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Zia Movahed,
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REVIEW

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In this article, Zia Movahed undertakes to provide evidence that Ibn Sina (Abu Ai al Husein ibn Abdallah Ibn Sina; Latinized as Avicenna; 973-1037 A.D.) pioneered some important results in modal logic, having anticipated the distinction between *de dicto* and de re propopsitions, upon which two formulae of Johannes Buridanus (ca. 1295 or 1300-1358 or 1360)[1] and two formulae of Ruth Charlotte Barcan Marcus (b. 1921).

Movahed begins (p. 248) by quoting from Rudolf Carnap's *Meaning and Necessity* [Carnap 1947, 196] to the effect that modal logic would be of little interest to logicians if it were restricted to propositional modal logic. Movahed notes that, contrary to Carnap's estimation, quantified modal logic proved to be of much greater interest than propositional modal logic (p. 248).

The chief interest in the various systems of quantified modal logics[2] , from the philosophical standpoint, says Movahed (p. 248), apart from the provability interpretation[3] , stems from these systems having become "a battleground for ongoing heated controversies over philosophical problems," such as the ontological status of possible worlds, necessity, existence, etc.

[1]See [Knuutila 1992] on Buridan on modal logic, in particular its connection with Aristotle's modal logic.

[2]A survey of the systems of quantified modal logics as the one in which Movahed is particularly interested in this study, is to be found in [Garson 1984].

[3]George Stephen Boolos (1940-1996) developed provability logic in the late 1970s and early 1980s as an application of modal logic to study of formal provability; see, *e.g.* [Boolos 1979; 1993].

The system in which Movahed is interested is one in which the Barcan formula,

$$\forall x \Box Fx \rightarrow \Box \forall x Fx$$

its converse[4],

$$\Box \forall x Fx \rightarrow \forall x \Box Fx$$

and the Buridan formula,

$$\Diamond \forall x Fx \rightarrow \forall x \Diamond Fx$$

are derivable. As Movahed notes (pp. 248-249), it was philosopher of religion Alvin Plantinga (b. 1932) who attributed the latter formula to Buridan [Plantinga 1974, 58][5].

The significance of Ibn Sina's anticipation of the Barcan formula, Movahed holds (p. 249), must be understood in the context of modern modal semantics, which is embedded in possible worlds semantics, and can be traced back to Leibniz. In modern terminology, a proposition P is *necessarily true* ($\Box P$) iff it is true in all possible worlds, is *possibly true* ($\Diamond P$) iff it is true in some possible worlds, and *impossible* iff it is true in no possible worlds. But these definitions, Movahed avers (p. 249), fail to distinguish between some different modal axioms. In this semantic, each of the following axioms of propositional modal logic are valid:

$$\Box P \rightarrow P$$

$$P \rightarrow \Box \Diamond P$$

$$\Box P \rightarrow \Box \Box P$$

$$\Diamond P \rightarrow \Box \Diamond P$$

[4]See [Marcus 1946].

[5]See [Sennet 1992] for an exposition and critical account of Plantinga's philosophical work.

To salvage the ability to distinguish among different modal axioms, Stig Kanger (1924-1988)[6] and Jaakko Hintikka (b. 1929)[7] , and independently Saul Aaron Kripke (b. 1940) [Kripke 1963][8] , introduced the binary relation of *relative possibility* over possible worlds by virtue of which, for every possible world α, there are worlds reachable from, or accessible to, α, and some which are not. We say that β *is accessible to* α and write $\alpha R \beta$ for the accessibility relation. This permits us to clarify the truth-functional definitions of modal operations as

$\Box P$ is true in world α iff P is true in every possible world β that is accessible to world α

$\Diamond P$ is true in world α iff P is true in some possible world β that is accessible to world α

Next Movahed sets out a system of axioms for quantified modal logic in which the Barcan formula, its converse, and the Buridan formula can be derived as theorems. Having done so, some passages are translated (p. 253) into English from Ibn Sina's [1970, 114-115, 116] *Kitab al-Ibara*, volume three of his *Kitab al-Shifa* (see [Ibn Sina 1954]), and interpreted to yield the Buridan formula, and the Barcan formula and its converse. Thus Ibn Sina's proposition "Every human being is possibly a writer" [Ibn Sina 1970, 114-115] is translated as "every human being x, possibly x is a writer" and symbolized as $\forall x \Diamond W(x)$; his proposition "Possibly every human being is a writer" is rendered $\Diamond \forall x W(x)$. Movahed then avers that, although Ibn Sina did not explicitly assert that $\forall x \Diamond W(x) \to \Diamond \forall x W(x)$, one can infer it, and that, furthermore, he can be understood from the context of his discussion that $\Diamond \forall x W(x)$ does not imply $\forall x \Diamond W(x)$ that he accepted what we call the Buridan formula, but not its converse. In accepting the Buridan formula and rejecting its converse, Ibn Sina was "within the wisdom" of modern quantified modal logic (p. 253).

[6]Kanger adopted the provability interpretation for modal logic; see especially [Kanger 1957]. Movahed fails to give a bibliographic citation to the relevant work of either Kanger or Hintikka.

[7]Kanger adopted the provability interpretation for modal logic; see especially [Kanger 1957]. Movahed fails to give a bibliographic citation to the relevant work of either Kanger or Hintikka.

[8]See [Beuchot 1982] for a discussion of Kripke's work in modal logic against the historical background of the modal logic of Aristotle and Aquinas. See [Rasmussen 1983] for an Aristotelian approach to Aquinas on modality. There are numerous studes of Aristotle's modal logic, prominent among them [Becker 1933], [McCall 1963], [Hintikka 1973], [Seel 1982]; [Patterson 1995], [Nortmann 1996], [Thom 1996], and [Johnson 2004].

According to Movahed, Ibn Sina in the next passage extended his discussion to universal negative propositions, and concluded that the distinction between $\Diamond\forall x\neg W(x)$ and $\forall x\Diamond\neg W(x)$ is the same as that between the universal affirmative propositions already treated, and likewise held that, whereas $\Diamond\forall x\neg W(x) \rightarrow \forall x\Diamond\neg W(x)$, again the converse does not hold.

Movahed then translates (p. 253) another passage from Ibn Sina's *al-Ibara* [Ibn Sina 1970, 116], according to which

> ...to say that: *some people possibly are not writers* is modally the same as saying that: *possibly some people are not writers*, and although one implies the other the meaning of the one may be opposite to the other.

This, says Movahed (p. 253), can be translated as

$$\exists x\Diamond\neg W(x) \leftrightarrow \Diamond\exists x\neg W(x)\,,$$

which is logically equivalent to

$$\forall x\Box W(x) \leftrightarrow \Box\forall x W(x)\,,$$

which is the conjunction of the Barcan formula (**BF**) and its converse (**CBF**) in a single biconditional. Thus, Movahed asserts (p. 253), Ibn Sina "discovers and endorses both **BF** and **CBF** while admitting that there are differrences of meaning between the antecedent and the consequent of each conditional."

Movahed offers a counterexample (pp. 253-254) to Ibn Sina's version of **BF**, namely that "*it may be possible that there should be things of a different species from any actual living organism, but it is not possible of any actual living organism that it should be of a different species*." But Movahed suggests (p. 254) that this is the sort of counterexample that Ibn Sina himself may have had in mind when asserting with respect to his presumed statement of the Barcan formula [Ibn Sina 1970, 116] that the meaning of "*some people are not writers* is modally the same as saying that: *some people possibly are not writers*, and ... one implies the other" is that "the meaning of the one may be opposite to the other." Movahed, however, mentions this counterexample and Ibn Sina's presumed intimation of it merely as a means of enlightening us of Ibn Sina's "far-reaching reflections on modal logic."

With all this preparation, Movahed is now ready to show(p. 254) how Ibn Sina descovered the *de dicto/de re* distinction. In considering $\Box\forall x\, Fx(\Diamond\forall x\, Fx)$, Ibn Sina, says Movahed, noted that the modal word quantifies the sentence preceded by the quantifier, and called this the "mode of the quantifier" (*jahat-e- soor*); and in $\forall x\,\Box Fx(\forall x\,\Diamond Fx)$, the modal word quantifies the predicate attributed to a thing or an object, and called this the "mode of predication" (*jahat-e-haml*). Ibn Sina's discovery was further discussed and developed over the next century by Islamic scholars, and then passed on to western European scholars in Latin translations of Ibn Sina's work. The *Kitab al-Shifa* appeared in Latin as *De Sufficientia*.[9] But medieval Latin-writing philosophers, influenced by Ibn Sina, were not typically given to attributions. They did, however, pick up and discuss modality *de re* (about thing) and *de dicto* (about sentence) in their own writings. Peter Abelard (1079-1142) may be said to have noticed the modal logic distinction *de re* versus *de dicto* ("*expositio per divisionem*" and "*expositio per compositionem*"). Some related questions to be considered, but ignored by Movahed, is whether Abelard was the first of the Latin philosophers to consider modality *de dicto* and modality *de re*; and, if so, whether he came upon it independently or extracted it from Ibn Sina or other Arabic writings. (This suggests an historiographical problem that goes far beyond our current consideration of any priority claims expounded by historians of logic and historians of mathematics. It is an issue which I shall return to following my exposition of Movahed's discussion of Ibn Sina's priority in discovering either modality *de dicto* and modality *de re* or in formulating, albeit in non-technical notational terms, the Barcan and Buridan formulae.) Among the few medieval western European philosophers who did provide attributions was Thomas Aquinas (1225-1274). Whether he mentioned Ibn Sina in this regard Movahed does not say. Instead our author remarks upon the close similarity

[9] The earliest extant codice containig Ibn Sina's *al-Ibara* in Latin translation as *De Sufficientia* is apparently [Ibn Sina 1508].

of the expression of Aquinas's treatise *De Modalibus*[10] (see [Aquinas 1976]) and Ibn Sina's, as translated by our author (pp. 252-253) in quoting Ibn Sina [1970, 114-115, 116]. In support of this similarity of expression between Ibn Sina on *jahat-e-soor* and *jahat-e-haml* and Aquinas on modal propositions *de re* and *de dicto*, Movahed merely quotes from [Kneale & Kneale 1962, 237] that a "modal proposition *de dicto* is always singular, since it has a dictum for its subject, whereas a modal proposition *de re* may be universal or particular according to the sign of quantity."

Regrettably, Movahed does not quote from, or even provide bibliographic information to, Aquinas's *De Modalibus*; so that we are left with having to depend upon Movahed's assertion of similarity of expression and the explanation in [Kneale & Kneale 1962, 237] of the difference between modal propositions *de re* and *de dicto*, or translate for ourselves the relevant texts of Ibn Sina and Aquinas.[11]

[10]For discussions of Aquinas on modality and various aspects of its applications by Aquinas, see, *e.g.* [Bochenski 1940], [Beuchot 1982], and [Arrias 2004]. [Robles Carcedo 1974] looks at the history of Aquinas's *De modalibus*, and [Rocca 1991] examines the role which the distinction between *res significata* and *modus significan* in Aquinas's epistemology, especially as it relates to theology, and we may consider this distinction in terms of the *de re/de dicto* distinction, or perhaps, more precisely, if we follw Ibn Sina, between *jahat-e-soor* and *jahat-e-haml*. [Knuutila 1993] is a discussion of modality in medieval philosophy. For a general account of modal theory by the medieval schoolmen, especially as it is applicable to metaphysics, see, *e.g.* [Friedman & Nielsen 2003]. The twentieth-century Thomist philosopher Jacques Maritain, a defender of classical (Aristotelian) logic, provides a contemporary study of modality from the Thomist perspective [Maritiain 1972]. [Trundle 1994; 1995 1996; 1999] and [Weingartner 1968] offer an interpretation of Aquinas's modal logic from the perspective of twenieth-century phenomenology and linguistic analysis, including in particular the question of the relation of Aquinas's thought on modal logic with Heidegger's and Wittgenstein's philosophy.

[11]Fortunately, there are many studies of Ibn Sina's influence on Aquinas, as well more generally of the influence of Islamic philosophers upon philosophers of the Latin West. [Gómez Nogales 1975] provides a bibliogrpahy devoted to investigations of the influence of Arabic philosophers on Aquinas, while [Siva 1974], *e.g.*, undertakes to evaluate that influence. [Rautzenberg 1930] is an investigation of Arabic Aristotelianism on the thought of Aquinas, and [Zedler 1956] examines Aquinas's exposition and evaluation of Ibn Sina, while [McGinnis 2005], *e.g.*, is more specifically concerned with Ibn Sina's influences on Aquinas's metaphysics. [Madkour 1933] is a discussion of medieval Arabic treatments of Aristotle's logic.

[Trundle 1994; 1995 1996; 1999] and [Weingartner 1968] offer an interpretation of Aquinas's modal logic from the perspective of twenith-century philosophy, Trundle considering the relation of Aquinas's thought on modal logic with Heidegger and Wittgenstein.

Movahed's assertion (p. 253) that Ibn Sina was "within the wisdom of modern quantified modal logic" is at best ambiguous. Nevertheless, it is clear from the abstract as well as from the text of the article itself, if not from the title of the article, that according to Movahed, Ibn Sina not only "anticipated" modality *de dicto* and *de re*, but was the first to formulate those notions. More patently clear and distinct is Movahed's outright assertion (p. 253) that, Ibn Sina "discovers and endorses both **BF** and **CBF**...." One problem, already suggested, is that Abelard could be said to have noticed the modal logic distinction *de re* versus *de dicto* ("*expositio per divisionem*" and "*expositio per compositionem*"), thereby developing a way to understand the Aristotelian "two Barbaras" problem. However, he claims that *de dicto* modalities are not real modalities. Apart from the question of whether or not, in disagreement with Aquinas and in evident disagreement with Ibn Sina, Abelard rejected the conception that *de dicto* modalities were real modalities, there is the open question of whether Abelard took the notions of *de dicto* and *de re* from Ibn Sina, and whether his *expositio per divisionem* and *expositio per compositionem* are legitimately interpretable in terms of *de dicto* and *de re* modalities.[12]

The kind of historical reconstruction undertaken here, in certain of its aspects apparently dependent upon Movahed's interpretations of what Ibn Sina and Aquinas meant and translated into modern symbolic notion raises the historiographically crucial issue of the justifiability of recasting historical documents in modern guise, and in particular the ticklish question of whether, in doing so, we are imposing upon the work of our predecessors ideas and understandings that they did not hold in their own time and context. There are historians of mathematics, such as Alexei Barabashev [Barabashev 1997], Izabella Bashmakova, and Yiannis Vandoulakis (see, *e.g.* [Bashmakova & Vandoulakis 1992]), who hold that there are good and legitimate reasons for rewriting the works of the past in terms of the mathematics of the present. There are, on the other side, those, such as Christian Thiel and Volker Peckhaus, who point out that we fail to do justice to ourselves or our predecessors in we do not examine the context in which those whose work we study operated; for we cannot then fully appreciate their contributions in light of what they knew and what the state of the mathematics was

[12]For discussions on Abelard's *dictum propositionis*, see *e.g.* [De Libera 1981] and [De Rijk 1975].

within which they achieved their own contributions.[13] Ivor Grattan-Guinness [1997, 7] went even further, arguing that when historians ask the question: 'How did we get here?', they ultimatley reach a far different answer than they would had they instead asked 'What happened in the past?' These two styles are not necessarily mutually exclusive. But by their nature the methodologies and goals of the search for one answer rather than another can easily lead to different results, since, in framing one question rather than another, we are asking after different facts. The danger of the first question, if in its pursuit one ignores the contextual situation in which our predecessors worked, is that we may well read into their mathematics what a certain piece of mathematics means to us from our own perspective, rather than what it meanto our predecessors and to those who created that piece of mathematics. By way of example [Anellis & Houser 1988, 7-8] wrote:

> Because Boolean algebras are distributive lattices, in fact complete distributive lattices, so that the concept of lattice is embedded and inherent in the concept of Boolean algebra, and because all of the apparatus necessary for finding lattices is clearly present in [Peirce 1880], the creators of Boolean algebra can be said to have introduced the notion of lattice. But it was Schröder [1890], not Boole or Peirce, who first brought together all of the Boolean concepts which he and Peirce had developed, requisite for his formulation of the first modern concept of lattice, which he called a Dualgruppe, and it was Dedekind [1895; 1897; 1900], applying this work of Schröder's to his own results in the theory of modules and ideals, who presented the first systematic account of lattices. Thus, the work of Boole, De Morgan, Peirce, and even Schröder, is *significant for the development* of lattice theory—for example their studies of the properties of duality in algebraic logic; but their work nevertheless cannot on that score alone be asserted to be the

[13] See, *e.g.* Volker Peckhaus's [Peckhaus 1989], in which he expresses concern that the "analytic-historical" approach to history of logic rather than the "contextual-historical" approach can be misleading.
Some of the hazards that can afflict the historiography of mathematics through what Peckhaus calls the "analytical-historical" approach and which can arise from asking what Grattan-Guinness [1997, 7] describes as the question "How did we get here?" without considering the question of "What happened in the past?" can be found in the examples given in [Anellis 1989].

> origin of the explicit and formal modern concept of the lattice.

In a similar vein, both Theodore Hailperin [1981] and Judy Green [1994] warn us against confusing the algebraic logic created by George Boole with what we recognize today as Boolean algebra.

In the absence of a more detailed account by Movahed, and in particular without an actual citation from Aquinas, we are left with a discomfort as to the general accuracy of our author's interpretation of Ibn Sina and of the question of Ibn Sina's priority in formulating the Buridan and Barcan formulae; or whether it is a case of reading a modern interpretation of a text that is not clearly present in Ibn Sina, or would have been accepted and understood by Ibn Sina in these modern terms. And we are left, without checking not only the various influences on Aquinas that might have provided the source for his treatment of modality and the *de re/de dicto* distinction, whether these were inspired by Ibn Sina or by some other researcher whose work Aquinas studied; and this becomes all the more worrisome without an actual and direct comparison by Movahed in this account of the texts in question of Ibn Sina and Aquinas, rather than the mere bald assertion of the close similarity of those texts. What is worse: Movahed fails even to provide a reference to some other, more detailed, study—should one exist—of the history of the origin and development by the medievals, and in particular by Ibn Sina, to which one can look for confirmation (or disconfirmation) of Movahed's thesis, or in answer to some of the other relevant questions which, I have suggested, Movahed's brief treatment raises in its reader. If this is really but an hypthesis, one should hope that some historian of logic would undertake to examine the issues in far greater detail, and either substantiate or contradict Movahed's thesis.

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