Check for updates

UNIQUENESS AND LOGICAL DISAGREEMENT (REVISITED)

Frederik J. ANDERSEN

ABSTRACT: This paper discusses the *Uniqueness Thesis*, a core thesis in the epistemology of disagreement. After presenting uniqueness and clarifying relevant terms, a novel counterexample to the thesis will be introduced. This counterexample involves *logical disagreement*. Several objections to the counterexample are then considered, and it is argued that the best responses to the counterexample all undermine the initial motivation for uniqueness.

KEYWORDS: the uniqueness thesis, rational uniqueness, logical disagreement, logical evidence, propositional justification, epistemic permissivism, peer disagreement

1. Introduction

The *Uniqueness Thesis* (henceforth denoted 'UT') concerns a relation between a body of evidence, a doxastic attitude, and a proposition.[1] Jonathan Matheson, a proponent of the thesis, defines UT as follows:

> (UT) For any body of evidence E and proposition $[p]$, E justifies at most one doxastic attitude toward $[p]$ (Matheson 2011, 360).

UT features frequently in the epistemology literature,[2] especially in the debate concerning peer disagreement—if two epistemic peers[3] disagree about a proposition p, is it then possible that they are both justified in their doxastic attitudes toward p? If UT is true, the answer is negative.

Importantly, there are in fact several non-equivalent definitions of UT in the literature. Thomas Kelly, for example, favors a formulation of UT saying that there

[1] This paper is largely based on Andersen (2020), but it includes several important corrections and additions.

[2] See for example Matheson (2011), Rosa (2012, 2016), Kelly (2014), White (2014), Kopec and Titelbaum (2016), Ross (2021), Kauss (2023).

[3] Roughly put, two agents in disagreement are epistemic peers when neither side is epistemically superior with respect to the target-proposition at hand, i.e., when the two are similar enough in all relevant factors such as evidence, track record, time constraints etc.

is *exactly one* justified doxastic attitude given a body of evidence (Kelly 2010, 119), while Matheson prefers *at most one*, as we have just seen. Matheson notes that in most cases there will be exactly one justified doxastic attitude given a body of evidence, but in some situations, there may be no justified doxastic attitude toward p whatsoever. This can arguably happen when one is not able to, or when it is simply not possible to, comprehend the proposition at hand.[4] If one takes (possible) comprehension of p to be a necessary condition for the existence of a justified doxastic attitude toward p, then it seems most reasonable to use Matheson's weaker definition of UT. Thus, this is what we will assume here.

Further, we will adopt Matheson's assumption that the term 'doxastic attitude' can only refer to the following three possibilities: *belief that* p; *disbelief that* p; and *suspension of judgement with respect to* p; i.e., the possibility space of attitudes that one can take toward a proposition p is exhausted by these three attitudes.[5]

Now, UT puts a constraint on the total number of doxastic attitudes that a body of evidence can justify toward a proposition. According to UT any body of evidence E justifies at most one doxastic attitude toward p. In other words, according to UT, there exists no body of evidence E such that E justifies both belief and disbelief toward p. Similarly, of course, the thesis implies that there exists no E such that E justifies both a (dis)belief in p and suspension of judgement with respect to p. In the paper titled '*The case for Rational Uniqueness*', Matheson makes two further clarifying remarks about UT:

> (UT) [...] makes no reference to individuals or times since (UT) claims (in part) that who possesses the body of evidence, as well as when it is possessed, makes no difference regarding which doxastic attitude is justified (if any) toward any particular proposition by that body of evidence (Matheson 2011, 360).[6]

---

[4] See Feldman (2006) for a motivation of this view.

[5] This assumption is common in the contemporary literature, see for example Kelly (2010), Matheson (2011), Rosa (2012), Titelbaum (2015, 2019). Note that some have argued that the doxastic attitude of *disbelief that p* is non-equivalent to that of *believing the negation of p*. See Smart (2021) for a recent argument. Unless otherwise stated we'll simply take disbelief that p and believing the negation of p as equivalent attitudes in what follows.

[6] Note that while Matheson's statement of UT doesn't make reference to individuals (i.e., cognizers or human agents) at all, some authors have presented versions of uniqueness that do. Consider for example Titelbaum and Kopec's tripartite distinction between propositional, attitudinal, and personal uniqueness (Titelbaum and Kopec 2019, 206). *Propositional Uniqueness*. Given any body of evidence and proposition, the evidence all-things-considered justifies either the proposition, its negation, or neither. *Attitudinal Uniqueness*. Given any body of evidence and proposition, the evidence all-things considered justifies at most one of the following attitudes

> (UT) concerns propositional justification, rather than doxastic justification. That is, the kind of justification relevant to (UT) is solely a relation between a body of evidence, a doxastic attitude, and a proposition. How individuals have come to have the doxastic attitudes they have toward the proposition in question will not be relevant to our discussion. Further, individuals can be propositionally justified in adopting attitudes toward propositions which they psychologically cannot adopt [...] Importantly, it is not a necessary condition for being justified in believing [p] that one be able to demonstrate that one is justified in believing (Matheson 2011, 360-361).

The first of these quotes states that according to UT a given body of evidence E justifies exactly the same doxastic attitude (if any) towards p, no matter the subject that assesses E and at what time this is done. In the second quote, Matheson distinguishes between *propositional* and *doxastic* justification, where the former is a relation between a body of evidence, a doxastic attitude, and a proposition, the latter concerns *how* a given individual came to adopt a specific doxastic attitude towards a proposition, i.e., doxastic justification is concerned with one's reasons for actually adopting a certain attitude toward p. Doxastic justification presumes that a given individual has a certain attitude toward p, and the question is then whether or not this individual has sufficiently good (epistemic) reasons to be justified in having that attitude.[7] When it comes to propositional justification, on the other hand, it is irrelevant whether any individual is ever concerned with p; the crux of propositional justification is that a justification-relation between a body of evidence, a doxastic attitude, and a proposition holds, not whether any individual realizes this. Understood in this way propositional justification refers to an external relation, and an individual can accordingly be propositionally justified in a doxastic attitude towards p even though this individual has not adopted the relevant attitude psychologically. And hence, it is not necessary for a subject to be able to demonstrate or defend this given attitude towards p in order for it to be propositionally justified. Matheson tells us that UT is a thesis concerning propositional justification rather than doxastic justification.

2. Clarifications

---

toward the proposition: belief, disbelief, or suspension. *Personal Uniqueness*. Given any body of evidence and proposition, there is at most one doxastic attitude that any agent with that total evidence is rationally permitted to take toward the proposition.

[7] For accounts of the epistemic *basing relation*, which is often taken to be relevant for doxastic justification, see for instance McCain (2014), Carter and Bondy (2019), Korcz (2021).

Before we move on to consider the announced counterexample to UT, let us pause to further specify what is meant by 'justification' and 'evidence' in the rest of the paper. We will deliberately stay on a high level of generality in order not to exclude too many accounts of justification and evidence from the later discussions in sections 3 and 4.

When using the term 'justification,' this use is restricted to the epistemic domain, we are not concerned with any practical issues whatsoever. So, in other words, our concern is with the justification of doxastic attitudes towards propositions. This kind of justification is taken to be regulated by epistemic norms, i.e., truth-conducive norms, and as indicated in §1, we are concerned with *propositional* justification rather than doxastic justification.[8]

Our use of the term 'evidence' assumes that we can all agree that evidence can stem from many different sources like direct visual perception, testimony from individuals or media, scientific experiments etc. The only constraints we will force on our understanding of evidence from the outset are: (1) evidence must be propositional (and thus truth-apt); (2) any piece of evidence must be true; (3) any piece of evidence must (at least in principle) be accessible to human beings; and (4) evidence should be supportive of doxastic attitudes, where 'support' may be interpreted probabilistically, but does not have to be.

(2) is arguably the most controversial among these four constraints. For our purposes, however, there is a very good reason for including this factivity condition. To see this, suppose that one could have false pieces of evidence in one's (total) body of evidence E. Then, given the further assumption that false evidence can support anything, we could easily have a situation where a true bit of evidence e_1 from E entails p and thus supports the belief that p, while a false bit of evidence e_2 from E entails not-p and thus supports disbelieving that p, making E inconsistent and "explosive." This would in effect trivialize the debate about UT; on this account of evidence UT is obviously false.[9] Hence, we should either accept that evidence is factive or we should deny that false evidence can support anything. For the rest of the paper we will take the first option.

3. The Argument from Logical Disagreement

[8] The literature on epistemic justification is vast, but prominent examples of theories of justification can be found in Goldman (1986), BonJour (1985), Feldman and Conee (1985), Alston (1989), Williamson (2000), Conee and Feldman (2004). Note also Littlejohn's tripartite division of epistemic justification which includes *personal* justification as well as doxastic and propositional (Littlejohn 2012, 5). According to Littlejohn, doxastic justification is sufficient for personal justification, but not *vice versa*.

[9] Thanks to Franz Berto for pressing this point about false evidence.

Consider now the following case against UT:

> **Logical Disagreement**. Two logicians, S_1 and S_2, are walking into an empty auditorium where they find a deduction written on a blackboard. S_1 and S_2 are simultaneously looking at the board. As it happens, S_1 is a classical logician, while S_2 is an intuitionist. Now, by definition, the deduction consists in a finite number of steps, so all steps of the deduction except for the conclusion C will serve as a common body of evidence E, i.e., a set of propositions that are represented in a language that both logicians fully comprehend. The central question is then whether E entails C. Suppose that conclusion C on line n is the result of applying DNE (double negation elimination) to not-not-C on line $n-1$.[10] As S_1 accepts classical logic, she also accepts the inference from not-not-C to C, while S_2 given her intuitionist convictions denies DNE as a general rule of inference and thus denies that C needs to come out supported by E.

In this case we have a situation in which two agents possess exactly (!) the same evidence (the propositions represented by lines $n-1$ on the blackboard), but they are justified in diverging doxastic attitudes toward the relevant proposition in question, namely C. We see that E justifies S_1 in her belief that C, while E justifies (at least) suspension of judgement regarding C for S_2 (as C is not necessarily supported by E). Thus, the case is a clear counterexample to UT as the number of attitudes that E justifies exceeds one.

Of course, as the reader will have noticed by now, the case is concerned with a special type of evidence, i.e., evidence of the completely formal type that we find in pure logic and mathematics. This means that the counterexample is narrow in the sense that it does not indicate the existence of counterexamples to UT among other types of evidence.[11] However, this will be completely irrelevant as

[10] Using standard notation that isn't meant to favor any logical tradition, DNE is an inference from $\Gamma \vdash \sim\sim \varphi$ to $\Gamma \vdash \varphi$, where 'Γ' denotes a set of sentences in a given language, '$\vdash$' denotes deducibility from left to right, '$\sim$' denotes a negation operator, and 'φ' picks out a single sentence of the language. Some readers may point out that it is underspecified in the case above whether S_1 and S_2 disagree over an *instance* or a *schema* of DNE. This is true, but it will not make a significant difference to the main argument of the paper. The crux is that the logicians genuinely disagree. For more elaborate discussions of genuine logical disagreement the reader should consult Hattiangadi (2018), Hjortland (2022), Andersen (2023b), Hattiangadi and Andersen (202X).

[11] However, some epistemologists have suggested that there are counterexamples to UT among other types of evidence. Consider, for example, a case where S_1 and S_2 discuss which football team will win the national league this season. Suppose that their discussion takes place the day before the final match day, and at this point of the season only two teams can win; either team A or team B (not both). Suppose further that the only evidence available to the subjects is a certain newspaper statistic, which shows the scores of the season so far. According to this statistic, team A is in front of team B by the smallest possible margin. Now, S_1 is convinced that team A will

long as we regard UT as a *general* epistemic principle. If the case holds, we will have a counterexample sufficient for rejecting UT.

Finally, before taking on some pressing objections to the Argument from Logical Disagreement, one further clarifying comment is called for. Note that the logical disagreement described above isn't simply a case where S_1 and S_2 are talking past each other because of equivocation about the meaning of the expression 'not', as Quine (1986, 81) would have it. The reason why we can rule this out is a certain "technique for arguing that an apparent conflict is a real one" due to Williamson (1988).[12] In (1982) Harris established that in a system of natural deduction with two different operators for negation—classical ('$\neg$') and intuitionist ('$\rightharpoonup$'), respectively—the biconditional $\neg\varphi \leftrightarrow \rightharpoonup \varphi$ becomes provable, for any formula φ. From this basis Williamson's technique requires us to ask whether (i) there are rules of inference governing both $\neg$ and $\rightharpoonup$, and (ii) whether such rules could allow classical and intuitionist logicians (like S_1 and S_2) to characterize negation as the unique operator obeying those rules (up to logical equivalence).

As it turns out, the answer to (i) is positive: both $\neg$ and $\rightharpoonup$ obey Ex Falso Quodlibet ('*EFQ*') and the Introduction Rule for Negation ('N_{Intro}'). Let φ, ψ be well-formed formulas. Then a monadic operator $\sim$ obeys EFQ, N_{Intro}, and N_{Elim}, just in case the following two schemas are valid:

$$\frac{\varphi \qquad \sim\varphi}{\psi}\ EFQ \qquad\qquad \frac{\begin{array}{c}(n)\\ \varphi\\ \vdots\\ \bot\end{array}}{\sim\varphi}\ (n)\ N_{Intro}$$

take the championship due to the statistical support for this (they are ahead at this point). However, S_2 suspends judgement about who will be the champions as team A leads with the smallest possible margin and it is still possible for team B to make it. In such a case the proponent of UT should say that at most one of the subjects' doxastic attitudes is justified, but one might argue that this is wrong. In such borderline cases it may seem that at least two out of three doxastic attitudes could be justified. If this is right, we have a counterexample to UT featuring another type of evidence, i.e., empirical data. Find similar borderline cases in (Kelly 2014, 299-300). For a recent discussion of (merely) statistical evidence and its role in epistemology, see Silva (2023).

[12] Note that our exhibition of Williamson's technique follows the order of presentation found in Rossi (2023). We follow Rossi's lead as his presentation of the material is very clear and detailed.

Here, numerals in brackets, i.e., (n), serve two distinct purposes: they mark discharged assumptions; and they indicate at which point in the derivation assumptions are discharged.

The answer to (ii) is also positive. EFQ and N_{Intro} are jointwise strong enough to define any monadic operator obeying them (up to logical equivalence). To see this, let $\sim_1$ and $\sim_2$ be any two monadic operators obeying EFQ and N_{Intro}. The following derivation establishes the deductive equivalence: $\vdash \sim_1 p \leftrightarrow \sim_2 p$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{(1)}{p} \quad \overset{(2)}{\sim_1 p}}{p}\,EFQ \qquad \dfrac{\overset{(1)}{p} \quad \overset{(2)}{\sim_1 p}}{\sim_2 p}\,EFQ}{\bot}}{\sim_2 p}\,(1)N_{Intro}}{\sim_1 p \to \sim_2 p}\,(2)\to_{Intro} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\overset{(3)}{p} \quad \overset{(4)}{\sim_2 p}}{p}\,EFQ \qquad \dfrac{\overset{(3)}{p} \quad \overset{(4)}{\sim_2 p}}{\sim_1 p}\,EFQ}{\bot}}{\sim_1 p}\,(3)N_{Intro}}{\sim_2 p \to \sim_1 p}\,(4)\to_{Intro}}{\sim_1 p \leftrightarrow \sim_2 p}\,\leftrightarrow I$$

As the answers to both (i) and (ii) are positive, Williamson (1988, 111) proposes a proof-theoretic argument showing that the disagreement between classical and intuitionist logicians over DNE is a genuine one, and not merely a verbal dispute. Summa: If there is only one monadic operator—up to logical equivalence—obeying both EFQ and N_{Intro}, then this must rule out the possibility that the classical and intuitionist logicians are merely talking past each other when disagreeing about whether it obeys DNE. Either the intuitionist is right and the classicist wrong (or *vice versa*). In any case, there cannot be a single logic with two negation operators only one of which obeys DNE.

4. Objections and Responses

As the case presented above will be very hard to accept for many readers (for various reasons), the rest of the paper aims to motivate the Argument from Logical Disagreement. The strategy here is simple. While discussing various objections to **Logical Disagreement**, it will become clear that the UT-proponent can only avoid the counterexample by undermining the initial motivation behind UT, i.e., explaining away the counterexample to UT will lead to an indirect defeat of the thesis. In the following, five objections to **Logical Disagreement** will be scrutinized (§§4.1-4.5). The first two will simply be rejected, the third will be found underdeveloped, and while the remaining two can actually explain away the counterexample to UT, this can only be done by undermining the motivation behind the principle.

4.1 Evidence is Contingent

> **Objection 1**. Even though the evidence E present in **Logical Disagreement** satisfies our four rudimentary constraints on evidence (cf. §2) as E is propositional, factive, accessible, and supportive, E is still not a genuine body of evidence. For only contingent propositions can be evidence. Thus, UT is not even applicable in **Logical Disagreement**.

First of all, there is no principle reason why necessary propositions such as the ones found in pure mathematics and logic cannot be counted as evidence. Propositions of logic and mathematics can clearly serve the supportive role of evidence very well, i.e., such propositions speak in favor of certain hypotheses in the strongest possible way (by entailment). Hence, if any proposition is able to justify a belief, it seems that pure logical or mathematical propositions are ideal candidates. Habit may dictate, perhaps leading back to acceptance of Hume's Fork, that some of us cannot see the point in taking purely formal premises of deductive arguments as evidence, but without further qualification this is obviously not a good argument for accepting such an exclusion in philosophical or scientific work. Moreover, accepting **Objection 1** leads to absurd consequences when we hold other plausible epistemic principles to be true. Take for example Timothy Williamson's principle $E = K$, i.e., evidence equals knowledge (2000, Chapter 9). If we accept that our evidence is coextensive with our knowledge, and that **Objection 1** holds, it directly follows that we cannot have pure mathematical or logical knowledge. To deny that we can and do have such knowledge would not only be absurd, it would be intellectual suicide.

4.2 Communication Breakdown

> **Objection 2**. The case **Logical Disagreement** misrepresents the interaction between classical logicians and intuitionists. Where the classical logician works with a philosophical presupposition of a realm of mathematical objects independent of the thinking subject (objects that obey the laws of classical logic and can stand in set-theoretic relations), this is radically different from the intuitionists who advocate for constructive methods and take mathematics to be about mental constructions. As a result of this schism, the two logicians in the proposed case would run into an insurmountable communication breakdown, i.e., the DNE-inference acceptable to the classical logician would not even be understandable to the intuitionist—it would be nonsense. To quote Brouwer: "*Let us now consider the concept: 'denumerably infinite ordinal number.' From the fact that this concept has a clear and well-defined meaning for both formalist and intuitionist, the former infers the right to create the 'set of all denumerably infinite ordinal numbers,' the power of which he calls aleph-one, a right not recognized by the intuitionist.*" (Brouwer, 1975). Something similar to what

> Brouwer describes in the interaction between diverse traditions in this quote occurs in **Logical Disagreement** with respect to DNE, i.e., the intuitionist does simply not comprehend the final step of the deduction on the blackboard. Thus, suspension of judgement is not a justified doxastic attitude for the intuitionist in this case; the supposed logical connection between E and C is gibberish to her. Rather, **Logical Disagreement** represents the kind of case where there is no justified doxastic attitude for the intuitionist to have. Hence, UT would be saved (at least the *at most one doxastic attitude*-version of the thesis). The case allows only one justified attitude, namely the attitude of the classical logician.

This objection overstates the divide between the classical and intuitionist traditions. Comprehension of classical logic is often presupposed in discussions of non-classical logical systems, e.g., as a metatheory. Indeed, it is stipulated in **Logical Disagreement** that the deduction found on the blackboard is written in a language that both logicians fully comprehend. We do not need more than noticing and appreciating this very stipulation in order to slide off the objection.

Further, we can strengthen this reply by noticing that it is not the case that when there is logical disagreement, one party has automatically misunderstood (or lacks) some concept. The disagreement may just be the result of one side having false beliefs. So, in **Logical Disagreement**, it need not be the case that the intuitionist (supposing that she got it wrong) lacks some concept about how negation works, or has misunderstood or changed its meaning. Negation means whatever it means, also in the intuitionist's mouth, she just has false beliefs about that meaning.[13]

4.3 Logical Monism

Now, let us turn to the more challenging objections.

> **Objection 3**. The evidence E does in fact justify exactly one doxastic attitude in **Logical Disagreement**, it is just that we do not know which attitude it is. For we do not know which logical theory is the "correct" model of logical consequence, but surely there is only one correct logic in the end. Thus, UT survives the case even though the logical disagreement between the classical logician and intuitionist leaves us in the dark with respect to which doxastic attitude is justified by E.[14]

This objection begs the question against *logical pluralists* (e.g., Beall & Restall-style), i.e., the view that there is more than one true (or correct) logic.[15] According

[13] A similar point is made by Williamson in (2007, Chapter 4).

[14] See, e.g., Griffiths and Paseau (2022) for a recent defense of *logical monism*.

[15] In principle, the objection also begs the question against *logical nihilism*, which is the extreme view that there is no true (or correct) logic at all (Russell 2018).

to logical pluralists, there is not always a single answer to the question whether a proposition p logically follows from a set of propositions (premises), in some cases there are more than one correct answer. A rough motivation for logical pluralism is that theories of classical logic, relevance logic, intuitionistic logic etc., all have a rightful place in formalizing and restraining logical inference as various important aspects of our pre-theoretic notion of logical consequence can be explicated by each of these approaches to logic.

Clearly, begging the question against the pluralist in this way merely relocates the tension from an infight between UT-supporters and -deniers to a clash between logical monism and pluralism, so it seems like a dissatisfying option. Of course, some UT-supporters might be happy to say that logical pluralism is false, and thus they will have a way to save their principle, but this strategy should be supported by strong independent reasons. It will not be enough for the UT-supporter to accept logical monism because it seems like the default position amongst mainstream epistemologists. Hence, **Objection 3** is underdeveloped as it stands, and UT-supporters opting for this way out have further work to do.

Developing the back and forth between logical monists and pluralists any further here would take us beyond the scope of this paper, but the reader can find some useful references in the footnote below.[16]

4.4 Splitting the Evidence

> **Objection 4**. As S_1 and S_2 belong to two opposing traditions in logic and don't accept the same rules of inference, it is actually not the case that they possess the same evidence in the situation described. Surely, considered just as a set of (formal) propositions, the evidence is the same for both subjects, but due to the subjects' diverse logical backgrounds the evidence splits in two. The case really presents both E and E^*, where the acceptable inference rules of classical logic are tacitly accepted to induce E and the rules of intuitionist logic are tacitly accepted to induce E^*. No set of (formal) propositions supports anything pre-theoretically. Choosing a logical theory is necessary to even generate *logical* evidence. Pre-theoretically, the question of which doxastic attitude is supported by a body of logical evidence is empty. Hence, **Logical Disagreement** is not a counterexample to UT since each body of evidence only justifies one doxastic attitude.

Prima facie, this objection seems to have something going for it. Indeed, it might save UT seen as a general epistemic principle since at most one doxastic attitude can be justified per body of evidence. However, at the same time it undermines the

[16] For more on logical pluralism in the Beall & Restall-style, see, e.g., (Beall and Restall, 2000, 2006). Other kinds of logical pluralism can be found in Carnap (2014), Shapiro (2014). For an extensive overview, see Russell (2019).

initial appeal of UT. For if we need to choose a logical theory in order to even generate logical evidence, we get a kind of relativism with respect to logical evidence. To illustrate, take an arbitrary set of (formal) propositions. This set does not constitute a unique body of logical evidence, as would be natural to suppose, instead it constitutes as many different bodies of logical evidence as there are acceptable logical theories.[17]

This moves our discussion away from evidence—as the central topic—to a discussion of acceptable theories instead, but no such discussion should be relevant to UT. UT should not be true only relative to preferred theory. For let us remind ourselves of how strong a thesis UT really is: it concerns all bodies of evidence, no matter what subject possesses it, and no matter the time.

The crucial point is that UT is supposed to motivate a certain response to peer disagreement, i.e., at most one peer can be justified in her doxastic attitude toward the target-proposition in such disagreements. But if logical evidence is relativized to preferred logical theory, the scope of UT is reduced drastically. You can now only share logical evidence with those from your own theoretical equivalence class, and there can be as many of those classes as there are acceptable logical theories. This kind of relativism is clearly not desirable for a UT-proponent, and thus saving UT using **Objection 4** turns out to be a Pyrrhic victory.[18]

---

[17] See also Andersen (2023a) for a recent discussion of *justification holism* versus *justification atomism* in the epistemology of logic, which is highly relevant to this issue.

[18] Other epistemologists have suggested that one way in which uniqueness might fail is if there is a plurality of methods (in a broad sense) which one could rationally use to generate evidence. Accordingly, the counterexample **Logical Disagreement** presented here, and our discussion about logical evidence being relativized to acceptable logical theories, might be subsumed under a broader style of argument against uniqueness, namely that UT fails because evidence (of various types) is relative to acceptable methods. For further discussion of this general style of argument, see for instance Hales (2014).

Note also how the issues surrounding logical evidence and uniqueness relate to some more established debates about permissible *epistemic standards* (Titelbaum and Kopec 2019). Plenty of formal epistemologists claim that a body of evidence supports a hypothesis only relative to a rational reasoning method, and since there are multiple, extensionally non-equivalent, rational reasoning methods available, there is not always an unambiguous fact of the matter about whether some evidence supports a particular hypothesis. Subjective Bayesianism, for example, could deny UT by appealing to legitimate differences in epistemic standards. In general, Bayesians hold that any rational agent's credences at a given time can be obtained by conditionalizing their *hypothetical prior* ('Cr_h') on their total evidence at that time. For a total body of evidence E and a hypothesis H, the evidence supports the hypothesis exactly when $Cr_h(H \mid E) > Cr_h(H)$. Here, facts about evidential support are relative to the hypothetical prior of the relevant agent, and we can plausibly think of an agent's hypothetical prior as capturing their epistemic standards. Some Objective Bayesians claim that there is a unique rational

However, some might hesitate to admit that **Objection 4** leads to evidential relativism regarding logical evidence, for it may be objected that E and E^* don't have the same epistemic status. There could be good and purely epistemic reasons for favoring E over E^* (or *vice versa*) the reply goes. As noted above, E is the body of evidence induced by the tacit acceptance of classical logic, while E^* is the result of tacitly accepting intuitionist logic, but surely logicians do not just accept any old theory of logic, they have epistemic reasons for accepting whatever theory they favor. Thus, S_1's *total* evidence pool may very well include evidence for accepting DNE, law of the excluded middle etc., which the intuitionist lacks. Similarly, S_2's *total* evidence pool may well include evidence for denying DNE, law of the excluded middle etc., which the classical logician does not have in her possession. Further, S_1's reasons may be better than S_2's ditto (or *vice versa*).

Although this worry is legitimate, it will not save UT. First, it is underspecified in the literature whether UT is meant to apply to the *total* bodies of evidence in this sense, i.e., including pieces of evidence supporting one's methods used to generate evidence. There are hints about the importance of evidence for evidence-generating methods in the literature on *deep disagreement*,[19] but usually such evidence is taken as background information, and thus not as included in whatever body of evidence is under consideration in standard disagreement cases. Thus, it is not clear what UT-proponents would say about cases involving such *total* bodies of evidence. Further, one could easily rewrite **Logical Disagreement** stipulating that the two logicians were (known) epistemic peers. Then, insofar as evidential symmetry is necessary for peerhood, this would exclude any evidence from the case besides the common evidence. Of course, one could then say that if S_1 is a classical logician and S_2 an intuitionist, they cannot be epistemic peers, but in that case, we are back to square one; logical evidence becomes relativized to your own theoretical equivalence class and relativism looms.

4.5 Individualistic versus Social Epistemology

Objection 5. UT is most plausibly defended as an *intra*-personal thesis, but **Logical Disagreement** is an inter-personal case. Thomas Kelly distinguishes between

---

hypothetical prior, so, in their case while evidential support is relative to the hypothetical prior there is still at most one rational hypothetical prior, and so UT is true. Yet some Subjective Bayesians claim that multiple hypothetical priors are rationally acceptable. Thus, for them, two rational agents could have different hypothetical priors, i.e., different epistemic standards, and end up in situations where the same body of evidence E supports a hypothesis H for one of them while it doesn't for the other.

[19] For detailed discussions of deep disagreement, see Lynch (2010, 2016), Kappel (2012, 2021), Ranalli (2020, 2021), Ranalli and Lagewaard (2022a, b).

intra-personal and *inter-personal* versions of UT (Kelly 2014, 307):

UT_{Intra} Given that my evidence is E, there is some doxastic attitude D that is the only fully rational doxastic attitude for me to take towards proposition p [...].[20]

UT_{Inter} Given evidence E, there is some doxastic attitude D that is the only fully rational doxastic attitude for anyone to take towards proposition p [...].[21]

Only UT_{Intra} holds as a general epistemic principle; not UT_{Inter}.

This objection saves UT as a general epistemic principle *intra-personally*, but as should be clear, it also completely undermines the core motivation for the thesis, which is social. Instead of relativizing evidence to acceptable theories or methods as in **Objection 4**, E is now relativized to subjects, and an even worse kind of relativism is unavoidable.

We should agree that UT_{Intra} is true. Take a perceptual case. If subject S clearly sees that there is a computer in front of her on the table and this visual perception constitutes her relevant evidence, then under normal circumstances there will be at most one justified doxastic attitude for her to adopt towards the proposition expressed by the sentence '*There is a computer on the table,*' i.e., S is justified in believing the proposition to be true (while either disbelieving or suspending judgement would be unjustified). Likewise, UT_{Intra} seems true in logic cases insofar as we assume the agent in play has accepted a certain logical theory (as the only correct one) in advance. This blocks cases where **Logical Disagreement** is reformulated as a single person-case with an eclectic logician who prefers neither the classical nor intuitionist tradition of logic, and yet is fully competent in both. Given our assumption, this logician cannot be intra-personally justified in more than one doxastic attitude towards a given p, e.g., the eclectic logician cannot be justified in a belief that p as well as a suspension of judgement with respect to p based on the same body of logical evidence.

However, as mentioned above, admitting that only UT_{Intra} is true comes with an unbearable cost for the UT-proponent. For with the embrace of this view, UT is no longer relevant to the peer disagreement debate which it was supposed to be central to. As UT_{Intra} is compatible with multiple doxastic attitudes being justified in cases of peer disagreement, the initial motivation behind UT is now completely lost. Thus, UT-proponents should not accept **Objection 5** as it indirectly undermines UT.

[20] Note that even though Kelly uses the term 'rational' instead of 'justified' in the quote above, it will not make any substantial difference for our purposes.
[21] See footnote 20.

5. Concluding Remarks

This paper has introduced a new counterexample to UT which involves logical disagreement. To legitimize this example and strengthen the case for it, we have shown that five different objections trying to save UT from **Logical Disagreement** fails. Two of the five objections were simply fended off, one needed further development to pose any real threat, while explaining away the counterexample with either one of the remaining two options resulted in an unbearable indirect defeat of the thesis. Hence, in the absence of successful objections to **Logical Disagreement**, the paper recommends that we hesitate in accepting UT as a general epistemic principle.[22]

References

Alston, W. P. (1989). *Epistemic Justification: Essays in Theory of Knowledge*. Cornell University Press.

Andersen, F. J. (2020). "Uniqueness and Logical Disagreement". *Logos & Episteme. An International Journal of Epistemology*, 11(1): 7–18.

———. (2023a). "Countering justification holism in the epistemology of logic: The argument from pre-theoretic universality". *Australasian Journal of Logic*.

———. (2023b). *Logical Disagreement—An Epistemological Study*. PhD thesis, University of St Andrews.

Beall, J. C. and Restall, G. (2000). "Logical pluralism". *Australasian Journal of Philosophy*, 78(4): 475–493.

———. (2006). *Logical pluralism*. Oxford University Press.

BonJour, L. (1985). *The structure of empirical knowledge*. Harvard University Press.

Brouwer, L. (1975). "Intuitionism and formalism". In *Philosophy and Foundations of Mathematics*, edited by A. Heyting, 123–138. Elsevier North-Holland.

Carnap, R. (2014). *Logical syntax of language*. Routledge.

Carter, J. A. and Bondy, P. (2019). *Well-founded belief: New essays on the epistemic basing relation*. Routledge.

Conee, E. and Feldman, R. (2004). *Evidentialism: Essays in epistemology*. Clarendon Press.

Feldman, R. (2006). "Epistemological puzzles about disagreement". In *Epistemology Futures*, edited by S. Hetherington, 216–36. Clarendon Press.

[22] Thanks to Greg Restall, Franz Berto, Jessica Brown, Klemens Kappel, and Alessandro Rossi, for written feedback on earlier versions of the paper. My research is supported by the Niels Bohr Foundation (Royal Danish Academy of Sciences) and the Royal Institute of Philosophy (London).

Feldman, R. and Conee, E. (1985). "Evidentialism". *Philosophical Studies*, 48(1): 15–34.

Goldman, A. I. (1986). *Epistemology and cognition*. Harvard University Press.

Griffiths, O. and Paseau, A. (2022). *One true logic*. Oxford University Press.

Hales, S. D. (2014). "Motivations for relativism as a solution to disagreements". *Philosophy*, 89(1): 63–82.

Harris, J. H. (1982). "What's so logical about the 'logical' axioms?" *Studia Logica*, 41: 159–171.

Hattiangadi, A. (2018). "Logical disagreement". In *Metaepistemology*, edited by C. McHugh, 88–106. Oxford University Press.

Hattiangadi, A. and Andersen, F. J. (202X). "Logical disagreement". In *The Oxford Handbook of Philosophy of Logic*. Oxford University Press.

Hjortland, O. T. (2022). "Disagreement about logic". *Inquiry*, 65(6): 660–682.

Kappel, K. (2012). "The problem of deep disagreement". *Discipline Filosofiche*, 22(2): 7–25.

———. (2021). "Higher order evidence and deep disagreement". *Topoi*, 40(5): 1039–1050.

Kauss, D. (2023). "A rational agent with our evidence". *Erkenntnis*.

Kelly, T. (2010). "Peer disagreement and higher order evidence". In *Disagreement*, edited by R. Feldman, and T.A. Warfield, 111-175, Oxford University Press.

———. (2014). "Evidence can be permissive". In *Contemporary debates in epistemology*, edited by M. Steup, J. Turri, and E. Sosa, 298–311. Wiley Blackwell.

Kopec, M. and Titelbaum, M. G. (2016). "The uniqueness thesis". *Philosophy Compass*, 11(4): 189–200.

Korcz, K. A. (2021). "The Epistemic Basing Relation". In *The Stanford Encyclopedia of Philosophy*, edited by E. N. Zalta. Metaphysics Research Lab, Stanford University, Spring 2021 edition.

Littlejohn, C. (2012). *Justification and the truth-connection*. Cambridge University Press.

Lynch, M. (2010). "Epistemic circularity and epistemic disagreement". In *Social epistemology*, edited by A. Haddock, P. D. Millar, and D. Pritchard, 262-277. Oxford University Press.

———. (2016). "After the spade turns: disagreement, first principles and epistemic contractarianism". *International Journal for the Study of Skepticism*, 6(2-3): 248–259.

Matheson, J. (2011). "The case for rational uniqueness". *Logos & Episteme. An International Journal of Epistemology*, 2(3): 359–373.

McCain, K. (2014). *Evidentialism and epistemic justification*. Routledge.

Quine, W. V. O. (1986). *Philosophy of logic*. Harvard University Press.

Ranalli, C. (2020). "Deep disagreement and hinge epistemology". *Synthese*, 197(11): 4975–5007.

———. (2021). "What is deep disagreement?" *Topoi*.

Ranalli, C. and Lagewaard, T. (2022a). "Deep disagreement (part 1): Theories of deep disagreement". *Philosophy Compass*, 17(12): e12886.

———. (2022b). "Deep disagreement (part 2): Epistemology of deep disagreement". *Philosophy Compass*, 17(12): e12887.

Rosa, L. (2012). "Justification and the uniqueness thesis". *Logos & Episteme. An International Journal of Epistemology*, 3(4):571–577.

———. (2016). "Justification and the uniqueness thesis again: A response to Anantharaman". *Logos & Episteme. An International Journal of Epistemology*, 7(1):95–100.

Ross, R. (2021). "Alleged counterexamples to uniqueness". *Logos & Episteme. An International Journal of Epistemology*, 12(2): 203–213.

Rossi, A. (2023). "From collapse theorems to proof-theoretic arguments". *Australasian Journal of Logic*, 20(1): 1-31.

Russell, G. (2018). "Logical nihilism: Could there be no logic?" *Philosophical Issues*, 28(1): 308-324.

———. (2019). "Logical pluralism". In *The Stanford Encyclopedia of Philosophy*, edited by E. N. Zalta. Metaphysics Research Lab, Stanford University, summer 2019 edition.

Shapiro, S. (2014). *Varieties of logic*. Oxford University Press.

Silva, P. (2023). "Merely statistical evidence: when and why it justifies belief". *Philosophical Studies* 180 (9): 2639-2664.

Smart, J. A. (2021). "Disbelief is a distinct doxastic attitude". *Synthese*, 198(12): 11797–11813.

Titelbaum, M. (2015). "Rationality fixed point (or: In defense of right reason)". In *Oxford Studies in Epistemology 5*, edited by T. S. Gendler, and J. Hawthorne, 253–294. Oxford University Press.

———. (2019). "Return to reason". In *Higher-order evidence. New essays*, edited by M. Skipper, and A. Steglich-Petersen, 226–245. Oxford University Press.

Titelbaum, M. G. and Kopec, M. (2019). "When rational reasoners reason differently". In *Reasoning: New Essays on Theoretical and Practical Thinking*, edited by M. B. Jackson, B. B. Jackson, 205–231. Oxford University Press.

White, R. (2014). "Evidence cannot be permissive". In *Contemporary debates in epistemology*, edited by M. Steup, J. Turri, and E. Sosa, 312–323. Wiley Blackwell.

Williamson, T. (1988). "Equivocation and existence". In *Proceedings of the Aristotelian Society* 88: 109–127.

———. (2000). *Knowledge and its Limits*. Oxford University Press.

———. (2007). *The philosophy of philosophy*. Wiley Blackwell.