Einstein's "true" discontinuity.
With an application to Zeno

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Received: 10.09.2007
Final version: 6.08.2008

BIBLID [0495-4548 (2008) 23: 63; pp. 339-349]

ABSTRACT: The question whether quantum discontinuity can or cannot provide an answer to Zeno's Paradoxes is reopened. It is observed that what is usually understood by the term "discontinuity", namely, Einstein's conception of the photon as described by himself and all others, is *unsuitable* to the task because, essentially, it reduces to the trivial 'discontinuity' of objects scattered in space. By contrast, quantization of energy *levels*, which are not in space but can only alternate in *time*, provide the right sort of discontinuity required. Discrete quantized orbits, corresponding to eigen-frequencies, are irreducible, and nothing is allowed to stand in-between them in satisfaction of the quantum postulate, furnishing the requisite, and so far missing, immediate *nextness* of a point to a certain other. In this way, Zeno's Runner need not postpone his first step indefinitely, always waiting upon an infinity of preceding steps, before it can be taken. There is now a point that *is* next to a point and so a step on that point, which *is* the first step. It follows that, if one kind of discontinuity, Einstein's, is incapable of offerring an answer to Zeno, while another kind can, the two are discrepant. One of them, the former, is not a kind of discontinuity properly so called at all, though evidently the consequence of one.

Keywords: Zeno, infinite divisibility, photons, quanta, space, quantized levels, discontinuity, indeterminacy, Planck, Einstein, Bohr.

1. The Family Resemblance View of QM

One of the main reasons why one cannot grasp "the meaning" of quantum mechanics (QM), is that there is no such thing as *the* meaning of QM. Its principles and theorems bifurcate so that, looking at them, one is strangely reminded of Wittgenstein's family resemblances (1968, § 66, ff), as the splitting varieties of use of what initially appeared to be a single word, yet now applied to different cases.

There are family resemblances between one derivation of the uncertainties and another, incompatible resemblances at that (Heisenberg 1958, p. 49, as *opposed* to Heisenberg 1949, pp. 21-23; and Heisenberg 1949 p. 21 as opposed to *op. cit.* p. 23!)[1], hence incompatible versions of complementarity, a family resemblance between Bohrian Wholeness, which is *proximal*, and Nonlocality, which is a *distant* wholeness, and between one QM and another! For the 'family' contains even this. The violation

[1] In his 1958, the photon "*pushes*(!) the electron" (p. 49), leading to $\Delta p \Delta q \geq h$. This 'derivation', due to (infamous) disturbance, whose interference alone prevents the joint determination of p and q, otherwise mutually compatible, is inaugurated in 1949, p. 21. But in that same work , two pages below, $\Delta p \Delta q \geq h$ is derived from de Broglie's relation, $p = h/\lambda$, where now p is determined by the *wave*, making the two variables *logically* incompatible instead. The sole resemblance between them is the resulting (?) $\Delta p \Delta q \geq h$ itself, which then, considering, is more of a resemblance by sheer accident than one by ties of family.

of Bell's Inequality implies that QM can either be local, *L*, but then not causal, –*C*, or causal, *C*, but then not local, –*L*. (Plotnitksy, 1994, p. 153). But not both at once. Of course, these two alternatives, QM as *L* & –*C* and QM as *C* & –*L*, are contradictories for both conjuncts involved, but they are both handsomely concealed under a single formalism; QM. One wonders, whether even a notion as flexible and permissible as that of family resemblances can make room even for that.

The family resemblance, if any, that I will be concerned with is even more generic than any of the previous. But certainly not less deserving of the "family" title (and its inherent ambiguity). We get a first taste of it in Bohm's following distinction, clear enough to place the matter in its proper perspective:

> Energy appeared to have a certain atomicity *both* in the form of light quanta *and* in the discrete allowed energy *levels* for matter. (Bohm 1957, p. 78)

This is the family resemblance, or lack of one, that I will subsequently investigate.

2. No nextness. The relevance of Zeno

In a number of recent works of mine (2003, 2004a, 2004b, 2007) I have been attempting to apply the concept of quantum discontinuity to Zeno's paradoxes, as a plausible solution. I will briefly state the reasons which prompted this approach. In the Stadium Paradox, sufficient to our purposes, Zeno lays down two premises:

[1] No distance can be crossed, unless all the (smaller) distances comprising it are crossed *first*. This is the premise of logical order.

[2] A distance is always comprised of (smaller) distances, which are infinite in number. This is the premise of infinite divisibility.

On their combined presence, Zeno validly proceeds to derive that

[3] No distance can be crossed. (Therefore, nothing moves.)

The premise of logical priority is of considerable importance to the cogency of the argument. I cannot eat five bites out of my steak, unless I eat the first bite first. This is a matter of strict logical order. The fifth bite has to "wait upon" the first one, as it were, and all of the preceding four, in other words, wait for its turn. Analogously, if an infinity of smaller steps has to interfere and precede, prior to taking any step, no step can be taken. And so no distance can be crossed, for any crossing of it will have to be indefinitely *postponed;* any crossing will have to wait upon a previous and that, in turn, upon a previous still —if there is no limit to their diminution. All prospective steps to take, if perpetually having to yield their place to preceding ones, and so on for ever, are simply steps whose own turn will never arrive. So no step can be taken. (So, nothing moves.)

Putting the point with distances, rather than with steps (a serious difference this, by all accounts —see below), if, before crossing any distance, one must first cross all the smaller distances contained in it, and these are infinite in number, either one can cross an infinity of distances, or one cannot cross any distance. Zeno rightfully denies the former possibility, and so is led to conclude that nothing moves.

The first thing that comes to mind in this strangulating setting, is to directly dispute, whether there *are* infinitely many smaller distances really contained in a distance, or, analogously, infinitely many smaller steps really contained in a step. Namely, to dispute Premise [2]. This is how Ancient Atomism came into being:

> A much more decisive impact of Zeno's work [...] is evident in the atomism of Leucippus and Democritus. (Kirk, Raven and Schofield 1983, p. 279)

And according to yet another, equally reliable source, "paradoxes such as Zeno's were used as arguments in favour of atomism" (Sorabji 1983, p. 337). So far so good. *Some* form of atomism is clearly a way out of the Paradoxes. But Ancient Atomism is basically an objection against the infinite divisibility of bodies. And Zeno's arguments are against the infinite divisibility of *processes*. A family resemblance there may well exist between them but, having seen what those are like, caution should be exercised. I might say that quantum discontinuity is a way out of the Paradoxes and you might be thinking, say, of Einstein's *photons*, when you hear me say this, since they too are a form of atomicity, though only *a* form of one, as Bohm's passage reveals. And then all further communication between us will abruptly break down. You simply cannot see, what possible relevance there is between Zeno's problem and the discontinuity of one photon to another. This is exactly what happened between Prof. Joseph Alper and myself:

> Zeno's paradox cannot be explained by quantum mechanics. More strongly, the paradox cannot even be *stated* in quantum mechanical terms.[2]

Different people mean different things by the same word, the eternal reason behind all family resemblances, if there ever was one. Einstein's conception of photon discontinuity does *not* have a relevance to Zeno. The problem is, however, that it does not have much relevance with Planck's quantum either, which was what *I* meant, when I spoke of discontinuity. This is the whole point of introducing Zeno's Paradoxes in the present connection, to begin with. If one kind of quantum discontinuity can offer a direct response to Zeno's Paradoxes, while another kind is simply irrelevant to them, one kind of quantum discontinuity is just irrelevant to the other or, more, too *unlike* the other to even belong to the same theory.

So we had best get clear, before proceeding, what sort of discontinuity is the one required for rebutting Zeno. Whether or not this is the one that QM provides, can then be decided in retrospection. A continuous line, as we ordinarily tend to conceive it, is a line which is packed with infinitely many points. This being the case, there is no point that is authentically next to any other. How *close*, then, is a point of such a line to its 'next'? Zeno's reply is that, if the line is continuous, i.e. infinitely divisible, there *is* no point that is authentically next to any other.

So the answer all but spells itself. Introduce a point that is really a next point. Then the Stadium Runner can naturally progress to such a point, the very first *stepping* stone, as it were, and thus move —discontinuously. Were I requested to start counting from the start, I would go "one, two, three &c". No problem. Yet were I requested to start

[2] Report on my work (2004b) for the *Journal of Philosophical Research*, October 11, 2006, signed by Joseph Alper, in which the author expresses his wish to receive my reaction to his comments. This is it.

counting *decimals* from the start, how exactly could I start? There *is* no place to start counting, if naturals yield their place to decimals. Zeno's Runner faces the exact same problem with taking a step, if motion is continuous. There is just no place for him to start. Hence, some notion of direct *nextness* is definitely required.[3] The question is, whether QM furnishes the requisite sort or some other, unsuitable. Or, for that matter, how many sorts it does furnish in the first place.

The historical evidence in favour of an atomistic reply to Zeno's riddles of motion is overwhelming. And the historical is, because the logical is. It makes sense to respond to them thus, provided one can make sense of them to begin with. In their search of a point that is really next to another point, McLaughlin and Miller (1992), and Harrison (1996) choose to quantize the *space*, which the Runner has to cross. I, by contrast, quantized the *motion* that he has to perform. But some family of discontinuity or other was mutually deemed to be relevant.

I chose discontinuity of motion from the family, because (a) the laws of spacetime *per se* were thus left intact, as intact indeed as QM preserves them (Gavroglou 1989, p. 551), (b) because discrete space points must be kept *separate*, which they cannot do, if there is no space in-between them to *keep* them separate, (c) because, if the Tortoise travels over *dimensionless* points, she will be crossing them in zero time and so the trailing Achilles would never overtake her in any case. I concluded that our sole option was to quantize Achilles's, or the Runner's, motion.

Zeno had a tacit assumption here, which has gone unnoticed by nearly every one concerned. He freely assumed that crossing a particular part of space must itself be conducted "*as*-space". Namely, only in ways matching the structure of space. This means that a body moving through space must either copy the rules of space, or else stay put. QM, which has quantized only the dynamical quantities, E and p, and their action yielding products, Et and pq, but not spacetime, is one definite evidence to the contrary. But there is one other version of the point, belonging to Henry Bergson, that is really worth quoting:

> The motion which Zeno attributes to Achilles would indeed be equivalent with the one he really performs, if and only if one has the right to treat motion, as he can treat *the distance travelled*, that is to say, as composable and decomposable at will. Once this paralogism is accepted, all the rest will but follow. (Bergson 1907; the greek translation, 1925)

So here is yet another philosopher, proposing that we preserve the undivided unity of motion, its "wholeness", as Bohr would put it, or its "duration", as Bergson himself would, in the long line of thinkers introducing atomism for confronting Zeno. Only Prof. Alper thinks that Zeno's paradoxes cannot even be *stated* in QM terms.

[3] Zeno construes, "if there is continuity, then there is no nextness" (and so no motion), as a logical truth. This is made apparent in his Paradox of Extension: "How can a line of finite length be divided into infinitely many parts of finite length? And how can a line made up of lengthless parts add up to a line that has length?" (in Harrison 1996, p. 273). There is hardly a more pressing demand for the introduction of Atomism than the previous. One cannot pile together lengthless parts, i.e. points, however many, and come up with a line that has length. In fact, Zeno's point is that contiguous points *cannot* make up a line. They just simply are *too close*, ultimately collapsing to one, solitary point. (Hence, nothing moves. For to move, you need two points.)

3. Einstein's "true discontinuity"

If there's any value in later Wittgenstein philosophy, with its language games and family resemblances, this lies in its warning against too much generality. In utter opposition to this tendency, physicists crave but for the theory which would unify the very universe, if possible. They have an eye only for unity, none for diversity. So they ignore it, even when they see it. Prof. Alper is no exception (op. cit.):

> Einstein did not reject discontinuity. In fact, it was Einstein —*not Planck*— who invented the idea of the photon and fully realized the discontinuity it entailed.

The difference is spotted but, presumably, transcended for the sake of greater unity. It is therefore instructive to see, how fully indeed did Einstein realize the discontinuity inherent in the idea of the photon:

> The mining of coal from a coal mine can be conducted in a continuous manner. The amount of coal excavated can increase or decrease in smaller and yet smaller amounts, without any restriction. But the number of coal miners, working in the mine, can only change in discontinuous fashion. It would be sheer absurdity to say that since yesterday the number of miners increased by 3,273. (Einstein & Infeld 1959)

This "fully realized" discontinuity (you may put the inverted comas over the latter word no less, if you so desire), is subsequently integrated thus:

> We can regard the mass of sand on a beach as continuous, even though the granular composition of the sand is evident. Yet in essence the mass of sand can vary only by integer multiples of the mass of a single grain. The mass of this grain would then be our *elementary quantum(!)*. From this example we see that the discontinuous character of a quantity hitherto considered as continuous, may be uncovered, if we augment the accuracy of our measurements. We must allow that certain physical quantities, so far considered as continuous, may turn out to consist of elementary quanta. (*Ibid.*)

What a full realization of discontinuity indeed. Quantum discontinuity, the one so fully realized, is declared tantamount to *people* 'discontinuity', or grains of sand discontinuity, or birds-in-a-flock discontinuity. So where's the problem? Quantization of energy is hardly any different than people discontinuity, or bird discontinuity.

What the problem is, I would think, is that people 'discontinuity', if that's the word for this unfortunate idea, or grain of sand discontinuity, or bird discontinuity, are one and all instances of magnitudes that are discrete by *definition*. Miners coming out of the mine are self-evidently discrete in number. In other words, discrete *a priori* or, if you prefer, necessarily discrete. Hence, their discreteness is an utter triviality. To compare this with energy discreteness, the anomaly of the century, is not *quite* to the point, is it? For energy discreteness is anything but self-evident and anything but discrete by definition. Energy should not have been discrete at all. If anything, it should have been *continuous* by definition.

It was this which prompted Bohr to speak of "the *irrationality* inherent in the quantum postulate" (1934, pp. 10, 19, 54), the irrationality that energy was found to occur only at certain values, rather than at any value, as every one (classically) expected that it should. For Bohr understood its meaning, as he also did the meaning of miners, grains of sand or birds, for which he never spoke in similar terms. If anything, he is contrasting between the two cases.

Let me then demonstrate, what Einstein has turned this amazing, and disconcerting, discovery into, to be approvingly cosigned by Prof. Alper. A photon-emitting source is connected with a timer, which controls their emission at calculable intervals of our choice. A certain initial interval t is succeeded by a second, $t/2$, a third, $t/4$, and so on. Then, given that the speed of photons is a constant, to these decreasing intervals of emission will correspond shorter and even shorter *distances* separating every photon from the next. Then, if the interval between a certain photon and the one succeeding it is reduced to a $t \rightarrow 0$, their distance will be a vanishing quantity no less. Is this anything like quantized *orbit* discontinuity, which was my sense in replying to Zeno? No. That discontinuity is not a vanishing quantity at all, is it?

Compare, then, the two members of the family; Planck's quantum with Einstein's, which two (thankfully) Prof. Alper has also contrasted. Radiation trapped in a cavity (also referred to as a "black body") produces a standing wave, so that Planck's law, $E = h\nu$, is a law fixing the range of possible eigen-frequencies of this closed wave while in the cavity, in other words, its higher harmonics, as is the case with any other, similar, harmonic oscillator (e.g. a guitar string). And these can only be integer multiples of the fundamental oscillation, since all to be inscribed within the rigid boundaries of the cavity (wall to wall), or the length of the string, into which they simply have to fit. All other frequencies, producing wavelengths whose sum does not add up to the exact boundary length, are mathematically precluded and so, via $E = h\nu$, are the energies, or "eigen-states", corresponding to them. If $E \approx \nu$, the energy of any harmonic oscillator, such as an atom, is quantized.

A reversal of Planck's reasoning, leading to the above frequency-energy association, may proceed in the following fashion. Let us allow that certain known absorption and emission phenomena are better understood, if we assume that they occur by discrete amounts only, a fact which Einstein also stresses (Einstein 1965, p. 368). Then given that such phenomena are observed only in the cases of harmonic oscillators, which manifest a comparable behaviour regarding the range of their permissible *frequencies*, those being analogously discrete, it all appears as if the energy of a certain radiation runs on exactly parallel lines with its frequency (rather than with its amplitude, as was classically assumed). Hence, there seems to exist an analogy here, namely, the ratio of E/ν must be an invariant. This is how the quantum of action came into being.

Now the term "quantized" does not apply to the eigen-frequencies themselves. On their own they are but a pure mathematical phenomenon, indeed, a classical phenomenon, known long before QM. But the energies corresponding to them, the eigen-states, are anything but classical. For, mathematically, there shouldn't be other frequencies besides the eigen-frequencies, in the first place. But, mathematically, and generally, there should always be other energies. What the quantum postulate excludes, therefore, in the introduction of such eigen-states, are other *states*.

This is no longer a "here-there" discontinuity. It is, by utter contrast, a "state-no *state"* discontinuity. And, at this point, why the full realization of discontinuity is the one in Einstein's conception, rather than Planck's, is becoming more and more obscure by the minute. Light quanta (a hazardous term, if there ever was one) are droplets of concentrated energy, existing, let us say, scattered in space. And their 'discontinuity' as trivial as that of any object standing apart from any other. E.g. miners or cars in the street. But Planck's quanta are quantized energy *levels*, namely, entities which are only high or low, but otherwise not to be found anywhere in space (see section 4). Only the object in possession of them is.

And these are energy differences, or levels, of a *single* oscillator. But a single, solitary oscillator, vibrating at discrete frequencies, and hence possessed of only discrete energies, though perfectly sufficient to the task, can no longer, in its solitude, be "fully discontinuous" with any other of its kin, or any other thing for that matter. And hence, on Einstenian standards, cannot be discontinuous at all. To introduce Einstein's 'full' concept of discontinuity, a discontinuity *other* than discrete level discontinuity, we need to introduce at least two entities, e.g. two photons which, at least on Einsteinian standards, are truly and fully discontinuous to one another, since spatially distinct. But to do this with our solitary oscillator, manifesting a discontinuity all of its own, would be to treat it as spatially distinct to... itself. Since it is not so distinct, let us suppose, Einstein's true and full concept of discontinuity is wholly inapplicable to it. But not so Planck's. True enough, there are full discontinuities and other kinds. The trick is to know which is which.

The former, resting on *variable* distances, is expendable and can be diminished at will. (See my photon-emitting device.) The latter can do neither. Its levels are fixed and immutable. And, what is worse, in-between 'discontinuities' in space, there is always *something;* namely, space. And hence existence. But in-between discontinuous levels there is... what? Space certainly isn't, and further smaller levels aren't either. So I would say it is the Void, had I meant to spook Prof. Alper.

Photon 'discontinuity' is harmless (classical!), because it can conceptually reach no further, than a number of things being separated by the mere space that stretches between them. But energy level discontinuity is hardly as harmless. For in-between such levels energy itself can no longer be even *conserved*. (Though it can, overall.) So Prof. Alper would much rather get the best of both worlds. Have true and full discontinuity together with the homey security of a merely spatial kind. And Einstein certainly offers him *abundant* assistance in this direction:

> Phenomena connected with the emission of light are more readily understood if one assumes that the energy of light is *discontinuously* distributed in *space*. In accordance with the assumption to be considered here, the energy of a light ray is not *continuously* distributed over an increasing *space* but consists of a finite number of energy quanta which are localized at *points in space*. (Einstein 1965 (1905), p. 368)

Need more? (Then see op. cit., p. 373) One light quantum, or photon, is over here, one other is over there, so they are truly and fully discontinuous, exactly like miners or cars in the street are. Or exactly like I and my aunt are, even if nobody has so far come to realize, how fully discontinuous the two of us really were.

What a 'discontinuity' indeed! The space between any two such photons, as my photon-device argument shows, can obviously be traversed *continuously*. For their distinctness, unlike quantum distinctness, is merely a spatial distinctness and hence such as sets no obstacles, or limitations, on how to cross it. So it can, in principle, be crossed by going through all the (infinite) points separating them, that is to say, continuously. What is more, for that exact same reason, there are no limits to how *small* the space that separates two photons, can be taken —my photon-device argument once again. We can view them drawn as close to one another or as far apart as we wish, eventually eliminating such "true and full" discontinuity altogether. Could any of all that happen to the other kind?

We know it cannot. For when de Broglie reobtained Bohr's quantum conditions as higher harmonics of the fundamental orbit, directly applying Planck's logic of standing waves to it, thus identifying each discrete electron orbit with a certain eigen-frequency, *the orbital distances thus resulting could no longer be crossed continuously*. Nor be made any *smaller* than the mathematics ever allowed. For if $E \approx \nu$, there can *be* no energy in-between quantized orbits, viewed as higher harmonics, any more than there can be any frequency between them. Whereupon, assigning an intermediate energy to the electron now, lying at an intermediate *post*, would be to simply contradict $E \approx \nu$; that is to say, QM. Consequently, the space separating any two successive such orbits could not be crossed at all, strictly speaking, nor be taken as arbitrarily reducible. The electron could only be *at* the orbits (viz. the eigen states) and only at *these* orbits. Not between them. All other, putative orbits, are not points that the electron can step on. Only the *next allowed* orbit is.

And this, I submit, is quantum nextness. Which *is* a way out of Zeno's paradoxes. The Runner's absolutely first and irreducible stepping stone. The Runner's first step need no longer be indefinitely postponed. There are no longer smaller distances, whose crossing must precede this crossing, which now is *the* crossing. He now has somewhere to go. It goes without saying, by contrast, that Einstein's full discontinuity is impotent to even touch Zeno's insights. His photons are separated by a crossable (= continuous) distance. There are still infinitely many possible places for the Runner to step on, all of them more and more proximal still, just before leaving the photon he stands on. So he cannot take the step. For there *is* no first step *to* take —yet. (See Barnes 1982, p. 262.) But real quantum states (orbits) are not separated by a similar, continuously crossable distance. The Runner, standing on any such orbit, can now take a quantum *leap*, bringing him to an immediate next orbit. There *is* now a first step to take, there now does exist a place, indeed a *unique* place, to step on: Any next orbit. No more infinite postponment.

How he gets to that place, on the other hand, when being nowhere in-between, I have no answer for, nor does QM. Or, rather, not *saying* how, is what QM is (Gavroglou 1989, p. 551). But it is not the nature of motion I tried to resolve by taking recourse to QM. Only its possibility. Understanding the former in the face of the Paradoxes,[4] *or* in the face of discontinuity, is a far far heavier task than that.

The fact that the Planck-Bohr type of quantization offers a definite way out of Zeno's paradoxes, while Einstein's *quasi*-quantization offers none, is evidence enough of their discrepancy. The present analysis, partly conducted in independence of the

[4] Try this: "A moving thing moves either in the place where it is [= so it won't be where it is], or in the place where it is not [= so it will be where it's not]. Hence, nothing moves" (Vlastos 1995, p. 206, brackets mine).

paradoxes, though guided by their intrinsic drive, corroborates this conclusion. Either there are two authentic, yet profoundly discrepant types of quantization, sharing but a faint family resemblance, as a weaker formulation of my thesis, or there is but one such, as a stronger formulation of it. This paper is quite comfortable with either. But, be that as it may, there is still one and only one type of *discontinuity*. And that is certainly not Einstein's, the sole Prof. Alper seems to have a grasp of. Were we to limit ourselves to it, we would have failed far worse, than just answering Zeno.

4. *Atoms of bodies and atoms of events: The horror vacui*

The case for the existence of atoms, a direct translation of Zeno's Paradox of Extension, has as follows: Suppose that a finite body has an infinite number of parts. Now these parts are either of a certain size or else of no size. But in the former case the body would be infinite in size and in the latter of zero size, both of which are contradictory to the assumption. Therefore a finite material body can only have a finite number of parts, that is to say atoms. Infinite divisibility of bodies and infinite divisibility of processes both employ the same type of principles and the same type of reductio ad absurdum on their force, up to a point. A finite step consisting of infinitely many smaller steps is as impossible as a finite body consisting of infinitely many smaller bodies. But this is where all affinity ends.

If the components of a body are divided and pulled apart, existence will not be negated. They will now be just so many separate bodies occupying separate parts of space, like I and my aunt also do —and *always* did. Given the principle that no two material bodies can occupy the same region of space, their separation, far from being a mystery, is a triviality. It leaves everything exactly as it was before, only *adding* to the number of separate bodies occupying separate regions of space. But when it comes to the division and pullling apart the components of an event, the latter developing in *time*, things take a dramatic turn. Reality itself will be interrupted:

> According to the quantum postulate, every change in the energy of the atom is the result of a complete transition between two of its stationary states. It was evident that no explanation of the indivisibility of the transition processes could be given within the frame of the *deterministic* description. (Bohr 1958, p. 87)

Discontinuities in time, therefore, as *opposed* to those of space (if there be any such), entail indeterminism in the strictest logical sense. An indivisible transition from stationary state 4 to stationary state 3 cannot be *resolved* into smaller units, e.g. 3,9999, 3.9998 &c, for the quantum that separates them is indivisible. But it cannot be performed in zero time either. Hence, during the transition *no energy can be ascribed to the electron* in consistency with the quantum postulate (Hooker 1971, p. 263). Such questions can never arise regarding the division of bodies. To therefore assume that an event developing by atomic steps in time can ever produce an analogy with a material body composed of atoms in space, betrays ignorance of the problem. Bohr was anything but ignorant of it, though some others certainly seem to be.

If the transition is indivisible, then for the entirety of its duration, the electron will be in *no state*. Is it similarly true to say that, were a body subdivided in just so many parts, it or they would likewise be in no state? The notion of indeterminacy cannot even apply to division in space. Hence, there is hardly a contrast greater than the one between discontinuities in time (events) and discontinuities in space (bodies). Real discontinuity, if one only knew where to look, cannot even be stated in terms of atomicity of bodies. It is the property of a change, not one of space.

And energy in physics is not a vector. Hence, determinations of space cannot have any bearing on it. Now would Prof. Alper acknowledge that energy, however poorly it may be quantized in the Planck-Bohr model, if compared to his "full Einsteinian discontinuity", is still quantized even so? I trust he would. Then, if energy is at all quantized, *and* unrelated to any spatial attribute, Einstein's true and full discontinuity, multiply described by himself as one that is "distributed in *space"* and "consisting of energy quanta at points in *space"*, in all, therefore, a thoroughly space-dependent discontinuity, is simply foreign to energy quantization properly so-called. The latter can only be cast in terms of level (or degree) discreteness, and cannot even relate to space in the connection required. Prof. Alper has therefore done well to contrast Planck's quantum to that of Einstein's, if only he knew, what he was in fact contrasting it to.

The sole point of contact between Einstein's light quanta, on the one hand, and the atomic structure of energy, on the other, as resulting from its quantization, is at best that of reason and consequence. It is *because* two successive and quantized energy levels yield as their difference a single limiting value, one indivisible quantum, that this quantum is released as a surplus of energy resulting from a quantized transition.

Now I am far from implying that this meeting point between Planck's and Einstein's quantization is a trivial matter. It is simply to say that, despite the conditioned reflexes of a considerable number of physicists, the "light quantum" notion as an istance or, worse, a model of discontinuity is mistaken. This is not what quantum discontinuity is. So we have to get our vocabulary correct. The indivisibility of an emitted photon is actually a *consequence* of quantization. Hence, not the prototype for it. It is derivative. Not primitive. And one cannot coherently afford to call a consequence of quantization *the* quantization. Perhaps, not even *a* quantization. Saying this would be no less absurd, or confused, than declaring the smoke to be the fire. Smoke may be a consequence of fire. But it is not the fire. Let alone *the* fire.

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