Mathematical Hygiene

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Abstract. This paper aims to bring together the study of normative judgments in mathematics as studied by the philosophy of mathematics and verbal hygiene as studied by sociolinguistics. Verbal hygiene (Cameron 1995) refers to the set of normative ideas that language users have about which linguistic practices should be preferred, and the ways in which they go about encouraging or forcing others to adopt their preference. We introduce the notion of mathematical hygiene, which we define in a parallel way as the set of normative discourses regulating mathematical practices and the ways in which mathematicians promote those practices. To clarify our proposal, we present two case studies from 17th century France. First, we exemplify a case of mathematical hygiene proper: Descartes' algebraic geometry and Newton's subsequent criticism of it, a case of *(im)purity* in mathematics. Then, we compare Descartes' and Newton's mathematical hygiene with verbal hygiene from this period, as exemplified by the work of the grammarian Claude Favre de Vaugelas (Ayres-Bennett 1987). We argue that these early modern normative discourses on mathematics and language respectively can be seen as emanating from a common socio-political program: the development of a new bourgeois intellectual class. We conclude that the study of mathematical hygiene has the potential to yield new understandings of the social aspects of mathematical practice, and that similarities between mathematical and verbal hygiene at certain time periods, such as 17th century France, open up a new area of inquiry at the borders of linguistics and the philosophy of mathematics.

1. Introduction

The goal of this paper is to bring together research from two apriori unrelated academic domains: the study of *normativity in mathematical practice* in the philosophy of mathematics and the study of *verbal hygiene* in sociolinguistics.

Verbal hygiene is, broadly speaking, "the urge to meddle in matters of language" (Cameron, 1995, vii). More specifically, this term refers to the set of normative ideas that language users have about which linguistic practices should be preferred, and the ways in which they go about encouraging or forcing others to adopt their preference.

The goal of this programmatic paper is to introduce the notion of *mathematical hygiene*, which we define parallely as the set of normative discourses regulating mathematical practices and the ways in which mathematicians promote those practices. We will also raise the question of the relation between mathematical and verbal hygienes, and tentatively suggest that the two can be linked. In order to support our proposals, we will present a case study of mathematical and verbal hygiene in 17th century France. First, we will exemplify a case of mathematical

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hygiene proper: Descartes' algebraic geometry and Newton's subsequent criticism of it, and we will show that both Descartes' and Newton's attitudes can be seen as shaped by their social context. Then, we will briefly compare Descartes' mathematical hygiene with verbal hygiene from this period, as exemplified by work of the grammarian Claude Favre de Vaugelas. We will argue that both Descartes' and Vaugelas' normative discourses on mathematics and language respectively can be seen as emanating from a common socio-political project: the development of a new bourgeois intellectual class. We will suggest that the way in which discourses about mathematics and about language contribute to this development of this political class is through the discursive construction of *personae* (identities or stereotypes). The importance of personae to understanding the social aspects of language use and interpretation is well-established (Agha, 2003; Eckert, 2008; D'Onofrio, 2018, 2019), as is the role that normative discourses play in the creation of these ideological constructs (Agha, 2003, 2006). To our knowledge, the role that such ideological constructs play in mathematical practice and in discourses about mathematics has yet to be explored.[1] We therefore believe that the study of mathematical hygiene has the potential to yield new understandings of the social aspects of mathematical practice, both currently and historically, and that similarities between mathematical and verbal hygiene at certain time periods, such as 17th century France, open up a new area of inquiry at the borders of linguistics and the philosophy of mathematics.

The paper is laid out as follows: in section 2, we review the notion of *verbal hygiene* and describe some of the findings of previous research in this area of linguistics. Based on this previous work, we identify a number of different kinds of normative discourses on linguistic practices in languages like English. Then, in section 3, we argue that we find the same classes of normative discourses on mathematical practice, albeit with certain differences that reflect the different subject matter. We argue that the coincidence between verbal hygiene discourses and discourses about one particular mathematical practice, the search for *(im)purity*, opens up the possibility that these two phenomena share a common source. In section 4 we present our case studies from 17th century France. We first exemplify some mathematical hygiene from that period, focusing particularly on Descartes and Newton. We then explore the similarities between mathematical and verbal hygiene in that period by comparing Descartes with the grammarian Vaugelas. Section 5 concludes and outlines some of the many open questions and directions for future research that our proposals have identified.

2. Verbal Hygiene

As observed by Cameron (1995) and Curzan (2014), verbal hygiene appears to be one of the universals of human linguistic practice : as soon as two ways of saying (roughly) the same thing become available to language users, some of those users will start formulating opinions about whether one grammatical variant is better than the other, and encouraging others to use what they consider to be the superior one. Research into verbal hygiene has shown that there exist a large range of normative discourses regulating linguistic practices, and that verbal hygiene practices vary across languages, time periods, and social contexts (Paffey, 2007; Cameron, 2013; Jones, 2013; Årman, 2021; Brown, 2022, among many others).

By far the best studied verbal hygiene practices are those associated with *prescriptivism*. Although linguists often adopt slightly different meanings of this term (see Joseph, 1987; Cameron, 1995; Trask, 1999; Millar, 1998; Curzan, 2014; Ayres-Bennett, 2020, among others), definitions of prescriptivism all turn around the idea that, when faced with two grammatical options that are

[1]Historians of science and mathematics have discussed how power struggles between scientific actors have contributed to the shaping of scientific and mathematical domains; see for instance Lorenat (2022). Our work differs in that we look specifically at the linguistic practices involved in discussions of values in mathematics.

used in a community, prescriptivists identify one of the two as superior and promote the use of the "better" variant over its alternative. Which grammatical variant is preferred, and the arguments provided to support this preference, can be varied; however, as the scholars mentioned above show, linguistic preferences and the reasoning behind them are not arbitrary; rather, they are almost always linked to the prescriptivist's politics. A very clear example of linguistic prescriptions being guided by political motivation is the *gender fair* (or *non-sexist*) language movement (see Sczesny et al. (2016) for a recent review). Non-sexist language attempts to reduce stereotyping and discrimination against women in language through, for example, proscribing the use of masculine expressions with general reference in favour of either gender neutral expressions (1-a) or expressions that include reference to women (1-b)[2].

(1) a. *The candidate should submit* ***their*** *CV* should be used instead of *The candidate should submit* ***his*** *CV.*
   b. *The candidate should submit* ***his or her*** *CV* should be used instead of *The candidate should submit* ***his*** *CV.*

The verbal hygiene represented by policies containing statements such as those in (1) make their political motivations perfectly explicit. However, in most cases of prescriptivism (and indeed verbal hygiene in general), how the normative statements about language link up with the authors' political motivations is often more subtle.

Milroy and Milroy (1999) study a variety of arguments put forward by verbal hygienists in the UK justifying the superiority of the use of standard pronunciations, words or syntactic constructions. They argue that these "complaints"[3] play a role in "legitimiz[ing] the norms of formal registers of Standard English rather than the norms of everyday spoken English" (Milroy and Milroy, 1999, 30), and, in doing so, legitimizing discrimination against English speakers who do not master or regularly employ Standard English. According to these authors, the relation between standard English prescriptivism and its political consequence is shown in (2).

(2) Relation between language ideology and discrimination (Milroy and Milroy, 1999, 35):
   (1) That there is one, and only one, correct way of speaking and/or writing the English language.
   (2) That deviation from this norm are illiteracies, or barbarisms, and that non-standard forms are irregular and perversely deviant.
   (3) That people ought to use the standard language and that it is quite right to discriminate against non-standard users, as such usage is a sign of stupidity, ignorance, perversity, moral degeneracy, etc.

Milroy & Milroy distinguish between two classes of normative discourses found in UK English. The first class, which they call Type 1 complaints, are those that are "implicitly legalistic and which are concerned with correctness, attack 'mis-use' of specific parts of the phonology, grammar, vocabulary of English (and in the case of written English 'errors' of spelling, punctuation, etc.)" (Milroy and Milroy, 1999, 30-31). As an example of a Type 1 argument, the Milroys cite Jonathan Swift's comment in *A Proposal for Correcting, Improving and Ascertaining the English*

[2]For a concrete example of a non-sexist prescriptive language policy, consider the one of the *Linguistic Society of America*: `https://www.linguisticsociety.org/resource/guidelines-inclusive-language`.

[3]The Milroys describe large portions of English verbal hygiene as being part of a complaint *tradition*, that has characterized meta-linguistic discourse on English throughout the ages, see Schneider (2003).

Tongue (1712: 2)[4]: "our Language is extremely imperfect; that its daily Improvements are by no means in proportion to its daily Corruptions; and the Pretenders to polish and refine it, have chiefly multiplied Abuses and Absurdities; and, that in many Instances, it offends against every Part of Grammar". Showing that Swift's view is far from isolated, they also provide examples from the 20th century that echo Swift's sentiments almost 250 years earlier.

Milroy & Milroy oppose Type 1 complaints with Type 2 complaints, which are "moralistic". According to these authors, "Type 2 complaints do not devote themselves to stigmatizing specific errors in grammar, phonology, and so on. They accept the fact of standardisation in the written channel, and they are concerned with clarity, effectiveness, morality and honesty in the public use of the standard language". The Milroys exemplify their Type 2 category using the complaints of George Orwell. According to these authors, Orwell is "in favour of 'demotic' speech. He mentions the stilted bookish language [...] and upper class accents of the news bulletins, and comments that this language is totally ineffective in communicating with the ordinary public" (Milroy and Milroy, 1999, 36). In other words, Orwell values *clarity* and *effectiveness*, and finds these properties to be lacking in the English that is spoken and written in formal contexts during his time. In his 1946 essay *Politics and the English Language*, Orwell studies a selection of passages of political writing and discusses how they all exemplify properties that he finds distasteful. He says[5],

> Each of these passages has faults of its own, but quite apart from avoidable ugliness, two qualities are common to all of them. The first is staleness of imagery; the other is lack of precision. The writer either has a meaning and cannot express it, or he inadvertently says something else, or he is almost indifferent as to whether his words mean anything or not. This mixture of vagueness and sheer incompetence is the most marked characteristic of modern English prose, and especially of any kind of political writing. As soon as certain topics are raised, the concrete melts into the abstract and no one seems able to think of turns of speech that are not hackneyed

From this passage, we can see that Orwell thinks that the language used by speakers should be more than just grammatically correct: according to him, it should also be *beautiful*, *vibrant*, *precise*, *competent*, and *concrete*. In addition to these evaluative properties, Orwell is perhaps most famous for his concern that language should be *honest*: this is a point that he illustrates through *Newspeak* in his novel *Nineteen Eighty Four* (1949); and, in *Politics and the English Language*, he comes out strongly against "pretentious diction", which he believes can be used to trick readers into giving more scientific authority to a statement than is warranted, or for subtly dignifying and even glorifying distasteful aspects of politics and war. He says,

> Words like phenomenon, element, individual (as noun), objective, categorical, effective, virtual, basis, primary, promote, constitute, exhibit, exploit, utilize, eliminate, liquidate, are used to dress up simple statements and give an air of scientific impartiality to biased judgments. Adjectives like epoch-making, epic, historic, unforgettable, triumphant, age-old, inevitable, inexorable, veritable, are used to dignify the sordid processes of international politics, while writing that aims at glorifying war usually takes on an archaic color, its characteristic words

[4]Jack Lynch's online edition, consultable here: `https://jacklynch.net/Texts/proposal.html`.

[5]*Politics and the English Language* is available online through the Orwell Foundation here: `https://www.orwellfoundation.com/the-orwell-foundation/orwell/essays-and-other-works/politics-and-the-english-language/`.

> being: realm, throne, chariot, mailed fist, trident, sword, shield, buckler, banner, jackboot, clarion.

Orwell's concern for honesty in language leads directly into his concern for linguistic *purity.* The passage above continues:

> Foreign words and expressions such as cul de sac, ancien regime, deus ex machina, mutatis mutandis, status quo, gleichschaltung, weltanschauung, are used to give an air of culture and elegance. Except for the useful abbreviations i.e., e.g., and etc., there is no real need for any of the hundreds of foreign phrases now current in English. Bad writers, and especially scientific, political and sociological writers, are nearly always haunted by the notion that Latin or Greek words are grander than Saxon ones, and unnecessary words like expedite, ameliorate, predict, extraneous, deracinated, clandestine, subaqueous and hundreds of others constantly gain ground from their Anglo-Saxon opposite numbers.

As the Milroys note (p. 37), like many English writers, grammarians and pundits in the late 19th and 20th centuries, Orwell advocated for purging words of foreign origin in favour of those with Anglo-Saxon roots. Based on the passage above, Orwell views purity to be in the service of honesty, and, indeed, his ideas in this area were subsequently further developed in the *Plain English* movement. However, linguistic purism, which often goes hand in hand with prescriptivism of a "standard" language (Brousseau, 2011), is often also associated with nation building and ethnic nationalism (Thomas, 1991; Deumert and Vandenbussche, 2003; Milroy, 2005; Spitzmüller, 2007; Hansen et al., 2018, among many others).

The Milroys identify two types of arguments justifying the prescription of a standard English grammatical variant over a non-standard one. However, we argue that at least one kind of complaint that they group under Type 2 deserves to be further distinguished: Orwell's appeal to the effectiveness of communication. The idea that one form of language should be preferred over another because it facilitates effective communication is an old one; as Cameron (1995) observes, it is "a theme harped on by verbal hygienists across the cultural and political spectrum." According to Cameron,

> It is the cliché trotted out by the Queen's English Society whenever its members meet to extol the virtues of standard English: if we didn't abide by a single standard communication would break down. It is the major argument of the racist and xenophobic US English language movement, which campaigns to outlaw languages other than English in public domains: we need a common language or the nation will fragment. Conversely, it is the theme of Orwell's 'Politics and the English language', according to which totalitarianism succeeds by preventing us from communicating our thoughts clearly, so that in time we cease to have any clear thoughts at all. Cameron (1995), p. 23.

Since the preference for language that enhances communication is not purely moralistic, we will put it into a third category of arguments: *functional* ones.

To sum up: when it comes to prescriptivism, we can distinguish three classes of arguments that have been put forward to justify prescribing a standard grammatical variant over a non-standard one: *legalistic* arguments, which reference correctness, *moralistic* arguments, which reference evaluative properties like beauty, clarity, and honesty, and *functional* arguments, which reference communication. At first glance, the first and third classes of arguments appear qualitatively different from the second: more objective, less dependant on social context. However, if we look at them a bit more closely, we see that verbal hygienists often adopt an intuitive personal understanding of what counts as "correct" language, one that can be very different from one verbal hygienist to another (see Poplack et al., 2015). The same often holds for *efficient communication.* This notion is seldomly defined in any detail, and in the rare case when language commentators do outline their model of communication, it often looks very different from the models adopted by professional linguists and philosophers. As such, it is likely that the functional "efficient communication" arguments are really just as politically motivated as the moralistic ones. As Cameron says,

> The speciousness of the arguments just put forward to justify particular verbal hygiene practices might lead us to suspect that the fetish of communication is really just a cover for some other obsession. We are back once again to the way verbal hygiene is not just about ordering language itself, but also exploits the powerful symbolism in which language stands for other kinds of order. – moral, social and political. Cameron (1995), p. 25.

She continues,

> The social analogue of a 'breakdown in communication' is a breakdown in cultural and political consensus, the eruption into public discourse of irreconcilable differences and incommensurable values. Thus the anxiety that gets expressed as 'if we don't obey the rules we won't be able to communicate' might equally be defined as an anxiety about moral relativism or social fragmentation. Cameron (1995), p. 25.

Thus, even what appears to be the most straightforward, politically neutral verbal hygiene arguments can often be linked to underlying concerns about how the world is or how it may become in the near future.

Prescriptivist discourses are not the only kind of verbal hygiene that we find. In fact, as a technical term used in linguistics, *prescriptivism* is most often discussed alongside its opposing perspective: *descriptivism*. *Descriptivism* (or *anti-prescriptivism*) is the view that all grammatical variants are equal, and, as such, there should be no preferences between them. Like prescriptivism, descriptivism is a view most often articulated with (non)standard language in mind: according to descriptivists, the variant identified as the *standard* one is not inherently better than the *non-standard* one, and it is nonsensical, and even problematic, to encourage the use of one over the other. Descriptivist/anti-prescriptivist discourse is found in many kinds of language commentary, but it is especially well developed in the practice of professional linguistics. Linguists have been very critical of prescriptivism: as Cameron says,

> The typical attitude to it among linguists runs the gamut from despair at prescribers' ignorance to outrage at their bigotry, and it is aptly if apocalyptically summed up in the title of a 1950 book by Robert Hall *Leave Your Language Alone*. Cameron (1995), p. 3.

As the quotation above suggests, one of the reasons that linguists adopt the descriptivist perspective is that they are opposed to the elitist, conservative political project described above that prescriptivism about the standard language supports (see for example Labov, 1970). In other words, there are political motivations behind the anti-prescriptivist discourses as well. Cameron also argues that professional linguists' fervent adoption of a descriptivist stance has served another political purpose: the prescriptive/descriptive "binarism sets the parameters of linguistics as a discipline. The very first thing that any student of linguistics learns is that 'linguistics is descriptive not prescriptive' – concerned, in the way of all science, with objective facts and not subjective value judgments" (Cameron, 1995, 5). This ideology of linguistics as a science can also be seen in how descriptivist linguists characterize the existence of multiple, equally good, grammatical variants as *natural* (Cameron, 1995, 4-5), and characterize those who attempt to interfere with the natural linguistic order (i.e. the prescriptivists) as *irrational*. For example, Cameron cites James Milroy (Milroy, 1992, 32) as saying that prescriptivists have attitudes that are "strongly expressed and highly resistant to rational examination". Thus, "the anti-prescriptivist 'leave your language alone' tradition within linguistics [. . .] mirrors the very same value-laden attitudes it seems to be criticizing" (Cameron, 1995, 3)[6].

In summary, in this section we reviewed two common types of verbal hygiene: *prescriptivism* and *anti-prescriptivism/descriptivism*. Prescriptivists view one form of language (usually the "standard" language) as superior and promote its use. Partially following the Milroys, we identified three types of arguments that have been developed to favour the standard, and we argued that the shape of these arguments can be traced to the political beliefs of the hygienists who make them. Following Cameron, we argued that anti-prescriptivists also make moralistic arguments to support their position, and that these arguments also appear to have political motivations. With these patterns from the linguistic domain in mind, we now turn to the mathematical domain.

3. Mathematical Hygiene

In mathematics too there are many different ways of doing things, and as with linguistic practices, this variety gives rise to diverse opinions about the *right* way to do things. A first such example has been observed by Tappenden, who notes that "mathematicians often set finding the 'right' / 'proper' / 'correct'/'natural' definition as a research objective, and success—finding 'the proper' definition—can be counted as a significant advance in knowledge" (Tappenden (2008), p. 256). Mathematicians formulate opinions about what definition is right, for instance in terms of which definitions are more "fruitful" (Tappenden (2012)), and express their opinions in their research publications and in more informal, though still public, venues for less-specialized audiences, such as MathOverflow (`https://mathoverflow.net/`), a website where users can ask mathematical questions, pose responses to such questions, and discuss the questions and responses. Briefly, the search for definitions in mathematics gives rise to rich normative discourses.

Another central activity in mathematical practice that admits considerable variety, and consequently rich normative discourses as well, is the solution of problems by means of proofs. This activity aims at the *relief of ignorance* concerning particular mathematical questions; for instance, whether there are infinitely many prime numbers. Each proof is typically just one of many that resolves the problem at which it is aimed (Dawson (2015), Ording (2019)). Among this variety of proofs, mathematicians have opinions about which ones are better than others. A minimal criterion of success for a proof is that it *justifies belief* in the solution's answer. To ensure this, mathematicians constrain their investigations to rigorous, logically sound proofs (Hamami (2014)). However, mathematicians frequently have aims besides justification in their solving activity. For instance, some mathematicians seek proofs that *explain* the theorems whose truth they justify (Steiner (1978), Kitcher (1989), Hafner and Mancosu (2005)). Others seek proofs that *construct* the objects or properties under investigation (Bishop (1967), Detlefsen (2003), Heinzmann and Atten (2022)). Yet others seek proofs that are *efficient* for humans to produce, follow and comprehend (Avigad (2003), Arana (2017)). These "ideals of proof", along with many others (Arana (2022), Morris (2021)), are among the reasons mathematicians offer for preferring one proof over another.

[6]It has also been argued that standard language prescriptivism has surreptitiously played a significant role in actual descriptivist practice (see Armstrong and Mackenzie, 2013).

In this paper we will exemplify our notion of *mathematical hygiene* using discourses for one such ideal of proof: the debates on (im)purity (Detlefsen and Arana (2011)). Roughly, a proof of a theorem is pure if it draws only on considerations "close" or "intrinsic" to that theorem. Mathematicians employ a variety of terms to identify pure proofs, though; alternatively, a pure proof is one that avoids what is "extrinsic", "extraneous", "distant", "remote", "alien", or "foreign" to the problem or theorem under investigation. Purity has been valued by many mathematicians since antiquity, and this value resonates still today in number theory, classical geometry, algebraic geometry, modern algebra, the foundations of mathematics, to name but a few cases. It has also been opposed by many others, agitating instead for impurity. Our goal here is to introduce the normative discourses about purity as a case of mathematical hygiene, in order to raise parallels between these discourses and those of verbal hygiene, in order to set up in a programmatic way an investigation into whether these two phenomena share a common source.

Before we do so, we believe it is important to make a distinction between verbal hygiene in a mathematical context and mathematical hygiene proper. The study of verbal hygiene (in mathematics) would study normative discourses on mathematical language, such as aspects of notation or the language used in mathematical proofs. On the other hand, what we are calling *mathematical hygiene* proper is not (obviously) about language; rather it is about the practice of mathematics. This being said, we will see later in this paper examples of cases where the two seem to bleed together, which, we believe, argues in favor of a general framework that unites them.

However, our main argument that a general framework which unites mathematical and verbal hygienes is appropriate is that, as we will now show, we find both prescriptivism (supported by legalistic, moralistic and functional arguments) and descriptivism in mathematical hygiene. As mentioned in the introduction, we will illustrate this point with arguments for and against purity in mathematics.

A first example of prescriptivism involving both legalistic and moralistic arguments comes from Newton. In the Lucasian Lectures on Algebra, he argues for purity based on "correctness" and "simplicity" and "elegance". He says:

> Equations are expressions belonging to arithmetical computation and in geometry properly have no place except in so far as certain truly geometrical quantities (lines, surfaces and solids, that is, and their ratios) are stated to be equal to others. Multiplications, divisions and computations of that sort have recently been introduced into geometry, but the step is ill-considered and contrary to the original intentions of this science: for anyone who examines the constructions of problems by the straight line and circle devised by the first geometers will readily perceive that geometry was contrived as a means of escaping the tediousness of calculation by the ready drawing of lines. Consequently these two sciences [arithmetical computation and geometry] ought not to be confused. The Ancients so assiduously distinguished them one from the other that they never introduced arithmetical terms into geometry; while recent people, by confusing both, have lost the simplicity in which all elegance of geometry consists. Cf. Newton (1972), p. 429.

As shown in this quotation, Newton makes a legalistic argument when he considers the use of algebra in geometry to "contrary to the original intentions of this science", and he makes moralistic arguments when he bemoans the loss of "simplicity" and "elegance" that are implied by, according to him, incorporating arithmetical computations into geometry.

Another example of a legalistic argument for mathematical hygiene comes from Bolzano. Bolzano famously criticized geometrical proofs of the intermediate value theorem—the theorem that for a real function f continuous on a closed bounded interval $[a, b]$, for every μ such that $f(a) < \mu < f(b)$, there is a ν such that $a < \nu < b$, with $f(\nu) = \mu$. He objected, in particular, to the use of the following proposition to prove this theorem: *Every continuous line of simple curvature of which some ordinate values are positive and some negative must intersect the x-axis at a point between the positive and negative ordinate values.* Bolzano described this as a "truth borrowed from geometry", and he regarded the borrowing as illicit. As he put it:

> It is an intolerable offense against *correct method* to derive truths of *pure* (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely applied (or special) part, namely, *geometry.* Have we not felt and recognized for a long time the incongruity of such μετάβασις εἰς ἄλλο γένος [kind-crossing]? Have we not already avoided this whenever possible in hundreds of other cases, and regarded this avoidance as a merit? Cf. Bolzano (1999), p. 228.

For Bolzano, an impure proof of the intermediate value theorem, by means of geometrical considerations, would be incorrect and incongruous.

We can also give an additional example of moralistic arguments for mathematical hygiene. The Briançon-Skoda Theorem[7] was proved in 1974 by Skoda using methods from complex geometry, but the theorem is usually judged to belong to commutative algebra. Algebraists then wondered if it could be proved by purely algebraic means. The situation has been described by two actors in the field in the following way:

> The proof given by Briançon and Skoda of this completely algebraic statement is based on a quite transcendental deep result of Skoda.... The absence of an algebraic proof has been for algebraists something of a scandal—perhaps even an insult—and certainly a challenge. Cf. Lipman and Teissier (1981), p. 97.

The authors of this article then give such an algebraic proof, thereby achieving the hygiene they sought. Their argument for doing so is moralistic: not having a purely algebraic proof is scandalous! That the article was published in an excellent but absolutely core mathematics journal, the *Michigan Mathematical Journal*, reveals how mathematical hygiene is an essential part of mathematical practice.

Although legalistic and moralistic arguments are common in the purity debates, they are often supported by more functional arguments[8]. As we described in section 2, *communication* is the most common basis for functional arguments in verbal hygiene. In mathematical hygiene, *epistemic value* appears to most often play this role. Why should we prefer pure (or impure) proofs? Because pure (or impure) proofs help us understand the theorem better. This view is clearly articulated by the mathematician Jean Dieudonné:

> An aspect of modern mathematics which is in a way complementary to its unifying tendencies... concerns its capacity for sorting out features which have become unduly entangled... It may well be that some will find this insistence on "purity" of the various lines of reasoning rather superfluous and pedantic; for my part, I

[7]The Briançon-Skoda Theorem says: let R be either the formal or convergent power series ring in d variables and let I be an ideal of R. Then $\overline{I^d} \subseteq I$, where $\bar{I}$ is the *integral closure* of an ideal I.

[8]See also Novaes (2019) for discussions of the relationship between (what we've been calling) moralistic and functional arguments in mathematics.

> feel that one must always try to understand what one is doing as well as one can and that it is good discipline for the mind to seek not only economy of means in working procedures but also to adapt hypotheses as closely to conclusions as is possible. Cf. Dieudonné (1969), p. 11.

In contrast, a bit earlier, Bourbaki view impurity as what contributes to the "profound intelligibility of mathematics" (cf. Bourbaki (1948), p. 37).[9]

Although the mathematical domain is rife with what we might call "mathematical prescriptivism", it is important to highlight that the descriptivist view is also well represented. For example, consider the following quotation by the number theorist Melvyn Nathanson:

> The theorems in this book are simple statements about integers, but the standard proofs require contour integration, modular functions, estimates of exponential sums, and other tools of complex analysis. This is not unfair. In mathematics, when we want to prove a theorem, we may use any method. The rule is "no holds barred." It is OK to use complex variables, algebraic geometry, cohomology theory, and the kitchen sink to obtain a proof. But once a theorem is proved, once we know that it is true, particularly if it is a simply stated and easily understood fact about the natural numbers, then we may want to find another proof, one that uses only "elementary arguments" from number theory. Elementary proofs are not better than other proofs, nor are they necessarily easy. Indeed, they are often technically difficult, but they do satisfy the aesthetic boundary condition that they use only arithmetic arguments. Cf. Nathanson (2000), pp. viii-ix.

Nathanson's view here is descriptivist in that he holds that pure (which he calls *elementary*) proofs are not better than impure proofs, although he admits that one may have preferences for aesthetic reasons.

In summary, in this short overview, we have argued that normative discourses on mathematical practice can be broken down into the same kinds of analytical categories that have been found to characterize normative discourses on linguistic practice, at least when it comes to the purity of methods debate. In the next section, we present a particular case where the parallels between mathematical hygiene and verbal hygiene appear to be even deeper: language and mathematics in 17th century France.

4. Case study: 17th century France

We start by outlining a case of mathematical hygiene, and then turn to a comparison between mathematical and verbal hygienes in 17th century France.

4.1. **Descartes vs Newton (mathematical hygiene).** Our case study will focus on the contrast between Descartes and Newton on the proper approach to geometrical problem solving. Though Newton's mathematical activity begins after the death of Descartes, Newton mastered Descartes' geometry as a student and steadily positioned himself against Descartes in mathematics and beyond.

Ancient geometrical practice consisted of two steps: firstly, the analysis of the problem, in which one takes the problem to have been solved and works backwards to find the first principles (axioms, definitions, common notions) on which that solution would rest; and secondly, its synthesis, in which one demonstrates the solution from these first principles in the style familiar

[9] The fact that Dieudonné was a member of the Bourbaki collective underlines how complex the politics of mathematical hygiene can be.

from Euclid. This bipartite division of geometrical problem solving thus involves a process of discovery followed by a process of justification.

In *La géométrie*[10], Descartes proposed to keep this bipartite division of geometrical problem solving, but he reversed the traditional view that synthetic, Euclidean-style proof gave the best kind of knowledge, emphasizing instead the epistemic importance of analysis. This reversal was powered by his new conceptualization of synthesis and analysis. Cartesian analysis consists in the reduction of a geometrical problem to an algebraic equation, where the equations represent the curves of the problem and their configuration. These equations could then be solved and their roots found by algebraic computations. Cartesian synthesis then consists in the construction of those roots by geometrical means such as ruler and compass. Such constructions were to be carried out according to rules that followed the traditional strictures of Euclidean practice, so that the result was a Euclidean-style demonstration. The analysis, by contrast, was algebraic and computational.

As indicated in the quotation from Newton in the previous section (p. 8), the application of algebra to geometry can be understood as an instance of *impurity.*[11]The computational and symbolic apparatus of algebra and the deductive and visual apparatus of geometry occupy distinct branches of mathematics according to a traditional view that even today only mathematicians would contest. Cases for and against the value of Cartesian geometry are thus instances of mathematical hygiene. In what follows, we will analyze Descartes' case for the value of his new geometry, and Newton's case against its value in favor of geometry in the mode of the ancients, in terms of hygiene.

Descartes makes the case for the value of his new algebraic geometry in moralistic and functional terms. His moralistic case for mixing algebra and geometry rests on its *methodicalness.* He emphasized that while the ancients gave analyses of geometrical problems to find their solutions, they had no *method* of analysis. This explains, he says, why ancient geometrical texts are haphazardly organized, without any clear sense of hierarchy between the propositions as enumerated in a text such as Apollonius' *Conics.* As he put it in *La géométrie*:

> I have given these very simple [means] to show that it is possible to construct all the problems of ordinary geometry by doing no more than the little covered in the four figures that I have explained. This is one thing that I believe the ancient mathematicians did not observe, for otherwise they would not have put so much labor into writing so many books in which the very sequence of the propositions shows that they did not have the true method of finding all, but rather gathered together those propositions that they had happened to encounter. Descartes (1902), p. 376.

By contrast, he held that he had found such a method in his algebraic analysis. Since curves were represented by algebraic equations, whose polynomials have degrees, the simplicity of a curve can be measured quantitatively, by the natural numbers measuring their degrees. Problems and analyses of problems too could then be classified by the simplicity of the curves involved in their formulations. Descartes' method thus organized geometrical problem solving systematically, so that a geometer could start with the simplest problems and move onto to more complex ones,

[10]The original edition is Descartes (1637). We will follow convention in referring to its pagination in Descartes (1902). The edition studied by the young Newton is Descartes (1661), the Latin translation prepared by Frans van Schooten.

[11]This case has been frequently cited as a case of (im)purity in the literature: for instance, in Kreisel (1980), p. 185; Grosholz (1991), p. 40; Detlefsen (2008), p. 184; Detlefsen and Arana (2011), pp. 2–3; Giovannini (2016), p. 37; and Avigad (2022), pp. 112–113.

and at the same time start with the simplest analyses of these problems and only employ more complex ones when necessary. The geometer could then try to employ solution methods no more complex than the problem they had set out to solve.

This systematicity led Descartes to make a functional argument for the value of his geometry. He recognized that the systematicity of his new method would reduce the need for unpredictable genius in geometric problem solving, and that this was itself valuable for students of his geometry. After sketching the method in book 1 of *La géométrie*, he remarks:

> But I do not stop to explain this in more detail, because I would take away from you the pleasure of learning it yourself, and the utility of cultivating your mind by practicing on it, which is in my opinion the main thing that one can get out of this science. Also I do not notice there anything so difficult that those who are a little versed in ordinary geometry and in algebra, and who are careful with everything that is in this treatise, cannot find. Descartes (1902), p. 374.

Descartes thus emphasized the value of his method for training the mind, in terms of its function.[12]

Descartes gives another kind of functional argument for the value of his new geometry. He claims that his algebraic method permitted the systematic discovery of solutions to problems. Though an exaggeration, Descartes maintained that his method could solve *any* geometrical problem. In a 1637 letter to Mersenne, Descartes gave evidence for the power of his geometry as follows:

> In the *Optics* and the *Meteorology* I merely tried to show that my method is better than the usual one; in my *Geometry*, however, I claim to have demonstrated this. Right at the beginning I solve a problem which according to the testimony of Pappus none of the ancients managed to solve. (End December 1637, Descartes (1897), p. 478)

The acid test of this accomplishment, known as the Pappus problem, can be stated as follows for the particular case of four lines (see Figure 1). Let L_i be four given lines in the plane and let θ_i be four given angles with those lines, respectively. For any point P in the plane, let d_i be the length of the segment from P to L_i making the angle θ_i with L_i. Pappus's problem is then to find the locus of points P such that the ratio $d_1 d_2 : d_3 d_4$ is equal to a given constant. (The statement and figure are taken from Bos (2001), pp. 272-3.)

Though the ancients knew solutions to Pappus' problem for particular cases such as the one just stated for four lines, Descartes went further and solved the problem for an arbitrary number of lines. He framed the value of his new geometry in terms of its function of permitting him to surpass the ancients by finding a general solution to this classical problem.

In addition to these arguments for mathematical hygiene, Descartes also gives arguments for mathematical hygiene in terms of its use of language. As such, these further arguments blur the line between mathematical hygiene and verbal hygiene in a mathematical context.

A first example of this comes again from *La géométrie*, as he lauds the novelty of his method in the following terms:

> Here I beg you to note in passing that the hesitation of the ancients to use arithmetic terms in geometry, which could only proceed from not seeing their

[12]This functional argument for impurity anticipates the functional argument given instead for *purity* by Dieudonné as discussed in Section 3.

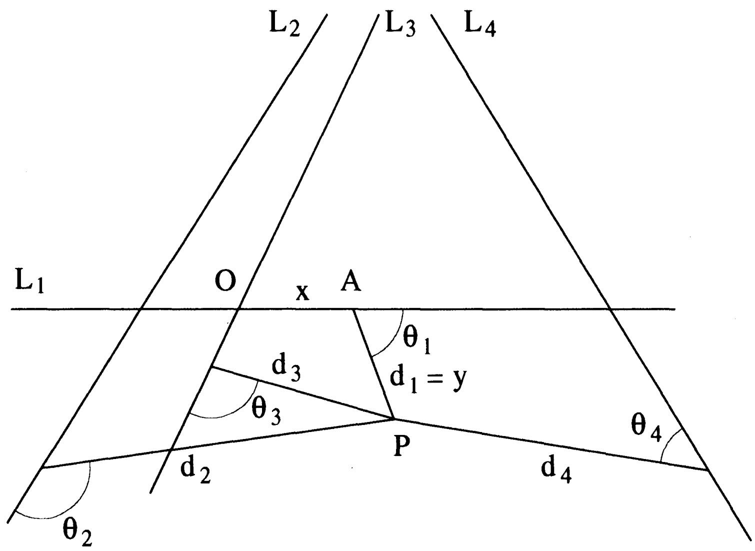

FIGURE 1. Pappus' problem in four lines

> relation clearly enough, caused a great deal of obscurity and difficulty in the way they explained themselves. Descartes (1902), p. 378.

Had the ancients known algebraic analysis, he writes, they would have used it, and their writing would have been accordingly clearer. This instance of mathematical hygiene is thus also an instance of verbal hygiene in mathematics as well: for the gain in using algebraic analysis is one of improved communication.

In a letter to Mersenne two years later, he explained that the link between algebra and geometry was even more profound:

> Those who know the conjunction that is between geometry and arithmetic, cannot doubt that all one can do by arithmetic can also be done by geometry; but to want to make this understood to whose who conceive of them as completely distinct sciences, this would be a waste of time [*oleum et operam perdere*]. 9 February 1639, Descartes (1898), p. 504.

Descartes seems to have thought that he had uncovered the unity of algebra and geometry, but that this unity eluded explanation in any terms that could be understood by a doubter. If Descartes is right, the application of algebra to geometry is in fact pure, for algebra and geometry belong ultimately to one and the same branch of mathematics. Our purpose here is not to adjudicate the purity of Cartesian geometry, though; it is simply to indicate the distance between Descartes' understanding of geometry and that of classicists like Newton.

Though Descartes preserved the bipartite structure of traditional geometrical practice between analysis and synthesis, in his published work he gave almost no attention to synthesis. He also did not give all the details of his analyses. In a letter to Mersenne in 1638, he explained why he chose to suppress such details:

> But the good thing is, touching on this question of Pappus, that I have only put into the construction and the entire demonstration, without including all the analysis, that which they imagine themselves that I have put in: of which they testify that they understand very little. But what deceives them is that I build it, as architects do buildings, only prescribing everything that needs to be done, and leaving the work of the hands to the carpenters and masons. They also do not know my demonstration, because I speak in it by a, b. This does not however make it in any way different from those of the ancients, except that by this way I can often put in a line what for them fills several pages, and for this reason it is incomparably clearer, easier and less prone to error. than theirs. For the analysis, I have omitted part of it, in order to leave some work for clever minds; for if I had given it to them, they would have boasted of having known it before for a long time, whereas now they cannot say anything about it until they discover what they do not know. 31 March 1638, Descartes (1898), p. 83.

Here again Descartes stresses the value of his new geometry for students, maintaining that he gives all the details of his analysis needed to indicate to the good student of his methods how to fill in the rest, but no more. By giving only the "blueprints" of his analyses, the rest is left for the mathematical equivalents of manual laborers. In addition, by writing in the symbolic notation of algebra, his analyses are shorter than they would be written in words, and this affords gains of clarity and security over classical methods. These are functional and moralistic arguments, blending mathematical hygiene with verbal hygiene in a mathematical context.

We turn now to Newton, who positioned himself from the early days of his mathematical career as an opposite of Descartes. In the 1670s, he responded to Descartes' work on the Pappus problem as follows:

> Descartes in regard to his accomplishment of this problem makes a great show as if he had achieved something so earnestly sought after by the Ancients and for whose sake he considers that Apollonius wrote his books on conics. With all respect to so great a man I should have believed that this topic remained not at all a mystery to the Ancients.... To reveal that this topic was no mystery to them, I shall attempt to restore their discovery by following in the steps of Pappus' problem. (From "Veterum Loca Solida Restituta", Newton (1971) pp. 274-277, translation from Guicciardini (2009), p. 93.)

While Descartes lauded the scope of his solution of the Pappus problem for any number of lines by his new algebraic analysis, Newton retorted that the ancients must also have had such a solution, even if no record of that could be found. He took upon himself the task of defending the honor of the ancients by seeking a purely geometrical solution to the Pappus problem. He emphasized that this would constitute a *restoration* of ancient knowledge rather than a new accomplishment, thus contrasting his conservative approach to Descartes' self-conscious modernity, even though the solution Newton found was also his own creation.

He published this solution to the Pappus problem in four lines in the *Principia* (1687), as Book I, Lemma 19, Corollary 2. After the solution, he commented:

> And thus there is exhibited in this corollary not an [analytical] computation but a geometrical synthesis, such as the ancients required, of the classical problem of four lines, which was begun by Euclid and carried on by Apollonius. Cf. Newton (1999), p. 484.

Though Descartes is not mentioned explicitly, the reference to a computational solution clearly indicates Newton's subject. Newton stresses again the restorative aspect of his solution, placing his work as part of the tradition of Euclid and Apollonius rather than the moderns.

Guicciardini has stressed how Newton's preference for the ancient mode of mathematical practice is consonant with his interests in ancient wisdom more generally. As he puts it:

> It should be noticed furthermore that, from the mid-1670s, Newton began looking at ancient texts not only for mathematical interests... It is striking that in the same years Newton began attributing to Jews, Egyptians and Pythagoreans a lost knowledge concerning alchemy, God and mathematics. It is plausible that in Newton's mind the restoration of the lost books of the ancient geometers of Alexandria was linked to his attempt to re-establish a *prisca sapientia*.... He viewed the history of mankind as a regress, a process of corruption, rather than a progress. In his alchemical, theological and chronological works, which he began composing in secrecy in the 1670s, the 'Moderns' are always depicted as inferior to the 'Ancients'. Newton's rejection of Cartesian algebra, his distancing himself from the analytical method of fluxions, and his interests in the geometrical works of Apollonius and Pappus are in resonance with other facets of his intellectual endeavour. Guicciardini (1999), pp. 30-32.

According to Guicciardini, Newton's talk of "restoring" ancient discoveries in geometry was part of a wider project to undo the regress of modernity. This restorative argument is thus a moralistic case for purity in mathematics.

In the passage from the Lucasian Lectures on Algebra that we quoted in the previous section, Newton wrote that "recent people, by confusing [algebra and geometry], have lost the simplicity in which all elegance of geometry consists." We observed that this too is a moralistic case for purity in geometry. Elegance is an aesthetic term commonly used in the seventeenth century to evaluate mathematics; for instance, Descartes invokes it in reference to the construction of a curve in a letter to Carcavi dated 17 August 1649 (AT V: 399). It suggests a simplicity of style, a minimum of adornment, drawing its value from the harmony of its organisation. In the seventeenth century elegance has neoclassical resonances, referring to the ancient style of writing mathematics. This is the sense in which Newton seems to have used it. To our knowledge there is no analysis of elegance in Newton's writings, only its reference to the style of the ancients. Since ancient geometers worked purely geometrically, Newton evaluates their purely geometrical pursuit of geometry in terms of its elegance.

He would return to this theme again and again, for instance in a comment on a text from 1698 stressing his vision "for restoring the Analysis of the Ancients which is more simple more ingenious & more fit for a Geometer than the Algebra of the Moderns", which leads to solutions "usually more simple & elegant then that which is forced from Algebra".[13] This moralistic case for hygiene in the form of pure geometry over Cartesian algebraic geometry recurs throughout Newton's mature life.

Other contemporaneous mathematical writers added more on the nature of elegance, though, and did not limit themselves to identifying elegant mathematics with the purely geometrical pursuits of the ancients. For instance, a review of Guisnée (1705) from 1730 says, in the context of an algebraic solution to a geometrical problem, that:

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[13] Newton is commenting on Antonius Hugo de Omerique's *Analysis Geometrica, sive Nova et Vera Methodus Resolvendi tam Problemata Geometrica quam Arithmeticas Quaestiones* (1698), Bodleian New College MS 361.2.2, f. 19r, cited and translated in Guicciardini (2009), p. 312.

> From this it follows that if an indeterminate problem of second degree can be resolved by two or more of the four curves, it is necessary to prefer the most simple one. This greater simplicity of the solution is a part of what one calls its *elegance*, the rest consists in drawing more immediately on what is given in the question, and bringing to it a minimum quantity of foreign and auxiliary principles. Cf. Académie Royale des Sciences (1730), p. 109.

Elegance here is again a matter of simplicity, but also a matter of purity, the avoidance of "foreign" elements. But the purity here is not the avoidance of algebra in geometry, but rather the avoidance of elements not "given in the question".[14]

Thus "elegance" is a contested aesthetic term in seventeenth century mathematics. For Newton it refers to the ancient way of doing mathematics, and is identified with their purely geometrical, synthetic proofs of geometrical theorems. For the reviewer of Guisnée, working instead within the Cartesian tradition, it refers to the avoidance of foreign elements even within an algebraic analysis of a geometrical problem. Both Newton and the Cartesians seek mathematical hygiene, preferring proofs of one sort over others; both seek in particular pure proofs; and indeed both justify their search for this type of hygiene in terms of elegance; but finally their hygiene projects differ.

Newton's employment of elegance against Cartesian geometry also sometimes blurs the distinction between mathematical hygiene proper and verbal hygiene in a mathematical context. Comparing his solution to the Pappus problem with Descartes', he writes:

> To be sure, their [the Ancients'] method is more elegant by far than the Cartesian one. For he [Descartes] achieved the result [the solution of the Pappus problem] by an algebraic calculus which, when transposed into words (following the practice of the Ancients in their writings), would prove to be so tedious and entangled as to provoke nausea, nor might it be understood. But they accomplished it by certain simple proportions, judging that nothing written in a different style was worthy to be read, and in consequence they were concealing the analysis by which they found their constructions. (From "Veterum Loca Solida Restituta", Newton (1971) p. 277, translation from Guicciardini (2009), p. 80.)

Newton compares the "elegance" of his geometrical solution, written in the style of the ancients, with the "tedious" algebraic solution of Descartes.

We will now discuss one last element to Newton's preference for the ancient style of writing mathematics. In an undated manuscript Newton (2022), Newton writes:

> The Ancients invented their Propositions by Analysis & Demonstrated them by synthesis, & admitted nothing into Geometry before it was demonstrated synthetically. I followed their example that the Propositions in that book might

[14]Here the author enunciates a type of purity that Hilbert would much later promote. In lectures on Euclidean geometry at Göttingen in 1898–1899, after noting that the planar Desargues theorem is typically proved using space, Hilbert remarks:

> Therefore we are for the first time in a position to put into practice *a critique of means of proof.* In modern mathematics such criticism is raised very often, where the aim is to preserve *the purity of method*, i.e. to prove theorems if possible using means that are suggested by the content of the theorem. Cf. Hilbert (2004), pp. 315–6.

On this type of purity, elsewhere (cf. Detlefsen and Arana (2011)) called the "topical" conception, Hilbert measures the purity of a proof by the extent to which it draws only on what belongs to the content of what is being proved. This echoes the conception here of purity as avoiding elements not "given in the question".

> be admitted into Geometry. For the glory of Geometry is its certainty & nothing is to be admitted into Geometry before it be made as certain plane & evident as art can make it.

The importance of synthetic demonstrations in geometry, to be given in purely geometrical rather than algebraic terms, is for Newton also a matter of their *certainty.* He seems to have to judged the deductive logical structure of Euclidean-style demonstrations from first principles to have the power to convince their readers with certainty of their conclusions, whereas analysis is only capable of showing how to discover a synthetic proof.

Let us take stock of what we have seen. Newton stresses:

- that geometers should do what the ancients did, because doing so restore their lost truth, and the ancients did geometry rather than algebra; and
- that pure geometry is elegant, in its simplicity, and algebra is complex and tedious; and
- that algebra is only analysis, discovery, and that only synthesis gives a proof, from first principles, for establishing the result with certainty.

By contrast, Descartes stresses:

- that algebra gives the power to solve all geometrical problems by bringing together algebra and geometry, which the ancients did not have; and
- that the ancients lacked method, whereas his algebraic techniques are methodical; and that this method can promote learning among well-versed students.

We have seen that Descartes and Newton thought synthesis and analysis were both valuable, but saw the roles of each differently. Descartes acknowledged the importance of proving results synthetically, but judged it best left as an exercise for "clever minds". Newton recognized the importance of analysis, since solutions have to be found somehow, and there is evidence that Newton sometimes used algebra to help with that. But he did not *publish* his analyses (though he sent them in correspondence sometimes).

Their differences are thus, at least in part, an issue of what gets written, and what is left to the reader. Their positions here reflect their agendas. Descartes is more inclined to reform the reader in getting them to do the work of discovery, and Newton is not. This agenda corresponds with Descartes' interest in creating among the new bourgeoisie and courtesans a new intellectual elite, active in the exercise of their own reason rather than imitating the ancients, who were not just the warrior elite of the past.[15] By contrast, Newton believed on religious grounds in a "elect" people who alone could know things rightly. Reflecting this, he did not publish but instead instructed small groups of disciples around him (cf. Force (1999), p. 244).

Descartes' agenda merits further attention because of its links with contemporaneous verbal hygiene projects that we will discuss in the next section. Here, we observe firstly that Descartes calls *La géométrie* an "essai". It is written in the first-person, with an autobiographical aspect. This is unusual in a mathematical text in any era. Descartes invites readers to work with the methods themselves, as competent geometers rather than as disciples, in contrast with Newton.

We note secondly that in *La géométrie* Descartes quotes at length Commandino's Latin translation of Pappus' description of the Pappus problem to show how cumbersome is the ancient style

[15]For more on this in relation to Descartes' general thought see chapter 8 of Taylor (1989), and for more in relation to his scientific and mathematical thought, see chapter 1 of Jones (2006).

of writing geometry, then reprises the Pappus problem in French quite briefly. He also lauded the brevity of his solution to the Pappus problem:

> In this way I think I have entirely fulfilled what Pappus tells us was sought by the ancients; and I will try to put the proof in few words, because it already bores me to write so much. Descartes (1902), p. 382.

This stress on *brevity* is a mark of the *honnête homme*, a type of *persona* that we will discuss next.

The *honnête homme* emerges as an ideal member of the court of Louis XIV. He is intelligent but also courageous and generous, without artifice. He aims to be a master of conversation, to entice others with his elegance and wit. He is a generalist so that he can converse with anyone about anything, able to adapt to the needs of his audience, never boring them with pedantry. Examples in the literature of the seventeenth century abound: for instance, Cléonte in Molière's *Le bourgeois gentilhomme* (1670) and Hippolyte in Racine's *Phèdre* (1677).

Three aspects of the *honnête homme* are particularly present in Descartes' geometrical work. We have already mentioned one, the aspiration toward brevity. A second, also already emphasized, is the push toward self-reliance on reason rather than on imitation of ancient masters. As Descartes put it in a late text:

> An *honnete homme* is not obligated to have read all books, nor to have learned with care everything all that is taught in the schools; and this would even be a fault in his education, if he had spent too much time on book learning. He has many other things to do during his life, the course of which must be so well-considered that the best part of it remains for him to carry out good acts, that he would have to be taught by his own reason if he learned only from it alone. (*La recherche de la vérité par la lumière naturelle*, 1640s, Descartes (1908), p. 495)

Thirdly, the honnête homme must above all be master of himself, in control of his passions. In his article on literary aspects of Descartes' geometry, Dominique Descotes has observed that Descartes sought to exemplify this *persona* in this respect, responding calmly in public to Fermat's attack on his method of tangents in 1638 (cf. Descotes (2005), p. 170).

We have shown how each pursued mathematical hygiene at least in part in order to further their wider ideological agendas. That's *not* to say that their pursuits of hygiene did not have agendas internal to mathematics proper — to cite just two examples, Bos (2001) and Guicciardini (2009) make precisely such cases — but rather to say that our understanding of their pursuits of hygiene is enriched by seeing how they arise also from their differing ideological agendas.

A remaining obstacle to our case, though, is the gap between the mathematical ideals propounded by Newton and Descartes, and their mathematical practices. In a review of volume 7 of Newton's mathematical papers, Michael Mahoney comments on one of Newton's attempts to reconstruct the purely geometrical analysis of the ancients, published only three hundred years later. Mahoney remarks that "despite his talents and his professed preference for classical, "pure" geometry, his discussion begins with references to equations and ends with the algebraic quadrature of curves. In short, Newton wrested briefly with past canons and then abandoned the struggle" (Mahoney (1977), p. 865). Mahoney continued by observing "Newton's relief at relinquishing the frock of classical expositor and interpreter, relentlessly stressing—and overstressing— the superiority of the ancient polished syntheses over the cruder 'algebraic' resolutions of more recent Cartesian geometry." We thus see Newton set aside his professed values for the algebraic analysis so lauded by his archenemy Descartes—though perhaps that is why Newton never published this work.

Descartes too failed to live up to his ideals. Descartes maintained that a geometrical problem was solved only when its solution was constructed synthetically. Algebra was to be used to analyse what curves should be employed in such constructions. Some curves, however, he judged insufficiently exact to be employable in geometrical construction, being instead mechanical in nature and thus incapable of giving clear and distinct knowledge of the sought solution. He thus took considerable effort in *La géométrie* to show that the curves he discovered by his algebraic analyses were indeed geometrical by his lights. Numerous commentators have observed that Descartes failed to do so adequately, because he did not prove that every curve defined by an algebraic equation could be traced by the kinds of instruments that he judged capable of tracing curves by continuous motions in a way that could be exactly known (see Grosholz (1991), p. 42; Bos (2001), pp. 404–5; Domski (2009), p. 125 and Domski (2022)). Indeed, such a proof would only be found in the 19th century (Kempe (1876)). Thus Descartes did not solve the Pappus problem in a way that met his own geometric ideals.

In this case study we have tried to show how the mathematical ideals propounded by Newton and Descartes in their pursuits of mathematical hygiene were informed by their wider ideological norms and projects. Their failures to achieve their own mathematical ideals, then, call into question their commitment to these ideals and thus to their relation to the ideological norms we have identified.

A general response to this calling into question is to observe that the failure to live up to an ideal does not necessarily impugn that ideal. As Mahoney observes, Newton may have gotten carried away in his lauding of synthetic methods in geometry, fueled by his all-too-human loathing of Cartesianism. Nevertheless, the pursuit of purely geometric solutions to geometric problems, avoiding algebra, remained an occupation of mathematics long after Newton (see Arana (2016)). As Emil Artin observed in his classic 1957 text *Geometric Algebra*:

> Many parts of classical geometry have developed into great independent theories. Linear algebra, topology, differential and algebraic geometry are the indispensable tools of the mathematician of our time. It is frequently desirable to devise *a course of geometric nature which is distinct from these great lines of thought*... (Artin (1957), p. v)

Artin's longing for purely geometric approaches to contemporary geometrical problems is, in part, Newton's heritage.

Similarly, Descartes' program, despite its failure, lived on. Emily Grosholz argues that Descartes never resolved the tension between geometry and algebra revealed by this failure (Grosholz (1991)). Mary Domski argues that in *Le monde* Descartes founds the geometricity of algebraically-defined curves on the relation between construction, motion, and intelligibility that he saw at play in God's creation of the world, so that the mathematical failure is patched by a metaphysical supposition (Domski (2009)). Henk Bos sees Descartes as aware of the shortcomings of his realization of his project, seeing an element of tragedy in this outcome. On this take, Descartes centered the algebraic aspect of his program as a reluctant concession to his failure to align geometry and algebra perfectly (Bos (2001), p. 406). Still, the mathematical power of Descartes' method remained patent, an epoch-changing achievement introducing the interplay of geometry and algebra into ordinary mathematical practice. As Bos remarks, "Descartes' canon carried so much conviction that in a relatively short time span it made mathematicians accept all algebraic curves and all problems leading to algebraic equations as solvable in principle and thereby of accidental interest only" (Bos (2001), p. 373). Even though Descartes' particular vision of how algebraic geometry was to be implemented was incomplete, the program was amenable to revision toward better realizations of its mathematical norms, and this was left to his successors, for instance Leibniz (Bos (1981), p. 332; Blåsjö (2017)). Indeed, the algebraic geometry that emerged from Descartes' program remains a core discipline of mathematics today.

About Descartes' case we can in fact show more. His failure to realize his mathematical ideals sheds more light on the importance of the ideological norms informing these ideals, in a surprising parallel with the *Meditations.* We stressed earlier the functional argument in *La géométrie* that "the utility of cultivating your mind by practicing on it. . . is in my opinion the main thing that one can get out of this science". Later, in the preface to the French edition of the *Principles of Philosophy*, he writes, "Finally, by the Géométrie, I claimed to demonstrate that I had found several things that had before been unknown, and thus to give occasion to believe that one could thereby still discover several others, in order to incite by this means all men to pursue the truth" (IXb 15–16). Whatever the value of geometrical knowledge is in itself, Descartes emphasizes that its *instrumental* value in training the mind, and in motivating people to seek truth, is even greater.[16] These instrumental values were to help cultivate a new intellectual elite among the emerging bourgeoisie class. We then have a new explanation for his quickness in moving past the mathematical gaps in his program. Impelled by the power of his ideology, he oversold his purely mathematical accomplishments.

This explanation parallels Descartes' strategy in the *Meditations*, where its epistemological arguments are similarly instrumentalized for a closely related ideological program. In this text, Descartes sets out to find what can be known clearly and distinctly, and how to ground all knowledge on that foundational knowledge. For this he employs his method of doubt, applying it in particular to sense perception. Teachers of the *Meditations* will note how difficult it is to convince students to take Cartesian doubt seriously, to think that its skepticism is a real possibility. Indeed, Descartes recognizes this difficulty. He notes how the arguments of the text call for readers "able and willing to meditate seriously with me, and to withdraw their minds from the senses and from all preconceived opinions. Such readers, as I well know, are few and far between" (AT VII 9, translation from Descartes (1984), p. 8). A goal of the *Meditations* is to help form such readers. True to their name, the *Meditations* are mental exercises in detaching ourselves from our senses. He stresses detachment from the senses in particular because of our *passivity* with respect to them. By contrast, we can control our *judgments* with respect to the data we passively receive from the senses, and the method of doubt is an exercise in taking such control. To practice this method is thus to train our rational self-control. Descartes believed that such rational self-control was a *moral* ideal, a type of intellectualized Stoicism (Taylor (1989)).

To follow the method of doubt, then, is to practice our ability to detach from our senses, and thereby to improve our moral character. At the same time, Descartes holds that the *evidence* of the arguments in the *Meditations* depends on our moral character. "[A]lthough the proofs I employ here are in my view as certain and evident as the proofs of geometry, if not more so," he writes, "it will, I fear, be impossible for many people to achieve an adequate perception of them, both because they are rather long and some depend on others, and also, above all, because they require a mind which is completely free from preconceived opinions and which can easily detach itself from involvement with the senses" (AT VII 4, translation from Descartes (1984), p. 5). Whether we will be moved to believe in the conclusions of these arguments depends on our

[16] See Jones (2006) for an account of Descartes' geometry as spiritual exercises meant to help cultivate the good life, though putting rather less emphasis on the algebraic dimensions of Descartes' accomplishment than we have.

having "a mind completely free from preconceived opinions" and a mind that that "can easily detach itself from involvement with the senses". As we improve our capacity to detach ourselves from the senses, we will come to find the arguments resulting from the method of doubt more convincing. The moral and epistemic ideals of the *Meditations* are thus intertwined.

So, we are claiming, are the ideological and mathematical ideals in Descartes' geometrical program. Practicing algebraic analysis in geometry cultivates the mind and incites the pursuit of truth by developing the researcher's mathematical confidence. Similarly, the more cultivated the mathematician becomes, stepping apart from preconceived opinions, the more readily she will understand "the conjunction that is between geometry and arithmetic" that algebra expresses—a conjunction whose instruction to the ignorant would otherwise be "a waste of time" (9 February 1639, Descartes (1898), p. 504).

In both Descartes' mathematics and epistemology, on our interpretation the value of their methods, and even the ability to see the correctness of their results, depend on reasons outside of mathematics and epistemology. In the case of the *Meditations*, these depend rather on the moral character of the reader-meditator, on their capacity to shed preconceived opinion and detach from their senses; that is to say, on extra-epistemological reasons. In the case of mathematics, the value of the algebraic analytic method, and the evidence of the fusion of geometry and algebra, on which the success of the algebraic method in geometrical problem solving rests, result from the cultivation of the researcher's mind, and her consequent ability to understand this fusion; that is to say, on extra-mathematical reasons. Even though, finally, Kempe's proof gives a reason internal to mathematics to accept the extensional equivalence of pointwise-constructible and algebraically definable curves, the intelligibility of Descartes' geometry did not require this proof, supplanted as it was from the outside in a characteristically Cartesian way.

4.2. **Descartes vs Vaugelas (verbal hygiene).** In the previous section we discussed the *honnête homme* persona and its importance for how Descartes conceptualized his mathematical work. It was also important in debates about verbal hygiene in the 17th century in France. To be more explicit, we can compare Descartes with his contemporary, Claude Favre de Vaugelas (1585-1650). Vaugelas was an influential grammarian who was known particularly for his *Remarques sur la langue française, utiles à ceux qui veulent bien parler et bien écrire* 'Remarks about the French language, useful to those who wish to speak well and write well', published in 1647. As discussed by Caron and Ayres-Bennett (2019), the *Remarques* was hugely inspirational to Vaugelas' colleagues and to generations of grammarians that followed, so much so that those who continued producing works of verbal hygiene in Vaugelas' vein are often called *les Remarqueurs* 'The Remarkers'.

At a basic level, *Remarques* laid out a method for learning the speech of the court, i.e. the home of the *honnête homme*. Thus, we hypothesize that what Descartes wanted to do by means of mathematics, Vaugelas and his followers wanted to do by means of language. Properly testing this hypothesis would require a full detailed sociolinguistic comparison of Descartes' (and his followers') and Vaugelas' (and his followers') writings. Unfortunately, this is out of the scope of this programmatic paper. However, we do think that there are reasons to believe that this hypothesis is on the right track. Firstly, Vaugelas' work is often considered to contribute to the constructed of the *honnête homme* persona by scholars of prescriptivism and purism in 17th century France. For instance, Ayres-Bennett (1987) describes Vaugelas' goals as follows:

> Ultimately then the linguistic is subsumed by the sociolinguistic, Vaugelas's theory of language depending on the desire to make communication as quick and

> easy as possible in order not to cause the listener any displeasure or make the interlocutor appear ridiculous, the worst possible fate to befall the *honnête homme*. Avoiding such ridicule would especially be the concern of the nouveau riche or the upper middle class man trying to rise in society and hoping to be accepted socially by the former nobility, and there are indications that such men did indeed read the *Remarques* as an aid to self-betterment. Ayres-Bennett (1987), p.6.

Secondly, many of Vaugelas' prescriptions value the same properties in language as Descartes values in mathematics. Consider the following example from Vaugelas' follow-up to *Remarques*: *Nouvelles remarques de M. de Vaugelas sur la langue françoise*, published posthumously in 1690.

> I say again that an infinity of people write like this; however, it's a mistake against the purity of the language, which wants us to say *La Philosophie sainte & la Philosophie profane défendent*, or instead, *La Philosophie saint & la profane défendent*. But I insist that the first is better: because we must always remember that our Language loves word repetitions very much, something that also contributes a lot to the clarity of language, which the French language shows to all the languages of the world. Also it generally does not delete anything: which is nevertheless a great elegance among the Greeks and Romans, but which often creates obscurity and equivocation.

In this passage, Vaugelas compares an elliptical construction *La Philosophie saint & la profane défendent* 'The saintly philosophy and the profane (one) defend' with the non-elliptical construction which repeats the noun: *La Philosophie saint & la Philosophie profane défendent* 'the saintly philosophy and the profane philosophy defend'. He states his preference for the non-elliptical construction and the reasons he provides to justify this choice are eerily similar to those given by Descartes: Vaugelas values clarity over elegance, something that, he claims, differentiates the modern French language from the languages of the Ancients.

5. Conclusion

In this paper, we have outlined a new line of inquiry straddling the boundaries of philosophy and linguistics: *mathematical hygiene*. While comparing the normative discourses on mathematics with those on another practice (art, sport etc.) would certainly be possible and probably interesting, language is a natural starting point since it is a common view in the philosophy of mathematics, among nominalists for instance, that mathematical knowledge is knowledge of language. We believe that further inquiries into normative discourses on mathematical practices, and how they may (or may not) parallel discourses on linguistic practices, could make meaningful contributions to open questions in both of the these fields. For the philosophy of mathematics: understanding the social context in which mathematical knowledge is produced, and how this context affects reasoning about mathematics and language in similar or different ways, could help better understand the nature of mathematical discoveries and, eventually, how to do better mathematics. For linguistics: understanding how language ideologies may (or may not) be reproduced in the mathematical domain could help better understand their formation. Although our case study was brief, we believe it serves as an enlightening jumping off point for linguists, philosophers and mathematicians who wish to learn more about the "urge to meddle in matters of mathematics".

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References

Académie Royale des Sciences (1730). *Histoire de l'Académie Royale des Sciences 1705*. Imprimerie Royale, Paris.

Agha, A. (2003). The social life of cultural value. *Language & Communication*, 23(3-4):231–273.

Agha, A. (2006). *Language and social relations*, volume 24. Cambridge University Press.

Arana, A. (2016). Imagination in mathematics. In Kind, A., editor, *The Routledge Handbook of Philosophy of Imagination*, chapter 34, pages 463–477. Routledge, London and New York.

Arana, A. (2017). On the alleged simplicity of impure proof. In Kossak, R. and Ording, P., editors, *Simplicity: Ideals of Practice in Mathematics and the Arts*, pages 207–226. Springer, Cham.

Arana, A. (2022). Idéaux de preuve : explication et pureté. In Arana, A. and Panza, M., editors, *Précis de philosophie de la logique et des mathématiques*, volume 2, chapter 9. Éditions de la Sorbonne, Paris.

Årman, H. (2021). Affects of verbal hygiene: the impact of language activism at a Swedish high school. *Language Policy*, 20(2):151–171.

Armstrong, N. and Mackenzie, I. (2013). *Standardization, ideology and linguistics*. Palgrave Macmillan, Basingstoke.

Artin, E. (1957). *Geometric Algebra*. Interscience Publishers, New York.

Avigad, J. (2003). Number theory and elementary arithmetic. *Philosophia Mathematica*, 11:257–284.

Avigad, J. (2022). Varieties of mathematical understanding. *Bulletin of the American Mathematical Society*, 59(1):99–117.

Ayres-Bennett, W. (1987). *Vaugelas and the Development of the French Language*, volume 23. MHRA.

Ayres-Bennett, W. (2020). From Haugen's codification to Thomas's purism: assessing the role of description and prescription, prescriptivism and purism in linguistic standardisation. *Language Policy*, 19(2):183–213.

Bishop, E. (1967). *Foundations of Constructive Analysis*. McGraw-Hill, New York.

Blåsjö, V. (2017). *Transcendental Curves in the Leibnizian Calculus*. Academic Press, Oxford.

Bolzano, B. (1999). Purely analytic proof of the theorem that between any two values which give results of opposite sign there lies at least one real root of the equation. In Ewald, W., editor, *From Kant to Hilbert*, volume 1, pages 227–248. Oxford University Press. Originally published in 1817; translated by S. Russ.

Bos, H. J. M. (1981). On the representation of curves in Descartes' Géométrie. *Archive for History of Exact Sciences*, 24(4):295–338.

Bos, H. J. M. (2001). *Redefining Geometrical Exactness.* Springer, New York.

Bourbaki, N. (1948). L'architecture des mathématiques. In Lionnais, F. L., editor, *Les grands courants de la pensée mathématique.* Éditions des Cahiers du Sud.

Brousseau, A.-M. (2011). Identités linguistiques, langues identitaires: synthèse. *Arborescences: revue d'études françaises*, (1):0–0.

Brown, L. (2022). Politeness as normative, evaluative and discriminatory: the case of verbal hygiene discourses on correct honorifics use in South Korea. *Journal of Politeness Research*, 18(1):63–91.

Cameron, D. (1995). *Verbal hygiene.* Routledge.

Cameron, D. (2013). The one, the many, and the other: Representing multi-and mono-lingualism in post-9/11 verbal hygiene. *Critical Multilingualism Studies*, 1(2):59–77.

Caron, P. and Ayres-Bennett, W. (2019). La norme et la prescription linguistique en France 1550-1720 L'exemple des remarqueurs. *Histoire Epistémologie Langage*, 41(2):41–66.

Curzan, A. (2014). *Fixing English: Prescriptivism and language history.* Cambridge University Press.

Dawson, Jr., J. W. (2015). *Why Prove it Again? Alternative Proofs in Mathematical Practice.* Birkhäuser, Heidelberg.

Descartes, R. (1637). La géométrie. In *Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences*, pages 297–413. Jan Maire, Leiden.

Descartes, R. (1659–1661). *Geometria.* Elzevirium, Amsterdam, second edition. Edited and translated from French to Latin by Frans van Schooten.

Descartes, R. (1897). *Oeuvres*, volume I. Léopold Cerf, Paris.

Descartes, R. (1898). *Oeuvres*, volume II. Léopold Cerf, Paris.

Descartes, R. (1902). *Oeuvres*, volume VI. Léopold Cerf, Paris.

Descartes, R. (1908). *Oeuvres*, volume X. Léopold Cerf, Paris.

Descartes, R. (1984). *The Philosophical Writings of Descartes*, volume 2. Cambridge University Press, Cambridge.

Descotes, D. (2005). Aspects littéraires de la *Géométrie* de Descartes. *Archives Internationales d'Histoire des Sciences*, 55(154):163–191.

Detlefsen, M. (2003). Constructive existence claims. In Schirn, M., editor, *Philosophy of Mathematics Today.* Oxford University Press.

Detlefsen, M. (2008). Purity as an ideal of proof. In Mancosu, P., editor, *The Philosophy of Mathematical Practice*, pages 179–197. Oxford University Press.

Detlefsen, M. and Arana, A. (2011). Purity of methods. *Philosophers' Imprint*, 11(2):1–20.

Deumert, A. and Vandenbussche, W. (2003). Standard languages. *Taxonomies and histories. Deumert & Vandenbussche (red.)*, pages 1–14.

Dieudonné, J. (1969). *Linear algebra and geometry.* Houghton Mifflin Co., Boston. Translation of **?**.

Domski, M. (2009). The intelligibility of motion and construction: Descartes' early mathematics and metaphysics, 1619–1637. *Studies in History and Philosophy of Science Part A*, 40(2):119–130.

Domski, M. (2022). Descartes' Mathematics. In Zalta, E. N. and Nodelman, U., editors, *The Stanford Encyclopedia of Philosophy.* Metaphysics Research Lab, Stanford University, Fall 2022 edition.

D'Onofrio, A. (2018). Personae and phonetic detail in sociolinguistic signs. *Language in Society*, 47(4):513–539.

D'Onofrio, A. (2019). Complicating categories: Personae mediate racialized expectations of non-native speech. *Journal of Sociolinguistics*, 23(4):346–366.

Eckert, P. (2008). Variation and the indexical field. *Journal of sociolinguistics*, 12(4):453–476.

Force, J. (1999). Newton, the "ancients," and the "modems". In Force, J. and Popkin, R., editors, *Newton and Religion*, pages 237–257. Kluwer Academic Publishers, Dordrecht.

Giovannini, E. N. (2016). Bridging the gap between analytic and synthetic geometry: Hilbert's axiomatic approach. *Synthese*, 193(1):31–70.

Grosholz, E. (1991). *Cartesian Method and the Problem of Reduction.* Oxford University Press, Oxford.

Guicciardini, N. (1999). *Reading the* Principia*: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736.* Cambridge University Press, Cambridge.

Guicciardini, N. (2009). *Isaac Newton on Mathematical Certainty and Method.* MIT Press, Cambridge, MA.

Guisnée, N. (1705). *Application de l'algebre a la geometrie.* Boudot and Quillau, Paris.

Hafner, J. and Mancosu, P. (2005). The varieties of mathematical explanation. In Mancosu, P., Jørgensen, K. F., and Pedersen, S. A., editors, *Visualization, explanation and reasoning styles in mathematics*, pages 215–250. Springer, Dordrecht.

Hamami, Y. (2014). Mathematical rigor, proof gap and the validity of mathematical inference. *Philosophia Scientiæ*, 18(1):7–26.

Hansen, K., Wypych, M., Bańko, M., and Bilewicz, M. (2018). Psychological determinants of linguistic purism: National identification, conservatism, and attitudes to loanwords. *Journal of Language and Social Psychology*, 37(3):365–375.

Heinzmann, G. and Atten, M. V. (2022). Révisionnisme mathématique : l'enjeu constructif. In Arana, A. and Panza, M., editors, *Précis de philosophie de la logique et des mathématiques*, volume 2, chapter 5. Éditions de la Sorbonne, Paris.

Hilbert, D. (2004). Grundlagen der Euklidischen Geometrie. In *David Hilbert's Lectures on the Foundations of Geometry, 1891–1902*, pages 221–395. Springer-Verlag, Berlin. Lectures on Euclidean Geometry (1898-99). Edited by Michael Hallett and Ulrich Majer.

Jones, M. (2006). *The Good Life in the Scientific Revolution: Descartes, Pascal, Leibniz, and the Cultivation of Virtue.* University of Chicago Press, Chicago.

Jones, R. H. (2013). Verbal hygiene in the Hong Kong gay community. *World Englishes*, 32(1):75–92.

Joseph, J. E. (1987). *Eloquence and power: The rise of language standards and standard languages*. Burns & Oates.

Kempe, A. (1876). On a general method of describing plane curves of the nth degree by linkwork. *Proceedings of the London Mathematical Society*, 7:213–216.

Kitcher, P. (1989). Explanatory unification and the causal structure of the world. In Kitcher, P. and Salmon, W., editors, *Scientific Explanation*, pages 410–505. University of Minnesota Press, Minneapolis.

Kreisel, G. (1980). Kurt Gödel. *Biographical Memoirs of Fellows of the Royal Society*, 26:149–224.

Labov, W. (1970). The logic of nonstandard English. In *Language and poverty*, pages 153–189. Elsevier.

Lipman, J. and Teissier, B. (1981). Pseudo-rational local rings and a theorem of Briançon-Skoda about integral closures of ideals. *Michigan Mathematical Journal*, 28:97–116.

Lorenat, J. (2022). An Okapi Hypothesis: Non-Euclidean Geometry and the Professional Expert in American Mathematics. *Isis*, 113(1):85–107.

Mahoney, M. S. (1977). Last Years at Cambridge: Review of *The Mathematical Papers of Isaac Newton. Vol. 7, 1691-1695* by Isaac Newton and D. T. Whiteside. *Science*, 196(4292):864–865.

Millar, S. (1998). Language prescription: A success in failure's clothing? In Hogg, R. and van Bergen, L., editors, *Historical Linguistics 1995: Selected papers from the 12th International Conference of Historical Linguistics*, page 177–188. John Benjamins, Amsterdam.

Milroy, J. (1992). *Linguistic variation and change: On the historical sociolinguistics of English*. Blackwell.

Milroy, J. (2005). Variability, language change, and the history of English. *International Journal of English Studies*, 5(1):1–11.

Milroy, J. and Milroy, L. (1999). *Authority in language: Investigating standard English*. Psychology Press.

Morris, R. L. (2021). The Values of Mathematical Proofs. In Sriraman, B., editor, *Handbook of the History and Philosophy of Mathematical Practice*, pages 1–32. Springer, Cham.

Nathanson, M. B. (2000). *Elementary methods in number theory*. Springer-Verlag, New York.

Newton, I. (1971). *The Mathematical Papers of Isaac Newton. Vol. IV: 1664–1666*. Cambridge University Press, Cambridge. Edited by D.T. Whiteside.

Newton, I. (1972). *The Mathematical Papers of Isaac Newton. Vol. V: 1683–1684*. Cambridge University Press, Cambridge. Edited by D.T. Whiteside.

Newton, I. (1999). *The Principia*. University of California Press, Berkeley. Translated by I. Bernard Cohen and Anne Whitman.

Newton, I. (2022). *Unarranged fragments, mostly relating to the dispute with Leibniz*. Cambridge University Library, Cambridge. MS Add. 3968, ff. 594r-619v. https://www.newtonproject.ox.ac.uk/view/texts/normalized/NATP00385.

Novaes, C. D. (2019). The beauty (?) of mathematical proofs. *Advances in experimental philosophy of logic and mathematics*, page 63.

Ording, P. (2019). *99 Variations on a Proof.* Princeton University Press, Princeton.

Paffey, D. (2007). Policing the Spanish language debate: verbal hygiene and the Spanish language academy (Real Academia Española). *Language Policy*, 6(3):313–332.

Poplack, S., Jarmasz, L.-G., Dion, N., and Rosen, N. (2015). Searching for standard French: The construction and mining of the *Recueil historique des grammaires du français*. *Journal of Historical Sociolinguistics*, 1(1):13–55.

Schneider, E. W. (2003). The dynamics of New Englishes: From identity construction to dialect birth. *Language*, 79(2):233–281.

Sczesny, S., Formanowicz, M., and Moser, F. (2016). Can gender-fair language reduce gender stereotyping and discrimination? *Frontiers in psychology*, 7:25.

Spitzmüller, J. (2007). Staking the claims of identity: Purism, linguistics and the media in post-1990 Germany. *Journal of Sociolinguistics*, 11(2):261–285.

Steiner, M. (1978). Mathematical explanation. *Philosophical Studies*, 34(2):135–151.

Tappenden, J. (2008). Mathematical concepts and definitions. In Mancosu, P., editor, *The Philosophy of Mathematical Practice.* Oxford University Press.

Tappenden, J. (2012). Fruitfulness as a theme in the philosophy of mathematics. *The Journal of Philosophy*, 109(1/2):204–219.

Taylor, C. (1989). *Sources of the Self.* Harvard University Press, Cambridge, MA.

Thomas, G. (1991). *Linguistic purism.* Longman Publishing Group.

Trask, R. L. (1999). *Key concepts in language and linguistics.* Psychology Press.

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