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On Some New Notions and Functions in Neutrosophic Topological Spaces

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Abstract: In this paper, we define the notion of neutrosophic semiopen (resp. preopen and α-open) functions and investigate relation among them. We give a characterization of neutrosophic α-open set, and provide conditions for a neutrosophic set to be a neutrosophic α-open set. We discuss characterizations of neutrosophic pre-continuous (resp. α-continuous) functions. We give a condition for a function of neutrosophic topological spaces to be a neutrosophic α-continuous function.

Keywords:neutrosophic α-open set;neutrosophic semiopen; neutrosophic preopen; neutrosophic pre-continuous ; neutrosophic α-continuous.

1 Introduction and Preliminaries

After the advent of the notion of fuzzy set by Zadeh[11], C. L. Chang [4] introduced the notion of fuzzy topological space and many researchers converted, among others, general topological notions in the context of fuzzy topology. The notion of intuitionistic fuzzy set introduced by Atanassov [1, 2, 3] is one of the generalizations of the notion of fuzzy set. Later, Coker [5] by using the notion of the intuitionistic fuzzy set, offered the useful notion of intuitionistic fuzzy topological space. Joung Kon Jeon et al.[7] introduced and studied the notions of intuitionistic fuzzy α-continuity and pre-continuity which we will investigate in the context of neutrosophic topology. After the introduction of the concepts of neutrosophy and neutrosophic set by F. Smarandache [[9], [10]], the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces were introduced by A. A. Salama and S. A. Alblowi[8].

In this paper, we define the notion of neutrosophic semiopen (resp. preopen and α-open) functions and investigate relation among them. We give a characterization of neutrosophic α-open set, and provide conditions for which a neutrosophic set is neutrosophic α-open. We discuss characterizations of neutrosophic precontinuous (resp. α-continuous) functions.

Definition 1.1. [6] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (briefly NTS) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X. Each neutrosophic supra set (briefly NS) which belongs to (X, T) is called a neutrosophic supra open set (briefly NSOS) in X. The complement $\overline{A}$ of a NSOS A in X is called a neutrosophic supra closed set (briefly IFSCS) in X.

Definition 1.2. [6] Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.3. [6] Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$.where r denotes the degree of membership value ,t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

2 Definitions

Definition 2.1. A neutrosophic set A in a neutrosophic topological space (X, T) is called

1) a neutrosophic semiopen set (briefly NSOS) if $A \subseteq Ncl(Nint(A))$.

2) a neutrosophic α-open set (briefly $N\alpha OS$) if $A \subseteq Nint(Ncl(Nint(A)))$.

3) a neutrosophic preopen set (briefly NPOS) if $A \subseteq Nint(Ncl(A))$.

4) a neutrosophic regular open set (briefly NROS) if $A = Nint(Ncl(A))$.

5) a neutrosophic semipreopen or β-open set (briefly $N\beta OS$) if $A \subseteq Ncl(Nint(Ncl(A)))$.

A neutrosophic set A is called neutrosophic semiclosed (resp. neutrosophic α-closed, neutrosophic preclosed, neutrosophic regular closed and neutrosophic β-closed) (briefly NSCS, NαCS, NPCS, NRCS and NβCS) if the complement of A is a neutrosophic semiopen (resp. neutrosophic α-open, neutrosophic preopen, neutrosophic regular open and neutrosophic β-open).

Example 2.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B, C, D and E in X as follows:
$A = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5})\rangle$,
$B = \langle x, (\frac{a}{0.5}, \frac{b}{0.55}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.55}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.55}, \frac{c}{0.5})\rangle$,
$C = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5})\rangle$. Then $T = \{0_N, 1_N, A, B, C\}$ is neutrosophic topology on X. Thus, (X, T) is neutrosophic topological space. Observe that $D = \langle x, (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5})\rangle$ is both semiopen and α-open in (X, T) and $E = \langle x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.6})\rangle$ is both preopen and β-open in (X, T).

Proposition 2.1. Let (X, T) be a neutrosophic topological space. If A is a neutrosophic α-open set then it is a neutrosophic semiopen set.

Proposition 2.2. Let (X, T) be a neutrosophic topological space. If A is a neutrosophic α-open set then it is a neutrosophic preopen set.

Proposition 2.3. Let A be a neutrosophic set in a neutrosophic topological spaces (X, T). If B is a neutrosophic semiopen set such that $B \subseteq A \subseteq Nint(Ncl(B))$, then A is a neutrosophic α-open set.

Proof. Since B is a neutrosophic semiopen set, we have $B \subseteq Ncl(Nint(B))$. Thus, $A \subseteq Nint(Ncl(B)) \subseteq Nint(Ncl(Ncl(Nint(B)))) = Nint(Ncl(Nint(B))) \subseteq Nint(Ncl(Nint(A)))$, and so A is a neutrosophic α-open set. □

Lemma 2.1. Any union of NS α-open sets (resp. neutrosophic preopen sets) is a NS α-open sets (resp., neutrosophic preopen sets).

The Proof is straightforward.

Proposition 2.4. A neutrosophic set A in a neutrosophic topological space X is neutrosophic α-open (resp. neutrosophic preopen) iff for every neutrosophic point $x_{r,t,s} \in A$, there exists a neutrosophic α-open set (resp. neutrosophic preopen set) $B_{x_{r,t,s}}$ such that $x_{r,t,s} \in B_{x_{r,t,s}} \subseteq A$.

Proof. If A is a neutrosophic α-open set (resp. neutrosophic preopen set), then we may take $B_{x_{r,t,s}} = A$ for every $x_{r,t,s} \in A$. Conversely assume that for every neutrosophic point $x_{r,t,s} \in A$, there exists a neutrosophic α-open set (resp., neutrosophic preopen set), $B_{x_{r,t,s}}$ such that $x_{r,t,s} \in B_{x_{r,t,s}} \subseteq A$. Then, $A = \cup\{x_{r,t,s} | x_{r,t,s} \in A\} \subseteq \cup\{B_{x_{r,t,s}} | x_{r,t,s} \in A\} \subseteq A$, and so $A = \cup\{B_{x_{r,t,s}} | x_{r,t,s} \in A\}$, which is a neutrosophic α-open set (resp. neutrosophic preopen set) by Lemma 2.1. □

Definition 2.2. Let f be a function from a neutrosophic topological spaces (X, T) and (Y, S). Then f is called

(i) a neutrosophic open function if $f(A)$ is a neutrosophic open set in Y for every neutrosophic open set A in X.

(ii) a neutrosophic α-open function if $f(A)$ is a neutrosophic α-open set in Y for every neutrosophic open set A in X.

(iii) a neutrosophic preopen function if $f(A)$ is a neutrosophic preopen set in Y for every neutrosophic open set A in X.

(iv) a neutrosophic semiopen function if $f(A)$ is a neutrosophic semiopen set in Y for every neutrosophic open set A in X.

Proposition 2.5. Let $(X, T), (Y, S)$ and (Z, R) be three neutrosophic topological spaces, let $f : (X, T) \to (Y, S)$ and $g : (Y, S) \to (Z, R)$ be functions. If f is neutrosophic open and g is neutrosophic α-open(resp., neutrosophic preopen), then $g \circ f$ is neutrosophic α-open(resp. neutrosophic preopen).

Proof. The Proof is straightforward. □

Proposition 2.6. Let (X, T) and (Y, S) are neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is neutrosophic α-open then it is neutrosophic semiopen.

Proof. Assume that f is neutrosophic α-open and let A be a neutrosophic open set in X. Then, $f(A)$ is a neutrosophic α-open set in Y. It follows from Proposition 2.1 that $f(A)$ is a neutrosophic semiopen set so that f is a neutrosophic semiopen function. □

Proposition 2.7. Let (X, T) and (Y, S) are neutrosophic topological spaces. If $f : (X, T) \to (Y, S)$ is neutrosophic α-open then it is neutrosophic preopen.

3 Neutrosophic Continuity

Definition 3.1. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). Then f is called a neutrosophic pre-continuous function if $f^{-1}(B)$ is a neutrosophic preopen set in X for every neutrosophic open set B in Y.

Proposition 3.1. For a function f from a neutrosophic topological spaces (X, T) to an (Y, S), the following are equivalent.

(i) f is neutrosophic pre-continuous.

(ii) $f^{-1}(B)$ is a neutrosophic preclosed set in X for every neutrosophic closed set B in Y.

(iii) $Ncl(Nint(f^{-1}(A))) \subseteq f^{-1}(Ncl(A))$ for every neutrosophic set A in Y.

Proof. $(i) \Rightarrow (ii)$ The Proof is straightforward.
$(ii) \Rightarrow (iii)$ Let A be a neutrosophic set in Y. Then $Ncl(A)$ is neutrosophic closed. It follows from (ii) that $f^{-1}(Ncl(A))$ is a neutrosophic preclosed set in X so that $Ncl(Nint(f^{-1}(A))) \subseteq Ncl(Nint(f^{-1}(Ncl(A)))) \subseteq f^{-1}(Ncl(A))$.
$(iii) \Rightarrow (i)$ Let A be a neutrosophic open set in Y. Then $\overline{A}$ is a neutrosophic closed set in Y, and so $Ncl(Nint(f^{-1}(\overline{A}))) \subseteq f^{-1}(Ncl(\overline{A})) = f^{-1}(A)$. This implies that $\overline{Nint(Ncl(f^{-1}(A)))} = Ncl(\overline{Ncl(f^{-1}(A))}) = Ncl(Nint(\overline{f^{-1}(A)})) = Ncl(Nint(f^{-1}(\overline{A}))) \subseteq f^{-1}(\overline{A}) = \overline{f^{-1}(A)}$, and thus $f^{-1}(A) \subseteq Nint(Ncl(f^{-1}(A)))$. Hence $f^{-1}(A)$ is a neutrosophic preopen set in X, and f is neutrosophic precontinuous. □

Definition 3.2. Let $x_{r,t,s}$ be a neutrosophic point of a neutrosophic topological space (X, T). A neutrosophic set A of X is called neutrosophic neighbourhood of $x_{r,t,s}$ if there exists a neutrosophic open set B in X such that $x_{r,t,s} \in B \subseteq A$.

Proposition 3.2. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). Then the following assertions are equivalent.

(i) f is a neutrosophic pre-continuous function.

(ii) For each neutrosophic point $x_{r,t,s} \in X$ and every neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a neutrosophic preopen set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$.

(iii) For each neutrosophic point $x_{r,t,s} \in X$ and every neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a neutrosophic preopen set B in X such that $x_{r,t,s} \in B$ and $f(B) \subseteq A$

Proof. $(i) \Rightarrow (ii)$ Let $x_{r,t,s}$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{r,t,s})$. Then there exists a neutrosophic open set B in Y such that $f(x_{r,t,s}) \in B \subseteq A$. Since f is a neutrosophic pre-continuous function, we know that $f^{-1}(B)$ is a neutrosophic preopen set in X and $x_{r,t,s} \in f^{-1}(f(x_{r,t,s})) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. Consequently (ii) is valid.
$(ii) \Rightarrow (iii)$ Let $x_{r,t,s}$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{r,t,s})$. The condition (ii) implies that there exists a neutrosophic preopen set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$ so that $x_{r,t,s} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (iii) is true.
$(iii) \Rightarrow (i)$ Let B be a neutrosophic open set in Y and let $x_{r,t,s} \in f^{-1}(B)$. Then $f(x_{r,t,s}) \in B$, and so B is a neutrosophic neighbourhood of $f(x_{r,t,s})$ since B is neutrosophic open set. It follows from (iii) that there exists a neutrosophic preopen set A in X such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$ so that $x_{r,t,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Applying Propostion 2.4 induces that $f^{-1}(B)$ is a neutrosophic preopen set in X. Therefore, f is a neutrosophic pre-continuous function. □

Definition 3.3. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). Then f is called a neutrosophic α-continuous function if $f^{-1}(B)$ is a neutrosophic α-open set in X for every neutrosophic open set B in Y.

Proposition 3.3. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S) that satisfies $Ncl(Nint(Ncl(f^{-1}(B)))) \subseteq f^{-1}(Ncl(B))$ for every neutrosophic set B in Y. Then f is a neutrosophic α-continuous function.

Proof. Let B be an neutrosophic open set in Y. Then $\overline{B}$ is a neutrosophic closed set in Y, which implies that from hypothesis that $Ncl(Nint(Ncl(f^{-1}(\overline{B})))) \subseteq f^{-1}(Ncl(\overline{B})) = f^{-1}(\overline{B})$. It follows that

$$\begin{aligned}
\overline{Nint(Ncl(Nint(f^{-1}(B))))} &= Ncl(\overline{Ncl(Nint(f^{-1}(B)))}) \\
&= Ncl(Nint(\overline{Nint(f^{-1}(B))})) \\
&= Ncl(Nint(Ncl(\overline{f^{-1}(B)}))) \\
&= Ncl(Nint(Ncl(f^{-1}(\overline{B})))) \subseteq f^{-1}(\overline{B}) \\
&= \overline{f^{-1}(B)}
\end{aligned}$$

so that $f^{-1}(B) \subseteq Nint(Ncl(Nint(f^{-1}(B))))$. This shows that $f^{-1}(B)$ is a neutrosophic α-open set in X. Hence, f is a neutrosophic α-continuous function. □

Proposition 3.4. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). Then the following assertions are equivalent.

(i) f is neutrosophic α-continuous.

(ii) For each neutrosophic point $x_{r,t,s} \in X$ and every neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a neutrosophic α-open set B in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$.

(iii) For each neutrosophic point $x_{r,t,s} \in X$ and every neutrosophic neighbourhood A of $f(x_{r,t,s})$, there exists a neutrosophic α-open set B in X such that $x_{r,t,s} \in B$ and $f(B) \subseteq A$

Proof. $(i) \Rightarrow (ii)$ Let $x_{r,t,s}$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{r,t,s})$. Then there exists a neutrosophic open set B in Y such that $f(x_{r,t,s}) \in B \subseteq A$. Since f is neutrosophic α-continuous, we know that $f^{-1}(B)$ is a neutrosophic α-open set in X and $x_{r,t,s} \in f^{-1}(f(x_{r,t,s})) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. Consequently (ii) is valid.
$(ii) \Rightarrow (iii)$ Let $x_{r,t,s}$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{r,t,s})$. The condition (ii) implies that there exists a neutrosophic α-open set B

in X such that $x_{r,t,s} \in B \subseteq f^{-1}(A)$ so that $x_{r,t,s} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (iii) is true.
$(iii) \Rightarrow (i)$ Let B be a neutrosophic open set in Y and let $x_{r,t,s} \in f^{-1}(B)$. Then $f(x_{r,t,s}) \in B$, and so B is a neutrosophic neighbourhood of $f(x_{r,t,s})$ since B is neutrosophic open set. It follows from (iii) that there exists a neutrosophic α-open set A in X such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$ so that $x_{r,t,s} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Applying Proposition 2.4 induces that $f^{-1}(B)$ is a neutrosophic α-open set in X. Therefore, f is a neutrosophic α-continuous function. □

Proposition 3.5. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). If f is neutrosophic α-continuous, then it is neutrosophic semi-continuous.

Proof. Let B be a neutrosophic open set in Y. Since f is neutrosophic α-continuous, $f^{-1}(B)$ is a neutrosophic semiopen set in X. It follows from Proposition 2.1 that $f^{-1}(B)$ is a neutrosophic semiopen set in X so that f is a neutrosophic semi-continuous function. □

Proposition 3.6. Let f be a function from a neutrosophic topological space (X, T) to a neutrosophic topological space (Y, S). If f is neutrosophic α-continuous, then it is neutrosophic pre-continuous.

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