty: (Because$_R$) fails to capture the asymmetry associated with some explanations, as we have seen in the tower-shadow scenario. In other words, if an explanation should intuitively be asymmetric, then (Because$_R$) is too permissive as it allows for intuitively incorrect converse explanations. We consider this

[12] "A cause is an object precedent and contiguous to another, and so united with it, that the idea of the one determines the mind to form the idea of the other, and the impression of the one to form a more lively idea of the other." (Hume (1739/1978, p. 170) One might wonder whether the temporal order of cause and effect is a properly non-epistemic condition in the context of Hume's work, but this is a question that need not concern us here.

[13] We deal with 'causation' in a follow-up paper.

difficulty to call for a complementation of Rott’s merely ‘informative’ because with an epistemically asymmetric because.

5.3 Using Belief Bases

So far the symmetry of Rott’s ‘because’ has been characterised within the original AGM belief revision theory, assuming validity of all *Gärdenfors postulates* as established in (Gärdenfors 1988, Ch. 3). This accords with the framework assumed in Rott’s analysis. Now we switch from the original AGM theory, which employs belief sets, to belief revision using belief bases (as outlined in Sects. 2.5–2.3) and Levi’s formulation of the Ramsey Test.

As regards revisions and contractions, belief bases behave somewhat differently if compared to belief sets.[14] Notably, recovery ($K^{-}5$), which is needed in the proof of Proposition 4, is not valid for belief base revisions.[15] Thus the question arises: could we resolve the symmetry problems of (Because$_R$) by resorting to belief base revisions instead of belief set revisions? The answer to this question is no – as the below proposition shows.

Proposition 5 *Assume a* (Because$_R$) *agent accepts all the formulas in* $K(H, <) = K(S)$ *for* $H = \{t, s, sh, t \wedge s \rightarrow sh\}$, *where the order of epistemic priority is* $t \sim s \sim sh < t \wedge s \rightarrow sh$. *Then* $t \overset{a}{\Rightarrow} sh \notin K_{>}(S)$ *and* $sh \overset{a}{\Rightarrow} t \in K_{>}(S)$.

As compared to Propositions 4, 5 makes things even worse for Rott’s analysis of because. Using belief bases, (Because$_R$) does not verify the desired direction ‘because of the tower there is a shadow’. Moreover, a (Because$_R$) agent is still committed to believe the undesired direction ‘because of the shadow there is a tower’. This shows that it is by no means a trivial task to find a Ramsey Test operator capturing the asymmetry of the tower-shadow example, even if we employ belief bases.

In sum, switching from belief sets to belief bases does not resolve the problem that Rott’s analysis of ‘because’ verifies the undesired direction that there is a tower because of the shadow. In the next section, we propose an alternative strengthening of (RT$_G$) that avoids this troublesome result and fares better in capturing asymmetric relations of relevance.

6 Another Ramsey Test Analysis of ‘Because’

6.1 Further Strengthening the Ramsey Test Semantics

Ramsey (1950, p. 247) expresses his idea about the semantics of conditionals as follows:

[14] See Hansson (1999) for a very comprehensive study of belief base revisions and contractions, including a detailed comparison to belief set revisions and contractions.

[15] Hansson (1999, Ch. 2) discusses the recovery postulate in detail. In Benthem (2015, p. 302) recovery is justifiedly called “the most conroversial” AGM postulate.

> In general we can say [...] that 'If ϕ then ψ' means that ψ is inferable from ϕ, that is, of course, from ϕ together with certain facts and laws not stated but in some way indicated by the context.

The inferability from lawlike generalisations and facts is thus tantamount to the acceptability of conditionals, as has been pointed out in Levi (2007, pp. 9–10)'s interpretation of Ramsey's approach to conditionals. Ramsey's test question is whether the consequent can be inferred from generalisations judged to be reliable and some facts that specify the boundary conditions or contextual knowledge so that the generalisations are applicable. A conditional 'If ϕ then ψ' is thus acceptable just in case the consequent is inferable from the antecedent, the atomic facts judged to be true, and the judged to be reliable generalisations. Notice that Ramsey's idea requires the retention of reliable generalisations. Otherwise, the conditional cannot be inferred. This is very much in line with the discussion of non-trivial implications of Sect. 5.1.

Inspired by Ramsey, our test question is: after the suspension of judgment on everything that entails antecedent and/or consequent, is an agent disposed to infer the consequent from the antecedent and the remaining background beliefs including the generalisations? This basic idea of our semantics may be expressed by the following evaluation recipe:

> First, suspend judgement about the antecedent and the consequent. Second, add the antecedent (hypothetically) to your stock of explicit beliefs. Finally, consider whether or not the consequent is entailed by your explicit beliefs.

Our basic idea is thus split into two steps. The first step consists in an 'agnostic move', i. e. our agent suspends acceptance and/or rejection of antecedent α and consequent γ with respect to her epistemic state.[16] The second step then consists in supposing or hypothesising the antecedent α, and checking whether the consequent γ is thereby inferred.

In order to render our idea precise, we introduce a belief function that helps us formally implement the agnostic move.

Definition 4 Belief Function

Let $\bot$ be some arbitrary classical contradiction, and ϕ a formula.

$$B(\phi) = \begin{cases} \phi & \text{if } \phi \in K \\ \neg\phi & \text{if } \neg\phi \in K \\ \bot & \text{otherwise.} \end{cases}$$

[16] The first step is reminiscent of Edmund Husserl (1913, §§31–33)'s Pyrrhonian epoché. This phenomenological epoché denotes the method of suspending or bracketing (German: *Einklammerung*) the acceptance status of one's beliefs about the world. We apply the Pyrrhonian idea with a—by far—smaller scope: we demand an agent to suspend her respective belief status of the particular antecedent and consequent under consideration. We call the bracketing or suspension of antecedent and consequent 'agnostic move' and credit Pyrrho by labelling our Strengthened Ramsey Test (SRT_P).

Now, we are in a position to present the core of our strengthened Ramsey Test semantics. Let $\gg$ be the conditional connective of the Strengthened Ramsey Test. Then our evaluation recipe can be formally expressed as follows:

$$\alpha \gg \gamma \in K_{>} \text{ iff } \alpha > \gamma \in K_{>} - (B(\alpha) \vee B(\gamma)) \text{ iff } \gamma \in K - (B(\alpha) \vee B(\gamma)) * \alpha. \tag{SRT_P}$$

The evaluation of $\alpha \gg \gamma$ consists of two steps. (i) The agnostic move is implemented by a contraction of the belief set K by $B(\alpha) \vee B(\gamma)$. The result is a new belief set K' such that $\neg\alpha, \alpha, \neg\gamma, \gamma \notin K'$ is guaranteed. Moreover, K' does neither contain $B(\alpha) \vee B(\gamma)$ nor $\neg B(\alpha) \vee \neg B(\gamma)$. The contraction by $B(\alpha) \vee B(\gamma)$ amounts to the agent's operation of suspending acceptance and/or rejection with respect to α and γ. We call the result of this contraction *the agnostic belief set* K'. (ii) $\alpha \gg \gamma \in K_{>}$ iff $\alpha > \gamma \in K'_{>}$ iff $\gamma \in K' * \alpha$. The second step requires for $\alpha \gg \gamma$ to be accepted that (RT_L) is satisfied for $\alpha > \gamma$ with respect to the agnostic belief set K' of step (i).

We noted that $\neg\alpha \notin K - B(\alpha) \vee B(\gamma)$. By the Levi identity, we obtain:

$$\gamma \in K - (B(\alpha) \vee B(\gamma)) * \alpha \text{ iff } \gamma \in K - (B(\alpha) \vee B(\gamma)) + \alpha. \tag{3}$$

Moreover, we have:

$$\gamma \in K - (B(\alpha) \vee B(\gamma)) + \alpha \text{ iff } K - (B(\alpha) \vee B(\gamma)), \alpha \vdash \gamma \tag{4}$$

where $\vdash$ is the provability relation of classical logic. We arrive thus at an alternative formulation of (SRT_P):

$$\alpha \gg \gamma \in K_{>} \text{ iff } K - (B(\alpha) \vee B(\gamma)), \alpha \vdash \gamma. \tag{5}$$

This formulation emphasises the inferential character of our strengthened Ramsey Test: $\alpha \gg \gamma$ means that γ is inferable from α together with the beliefs in $K - (B(\alpha) \vee B(\gamma))$.

Interestingly, (SRT_P) validates (Absurdity) if we employ belief sets. Suppose $\alpha, \gamma \in K$. Applying step (i) yields $\alpha, \gamma \notin K'$. However, by the recovery postulate, $(\alpha \vee \gamma) \rightarrow \gamma \in K'$ and thus, by closure, $\alpha \rightarrow \gamma \in K'$. Hence, $\gamma \in K - (\alpha \vee \gamma) * \alpha$.

In contrast, (SRT_P) invalidates (Absurdity) if we employ belief bases. For, then, recovery is not satisfied any more. Let $S = (H, <)$ be an epistemic state. Then, it is an open question whether or not $\gamma \in K(S') * \alpha$, and thus whether or not $\alpha \gg \gamma \in K_{>}(S)$.

Using belief bases provides Ramsey's semantics of conditionals a transparent meaning: $\alpha \gg \gamma$ means that γ is inferrable from α together with the 'facts and laws not stated' in the conditional, but explicitly stored in the agnostic epistemic state S'. In other words, our agent will accept $\alpha \gg \gamma \in K_{>}(S)$ only if she is disposed to (classically) infer γ from α together with the literals and generalisations stored in the agnostic epistemic state S'.

According to Ramsey's quote in Sect. 3.1 'two people' can disagree when arguing 'If p, will q?', even if both believe $\neg p$ for certain. Our (SRT_P) clarifies the sense in which $p \gg q \in K(S)$ and $p \gg \neg q \in K(S)$ are contradictories: the 'laws or hypotheses' of S must be different. It could be that two agents accept the same facts while they accept different generalisations. Hence, they would not have a dispute about facts but about how to revise the beliefs. If for 'either party' $\neg p \in K(S)$, then $K(S') * p$ entails either q or $\neg q$ or none of $q, \neg q$. A consistent epistemic agent cannot accept both $p \gg q$ and $p \gg \neg q$. Based on different sets of generalisations, however, two agents may well have different inferential dispositions. This is Ramsey's wisdom wherefore he speaks about 'two people'.

Let us compare Rott's (SRT$_R$) and (Because$_R$) with our (SRT_P). Our semantics is closer to Gärdenfors's (RT$_G$) than Rott's is. The only difference, apart from using the formulation (RT_L), to (RT$_G$) consists in bracketing the epistemic status of antecedent and consequent. After this suspension of judgement, (RT$_G$) is applied in the standard way. The bracketing of the epistemic status in the agnostic epistemic state may be seen as a further strengthening of (RT$_G^-$) in the sense that not only the consequent is contracted from the belief set, but also the antecedent. Almost ironically, we solve problems of excessive symmetry by a 'more' symmetric contraction as compared to (RT$_G^-$). The additional epistemic suspension of the antecedent is the reason in virtue of which our semantics does not require a contrary-to-fact-supposition, but nevertheless expresses a relation of positive relevance. In contrast, (Because$_R$) requires a counterfactual supposition in view of cases in which the antecedent α remains in $K - \gamma$. For, then $K - \gamma = (K - \gamma) * \alpha$, and so $\gamma \notin (K - \gamma) * \alpha$. Without a contrary-to-fact-supposition, our semantics does not rely on a notion that structurally resembles a counterfactual notion of causal dependence.

6.2 Another Analysis of 'Because'

By Ramsey's view on the relation between 'if' and 'because' and Rott's constraint on universal pro-conditionals, we can read our (SRT_P) 'if' as 'because' in the case when $\alpha, \gamma \in K$. Thus, we obtain the following analysis:

$$\alpha \overset{P}{\Rightarrow} \gamma \in K_{>} \text{ iff } \alpha \gg \gamma \in K_{>} \text{ and } \alpha, \gamma \in K. \qquad (\text{Because}_P)$$

To avoid well-known paradoxes involving tautologies, we may furthermore require that the consequent of an explanatory relation is contingent. That is, we may require that $\gamma \notin Cn(\emptyset)$. But we shall not further explore paradoxes with logical truths surrounding conditionals and explanatory relations as this is a different topic.

Arguably, the semantics of (Because$_P$) is simpler than that of (Because$_R$) insofar as it does not rest on counterfactual suppositions. In comparison to (RT$_G^-$), our semantics puts more emphasis on the inference relation between antecedent (plus context knowledge) and consequent by bracketing the epistemic status of the antecedent *and* the consequent—which is in the spirit of Ramsey's ideas about conditionals. Moreover, our semantics solves the class of symmetry problems associated with the tower-shadow scenario of Sect. 5.2, as we shall see in the next section.

6.3 Symmetry Problems Resolved

We reconsider now the tower-shadow scenario with respect to our analysis of ‘because’. In the original AGM framework, we will see that our analysis does not provide the desired asymmetry, because of the recovery postulate governing the belief set. If we use belief bases, however, we can show (1) that our semantics for ‘because’ validates the intuitively correct (explanatory) because statement, and (2) that our semantics invalidates the intuitively incorrect, converse because statement.

Proposition 6 *Assume a* (Because$_P$) *agent accepts all facts and the single, more entrenched generalisation of the tower-shadow scenario*, i.e. $t, s, sh, t \wedge s \rightarrow sh \in K$. *Then* $t \overset{P}{\Rightarrow} sh \in K_>$ *and* $sh \overset{P}{\Rightarrow} t \in K_>$.

The proposition shows that (Because$_P$) validates (with respect to K) that ‘because there is a tower, there is a shadow’, as desired. However, in the original AGM framework, our semantics validates (with respect to K) also the undesired direction ‘because there is a shadow, there is a tower’.

Let us move on to belief bases. The following proposition shows the asymmetry we were looking for.

Proposition 7 *Assume a* (Because$_P$) *agent accepts all the formulas in* $K(H, <) = K(S)$ *for* $H = \{t, s, sh, t \wedge s \rightarrow sh\}$, *where* $t, s, sh < t \wedge s \rightarrow sh$ *and* $t \sim s \sim sh < t \wedge s \rightarrow sh$. *Then* $t \overset{P}{\Rightarrow} sh \in K_> (S)$, *but* $sh \overset{P}{\Rightarrow} t \notin K_> (S)$.

The proposition shows that (Because$_P$) validates (with respect to $K(S)$) that ‘because there is a tower, there is a shadow’, as desired. Using belief bases, our semantics invalidates (with respect to $K(S)$) the undesired direction ‘because there is a shadow, there is a tower’.

The result is reasonable since a shadow may be cast by various things. It does not have to be a tower. To this claim, one may object that the shadow cast by *this* tower has a particular shape that is normally only produced by the very tower. This objection presupposes that an agent can uniquely infer the antecedent from the consequent. But there are frequently occurring examples where the agent is not able to do this. Here is such an example: person A sees person B taking poisonous arsenic, which leads to the death of B. Once A has suspended judgement about B’s poisoning himself and his death, the assumption of B’s taking arsenic lets A infer B’s death, but the assumption of B’s death does not allow A to infer B’s intake of arsenic.

We note that the generalisation $t \wedge s \rightarrow sh$ figures as ‘directed inference ticket’ when using belief bases in virtue of the absence of recovery. In general, it is easy to show that an implication $\alpha \rightarrow \gamma$ is ‘non-trivially’ in a belief base H iff $\alpha \gg \gamma \in K_> (H, <)$, where α, γ are literals or conjunctions thereof. In contrast to $\overset{a}{\Rightarrow}$, the ‘non-triviality’ of an implication $\alpha \rightarrow \gamma \in H$ is not sufficient for the acceptance of $\gamma \gg \alpha \in K_> (H, <)$.

In the original AGM framework, (Because$_P$) succumbs to the same class of symmetry problems as Rott’s analysis. In contrast to (Because$_R$), however, our semantics provides the desired asymmetry, but only if we use belief bases. The transition to belief bases seems necessary to break the symmetry that derives from the recovery postulate.[17] We conclude that, using belief bases, our strengthened Ramsey Test semantics solves the class of symmetry problems characterised by the tower-shadow scenario. The idea behind (Because$_P$) is thus able to capture these asymmetric relations of relevance.

6.4 Non-triviality

A note on *triviality* is in order here. As is well known, Gärdenfors (1986, 1988, Ch. 7) has shown that his version of the Ramsey Test implies, in the context of the full set of AGM postulates, that there are only *trivial* belief revision systems. (The precise meaning of triviality need not concern us here.) Therefrom, he concluded that either the Ramsey Test or a rationality postulate called *preservation*

$$\text{if } \neg\alpha \notin K \text{ and } \beta \in K, \quad \text{then} \quad \beta \in K * \alpha \qquad (K^{*}P)$$

has to be given up (where β may well be a conditional). Does our strengthened Ramsey Test fall prey to the triviality theorem? It does not. For here is a counterexample to ($K^{*}P$). Suppose $K = Cn(q \rightarrow \neg r)$. Hence, $q \gg \neg r \in K_{>}$. Now, let us revise K (consistently) with $q \rightarrow r$ such that (i) $\neg q < \neg q \vee r$ for the beliefs of $K' = K * (q \rightarrow r) = Cn(\{q \rightarrow r, q \rightarrow \neg r\})$. Using $(G-)$, we can infer from (i) that $(q \rightarrow r) \in K' - (\neg q \vee \top)$, where $\top$ stands for a tautology. Hence, $(q \rightarrow \neg r) \notin K' - (\neg q \vee \top)$. By the definition of $\gg$, this implies that $q \gg \neg r \notin K'_{>}$. Hence, ($K^{*}P$) is violated. (For belief bases, an analogous result can easily be obtained with the same formulas). Our semantics of $\gg$ is therefore non-trivial in the sense that a crucial premise of Gärdenfors' triviality theorem is violated.

Gärdenfors's triviality result forces us to chose between preservation (for conditionals) and the Ramsey Test. Our semantics for $\gg$ does not validate preservation in the first place. Hence, we can side with the Ramsey Test without falling prey to the triviality result.

6.5 Note on Package Contraction

We have expressed the suspension of judgement about the antecedent α and the consequent γ using a contraction by the disjunction $\alpha \vee \gamma$. We should acknowledge, however, that the suspension of judgement can also be expressed by an operation called *package contraction*. This operation contracts a belief set K by another belief set A. Such contractions can be determined using ideas about partial meet revision, which are based on the notion of a *package remainder* (Fuhrmann and Hansson 1994, Sect. 8):

Definition 5 $K \perp A$

Let K and A be two sets of formulas. $B \vdash A$ means that B entails at least one member of A. $K' \in K \perp A$ iff

(1) $K' \subseteq K$

[17] This finding might cast doubt on the validity of the recovery postulate from a different angle. For the usual criticisms see, for instance, Hansson (1991) and Makinson (1987).

(2) $K' \nvdash A$
(3) there is no K'' such that $K' \subset K'' \subseteq K$ and $K'' \vdash A$.

A selection function for $K \perp A$ can then be invoked, as explained in Sect. 2.4. (The notion of a selection function applies to remainder sets of belief sets and belief bases.)

Why did we not chose package contractions to define the suspension of judgement about the antecedent and the consequent? This question is easy to answer if we work with belief bases and adopt the conventions of Sect. 2.5 as well as Definition 2, (Def σ), and (PMBC) for belief base contractions. Let us assume these definitions (i. e. Definition (2), (Def σ) and (PMBC)) also for belief base package contractions. On these conditions, it is easy to show that $(H, <) - \alpha \vee \gamma = (H, <) - \{\alpha, \gamma\}$. So it does not make a difference which operation is used. For simplicity, we chose the contraction by $\alpha \vee \gamma$ as opposed to the package contraction by $\{\alpha, \gamma\}$.

If we do not work with belief bases or deviate from the conventions in Sect. 2.5, using a package contraction by $\{\alpha, \gamma\}$ rather than a contraction by $\alpha \vee \gamma$ may well have unintended consequences. Suppose the strengthened Ramsey Test conditional $\gg_p$ is defined using a package contraction. Further, assume $\alpha, \gamma \in K(S)$ and $\alpha \rightarrow \gamma$ has high epistemic priority, and is epistemically superior to both α and γ. (Epistemic priority may be spelled out in terms of an ordering among the members of a belief base, an entrenchment ordering, or an ordering of subsets of K that defines a selection function σ.) Then, it is reasonable to expect that $\alpha \gg_p \gamma$. For, there is an inferential connection between α and γ that is based on a generalisation with high epistemic priority. Now, suppose that $\alpha \vee \gamma$ has even higher epistemic priority than $\alpha \rightarrow \gamma$. Suppose, for contradiction, $\alpha \gg_p \gamma$. By the deduction theorem, this implies that (i) $\alpha \rightarrow \gamma \in K(S')$, where $S' = S - \{\alpha, \gamma\}$. Because of the high priority of $\alpha \vee \gamma$, we have $\alpha \vee \gamma \in K(S')$ and so (ii) $\neg\alpha \rightarrow \gamma \in K(S')$. Since $K(S')$ is closed under classical logic, (i) and (ii) imply that $\gamma \in K(S')$. This, however, contradicts $S' = S - \{\alpha, \gamma\}$. Hence, $\alpha \gg_p \gamma \notin K_>(S)$.

The underlying problem is that, if $\alpha \vee \gamma$ has epistemic priority over $\alpha \rightarrow \gamma$ (while we do believe $\alpha \rightarrow \gamma$ quite firmly), the package contraction by $\{\alpha, \gamma\}$ forces us to give up $\alpha \rightarrow \gamma$. We avoid this problem if we define the suspension of judgement via a contraction by $\alpha \vee \gamma$.

7 Generalising the Tower-Shadow Scenario

7.1 Conjunctive and Disjunctive Scenarios

We have spent quite a bit of time investigating the explanatory directions of the famous tower-shadow asymmetry. It proved anything but trivial to capture these directions in a Ramsey Test framework. Our solution to this problem is of course intended to work not only for a single example, but for a wider class of explanatory relations. Let us therefore specify further classes of explanatory relations that are well captured by (Because$_P$).

Suppose our prioritised belief base consists of two levels: an upper level G of generalisations and a lower level L of literals, as explained in Sect. 2.5. Further, let us distinguish between different types of generalisation:

$$\alpha_1 \wedge \cdots \wedge \alpha_n \rightarrow \gamma \tag{C}$$

$$\alpha_1 \vee \cdots \vee \alpha_n \rightarrow \gamma \tag{D}$$

where $\alpha_1, \ldots, \alpha_n$ are literals. We say that a generalisation of type (C) represents a conjunctive explanatory scenario, whereas a generalisation of type (D) represents a disjunctive explanatory scenario.[18] It seems as if these generalisations give rise to corresponding explanatory relations, in the sense of the present Ramsey Test analysis of 'because'. Suppose δ is a generalisation of type (C). Then, 'γ because of α_i' is verified by an epistemic state $(H, <)$ if (i) $\delta \in G$, and $\alpha_1, \ldots, \alpha_n \in L$. Suppose δ is a generalisation of type (D). Then, 'γ because of α_i' is verified by $(H, <)$ if (i) $\delta \in G$, and $\alpha_i \in L$. Recall that $H = G \cup L$.

These explanatory relations do in fact hold for a large class of conjunctive and disjunctive explanatory scenarios, but there are exceptions. Suppose a lit match that is dropped and lightning individually suffice to bring about a forest fire, on condition that oxygen is present. Further, assume that both a lit match has actually been dropped and lightning has actually occurred. So, there is a forest fire. Intuitively, we would endorse 'there is a forest fire because of lightning' and 'there is a forest fire because of the lit match'. The example can be formalised by the following prioritised belief base:

$m \wedge o \rightarrow f, \quad l \wedge o \rightarrow f$
m, l, o, f

where the propositional constants have the following natural language interpretations. m: a lit match has been dropped in the forest. l: there is lightning with electrical discharges to the ground of the forest. f: there is a forest fire. o: oxygen is present.

Let us test for $l \gg f$. $(H, <) - (l \vee f) = (\{m \wedge o \rightarrow f, l \wedge o \rightarrow f\}, <')$. Hence, $f \notin (K(H, <) - (l \vee f)) + l$. Therefore, $l \gg f \notin K_>(H, <)$. So our Ramsey Test analysis of 'because' does not verify 'there is a forest fire because of lightning', which is counterintuitive.

The present example is a variant of a scenario of overdetermining causes in the literature on actual causation (Halpern and Pearl 2005, Sec. 3). So we can describe this example as one of overdetermining causal explanations. It is easy to see that the problem in question arises not just because of the overdetermination structure but because of the combination of this structure with a background condition that is needed for the two explanations of the forest fire. We try to solve this problem by drawing on ideas about causal graphs, as introduced in the literature on actual causation (cf. Halpern and Pearl 2005).

[18] The distinction between conjunctive and disjunctive explanatory scenarios is taken from an analogous distinction in the literature on actual causation [cf. Halpern and Pearl (2005, Sec. 3)]. Disjunctive scenarios amount to cases of overdetermination if more than one of the antecedent conditions is satisfied.

Let us view the members of a belief base H with two levels G and L in terms of an undirected graph in the following way. The propositional constants of the literals in L are represented by nodes, while any two nodes whose propositional constants occur together in some generalisation in G are connected by an edge. We can derive the literal γ from the literal α, using certain generalisations in G, only if these literals are connected by a path. Let us call such a path *explanatory* iff there is a derivation of γ from α that uses generalisations of G and, possibly, also literals of L. In the case of a scenario of overdetermination, we have two different explanatory paths with two different literals α and α'. Arguably, for α to explain γ, it suffices if there is a subgraph that contains one explanatory path from α to γ. This view is analogous to the widely shared intuition that overdetermining causes are proper causes.

Drawing on this picture of explanatory paths, we can account for overdetermining explanations by weakening our strengthened Ramsey Test:

$$\begin{aligned} &\alpha \gg_s \gamma \in K_>(H,<) \text{ iff there are } (H',<') \text{ and } L^- \in H \text{ s.t.} \\ &(H',<') = (H,<) - \bigvee L^-, \ \ \alpha \gg \gamma \in K_>(H',<'), \text{ and} \\ &L^- \text{ is a possibly empty set of literals.} \end{aligned} \tag{$SRT_{P'}$}$$

$\bigvee A$ designates an arbitrary disjunction of the members of the set A of formulas. This translates directly to our analysis of 'because':

$$\begin{aligned} &\text{Because } \alpha,\gamma \text{ (relative to } K(H,<)) \text{ iff} \\ &\alpha,\gamma \in K(H,<) \text{ and } \alpha \gg_s \gamma \in K_>(H,<). \end{aligned} \tag{Because$_{P'}$}$$

The motivation for this refinement may be summarised as follows: to capture explanatory relations in terms of inferential connections between literals, it is sometimes necessary to ignore explanatory paths that are parallel to the one under consideration. Fortunately, there is no need to specify the meaning of 'sometimes' in this justification. In the literature on actual causation it is a common strategy to identify *active causal paths* in terms of a subset of the nodes of a given causal graph (Hitchcock 2007; Halpern and Pearl 2005). The index 's' in $\gg_s$ stands for 'subset', thus indicating that a subset of H suffices as background theory for α to be inferentially relevant for γ.

This refined analysis of 'because' solves our problem. For this to be seen, observe that $(H,<)$ contracted by $\bigvee\{m\}$ yields $(H',<') =$

$l \wedge o \rightarrow f, \quad m \wedge o \rightarrow f$
l, o, f

and $l \gg f \in K_>(H',<')$. In the next section, we shall see that (Because$_{P'}$) works for any combination of conjunctive and disjunctive scenarios. That is, we can combine generalisations of type (C) and (D), in an arbitrary way, to form explanatory paths.

7.2 Inferential Ramsey Test Explanations

Our Ramsey Test analysis of ‘because’, combined with belief bases, qualifies as an inferential approach to explanation. For, it is essential to this analysis that the explanandum can be inferred from the explanans, in the context of certain generalisations and possibly further background conditions. In this section, we shall specify which explanatory inferential relations are captured by our analysis, given the members of the belief base satisfy the conventions of Sect. 2.5. Thereby, we characterise a large class of explanatory relations for which our analysis of ‘because’ works correctly and completely.

We confine ourselves to explanatory relations between presumed facts that are expressed by literals. An inferential explanation of this type may be characterised as follows:

Definition 6 Inferential explanation of γ by α

We say that α inferentially explains γ – in the eyes of an agent a – iff there are G and L such that

(1) G is a set of generalisations
(2) L is a set of literals
(3) α and γ are literals and believed to be true by a
(4) all members of $G \cup L$ are believed to be true by a
(5) $G \cup L, \alpha \vdash \gamma$
(6) $G \cup L \nvdash \gamma$.

Figuratively speaking, we can say that α explains γ iff there is an inferential path from α to γ such that α is an essential premise of this path, and all premises are believed to be true. Our analysis of ‘because’ by (Because$_{P'}$) captures precisely this inferential understanding of an explanation:

Proposition 8 *Let α and γ be literals. Epistemic states are represented by prioritised belief bases with two levels: an upper level G of generalisations and a lower level L of literals, as explained in Sect.* 2.5. *A* (Because$_{P'}$) *agent accepts ‘γ because of α’ with respect to $(H, <)$ iff α inferentially explains γ—in the sense of Definition* 6*—in the eyes of the agent accepting all members of H.*

One must wonder, however, whether (Because$_{P'}$) is strictly asymmetric in the sense that ‘γ because of α’ implies that ‘α because of γ’ does not hold. This is not so. Symmetric explanations can be constructed if we have $G \cup L, \alpha \vdash \gamma$ and $G \cup L, \gamma \vdash \alpha$.

Should we therefore further strengthen our semantics so as to yield a strictly asymmetric conditional? Is our common sense and scientific notion of ‘because’ asymmetric in the sense that ‘γ because of α’ always precludes ‘α because of γ’? While it is difficult to provide a clear-cut example of a properly symmetric explanation, we hesitate to answer this question in the affirmative. We could enforce strict asymmetry, of course, by simply defining $\alpha \gg_a \gamma$ iff $\alpha \gg_s \gamma$ and $\gamma \not\gg_s \alpha$. This would rule out a number of further cases, such as inferential relations that are based on definitions or mathematical laws.

The question of whether explanatory relations are strictly asymmetric is strongly related to research in philosophy of science on explanation and causation. It is an open question whether or not any explanation must be causal, as suggested by Woodward (2003). If so, then there is good reason to suppose that explanatory relations must be asymmetric, given that causation is an asymmetric relation. Our analysis of ‘because’ thus becomes intertwined with an analysis of causation. Future research must show how our strengthened Ramsey Test semantics can be exploited for a fully fledged account of scientific explanation.

8 Conclusion

We strengthened Gärdenfors’s Ramsey Test semantics for conditionals in a way which is well-motivated by Ramsey’s original remarks. Like Rott’s Strong Ramsey Test, but unlike Gärdenfors Ramsey Test semantics, our semantics avoids the absurdity that any two accepted formulas constitute an accepted because sentence. However, Rott’s analysis of ‘because’ is susceptible to symmetry problems, as was shown by Proposition 3 and the tower-shadow scenario. Using belief bases, we could show that our strengthened Ramsey Test semantics captures the asymmetry of the tower-shadow scenario in an intuitively correct manner: the presence of the tower explains the presence of the shadow, but not vice versa.

We moved on to generalising the tower-shadow scenario by characterising the beliefs on the basis of which ‘γ because of α’ is verified by an epistemic state, according to our analysis. This generalisation reveals that our analysis of ‘because’ is not strictly asymmetric. At least for causal explanatory relations in scientific language, a strictly asymmetric analysis of ‘because’ seems to be called for. We envision to achieve such an analysis by an epistemic analysis of causation that is likewise based on the present variant of a strengthened Ramsey Test.

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Appendix: Proofs

Proposition 2 *Let K be a non-absurd belief set and γ a non-tautology. Then* (UPC) *and $\alpha, \gamma \in K$ implies* (Because$_R$).

Proof The proof presents the simplification of (UPC) to (Because$_R$). Assume $K \neq K_\perp$ (K is not the absurd belief set) and γ is not a tautology. Further, suppose $\alpha, \gamma \in K$. Then, by Gärdenfors’s contraction postulate $(K^{-}4)$, $\gamma \notin K - \gamma$.[19] Moreover, since $\gamma \in K$ and $K \neq K_\perp$, $\neg\gamma \notin K$. By $(K^{-}2)$ this implies that $\neg\gamma \notin (K - \gamma)$.

[19] For the list of AGM belief contraction postulates, see for example Gärdenfors (1988, pp. 61–62).

By Proposition 1, $\gamma \notin (K - \gamma)$ implies $\gamma \notin (K - \gamma) * \alpha$ or $\gamma \notin (K - \gamma) * \neg\alpha$, and $\neg\gamma \notin (K - \gamma)$ implies $\neg\gamma \notin (K - \gamma) * \alpha$ or $\neg\gamma \notin (K - \gamma) * \neg\alpha$. Hence, (i) $\gamma \notin (K - \gamma) * \alpha$ or $\gamma \notin (K - \gamma) * \neg\alpha$ and (ii) $\neg\gamma \notin (K - \gamma) * \alpha$ or $\neg\gamma \notin (K - \gamma) * \neg\alpha$. (i) implies that (iii), if $\gamma \in (K - \gamma) * \alpha$, then $\gamma \notin (K - \gamma) * \neg\alpha$. (ii) implies that (iv) if $\neg\gamma \in (K - \gamma) * \neg\alpha$, then $\neg\gamma \notin (K - \gamma) * \alpha$. From (iii), (iv), and (UPC), we can infer (Because$_R$). □

Proposition 3 *Let α and γ be literals or conjunctions of literals. Further, $\alpha, \gamma \in K$. Suppose that $\alpha \to \gamma$ is a non-trivial implication in K and Assumption 1 holds for $\alpha \to \gamma$. Then, $\alpha \overset{a}{\Rightarrow} \gamma \in K$ and $\gamma \overset{a}{\Rightarrow} \alpha \in K$.*

Proof ($\alpha \overset{a}{\Rightarrow} \gamma \in K$) Suppose $\alpha \to \gamma$ is a non-trivial implication in K. Then, by Definition 3 and Assumption 1, $\alpha \to \gamma$ is a non-trivial implication in $K - \gamma$. By Definition 3, $(K - \gamma) - \neg\alpha \vdash \alpha \to \gamma$. We obtain, by the deduction theorem, $(K - \gamma) - \neg\alpha, \alpha \vdash \gamma$, which we can rewrite as $((K - \gamma) - \neg\alpha) + \alpha \vdash \gamma$. By the Levi identity, we obtain $\gamma \in (K - \gamma) * \alpha$. Using $\alpha, \gamma \in K$, we can infer therefrom that $\alpha \overset{a}{\Rightarrow} \gamma \in K$.

($\gamma \overset{a}{\Rightarrow} \alpha \in K$) Suppose $\alpha \to \gamma$ is a non-trivial implication in K. Then, by Assumption 1, $\alpha \to \gamma$ is a non-trivial implication in $K - \alpha$. By contraposition, $\neg\gamma \to \neg\alpha$ is a non-trivial implication in $K - \alpha$. By Definition 3, $\neg\gamma \to \neg\alpha$ is a non-trivial implication in $(K - \alpha) - \gamma$. Via the deduction theorem, we obtain $(K - \alpha) - \gamma, \neg\gamma \vdash \neg\alpha$. By the Levi identity, we obtain $\neg\alpha \in (K - \alpha) * \neg\gamma$. Using $\alpha, \gamma \in K$, we can infer therefrom that $\gamma \overset{a}{\Rightarrow} \alpha \in K$. □

Proposition 4 *Assume a* (Because$_R$) *agent accepts all facts and the generalisation of the tower-shadow scenario*, i. e. $t, s, sh, t \wedge s \to sh \in K$, *where the order of epistemic entrenchment is $t, s, sh < t \wedge s \to sh$. Then, $t \overset{a}{\Rightarrow} sh \in K$ if $t \leq sh$ and $sh \overset{a}{\Rightarrow} t \in K$ if $sh \leq t$.*

Proof By (Because$_R$):

$$t \overset{a}{\Rightarrow} sh \in K \quad \text{iff} \quad sh \in (K - sh) * t \text{ or } \neg sh \in (K - sh) * \neg t$$
$$\text{and} \quad t, sh \in K.$$

$t, s, sh \in K$ holds by assumption. We show that (a) $t \overset{a}{\Rightarrow} sh \in K$ if $t \leq sh$. Let us assume $t \leq sh$. By (G-), (EE2), (EE1), this implies (i) $t \notin K - sh$. By the recovery postulate

$$\text{If } \alpha \in K, \quad \text{then} \quad K \subseteq (K - \alpha) + \alpha \qquad (K^{-}5)$$

$t \in (K - sh) * sh$. By the Levi identity, the deduction theorem, and (i), this implies that $sh \to t \in (K - sh)$. Hence, $\neg t \to \neg sh \in (K - sh)$. Using $t \notin K - sh$ and the Levi identity, we can infer therefrom that $\neg sh \in (K - sh) * \neg t$. This entails (a) in light of (Because$_R$).

The proof of (b) $sh \overset{a}{\Rightarrow} t$ if $sh \leq t$ is completely analogous to that of (a). □

Proposition 5 *Assume a* (Because$_R$) *agent accepts all the formulas in* $K(H, <) = K(S)$ *for* $H = \{t, s, sh, t \wedge s \rightarrow sh\}$, *where the order of epistemic priority is* $t \sim s \sim sh < t \wedge s \rightarrow sh$. *Then* $t \overset{a}{\Rightarrow} sh \notin K_>(S)$ *and* $sh \overset{a}{\Rightarrow} t \in K_>(S)$.

Proof $t \overset{a}{\Rightarrow} sh$ and $sh \overset{a}{\Rightarrow} t$ remain well-defined, when replacing K with $K(S)$:

$$t \overset{a}{\Rightarrow} sh \in K_>(S) \text{ iff } sh \in K((S - sh) * t) \text{ or } \neg sh \in K((S - sh) * \neg t)$$
$$\text{and } t, sh \in K(S)$$
$$sh \overset{a}{\Rightarrow} t \in K_>(S) \text{ iff } t \in K((S - t) * sh) \text{ or } \neg t \in K((S - t) * \neg sh)$$
$$\text{and } sh, t \in K(S).$$

$t \overset{a}{\Rightarrow} sh \notin K_>(S)$, where $S = (H, <)$: $t, sh \in K(S)$ is satisfied by assumption. By Definition 1, the remainder set $H \bot sh$ contains three sets, $H' = \{t \wedge s \rightarrow sh, s\}$, $H'' = \{t \wedge s \rightarrow sh, t\}$, and $H''' = \{s, t\}$. By Definition 2, $H' \leq H''$ and $H'' \leq H'$, but $H' \not\leq H'''$. Hence, by (Def σ), both H' and H'' are selected for the partial meet base contraction (PMBC) which yields $H - sh = \bigcap \sigma(H \bot sh) = \{t \wedge s \rightarrow sh\}$.

By (PMBR), $(H - sh) * t = \bigcap \sigma((H - sh) \bot \neg t) + t$. Since $\neg t \notin Cn(H - sh)$, by Definition 1, $H - sh$ is the unique member of $(H - sh) \bot \neg t$. By Definition 2, $(H - sh) \leq (H - sh)$ and by (Def σ), the partial meet base contraction (PMBC) yields $(H - sh) - \neg t = \bigcap \sigma((H - sh) \bot \neg t) = \{t \wedge s \rightarrow sh\}$. Notice that when $\neg t \notin H - sh$, then $(H - sh) - \neg t = H - sh$. By (H + α), $(H - sh) * t = \{t \wedge s \rightarrow sh\} \cup \{t\}$. Hence, $(S - sh) * t = ((H - sh) * t, <')$, where $<'$ is such that generalisations have strict priority over literals. Then $sh \notin K((S - sh) * t)$.

By (PMBR), $(H - sh) * \neg t = \bigcap \sigma((H - sh) \bot t) + \neg t$. By similar reasoning as above, $t \notin H - sh$ and thus $(H - sh) - t = H - sh$. By (H + α), $(H - sh) * \neg t = \{t \wedge s \rightarrow sh\} \cup \{\neg t\}$. Hence, $(S - sh) * \neg t = ((H - sh) * \neg t, <')$, where $<'$ is such that generalisations have strict priority over literals. Then $\neg sh \notin K((S - sh) * \neg t)$.

$sh \overset{a}{\Rightarrow} t \in K_>(S)$, where $S = (H, <)$: $t, sh \in K(S)$ is satisfied by assumption. By Definition 1, the remainder set $H \bot t$ contains only $H' = \{t \wedge s \rightarrow sh, s, sh\}$. By Definition 2, $H' \leq H'$ and by (Def σ), the partial meet base contraction (PMBC) yields $(H - t) = \bigcap \sigma(H \bot t) = \{t \wedge s \rightarrow sh, s, sh\}$. By similar reasoning, $(H - t) - sh = \{t \wedge s \rightarrow sh, s\}$. By (H + α), $((H - t) - sh) + \neg sh = \{t \wedge s \rightarrow sh, s, \neg sh\}$, which is by (PMBR) $(H - t) * \neg sh$. Hence, $(S - t) * \neg sh = ((H - t) * \neg sh, <')$, where $<'$ is such that generalisations have strict priority over literals. Then $\neg t \in K((S - t) * \neg sh)$. □

Proposition 6 *Assume a* (Because$_P$) *agent accepts all facts and the single, more entrenched generalisation of the tower-shadow scenario,* i. e. $t, s, sh, t \wedge s \rightarrow sh \in K$, *where the order of epistemic entrenchment is* $t \sim s \sim sh < t \wedge s \rightarrow sh$. *Then* $t \overset{P}{\Rightarrow} sh \in K_>$ *and* $sh \overset{P}{\Rightarrow} t \in K_>$.

Proof $t \overset{P}{\Rightarrow} sh \in K_>$: The agnostic belief set is $K' = K - (t \vee sh)$. By (G-), $t, s, sh \notin K'$, but by *recovery* (i) $(t \vee sh) \rightarrow sh \in K'$. By assumption, $\neg t \notin K$ and so by (K^-2), (ii) $\neg t \notin K'$. Using $t \vdash t \vee sh$ and the Levi identity, we can infer from (i) and (ii) that $sh \in K' * t$.

$sh\,{}^{P}\!\!\Rightarrow t \in K_{>}$: The agnostic belief set is, again, $K' = K - (t \vee sh)$ such that $t, s, sh \notin K'$, but by *recovery* $(t \vee sh) \rightarrow t \in K'$. Using $sh \vdash t \vee sh$, we infer that $t \in K' * sh$. □

Proposition 7 *Assume a* (Because$_P$) *agent accepts all the formulas in* $K(H, <) = K(S)$ for $H = \{t, s, sh, t \wedge s \rightarrow sh\}$, *where* $t, s, sh < t \wedge s \rightarrow sh$ *and the order of epistemic priority is* $t \sim s \sim sh < t \wedge s \rightarrow sh$. *Then* $t\,{}^{P}\!\!\Rightarrow sh \in K_{>}(S)$, *but* $sh\,{}^{P}\!\!\Rightarrow t \notin K_{>}(S)$.

Proof $t\,{}^{P}\!\!\Rightarrow sh \in K_{>}(S)$, where $S = (H, <)$: $t, sh \in K(S)$ is satisfied by assumption. By Definition 1, the remainder set $H \perp (t \vee sh)$ contains only $H'' = \{t \wedge s \rightarrow sh, s\}$. By Definition 2, $H'' \leq H''$ and by (Def σ), the partial meet base contraction (PMBC) yields the agnostic belief base $H' = H - (t \vee sh) = \bigcap \sigma(H \perp (t \vee sh)) = \{t \wedge s \rightarrow sh, s\}$.

By (PMBR), $H' * t = \bigcap \sigma(H' \perp \neg t) + t$. Since $\neg t \notin Cn(H')$, by Definition 1, H' is the only member of $H' \perp \neg t$. By Definition 2 and (Def σ), $\bigcap \sigma(H' \perp \neg t) = H'$. By (H + α), $H' + t = \{t \wedge s \rightarrow sh, s\} \cup \{t\}$. Hence, $S' * t = (H' + t, <')$, where $<'$ is such that generalisations have strict priority over literals. Then $sh \in K(S' * t)$ so that $t \gg sh \in K_{>}(S)$.

$sh\,{}^{P}\!\!\Rightarrow t \notin K_{>}(S)$, where $S = (H, <)$: $t, sh \in K(S)$ is satisfied by assumption. The agnostic belief base is again $H' = H - (t \vee sh) = \{t \wedge s \rightarrow sh, s\}$. By (PMBR), $H' * sh = \bigcap \sigma(H' \perp \neg sh) + sh$. Since $\neg sh \notin Cn(H')$, by Definition 1, H' is the only member of $H' \perp \neg sh$. By Definition 2 and (Def σ), $\bigcap \sigma(H' \perp \neg t) = H'$. By (H + α), $H' + sh = \{t \wedge s \rightarrow sh, s\} \cup \{sh\}$. Hence, $S' * sh = (H' + sh, <')$, where $<'$ is such that generalisations have strict priority over literals. Then $t \notin K(S' * sh)$ so that $sh \gg t \notin K_{>}(S)$. □

Proposition 8 *Let* α *and* γ *be literals. Epistemic states are represented by prioritised belief bases with two levels: an upper level G of generalisations and a lower level L of literals, as explained in Sect. 2.5. A* (Because$_{P'}$) *agent accepts '*γ *because of* α*' with respect to* $(H, <)$ *iff* α *inferentially explains* γ *– in the sense of Definition 6 – in the eyes of the agent accepting all members of H.*

Proof Suppose (i) γ because of α is verified by an epistemic state $(H, <)$ (in the sense of (Because$_{P'}$)). Let G be the set of generalisations of H, while L is the set of literals of H. Hence, (ii) there are $(H'', <'')$ and L^{-} such that $(H', <') = (H, <) - \bigvee L^{-}$ and $\alpha \gg \gamma \in K_{>}(H', <')$. Therefore, (iii) there is $(H'', <'') = (H', <') - \alpha \vee \gamma$ such that $H'' = G'' \cup L''$, $G'' = G \cap H''$, and $L'' = L \cap H''$. Hence, the pair (G'', L'') satisfies conditions (1), (2), and (4) for an agent who accepts all members of H. Moreover, (ii) and (iii) imply that $G'' \cup L'', \alpha \vdash \gamma$. Hence, Condition (5) is satisfied as well for (G'', L''). Finally, Condition (6) holds for (G'', L'') because of (ii) and (iii). (i) implies that Condition (3) of Definition 6 is satisfied for an agent who accepts all members of H. Hence, all conditions of this definition are satisfied for such an agent. Thus, α inferentially explains γ—in the sense of Definition 6—in the eyes of an agent who accepts all members of H.

For the other direction, suppose (i) α inferentially explains γ in the eyes of an agent a, in the sense of Definition 6. Hence, there is a set G of generalisations and a set L of literals such that conditions (1)–(6) of Definition 6 are satisfied for a. (ii) $H := G \cup L \cup \{\alpha\}$. $<$ is such that generalisations are prioritised over literals. Obviously, (iii) $\alpha \in K(H)$. By Condition (5) of Definition 6, (iv) $\gamma \in K(H, <)$. We show that $\alpha \vee \gamma \notin Cn(G \cup L)$. Suppose, for contradiction, $\alpha \vee \gamma \in Cn(G \cup L)$. This implies that (v) $\neg\alpha \rightarrow \gamma \in Cn(G \cup L)$. By Condition (5) of Definition 6, we know that (vi) $\alpha \rightarrow \gamma \in Cn(G \cup L)$. Since $\alpha \vee \neg\alpha \in Cn(G \cup L)$, (v) and (vi) imply that $\gamma \in Cn(G \cup L)$. This contradicts Condition (6) of Definition 6. Hence, $\alpha \vee \gamma \notin Cn(G \cup L)$. Therefore, $\sigma((H, <)\bot\alpha \vee \gamma) := \{G \cup L\}$, where σ is defined by (Def σ) and Definition 2. Using Condition (5) of Definition 6, we can infer therefrom that $\alpha \gg \gamma \in K_{>}(H, <)$. Using (iii) and (iv), we can infer therefrom that 'γ because of α' is verified by the epistemic state $(H, <)$ (in the sense of (Because$_{P'}$)). $\square$

References

Alchourrón, M. A., Gärdenfors, P., & Makinson, D. (1985). On the logic of theory change: Partial meet contraction functions and their associated revision functions. *Journal of Symbolic Logic*, *50*, 510–530.

Bradley, R. (2007). A defence of the Ramsey Test. *Mind*, *116*(461), 1–21.

Brewka, G. (1991). Belief revision in a framework for default reasoning. In *Proceedings of the workshop on the logic of theory change* (pp. 602–622). London: Springer.

Chandler, J. (2013). Transmission failure, AGM-style. *Erkenntnis*, *78*(2), 383–398.

Dalal, M. (1988). Investigations into a theory of knowledge base revisions: Preliminary report. In *Proceedings of the 7th national conference on artificial intelligence (AAAI-88), St. Paul* (pp. 475–479).

Fuhrmann, A., & Hansson, S. O. (1994). A survey of multiple contractions. *Journal of Logic, Language and Information*, *3*(1), 39–75.

Gärdenfors, P. (1978). Conditionals and changes of belief. In I. Niiniluoto & R. Tuomela (Eds.), *The logic and epistemology of scientific change* (Vol. 30, pp. 381–404) Acta philosophica fennica Amsterdam: North-Holland.

Gärdenfors, P. (1986). Belief revisions and the Ramsey Test for conditionals. *The Philosophical Review*, *95*(1), 81–93.

Gärdenfors, P. (1988). *Knowledge in flux*. Cambridge, Mass: MIT Press.

Gärdenfors, P., & Makinson, D. (1988). Revision of knowledge systems using epistemic entrenchment. In M. Vardi (Ed.), *TARK' 88—proceedings of the second conference on theoretical aspects of reasoning about knowledge* (pp. 83–95). Los Altos: Morgan and Kaufmann.

Grove, A. (1988). Two modellings for theory change. *Journal of Philosophical Logic*, *17*, 157–170.

Halpern, J. Y., & Pearl, J. (2005). Causes and explanations: A structural-model approach. Part I: Causes. *British Journal for the Philosophy of Science*, *56*(4), 843–887.

Hansson, S. O. (1991). Belief contraction without recovery. *Studia Logica*, *50*(2), 251–260.

Hansson, S. O. (1992). In defense of the Ramsey Test. *Journal of Philosophy*, *89*(10), 522–540.

Hansson, S. O. (1999). *A textbook of belief dynamics. Theory change and database updating*. Dordrecht: Kluwer.

Hitchcock, C. (2007). Prevention, preemption, and the principle of sufficient reason. *Philosophical Review*, *116*(4), 495–532.

Hume, D. A. (1739/1978). Treatise of human nature. Oxford: Clarendon Press.

Husserl, E. (Ed.). (1913). Ideen zu einer reinen Phänomenologie und Phänomenologischen Philosophie I. In *Jahrbuch für Philosophie und phänomenologische Forschung* (Vol. 1, pp. 1–323). Halle a.d.S.: Max Niemeyer.

Leitgeb, H. (2010). Ramsey Test without triviality. *Notre Dame Journal of Formal Logic*, *51*(1), 21–54.

Levi, I. (2007). *For the sake of the argument: Ramsey test conditionals, inductive inference and nonmonotonic reasoning*. Cambridge: Cambridge University Press.
Levi, I. (1988). Iteration of conditionals and the Ramsey Test. *Synthese*, *76*(1), 49–81.
Lewis, D. (1973a). *Counterfactuals*. Oxford: Blackwell.
Lewis, D. (1973b). Causation. *Journal of Philosophy*, *70*(17), 556–567.
Makinson, D. (1987). On the status of the postulate of recovery in the logic of theory change. *Journal of Philosophical Logic*, *16*(4), 383–394.
Ramsey, F. P. (1950). General propositions and causality. In R. B. Braithwaite (Ed.), *Foundations of mathematics and other logical essays* (pp. 237–257). New York: Humanities Press.
Rott, H. (1986). Ifs, though, and because. *Erkenntnis*, *25*(3), 345–370.
Rott, H. (2011). Reapproaching Ramsey: Conditionals and iterated belief change in the spirit of AGM. *Journal of Philosophical Logic*, *40*, 155–191.
Spohn, W. (2006). Causation: An alternative. *British Journal for the Philosophy of Science*, *57*(1), 93–119.
Stalnaker, R. (1968). A theory of conditionals. In N. Rescher (Ed.), *Studies in logical theory* (pp. 98–112)., American philosophical quarterly monograph series Oxford: Blackwell.
van Benthem, J. (2008). Logic and reasoning: Do the facts matter? *Studia Logica*, *88*, 67–84.
van Benthem, J., & Smets, S. (2015). Dynamic logics of belief change. In J. Y. Van Ditmarsch, et al. (Eds.), *Handbook of logics for knowledge and belief* (pp. 299–368). Oxford: College Publications.
Van Fraassen, B. (1980). *The scientific image*, Clarendon Library of Logic and Philosophy Oxford: Clarendon Press.
Woodward, J. (2003). *Making things happen: A theory of causal explanation*. Oxford: Oxford University Press.