A Correspondence between Temporal Description Logics

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ABSTRACT. *In this paper, we investigate the relationship between two decidable interval-based temporal description logics that have been proposed in the literature, $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$. Although many aspects of these two logics are quite similar, the two logics suggest two rather different paradigms for representing temporal conceptual knowledge. In this paper, we exhibit a reduction from $\mathcal{TL}$-$\mathcal{ALCF}$ concepts to $\mathcal{ALCF}(A)$ concepts that serves two purposes: first, it nicely illustrates the relationship between the two knowledge representation paradigms; and second, it provides a tight* PSPACE *upper bound for $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiabiliy, whose complexity was previously unknown.*

KEYWORDS: *description logics, temporal logics, computational complexity.*

1. Introduction

Description Logics (DLs) are a family of logic-based formalisms for representing and reasoning on conceptual knowledge, which have over the the last 20 years been successfully applied to a large number of application problems [BAA 03b]. Important characteristics of description logics are high expressivity together with sound, complete and terminating reasoning algorithms. Although expressive DLs typically have a rather high theoretical complexity (often EXPTIME-complete), highly optimized reasoners, such as FaCT [HOR 00], RACER [HAA 01], and DLP [PAT 99], have been developed and exhibit a quite impressive performance on real applications [BAA 03b].

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Temporal extensions of logic formalisms are relevant to capture the evolving behavior of dynamic domains, and they have been extensively considered in both artificial intelligence and theoretical computer science. In particular, temporal logical formalisms have been studied and applied in areas such as specification and verification of computer programs [PNU 86, EME 90], temporal information systems [GAB 94, CHO 98, CHO 03], planning and natural language [ALL 91, ALL 94, BEN 95].

Since the incorporation of temporal aspects also plays an important role in many application areas of description logics such as reasoning about temporal database schemas [ART 99b, ART 02, ART 03] and reasoning about actions and plans [ART 98, ART 99a], in the last years an increasing interest in temporal description logic (TDL) could be observed—see [ART 01, BAA 03a] for a survey. When constructing a temporal description logic, one of the most important decisions to be made is whether time points or time intervals should be used as the underlying temporal primitive [ART 01]. As known from temporal logic and other areas of artificial intelligence, this decision has a severe impact on the expressiveness and computational properties of the resulting logic [GAB 94, GAB ar, GOR 03b, GOR 03a]. In DL research, both routes haven been taken as witnessed by a series of papers on point-based TDLs [SCH 93, WOL 98, WOL 99, STU 01, LUT 01, ART 02, GAB 03], and a number of papers on interval-based ones [SCH 90, ART 94, BET 97, ART 98, LUT ar].

Interval-based TDLs have the advantage that they provide an attractive temporal expressivitiy much richer than the expressivity of point-based TDLs. On the other hand, the computational behavior of interval-based TDLs is often problematic: even very basic formalisms often turn out to be undecidable. An important example is the interval-based TDL proposed by Schmiedel [SCH 90], which is very natural but undecidable since it contains Halpern and Shoham's (undecidable) interval-based temporal logic—called $\mathcal{HS}$ in the following—as a fragment [VEN 90, HAL 91]. Due to these computational problems, one of the prime goals of this research area has been to identify decidable TDLs that are expressive enough to allow the representation of temporal conceptual knowledge in relevant application areas.

In this paper, we are concerned with two decidable interval-based TDLs: $\mathcal{TL}$-$\mathcal{ALCF}$ [ART 98], and $\mathcal{ALCF}(\mathcal{D})$ [LUT 02a]. $\mathcal{TL}$-$\mathcal{ALCF}$ is close in spirit to Schmiedel's undecidable temporal DL, and thus also to the temporal logic $\mathcal{HS}$. It was developed for reasoning about actions and plans [ART 98, ART 99a], and is well-suited for application domains in which objects have properties that vary over time. For example, in $\mathcal{TL}$-$\mathcal{ALCF}$ we can describe the evolution of students using the following concept:

$$\diamond(\mathtt{x},\mathtt{y})(\mathtt{y}\ \mathtt{s}\ \mathtt{x}).(\mathtt{Student}@\mathtt{x} \sqcap \mathtt{Bachelor\text{-}Student}@\mathtt{y}).$$

Here, $\mathtt{x}$ and $\mathtt{y}$ denote time intervals and $(\mathtt{y}\ \mathtt{s}\ \mathtt{x})$ states that $\mathtt{x}$ and $\mathtt{y}$ begin at the same time point, but $\mathtt{y}$ ends before $\mathtt{x}$. Thus, the described persons are students for some time interval $\mathtt{x}$ and bachelor students for some initial sub-interval $\mathtt{y}$ of $\mathtt{x}$ (since they become master's students or PhD students afterwards, which is not modeled for simplicity).

$\mathcal{TL}$-$\mathcal{ALCF}$ is equipped with a rather rich language for expressing temporal relationships that is based on the-well known Allen relations for expressing the possible relationships between time intervals [ALL 83]. In [ART 98], Artale and Franconi show that concept satisfiability and subsumption, the fundamental reasoning tasks in description logics, are both decidable for $\mathcal{TL}$-$\mathcal{ALCF}$. To do this, they use algorithms that first convert concepts into a certain normal form by means of a quite complex syntactic rewriting, and then apply two classical reasoning procedures, one developed for temporal constraint networks and one for description logics, to reason on the normalized concept.

The second description logic addressed in this paper, $\mathcal{ALCF}(\mathcal{D})$, is not a temporal DL in its general form. Rather, it is equipped with so-called concrete domains, which are used for representing qualities of real-world entities that are of a "concrete nature": e.g. lengths, weights, temperatures, durations, and spatial extensions [LUT 03]. The concrete domain $\mathcal{D}$ of $\mathcal{ALCF}(\mathcal{D})$ is not fixed, but rather can $\mathcal{ALCF}(\mathcal{D})$ be "instantiated" with a number of different concrete domains. In [LUT 97, LUT ar], it is shown that a concrete domain A based on temporal intervals and the Allen relations yields an instantiation $\mathcal{ALCF}(A)$ of $\mathcal{ALCF}(\mathcal{D})$ that is well-suited for interval-based reasoning with temporal knowledge.

The paradigm underlying the representation of temporal conceptual knowledge with $\mathcal{ALCF}(A)$ is quite different from the one of $\mathcal{TL}$-$\mathcal{ALCF}$. While $\mathcal{TL}$-$\mathcal{ALCF}$ is well-suited for reasoning about objects whose properties vary over time, in $\mathcal{ALCF}(A)$ objects are associated with a fixed temporal extension that can be understood as their lifespan, and during which all of their properties remain constant. It is then possible to enforce temporal constraints on the lifespans of related objects. For example, we can define a summer semester as a semester which is properly contained in some year (in contrast to winter semesters, which overlap two years):

$$\mathsf{Semester} \sqcap \exists \mathsf{in\text{-}year}.\mathsf{Year} \sqcap \exists time, (\mathsf{in\text{-}year} \circ time).\mathsf{during}.$$

The first conjunct states that the described objects are semesters, whereas the second conjunct states that semesters are related to the year in which they take place via the functional relation in-year. Finally, the last conjunct says that the lifespan of the semester is properly contained in the lifespan of the associated year. It has been shown that satisfiability and subsumption of $\mathcal{ALCF}(A)$-concepts is decidable and PSPACE-complete [LUT 02c].

Intuitively, the two TDLs $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$ are closely related: they both allow the representation of temporal conceptual knowledge based on time intervals, and they both contain the non-temporal DL $\mathcal{ALCF}$ as a proper fragment. Nevertheless, the different underlying paradigms make it surprisingly hard to relate the expressive power of the two logics. The purpose of the current paper is twofold:

1) Understand the relationship between $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$ in terms of their expressivity;

2) Provide a tight PSPACE complexity bound for concept satisfiability in $\mathcal{TL}$-$\mathcal{ALCF}$.

More precisely, we start with $\mathcal{TL}$-$\mathcal{ALCF}$ and show, on an intuitive level, how $\mathcal{TL}$-$\mathcal{ALCF}$ concepts can be translated into $\mathcal{ALCF}(A)$ concepts that have the same meaning. This shows how the gap between the two knowledge representation paradigms of $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$ can be bridged. Then, we formalize the translation by polynomially reducing $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability to $\mathcal{ALCF}(A)$ concept satisfiability. Due to the known PSPACE complexity of $\mathcal{ALCF}(A)$, this yields a PSPACE upper bound for $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability, which is tight. An additional advantage of the reduction is that "practicable" reasoning becomes available for $\mathcal{TL}$-$\mathcal{ALCF}$. Indeed, all modern DL reasoners such as the ones initially mentioned are based on tableau-style reasoning procedures [BAA 00]. For the logic $\mathcal{ALCF}(A)$, such a procedure has been developed in [LUT 02c]. In contrast, no (terminating) tableau-style algorithms have yet been found for logics of the $\mathcal{TL}$-$\mathcal{ALCF}$ family. Via our translation, the $\mathcal{ALCF}(A)$ decision procedure can be used for $\mathcal{TL}$-$\mathcal{ALCF}$, thus replacing the less practicable reasoning methods based on syntactic rewriting.

This paper is organized as follows: in Section 2, we introduce the syntax and semantics of $\mathcal{TL}$-$\mathcal{ALCF}$, together with a running example. In Section 3, we give the syntax and semantics of $\mathcal{ALCF}(A)$, and show how this logic is able to express the example introduced in Section 2. Based on this example translation, we discuss how the different paradigms of temporal-conceptual knowledge representation underlying $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$ are related. The translation is made precise in Section 4, where we use the ideas of Section 3 to reduce $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability to $\mathcal{ALCF}(A)$ concept satisfiability. In this way, we demonstrate the generality of the translation technique proposed in Section 3 and obtain a PSPACE-completeness result for $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability. Section 5 makes some conclusions and shows future directions.

2. The logic $\mathcal{TL}$-$\mathcal{ALCF}$

The temporal description logic $\mathcal{TL}$-$\mathcal{ALCF}$ [ART 94, ART 98] can be viewed as a combination of the non-temporal description logic $\mathcal{ALCF}$ [HOL 90] with the interval-based temporal logic $\mathcal{HS}$ of Halpern and Shoham [HAL 91]. However, to obtain decidable reasoning problems, $\mathcal{TL}$-$\mathcal{ALCF}$ allows only existential temporal quantifiers, but no universal temporal quantifiers—thus including only a fragment of $\mathcal{HS}$. Technically, $\mathcal{TL}$-$\mathcal{ALCF}$ can be regarded as a decidable fragment of first-order interval temporal logic.

The combinatory character of $\mathcal{TL}$-$\mathcal{ALCF}$ is reflected by its syntax, which is divided into the temporal part $\mathcal{TL}$ and the atemporal part $\mathcal{ALCF}$. We fix countably infinite and pairwise disjoint sets of *atomic concepts*, *roles*, *features*, *parametric features*, and *temporal variables*. Then, $\mathcal{TL}$-$\mathcal{ALCF}$ *concepts* are built following the syntax rules in Figure 1. In the figure and throughout this paper, we use

– A to denote atomic concepts,

– C, D to denote (temporal) $\mathcal{TL}$-$\mathcal{ALCF}$ concepts,

$\mathcal{TL}$	C, D	$\rightarrow$	$E \mid$	(non-temporal concept)
			$C \sqcap D \mid$	(conjunction)
			$C@X \mid$	(qualifier)
			$C[Y]@X \mid$	(substitutive qualifier)
			$\diamond(\overline{X})\overline{Tc}.C$	(temporal quantifier)
	Tc	$\rightarrow$	$(X\ r\ Y)$	(temporal constraint)
	$\overline{Tc}$	$\rightarrow$	$Tc \mid Tc\ \overline{Tc}$	
	r, s	$\rightarrow$	$r\ ,\ s \mid$	(disjunction)
			$\mathsf{s} \mid \mathsf{mi} \mid \mathsf{f} \mid \ldots$	(Allen's relations)
	X, Y	$\rightarrow$	$\sharp \mid \mathrm{x} \mid \mathrm{y} \mid \ldots$	(temporal variables)
	$\overline{X}$	$\rightarrow$	$X \mid X\ \overline{X}$	
$\mathcal{ALCF}$	E, F	$\rightarrow$	$A \mid$	(atomic concept)
			$\neg E \mid$	(complement)
			$E \sqcap F \mid$	(conjunction)
			$E \sqcup F \mid$	(disjunction)
			$\forall R.E \mid$	(universal quantifier)
			$\exists R.E \mid$	(existential quantifier)
			$p : E \mid$	(selection)
			$p{\downarrow}q \mid$	(agreement)
			$p{\uparrow}q \mid$	(disagreement)
			$p{\uparrow}$	(undefinedness)
	p, q	$\rightarrow$	$f \mid$	(atomic feature)
			$\star g \mid$	(atomic parametric feature)
			$p \circ q$	(path)

Figure 1. *Syntax rules for the description logic $\mathcal{TL}$-$\mathcal{ALCF}$*

– E, F to denote (non-temporal) $\mathcal{ALCF}$ concepts,

– R to denote roles,

– f to denote (non-parametric) features,

– $\star g$ to denote parametric features,

– p and q to denote *paths*, i.e. finite sequences $\gamma_1 \circ \cdots \circ \gamma_k$, where each γ_i is a feature or a parametric feature,

– X, Y to denote temporal variables, and

– r, s to denote (Allen's) interval relations.

The $\star$ symbol is not intended as an operator, but only used to distinguish parametric from non-parametric features. For the basic temporal interval relations, Allen's notation [ALL 83] is used: before (b), meets (m), during (d), overlaps (o), starts (s), finishes (f), equal (=), after (a), met-by (mi), contains (di), overlapped-by (oi), started-by (si), and finished-by (fi).

Due to the wealth of expressive means, a first encounter with $\mathcal{TL}$-$\mathcal{ALCF}$'s syntax can be slightly confusing. We will give some intuitive examples after introducing the semantics. However, an in-depth introduction to knowledge representation with $\mathcal{TL}$-$\mathcal{ALCF}$ is out of the scope of this paper, and we refer the interested reader to [ART 98]. We should also like to note that the purpose of many of $\mathcal{TL}$-$\mathcal{ALCF}$'s operators is to allow an intuitive representation of temporal knowledge. Technically, they can be viewed as syntactic sugar: $\mathcal{TL}$-$\mathcal{ALCF}$ concepts can be converted into equivalent ones in a quite convenient normal form, which is introduced in Section 4.

The core of the temporal part of $\mathcal{TL}$-$\mathcal{ALCF}$ is constituted by the temporal existential quantifier "$\diamond$" and by the "@" operator. The $\diamond$ operator introduces temporal variables that stand for time intervals, and relates such variables via temporal constraints based on the Allen relations. Then the @ operator allows to specify which concepts are "true" at intervals denoted by temporal variables. The special temporal variable $\sharp$, usually called now, is intended as the reference interval and cannot be bound by the temporal quantifier ($\diamond$). Thus, $\sharp$ is a free temporal variable in each $\mathcal{TL}$-$\mathcal{ALCF}$ concept in which it occurs. In the following, we only admit concepts that have no variables except $\sharp$ as their free variable.

$\mathcal{TL}$-$\mathcal{ALCF}$ is provided with a two-dimensional semantics, which is defined in several steps. We start with assuming a temporal structure $\mathcal{T} = (\mathcal{P}, <)$, where $\mathcal{P}$ is a set of time points and $<$ is a linear, unbounded, and dense order on $\mathcal{P}$. The *interval set* of a structure $\mathcal{T}$ is defined as the set $\mathcal{T}^\star_<$ of all closed proper intervals $[u, v] \doteq \{x \in \mathcal{P} \mid u \leq x \leq v, u \neq v\}$ in $\mathcal{T}$. An *interpretation* $\mathcal{I} \doteq \langle \mathcal{T}^\star_<, \Delta^\mathcal{I}, \cdot^\mathcal{I} \rangle$ consists of

– a set $\mathcal{T}^\star_<$ (the *interval set* of the selected temporal structure $\mathcal{T}$),

– a set $\Delta^\mathcal{I}$ (the *domain* of $\mathcal{I}$), and

– a function $\cdot^\mathcal{I}$ (the *interpretation function* of $\mathcal{I}$), which gives a meaning to atomic concepts, roles, features and parametric features:

$$A^{\mathcal{I}} \subseteq \mathcal{T}^{\star}_{<} \times \Delta^{\mathcal{I}}; \quad R^{\mathcal{I}} \subseteq \mathcal{T}^{\star}_{<} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}};$$

$$f^{\mathcal{I}} : (\mathcal{T}^{\star}_{<} \times \Delta^{\mathcal{I}}) \overset{partial}{\longmapsto} \Delta^{\mathcal{I}}; \quad \star g^{\mathcal{I}} : \Delta^{\mathcal{I}} \overset{partial}{\longmapsto} \Delta^{\mathcal{I}}$$

Note the relationship between roles, features, and parametric features: first, features are simply roles that are required to be functional; second, parametric features differ from features in being independent from time, i.e., they are (temporally) *global* functional roles.

$$(\mathsf{s})^{\mathcal{E}} = \{([u,v],[u_1,v_1]) \in \mathcal{T}^{\star}_{<} \times \mathcal{T}^{\star}_{<} \mid u = u_1 \wedge v < v_1\}$$

...(similarly for the other Allen relations)

$$\begin{aligned}
(r\textbf{,}\ s)^{\mathcal{E}} &= r^{\mathcal{E}} \cup s^{\mathcal{E}} \\
\langle \overline{X}, \overline{Tc} \rangle^{\mathcal{E}} &= \{\mathcal{V} : \overline{X} \mapsto \mathcal{T}^{\star}_{<} \mid \forall (X\ r\ Y) \in \overline{Tc}\textbf{.}\ (\mathcal{V}(X), \mathcal{V}(Y)) \in r^{\mathcal{E}}\}
\end{aligned}$$

$$\begin{aligned}
A^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \Delta^{\mathcal{I}} \mid (t,a) \in A^{\mathcal{I}}\} \\
(\neg C)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} \\
(C \sqcap D)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} \cap D^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} \\
(C \sqcup D)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} \cup D^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} \\
(\forall R.C)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \Delta^{\mathcal{I}} \mid \forall b.(a,b) \in R^{\mathcal{I}}_t \Rightarrow b \in C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}\} \\
(\exists R.C)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a,b) \in R^{\mathcal{I}}_t \wedge b \in C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}\} \\
(p : C)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \mathit{dom}\, p^{\mathcal{I}}_t \mid p^{\mathcal{I}}_t(a) \in C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}\} \\
(p{\downarrow}q)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \mathit{dom}\, p^{\mathcal{I}}_t \cap \mathit{dom}\, q^{\mathcal{I}}_t \mid p^{\mathcal{I}}_t(a) = q^{\mathcal{I}}_t(a)\} \\
(p{\uparrow}q)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \mathit{dom}\, p^{\mathcal{I}}_t \cap \mathit{dom}\, q^{\mathcal{I}}_t \mid p^{\mathcal{I}}_t(a) \neq q^{\mathcal{I}}_t(a)\} \\
(p{\uparrow})^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \Delta^{\mathcal{I}} \setminus \mathit{dom}\, p^{\mathcal{I}}_t \\
(C@X)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= C^{\mathcal{I}}_{\mathcal{V},\mathcal{V}(X),\mathcal{H}} \\
(C[Y]@X)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}\cup\{Y\mapsto\mathcal{V}(X)\}} \\
(\diamond(\overline{X})\overline{Tc}.C)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}} &= \{a \in \Delta^{\mathcal{I}} \mid \exists \mathcal{W}\textbf{.}\ \mathcal{W} \in \langle \overline{X}, \overline{Tc} \rangle^{\mathcal{E}}_{\mathcal{H}\cup\{\sharp\mapsto t\}} \wedge a \in C^{\mathcal{I}}_{\mathcal{W},t,\emptyset}\} \\
R^{\mathcal{I}}_t &= \{(a,b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid (t,a,b) \in R^{\mathcal{I}}\} \\
f^{\mathcal{I}}_t(a) = b &\text{ iff } f^{\mathcal{I}}(t,a) = b \\
(\gamma \circ q)^{\mathcal{I}}_t(a) = b &\text{ iff } q^{\mathcal{I}}_t(\gamma^{\mathcal{I}}_t(a)) = b \\
\star g^{\mathcal{I}}_t &= \star g^{\mathcal{I}}
\end{aligned}$$

Figure 2. *The $\mathcal{TL}$-$\mathcal{ALCF}$ semantics*

The second step in defining $\mathcal{TL}$-$\mathcal{ALCF}$'s semantics consists of dealing with temporal constraint networks that occur inside the $\diamond$ operator. These networks are one of the most common formalisms for temporal reasoning in AI, see e.g. [ALL 83, VIL 90, NEB 95]. Formally, a *temporal constraint network* is a labeled directed graph $\langle \overline{X}, \overline{Tc} \rangle$, where $\overline{X}$ is a set of variables representing the nodes and $\overline{Tc}$ is a set of temporal constraints representing the labeled edges as defined in Figure 1. The semantics of temporal constraint networks is defined using *variable assignments*, i.e. total functions $\mathcal{V} : \overline{X} \mapsto \mathcal{T}^{\star}_{<}$ associating an interval to each temporal variable from a set $\overline{X}$. As defined by the *temporal interpretation function* $\cdot^{\mathcal{E}}$ in the upper half of Figure 2, an interpretation of a temporal constraint network is a set of variable assignments that satisfy the temporal constraints. The notation $\langle \overline{X}, \overline{Tc} \rangle^{\mathcal{E}}_{\{x_1 \mapsto t_1, x_2 \mapsto t_2, \ldots\}}$, used to interpret concept expressions in the next step, denotes the subset of $\langle \overline{X}, \overline{Tc} \rangle^{\mathcal{E}}$ where the variable x_i is mapped to the interval value t_i.

We can now perform the last step of defining $\mathcal{TL}$-$\mathcal{ALCF}$'s semantics. The *interpretation* $C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}$ of a $\mathcal{TL}$-$\mathcal{ALCF}$ concept C with free variables $x_1, \ldots, x_k$ (possibly including $\sharp$) is based on

– a variable assignment $\mathcal{V}$ such that $x_1, \ldots, x_k$ are in the domain of $\mathcal{V}$,

– an interval $t \in \mathcal{T}^{\star}_{<}$, and

– an *assignment constraint* $\mathcal{H} = \{y_1 \mapsto t_1, \ldots\}$ with y_i variable and $t_i \in \mathcal{T}^{\star}_{<}$.

The exact details of defining the interpretation of $\mathcal{TL}$-$\mathcal{ALCF}$ concepts can be found in the lower part of Figure 2.

Intuitively, the interpretation $C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}$ of a $\mathcal{TL}$-$\mathcal{ALCF}$ concept C is the set of elements of the domain which are of type C at the time interval t, with the assignment for the free temporal variables in C given by $\mathcal{V}$ (c.f. the definition of $(C@X)^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}$) and with the assignment of variables in the scope of the outermost temporal quantifiers constrained by $\mathcal{H}$. The *natural interpretation function* $C^{\mathcal{I}}_{t}$, being equivalent to the interpretation function $C^{\mathcal{I}}_{\mathcal{V},t,\mathcal{H}}$ with any $\mathcal{V}$ such that $\mathcal{V}(\sharp) = t$, and $\mathcal{H} = \emptyset$, is introduced as an abbreviation. An interpretation $\mathcal{I}$ is a *model* for a concept C if, for some $t \in \mathcal{T}^{\star}_{<}$, $C^{\mathcal{I}}_{t} \neq \emptyset$. If a concept has a model, then it is *satisfi able*, otherwise it is *unsatisfi able*.

We will now informally discuss the intended *meaning* of $\mathcal{TL}$-$\mathcal{ALCF}$ concepts. As already noted, a central role is played by the temporal existential quantifier "$\diamond$" and the temporal qualification operator "@". For example, to represent all the objects that satisfy a concept C at a time interval that is after the "current interval", we can write

$$\diamond(x)(x \ \mathsf{a} \ \sharp).(C@x).$$

Here, the $\diamond$ operator introduces the new variable x and ensures that the time interval it denotes is located after the current interval $\sharp$. Then, the @ operator "evaluates" C at x thus ensuring that C holds at the time interval denoted by x.

Let us now consider some more interesting examples from the well-known blocks world domain. First, we define a concept representing the action of stacking a block on top of another block[1].

$$\texttt{Basic-Stack} = \Diamond(x\ y)(x\ \mathsf{m}\ \sharp)(\sharp\ \mathsf{m}\ y).\ \big((\star\texttt{BLOCK} : \texttt{OnTable})@x \sqcap (\star\texttt{BLOCK} : \texttt{OnBlock})@y\big)$$

Basic-Stack denotes any action involving a $\star$BLOCK that was once OnTable and then OnBlock. The parametric feature $\star$BLOCK plays the role of *formal* parameter of the action, mapping any individual action of type Basic-Stack to the block to be stacked, independently from time. The $\sharp$ interval can be understood as the occurring time of the stacking action. The temporal constraints $(x\ \mathsf{m}\ \sharp)$ and $(\sharp\ \mathsf{m}\ y)$ state that the interval x should meet the interval $\sharp$—the occurrence interval of the action type Basic-Stack—and that $\sharp$ should meet y.

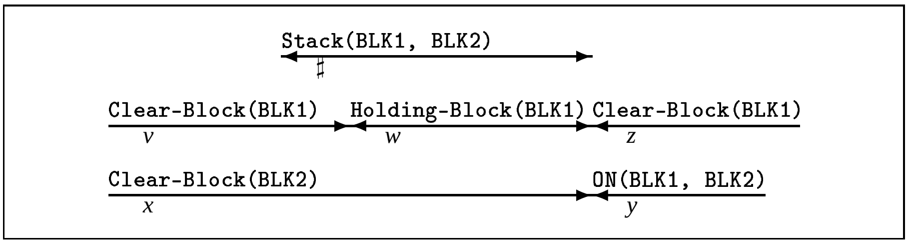

Figure 3. *Temporal dependencies in the defi nition of the* Stack *action*

To illustrate the expressive power of other $\mathcal{TL}$-$\mathcal{ALCF}$ constructors, let us now refine the Basic-Stack example. Figure 3 shows the temporal configuration induced by the stacking action in some more detail: a stacking action involves two blocks—BLK1 and BLK2—which should be both clear at the beginning; the central part of the action consists of grasping one block; at the end, the blocks are one on top of another, and the bottom one is no longer clear. The formal definition of the action Stack is:

$$\begin{aligned}\texttt{Stack} = \Diamond(x\ y\ z\ v\ w)\ & (x\ \mathsf{fi}\ \sharp)(y\ \mathsf{mi}\ \sharp)(z\ \mathsf{mi}\ \sharp)(v\ \mathsf{o}\ \sharp)(w\ \mathsf{f}\ \sharp)(w\ \mathsf{mi}\ v).\\ & \big((\star\texttt{BLOCK2} : \texttt{Clear-Block})@x \sqcap (\star\texttt{BLOCK1}\circ\texttt{ON}\downarrow\star\texttt{BLOCK2})@y \sqcap\\ & (\star\texttt{BLOCK1} : \texttt{Clear-Block})@v \sqcap (\star\texttt{BLOCK1} : \texttt{Holding-Block})@w \sqcap\\ & (\star\texttt{BLOCK1} : \texttt{Clear-Block})@z\big)\end{aligned}$$

Apart from providing a more fain-grained modeling, the new definition of stacking uses the feature agreement constructor: $(\star\texttt{BLOCK1}\circ\texttt{ON}\downarrow\star\texttt{BLOCK2})@y$ indicates that, at interval y, the object ON which $\star$BLOCK1 is placed is $\star$BLOCK2. Note that the world states described at the intervals denoted by v, w, z are the result of an action of *grasping* a previously clear block:

1. In this paper, equalities are used only for introducing names for complex concepts. Such equalities are thus not intended to denote so-called TBoxes, which are frequently used with description logics. Please refer to Section 5 for a brief discussion of reasoning under TBoxes.

$$\texttt{Grasp} \;=\; \Diamond(x\;w\;z)\;\;(x \;\mathsf{o}\; \sharp)(w \;\mathsf{f}\; \sharp)(w \;\mathsf{mi}\; x)(z \;\mathsf{mi}\; \sharp).\\ \big((\star\texttt{BLOCK1} : \texttt{Clear-Block})@x \sqcap\; (\star\texttt{BLOCK1} : \texttt{Holding-Block})@w \sqcap\\ (\star\texttt{BLOCK1} : \texttt{Clear-Block})@z\big)$$

The Stack action can be redefined by making use of the Grasp action:

$$\texttt{Stack} \;=\; \Diamond(x\;y\;u\;v)\;\;(x \;\mathsf{fi}\; \sharp)(y \;\mathsf{mi}\; \sharp)(u \;\mathsf{f}\; \sharp)(v \;\mathsf{o}\; \sharp).\\ \big((\star\texttt{BLOCK2} : \texttt{Clear-Block})@x \sqcap (\star\texttt{BLOCK1}\circ\texttt{ON}\downarrow\star\texttt{BLOCK2})@y \sqcap\\ (\texttt{Grasp}[x]@v)@u\big)$$

The temporal substitutive qualifier $(\texttt{Grasp}[x]@v)$ *renames* within the defined Grasp action the variable x to v. Thus, it is a way of establishing a coreference between two temporal variables ensuring that the temporal constraints peculiar to the renamed variable x are inherited by the substituting interval v. Furthermore, the effect of temporally qualifying the grasping action at u is that the $\sharp$ variable associated to the grasping action—referring to the occurrence time of the action itself—is bound to the interval denoted by u. Because of this binding on the occurrence time of the grasping action, the $\sharp$ variable in the grasping action and the $\sharp$ variable in the stacking action denote different time intervals, so that the grasping action occurs at an interval finishing the occurrence time of the stacking action.

3. The logic $\mathcal{ALCF}(A)$

As noted in the introduction, the temporal description logic $\mathcal{ALCF}(A)$ is obtained by taking the logic $\mathcal{ALCF}(\mathcal{D})$, which provides for concrete domains, and instantiating it with a concrete domain A that is based on time intervals and the Allen interval relations [LUT 97, LUT 02c, LUT ar]. For the sake of brevity, we do not introduce $\mathcal{ALCF}(\mathcal{D})$ in general (see, e.g. [LUT 02c]), but rather define it's specialization $\mathcal{ALCF}(A)$ right away.

The syntax of $\mathcal{ALCF}(A)$ is obtained from the syntax of $\mathcal{ALCF}$ as given in Figure 1 by making the following modifications:

– $\mathcal{ALCF}(A)$ does not provide parametric features.

– $\mathcal{ALCF}(A)$ is equipped with a new sort of feature, so called *temporal features*.

– The temporal part of $\mathcal{ALCF}(A)$ is integrated into the language by adding the *temporal concept constructor*:

$$E, F \;\rightarrow\; \exists p_1, p_2.r,$$

where r is one of the Allen relations, and $p_1, \ldots, p_n$ are *temporal paths*—sequences $\gamma_1 \circ \cdots \circ \gamma_k \circ h$ with $\gamma_1, \ldots, \gamma_k$ features, and h a temporal feature.

In contrast to $\mathcal{TL}$-$\mathcal{ALCF}$, the semantics of $\mathcal{ALCF}(A)$ is not a multi-dimensional one, but rather it is very close to "classical" description logics semantics. To introduce it, we again fix a linear, unbounded, and dense temporal structure $\mathcal{T} = (\mathcal{P}, <)$—this structure is assumed to be the same as in the $\mathcal{TL}$-$\mathcal{ALCF}$ case. Then, an $\mathcal{ALCF}(A)$ *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a set $\Delta^{\mathcal{I}}$(the *domain*), and an interpretation function $\cdot^{\mathcal{I}}$ that assigns a meaning to atomic concepts, roles, features, and temporal features:

$$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}; \quad R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}};$$
$$f^{\mathcal{I}} : \Delta^{\mathcal{I}} \overset{partial}{\longmapsto} \Delta^{\mathcal{I}}; \quad h^{\mathcal{I}} : \Delta^{\mathcal{I}} \overset{partial}{\longmapsto} \mathcal{T}^{\star}_{<}$$

If $p = q \circ h$ is a temporal path, then $p^{\mathcal{I}}$ is defined as $h^{\mathcal{I}}(q^{\mathcal{I}}(\cdot))$, where the meaning of atemporal paths is defined as in Figure 2. Apart from the temporal concept constructor, the interpretation of complex concepts is also determined by Figure 2—just omit the three temporal indices. The semantics of the new temporal concept constructor is given as follows:

$$(\exists p_1, p_2.r)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists t_1, t_2 \in \mathcal{T}^{\star}_{<}\colon\ (a, t_1) \in p_1^{\mathcal{I}} \wedge (a, t_2) \in p_2^{\mathcal{I}} \wedge (t_1, t_2) \in r^{\mathcal{E}}\},$$

where $r^{\mathcal{E}}$ is defined as in Figure 2.

Before discussing the intuitions behind $\mathcal{ALCF}(A)$, let us adopt two conventions: first, we will use parametric feature names of $\mathcal{TL}$-$\mathcal{ALCF}$ as non-temporal feature names in $\mathcal{ALCF}(A)$. Thus, we may write e.g. $\star$`BLOCK` in an $\mathcal{ALCF}(A)$ concept to denote a (non-temporal) feature. Second, in the following we will only need a single temporal feature which will be denoted with $time$.

Comparing the semantics of $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$, the main difference is that $\mathcal{TL}$-$\mathcal{ALCF}$'s semantics is two-dimensional (i.e. based on the product of the domain and the set of time intervals), while $\mathcal{ALCF}(A)$'s semantic is not. The consequences of this difference can be summarized as follows:

– in $\mathcal{TL}$-$\mathcal{ALCF}$, a domain element may be in the extension of a concept *only w.r.t. a given time interval*; moreover, objects are not associated with a "life span", but rather exist at any given time interval.

– in $\mathcal{ALCF}(A)$, concept membership of domain elements is independent of time; moreover, objects are associated with a *unique life span* via the $time$ feature.[2]

The semantic difference induces two different paradigms for the representation of temporal conceptual knowledge. If the aim is to talk about "eternal" objects whose properties vary over time, then $\mathcal{TL}$-$\mathcal{ALCF}$ seems like a natural choice. On the other hand, if we want to reason about temporal entities that are associated with a unique temporal extension, then using $\mathcal{ALCF}(A)$ is the better approach.

2. Or with multiple time spans if we admit more than one temporal feature. This can be very useful: consider e.g. the introduction of distinct temporal features for the life time, the youth, the work time, etc. However, in the context of $\mathcal{TL}$-$\mathcal{ALCF}$ we prefer to stick to a single temporal feature.

Despite these differences, there exists a close and natural relationship between the two temporal description logics $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$. To get a first idea, let us represent the basic stack action from Section 2 in the framework of $\mathcal{ALCF}(A)$:

$$\begin{aligned}\texttt{Basic-Stack} \;\doteq\;& \texttt{step}_1 : (\star\texttt{BLOCK} : \texttt{OnTable}) \sqcap \\ & \texttt{step}_2 : (\star\texttt{BLOCK} : \texttt{OnBlock}) \sqcap \\ & \exists(\texttt{step}_1 \circ time), (\texttt{step}_\sharp \circ time).\mathsf{m} \sqcap \\ & \exists(\texttt{step}_\sharp \circ time), (\texttt{step}_2 \circ time).\mathsf{m}\end{aligned}$$

The concept states that any `Basic-Stack` is related to three objects via the features $\texttt{step}_1, \texttt{step}_2,$ and$\texttt{step}_\sharp$. These objects describe the basic stack action at different time intervals – with $\texttt{step}_\sharp$ representing the occurring time of the action. For each step, a corresponding time interval is associated by the $time$ feature. The relation between these time intervals is described using the temporal concept constructor and resembles the temporal network in the $\mathcal{TL}$-$\mathcal{ALCF}$ definition of the basic stack. In $\texttt{step}_1$, the $\star$BLOCK is `OnTable`, and in $\texttt{step}_2$ it is `OnBlock`. This situation is illustrated in Figure 4.

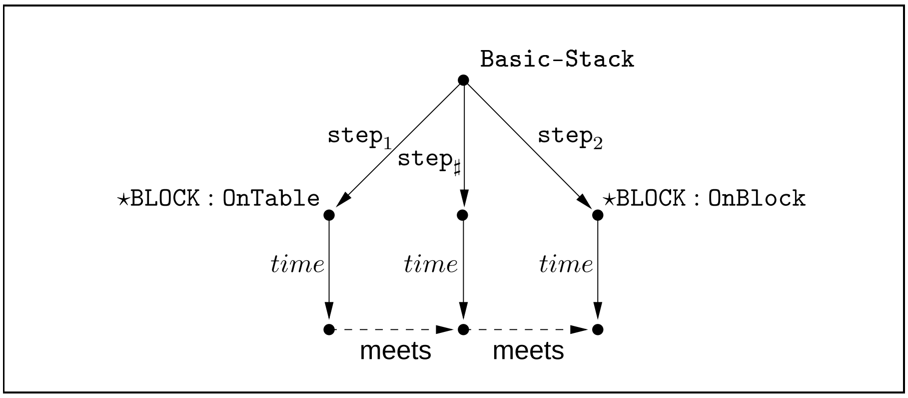

Figure 4. *Model of the $\mathcal{ALCF}(A)$* `Basic-Stack`

Comparing the two definitions of `Basic-Stack`, their main difference can be characterized as follows: in the $\mathcal{TL}$-$\mathcal{ALCF}$ definition, the basic stack is represented by a single logical object, whose properties are defined separately for each temporal interval. To the contrary, in $\mathcal{ALCF}(A)$, the basic stack is represented by a logical "meta-object" (the `Basic-Stack` object itself in the above concept definition) and a set of additional logical "temporal-facet" objects (the $\texttt{step}_\texttt{i}$ successors of the `Basic-Stack` meta-object), each of which has unique properties and represents the basic stack at a unique time interval.

To reduce satisfiability of $\mathcal{TL}$-$\mathcal{ALCF}$ concepts to satisfiability of $\mathcal{ALCF}(A)$ concepts, we exploit the idea suggested by this simple example: the translation must be such that one domain element in models of the $\mathcal{TL}$-$\mathcal{ALCF}$ concepts corresponds to a number of domain elements in models of its $\mathcal{ALCF}(A)$ translation, i.e. one meta-object together with a number of temporal-facet objects that represent the single $\mathcal{TL}$-$\mathcal{ALCF}$ domain element at different time intervals. An additional difficulty is to preserve the temporal invariance of parametric features. As illustrated in the next section, this problem is solved by using the feature agreement constructor of $\mathcal{ALCF}(A)$.

4. The reduction

This section presents the reduction of $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability to $\mathcal{ALCF}(A)$ concept satisfiability. To simplify matters, we will only consider $\mathcal{TL}$-$\mathcal{ALCF}$ concepts in so-called *existential normal form (ENF)*. In this normal form, the only temporal operator that may occur is a single "$\diamond$" operator, i.e. $\mathcal{TL}$-$\mathcal{ALCF}$ concepts in ENF are of the form

$$C = \diamond(\overline{X})\overline{Tc}.Q_0 \sqcap Q_1@X_1 \sqcap \ldots \sqcap Q_n@X_n, \qquad (*)$$

where $\overline{X} = \{X_1, \ldots, X_n\}$ and each Q_i is an (atemporal) $\mathcal{ALCF}$ concept. Additionally, we assume that the $\mathcal{TL}$-$\mathcal{ALCF}$ concepts $Q_0, \ldots, Q_n$ are in *negation normal form (NNF)*, i.e. that negation occurs only in front of concept names. In this case, we will simply say that the concept C is in *normal form (NF)*. As the following proposition shows, normal form can be assumed without loss of generality.

$\neg\top \rightarrow \bot$	$\neg\bot \rightarrow \top$
$\neg(C \sqcap D) \rightarrow \neg C \sqcup \neg D$	$\neg(C \sqcup D) \rightarrow \neg C \sqcap \neg D$
$\neg(\forall R.C) \rightarrow \exists R.\neg C$	$\neg(\exists R.C) \rightarrow \forall R.\neg C$
$\neg\neg C \rightarrow C$	$\neg(p : C) \rightarrow p\uparrow \sqcup\ p : \neg C$
$\neg(p{\downarrow}q) \rightarrow p\uparrow \sqcup\ q\uparrow \sqcup\ p{\uparrow}q$	$\neg(p{\uparrow}q) \rightarrow p\uparrow \sqcup\ q\uparrow \sqcup\ p{\downarrow}q$
$\neg(p\uparrow) \rightarrow p : \top$	

Figure 5. *NNF rewrite rules*

PROPOSITION 1 (EQUIVALENCE OF NF). — *Every $\mathcal{TL}$-$\mathcal{ALCF}$ concept C can be converted in polynomial time into an equivalent concept in normal form.*

PROOF. — In [ART 98], it is shown that every $\mathcal{TL}$-$\mathcal{ALCF}$ concept can be converted in polynomial time to an equivalent one in ENF. We can then convert the $Q_0, \ldots, Q_n$ to NNF by exhaustively applying the rewrite rules in Figure 5. Note that this takes only polynomial time and the length of the resulting concept is polynomial in the length of the original concept. ■

Let C be a $\mathcal{TL}$-$\mathcal{ALCF}$ concept in NF of the form $(*)$. To translate C into an equi-satisfiable $\mathcal{ALCF}(A)$ concept $\Psi(C)$, we introduce the new features $f_0, \ldots, f_n$ (corresponding to the $\mathtt{step}_i$ features in Section 3), the new concept names $A_{i,j}$ for all $0 \leq i, j \leq n$, and the new concrete feature $time$. We assume w.l.o.g. that these features and concept names are not used in the $\mathcal{TL}$-$\mathcal{ALCF}$ concept C. For the remainder of this section, we use the symbol f to denote features that are distinct from the reserved features $f_0, \ldots, f_n$, parametric features are denoted by $\star g$, and γ denotes features that may or may not be parametric, but are distinct from the reserved features. To define the concept $\Psi(C)$, we need to define a number of auxiliary concepts. To start with, we need a mapping from $\mathcal{TL}$-$\mathcal{ALCF}$'s temporal constraint networks to $\mathcal{ALCF}(A)$ concepts.

DEFINITION 2 (TRANSLATION OF TEMPORAL NETWORKS). — *Let $\overline{\mathcal{T}c}$ be a temporal constraint network for the set of variables $\overline{X} = \{X_0, X_1, \ldots, X_n\}$, where $X_0 = \sharp$. For each temporal constraint $(X\ r\ Y) \in \overline{\mathcal{T}c}$, we define an $\mathcal{ALCF}(A)$ concept, $\alpha(X\ r\ Y)$, as follows:*

$$\alpha(X\ r\ Y) := \exists (f_i \circ time), (f_j \circ time).r \quad \textit{if } X = X_i \textit{ and } Y = X_j.$$

Then, the translation $\alpha(\overline{\mathcal{T}c})$ of $\overline{\mathcal{T}c}$ is defined as follows:

$$\alpha(\overline{\mathcal{T}c}) := \bigsqcap_{(X r Y) \in \overline{\mathcal{T}c}} \alpha(X\ r\ Y).$$

The remaining auxiliary concepts—$\Gamma_C, \Omega, \Omega'$—are defined in Figure 6. In the definition of Γ_C, we use $\mathsf{feat}(C)$ to denote the set of all features (either non-parametric or parametric) in C, and $\mathsf{rol}(C)$ to denote the set of all role names in C.

DEFINITION 3 (TRANSLATION OF $\mathcal{TL}$-$\mathcal{ALCF}$ CONCEPTS). — *Given a $\mathcal{TL}$-$\mathcal{ALCF}$ concept C, its $\mathcal{ALCF}(A)$ translation, $\Psi(C)$, is defined as:*

$$\Psi(C) := \alpha(\overline{\mathcal{T}c}) \sqcap \Gamma_C \sqcap \Omega \sqcap \Omega'.$$

Before giving a formal proof of the fact that $\Psi(C)$ and C are equi-satisfiable, let us briefly discuss the various concepts used in the reduction. The concept Ω enforces the existence of $n+1$ temporal-facet objects as f_i-successors of the root object (i.e. the object that satisfies the reduction concept $\Psi(C)$). Thus, this root object is a meta-object in the sense of the previous section. Furthermore, Ω ensures that, for each i, the temporal facet that is an f_i-successor must be a member of $\Psi_i(Q_i)$. The purpose of the Ψ_i translation used here is to insert the f_i features after each feature and role name used in Q_i. This is necessary since not only the root object, but also all other objects are composed of a meta-object and $n+1$ temporal facets.
The concept $\alpha(\overline{\mathcal{T}c})$ associates a $time$ feature to each temporal-facet of the root meta-object ensuring that the values of such $time$ features satisfy all constraints in $\overline{\mathcal{T}c}$. It is interesting to note that *only* the f_i successors of the root meta-object are equipped with

$$\Omega := f_0 : \Psi_0(Q_0) \sqcap \cdots \sqcap f_n : \Psi_n(Q_n)$$

$$\Omega' := \bigsqcap_{0 \le i < j \le n} (\exists (f_i \circ \mathit{time}), (f_j \circ \mathit{time}). =) \rightarrow A_{i,j}$$

$$\begin{aligned}\Gamma_C := & \bigsqcap_{0 \le i < j \le n} A_{i,j} \rightarrow f_i \downarrow f_j \sqcap \\ & \bigsqcap_{0 \le i < j \le n} A_{i,j} \rightarrow \Big(\bigsqcap_{0 \le k \le n} \big(\bigsqcap_{\gamma \in \mathsf{feat}(C)} (f_k : \gamma \uparrow \sqcup f_k : \gamma : A_{i,j}) \sqcap \\ & \qquad \bigsqcap_{R \in \mathsf{rol}(C)} f_k : \forall R. A_{i,j}) \big) \sqcap \\ & \bigsqcap_{\star g \text{ used in C}} \Big((\sqcap_{j=0}^{n} f_j \circ \star g \uparrow) \sqcup (\sqcap_{j=1}^{n} (f_0 \circ \star g) \downarrow (f_j \circ \star g)) \Big)\end{aligned}$$

$$\begin{aligned}\Phi_i(\gamma) &:= \gamma \\ \Phi_i(p \circ \gamma) &:= \Phi_i(p) \circ f_i \circ \gamma\end{aligned}$$

$$\begin{aligned}\Gamma_C^i(\gamma) &:= \gamma : \Gamma_C \\ \Gamma_C^i(\gamma \circ p) &:= \gamma : (\Gamma_C \sqcap f_i : \Gamma_C^i(p))\end{aligned}$$

$$\begin{aligned}\Psi_i(A) &:= A \\ \Psi_i(\neg A) &:= \neg A \\ \Psi_i(D \sqcap E) &:= \Psi_i(D) \ \sqcap \ \Psi_i(E) \\ \Psi_i(D \sqcup E) &:= \Psi_i(D) \ \sqcup \ \Psi_i(E) \\ \Psi_i(\exists R.D) &:= \exists R.(\Gamma_C \sqcap f_i : \Psi_i(D)) \\ \Psi_i(\forall R.D) &:= \forall R.(f_i : \Psi_i(D)) \\ \Psi_i(p : D) &:= \Phi_i(p) : f_i : \Psi_i(D) \sqcap \Gamma_C^i(p) \\ \Psi_i(p \downarrow q) &:= \Phi_i(p) \downarrow \Phi_i(q) \sqcap \Gamma_C^i(p) \sqcap \Gamma_C^i(q) \\ \Psi_i(p \uparrow q) &:= \Phi_i(p) \uparrow \Phi_i(q) \sqcap \Gamma_C^i(p) \sqcap \Gamma_C^i(q) \\ \Psi_i(p \uparrow) &:= \Phi_i(p) \uparrow\end{aligned}$$

Figure 6. *Defi nition of auxiliary concepts*

time intervals via the $time$ feature. As we said before, all successors of such temporal-facet objects implicitly "inherit" the same temporal structure via the Ψ_i translation.
The concept Γ_C serves two purposes. First, the last row of Γ_C uses feature agreements to ensure that parametric features are independent from time, i.e. if two $\mathcal{ALCF}(A)$ domain elements d_1 and d_2 represent two temporal-facet of the same meta-object, then d_1 and d_2 should have the same successor for each parametric feature. Second, together with the Ω' concept and translations Ψ_i and Γ^i_C, Γ_C ensures that if, for a given meta-object, two variables X_i and X_j denote the *same* time interval, then for each successor of such meta-object both f_i and f_j features coincide. The latter is necessary since, in $\mathcal{TL}$-$\mathcal{ALCF}$ models, a domain element together with a time interval *uniquely* identifies concept membership, role membership, etc.

Proof of correctness

Throughout the proofs, we will write $\mathsf{sub}(C)$ to denote the set of all subconcepts of the concept C, including C itself. We now establish the correctness of our reduction. For the sake of clarity, it is split into two propositions.

PROPOSITION 4. — *Let C be a $\mathcal{TL}$-$\mathcal{ALCF}$ concept in normal form. Then $\mathcal{ALCF}(A)$ satisfiability of $\Psi(C)$ implies $\mathcal{TL}$-$\mathcal{ALCF}$ satisfiability of C.*

PROOF. — Let $\mathcal{I}$ be a model of $\Psi(C)$, and let $d_C \in \Psi(C)^{\mathcal{I}}$. We define Δ^* to be the smallest subset of Δ satisfying the following properties:

1) $d_C \in \Delta^*$;

2) if $d \in \Delta^*$, $d' \in \Delta^{\mathcal{I}}$, $0 \le i \le n$, and $\exists R.(\Gamma_C \sqcap f_i : \Psi_i(D)) \in \mathsf{sub}(\Psi(C))$ such that

- $f_i^{\mathcal{I}}(d) \in (\exists R.(\Gamma_C \sqcap f_i : \Psi_i(D)))^{\mathcal{I}}$,
- $(f_i^{\mathcal{I}}(d), d') \in R^{\mathcal{I}}$, and
- $d' \in (\Gamma_C \sqcap f_i : \Psi_i(D))^{\mathcal{I}}$,

then $d' \in \Delta^*$;

3) if $d \in \Delta^*$, $d_1, \ldots, d_k \in \Delta^{\mathcal{I}}$, $0 \le i \le n$, $X \in \mathsf{sub}(\Psi(C))$, and p^* is a path such that

- $d = d_1$,
- X is of the form $\Phi_i(p) : f_i : \Psi_i(D) \sqcap \Gamma^i_C(p)$, and $p^* = p$,
  X is of the form $\Phi_i(p) \downarrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$, and $p^* \in \{p, q\}$, or
  X is of the form $\Phi_i(p) \uparrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$, and $p^* \in \{p, q\}$,
- $f_i^{\mathcal{I}}(d) \in X^{\mathcal{I}}$,
- $p^* = \gamma_1 \circ \cdots \circ \gamma_{k-1}$, and
- $(f_i^{\mathcal{I}}(d_\ell), d_{\ell+1}) \in \gamma_\ell^{\mathcal{I}}$ for $1 \le \ell < k$,

then $d_1, \ldots, d_k \in \Delta^*$.

Obviously, the sub-interpretation of $\mathcal{I}$ induced by Δ^* is rooted by d_C. Moreover, it is not hard to show that $\Delta^* \subseteq \Gamma_C^{\mathcal{I}}$:

– $d_C \in \Gamma_C^{\mathcal{I}}$ by definition of $\Psi(C)$.

– if $d' \in \Delta^*$ due to Property 2, then $d' \in \Gamma_C$ by choice of d';

– Let $d_1, \ldots, d_k \in \Delta^*$ due to Property 3. Using the definition of the concept $\Gamma_C^i(p)$ and the fact that $f_i^{\mathcal{I}}(d_1) \in \Gamma_C^i(p^*)$, it is easily verified that $d_j \in \Gamma_C^{\mathcal{I}}$ for $1 \leq j \leq k$.

We now define a $\mathcal{TL}$-$\mathcal{ALCF}$ interpretation $\mathcal{J}$. For convenience, we set

$$t_i := time^{\mathcal{I}}(f_i^{\mathcal{I}}(d_C)), \text{ for } 0 \leq i \leq n$$

and use this abbreviation for the remainder of the proof. Now set

$$\begin{array}{rcl}
\Delta^{\mathcal{J}} & := & \Delta^* \\
A^{\mathcal{J}} & := & \{(t,d) \mid d \in \Delta^{\mathcal{J}}, t = t_i, \text{ and } f_i^{\mathcal{I}}(d) \in A^{\mathcal{I}} \text{ for some } i \leq n\} \\
R^{\mathcal{J}} & := & \{(t,d,d') \mid d,d' \in \Delta^{\mathcal{J}}, t = t_i, \text{ and } (f_i^{\mathcal{I}}(d), d') \in R^{\mathcal{I}} \text{ for some } i \leq n\} \\
f^{\mathcal{J}} & := & \{(t,d,d') \mid d,d' \in \Delta^{\mathcal{J}}, t = t_i, \text{ and } f^{\mathcal{I}}((f_i^{\mathcal{I}}(d)) = d' \text{ for some } i \leq n\} \\
\star g^{\mathcal{J}} & := & \{(d,d') \mid d,d' \in \Delta^{\mathcal{J}} \text{ and } \star g^{\mathcal{I}}(f_0^{\mathcal{I}}(d)) = d'\}
\end{array}$$

We now prove some important properties of $\mathcal{J}$. Note that the first property implies that the interpretation of non-parametric features in $\mathcal{J}$ is functional as required (the interpretation of parametric features is obviously also functional, but does not depend on the following property).

1) For all $d \in \Delta^*$ and all i, j with $t_i = t_j$, we have $f_i^{\mathcal{I}}(d) = f_j^{\mathcal{I}}(d)$.

Proof: By definition of Ω', $t_i = t_j$ implies $d_C \in A_{i,j}$. Since Δ^* is rooted by d_C and $\Delta^* \subseteq \Gamma_C^{\mathcal{I}}$, the definition of Γ_C (second/third line) yields that $\Delta^* \subseteq A_{i,j}^{\mathcal{I}}$. Again by definition of Γ_C (first line) and since $\Delta^* \subseteq \Gamma_C^{\mathcal{I}}$, this implies $f_i^{\mathcal{I}}(d) = f_j^{\mathcal{I}}(d)$ for all $d \in \Delta^*$.

2) For all $d \in \Delta^*$, all i with $1 \leq i \leq n$, and all parametric features $\star g$, either $\star g^{\mathcal{I}}(f_i^{\mathcal{I}}(d))$ and $\star g^{\mathcal{I}}(f_0^{\mathcal{I}}(d))$ are both undefined, or $\star g^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = \star g^{\mathcal{I}}(f_0^{\mathcal{I}}(d))$.

The proof is easy by considering the fact that $\Delta^* \subseteq \Gamma_C^{\mathcal{I}}$ together with the last line of the definition of Γ_C.

3) Let $d, d' \in \Delta^*$, p be a path not containing the features $f_1, \ldots, f_n$, and $0 \leq i \leq n$. Then $d' \in (\Phi_i(p))^{\mathcal{I}}(f_i^{\mathcal{I}}(d))$ iff $p_{t_i}^{\mathcal{J}}(d) = d'$.

The proof is by induction on the length of p. For the induction start, let p be of length one, i.e. $p = \gamma$. Then $\Phi_i(p) = \gamma$.

First, assume that γ is a non-parametric feature. Then, $d' = \gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d))$ implies $(t_i, d, d') \in \gamma^{\mathcal{J}}$ by definition of $\mathcal{J}$, and thus the "only if" direction holds. For the "if" direction, assume that $(t_i, d, d') \in \gamma^{\mathcal{J}}$. Then there is a j with $0 \leq j \leq n$ such that $t_i = t_j$ and $\gamma^{\mathcal{I}}(f_j^{\mathcal{I}}(d)) = d'$. By Property 1, we have $f_j^{\mathcal{I}}(d) = f_i^{\mathcal{I}}(d)$ and thus $\gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d'$ as required.

Let now γ be a parametric feature. Then, $\gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d'$ iff (by Property 2) $\gamma^{\mathcal{I}}(f_0^{\mathcal{I}}(d)) = d'$ iff $\gamma^{\mathcal{J}}(d) = d'$.

Now for the induction step. Let $p = q \circ \gamma$. Then $\Phi_i(p) = \Phi_i(q) \circ f_i \circ \gamma$. Assume that $(\Phi_i(p))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d'$. Then there is a d'' with $(\Phi_i(q))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d''$ and $\gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d'')) = d'$. By IH, we obtain $q_{t_i}^{\mathcal{J}}(d) = d''$. To prove that $p_{t_i}^{\mathcal{J}}(d) = d'$, it thus remains to show that $\gamma_{t_i}^{\mathcal{J}}(d'') = d'$. In both the non-parametric and the parametric case, this can be done exactly as in the induction start (using the fact that $\gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d'')) = d'$).

Vice-versa, assume that $p_{t_i}^{\mathcal{J}}(d) = d'$. Then there is a d'' such that $q_{t_i}^{\mathcal{J}}(d) = d''$ and $\gamma_{t_i}^{\mathcal{J}}(d'') = d'$. By IH, the former yields $(\Phi_i(q))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d''$. It thus remains to show that $\gamma^{\mathcal{I}}(f_i^{\mathcal{I}}(d'')) = d'$, which can again be done as in the induction start (using the fact that $\gamma_{t_i}^{\mathcal{J}}(d'') = d'$).

We now prove the following, central claim:

CLAIM 5. — For all $d \in \Delta^{\mathcal{J}}$, $0 \leq i \leq n$, and $D \in \mathsf{sub}(C)$, we have that $f_i^{\mathcal{I}}(d) \in \Psi_i(D)^{\mathcal{I}}$ implies $d \in D_{t_i}^{\mathcal{J}}$.

This claim easily yields the desired result: since C is in normal form, it is of the form

$$C = \Diamond(\overline{X})\overline{\mathit{Tc}}.Q_0 \sqcap Q_1@X_1 \sqcap \ldots \sqcap Q_n@X_n,$$

with $\overline{X} = \{X_1, \ldots, X_n\}$. We define a variable assignment $\mathcal{W}$ for by setting $\mathcal{W}(X_i) := t_i$ for $1 \leq i \leq n$. Since $d_C \in \alpha(\overline{\mathit{Tc}})^{\mathcal{I}}$, we have $\mathcal{W} \in \langle \overline{X}, \overline{\mathit{Tc}} \rangle_{\sharp \mapsto t_0}^{\mathcal{E}}$. Using the claim, it is then readily verified that

$$d_C \in (Q_0 \sqcap Q_1@X_1 \sqcap \ldots \sqcap Q_n@X_n)_{\mathcal{W}, t_0, \emptyset}^{\mathcal{J}}.$$

Thus, $d_C \in C_{t_0}^{\mathcal{J}}$ and C is $\mathcal{TL}$-$\mathcal{ALCF}$ satisfiable as required. The proof of the claim is by structural induction:

– D is a concept name. Then, $\Psi_i(D) = A$ and $(t_i, d) \in A^{\mathcal{J}}$ is an immediate consequence of the definition of $\mathcal{J}$.

– $D = \neg A$ (A is a concept name since C is in NNF). Then, $\Psi_i(D) = \neg A$. Suppose $f_i^{\mathcal{I}}(d) \notin A^{\mathcal{I}}$ and $(t_i, d) \in A^{\mathcal{J}}$. Then there is a j with $0 \leq j \leq n$ such that $t_i = t_j$ and $f_j^{\mathcal{I}}(d) \in A^{\mathcal{I}}$. By Property 1, we have $f_j^{\mathcal{I}}(d) = f_i^{\mathcal{I}}(d)$. Thus, $f_i^{\mathcal{I}}(d) \in A^{\mathcal{I}}$, which is a contradiction.

– $D = D_1 \sqcap D_2$. Easy using IH and the semantics.

– $D = D_1 \sqcup D_2$. Easy using IH and the semantics.

– $D = \exists R.E$. Then, $\Psi_i(D) = \exists R.(\Gamma_C \sqcap f_i : \Psi_i(E))$. Since $f_i^{\mathcal{I}}(d) \in \Psi_i(D)^{\mathcal{I}}$, there is a d' such that $(f_i^{\mathcal{I}}(d), d') \in R^{\mathcal{I}}$ and $d' \in (\Gamma_C \sqcap f_i : \Psi_i(E))^{\mathcal{I}}$. By definition of Δ^*, we thus have $d' \in \Delta^*$. Moreover, $f_i^{\mathcal{I}}(d') \in \Psi_i(E)^{\mathcal{I}}$. By definition of $R^{\mathcal{J}}$, we obtain $(t_i, d, d') \in R^{\mathcal{J}}$. By IH and since $f_i^{\mathcal{I}}(d') \in \Psi_i(E)^{\mathcal{I}}$, we get $d' \in E_{t_i}^{\mathcal{J}}$. Thus, $d \in D_{t_i}^{\mathcal{J}}$.

– $D = \forall R.E$. Then, $\Psi_i(D) = \forall R.(f_i : \Psi_i(E))$. Let $(t_i, d, d') \in R^{\mathcal{J}}$. Then there is a j with $0 \leq j \leq n$ such that $t_i = t_j$ and $(f_j^{\mathcal{I}}(d), d') \in R^{\mathcal{I}}$. By Property 1, we have $f_j^{\mathcal{I}}(d) = f_i^{\mathcal{I}}(d)$ and hence $(f_i^{\mathcal{I}}(d), d') \in R^{\mathcal{I}}$. Since $f_i^{\mathcal{I}}(d) \in \Psi_i(D)^{\mathcal{I}}$, we thus have $d' \in (f_i : \Psi_i(E))^{\mathcal{I}}$ and $f_i^{\mathcal{I}}(d') \in \Psi_i(E)^{\mathcal{I}}$. Thus, IH yields $d' \in E_{t_i}^{\mathcal{J}}$ as required.

– $D = p : E$. Then, $\Psi_i(D) = \Phi_i(p) : f_i : \Psi_i(E) \sqcap \Gamma^i_C(p)$. Since $f_i^{\mathcal{I}}(d) \in \Psi_i(D)$, there is a $d' \in \Delta^{\mathcal{I}}$ such that $(\Phi_i(p))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d$ and $f_i^{\mathcal{I}}(d) \in \Psi_i(E)^{\mathcal{I}}$. By Property 3, we thus have $p^{\mathcal{J}}_{t_i}(d) = d'$, and IH yields $d' \in E^{\mathcal{J}}_{t_i}$. Summing up, we obtain $d \in D^{\mathcal{J}}_{t_i}$.

– $D = p \downarrow q$. Then, $\Psi_i(D) = \Phi_i(p) \downarrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$. Since $f_i^{\mathcal{I}}(d) \in \Psi_i(D)^{\mathcal{I}}$, there is a $d' \in \Delta^{\mathcal{I}}$ such that

$$(\Phi_i(p))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = (\Phi_i(q))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d.$$

By definition of Δ^*, we have $d' \in \Delta^*$. By Property 3, we have $p^{\mathcal{J}}_{t_i}(d) = q^{\mathcal{J}}_{t_i}(d) = d'$, and thus $d \in D^{\mathcal{J}}_{t_i}$ as required.

– $D = p \uparrow q$. Then, $\Psi_i(D) = \Phi_i(p) \uparrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$. Analogous to the previous case.

– $D = p\uparrow$. Then, $\Psi_i(D) = \Phi_i(p)\uparrow$. Let $f_i^{\mathcal{I}}(d) \in \Psi_i(D)^{\mathcal{I}}$, and assume that $d \notin D^{\mathcal{J}}_{t_i}$. Then there is a $d' \in \Delta^{\mathcal{J}}$ such that $p^{\mathcal{J}}_{t_i}(d) = d'$. By Property 3, we thus have $(\Phi_i(p))^{\mathcal{I}}(f_i^{\mathcal{I}}(d)) = d'$, which is a contradiction.

■

PROPOSITION 6. — *Let C be a $\mathcal{TL}$-$\mathcal{ALCF}$ concept in normal form. Then $\mathcal{TL}$-$\mathcal{ALCF}$ satisfiability of C implies $\mathcal{ALCF}(A)$ satisfiability of $\Psi(C)$.*

PROOF. — Let $\mathcal{J}$ be a $\mathcal{TL}$-$\mathcal{ALCF}$ model of C and let $d_C \in \Delta^{\mathcal{J}}$ and $t_0 \in \mathcal{T}^{\star}_{<}$ such that $d_C \in C^{\mathcal{J}}_{t_0}$. Let

$$C = \Diamond(\overline{X})\overline{Tc}.Q_0 \sqcap Q_1@X_1 \sqcap \ldots \sqcap Q_n@X_n$$

with $\overline{X} = \{X_1, \ldots, X_n\}$. By the semantics, there exists a variable assignment $\mathcal{W} \in \langle \overline{X}, \overline{Tc}\rangle^{\mathcal{E}}_{\sharp \to t_0}$ such that

$$d_C \in (Q_0 \sqcap Q_1@X_1 \sqcap \ldots \sqcap Q_n@X_n)^{\mathcal{J}}_{\mathcal{W},t_0,\emptyset}. \qquad (*)$$

In the remainder of this proof, we use $X_0 = \sharp$, and t_i to denote $\mathcal{W}(X_i)$, for $1 \le i \le n$. We now construct an $\mathcal{ALCF}$-interpretation $\mathcal{I}$:

$$\begin{array}{rcl}
\Delta^{\mathcal{I}} & := & \Delta^{\mathcal{J}} \cup \{(d, t_i) \mid d \in \Delta^{\mathcal{J}} \text{ and } 0 \le i \le n\} \\
A^{\mathcal{I}} & := & \{(d, t) \mid (t, d) \in A^{\mathcal{J}} \text{ and } t = t_i \text{ for some } i \le n\} \\
f^{\mathcal{I}} & := & \{((d, t), d') \mid (t, d, d') \in f^{\mathcal{J}} \text{ and } t = t_i \text{ for some } i \le n\} \\
\star g^{\mathcal{I}} & := & \{((d, t_i), d') \mid (d, d') \in \star g^{\mathcal{J}} \text{ and } 0 \le i \le n\} \\
R^{\mathcal{I}} & := & \{((d, t), d') \mid (t, d, d') \in R^{\mathcal{J}} \text{ and } t = t_i \text{ for some } i \le n\} \\
time^{\mathcal{I}} & := & \{((d_C, t_i), t_i) \mid 0 \le i \le n\} \\
A^{\mathcal{I}}_{j,\ell} & := & \begin{cases} \Delta^{\mathcal{J}} & \text{if } t_j = t_\ell \\ \emptyset & \text{otherwise} \end{cases} \\
f_i^{\mathcal{I}} & := & \{(d, (d, t_i)) \mid d \in \Delta^{\mathcal{J}} \text{ and } 0 \le i \le n\}
\end{array}$$

for all concept names $A, A_{i,j}$, non-parametric features f, reserved features f_i, parametric features $\star g$, role names R, and $j, \ell \in \{0, \dots, n\}$. We show that $d_C \in \Psi(C)^{\mathcal{I}}$. To this end, it is readily verified that $d_C \in (\alpha(\overline{\mathcal{T}c}) \sqcap \Gamma_C \sqcap \Omega')^{\mathcal{I}}$. It thus remains to show that $d_C \in \Omega^{\mathcal{I}}$. This is obviously an immediate consequence of $(*)$, $\mathcal{I}$'s interpretation of the f_i features, and the following claim:

CLAIM 7. — For all $d \in \Delta^{\mathcal{J}}$, $0 \leq i \leq n$, and $D \in \mathsf{sub}(C)$, we have that $d \in D^{\mathcal{J}}_{t_i}$ implies $(d, t_i) \in \Psi_i(D)^{\mathcal{I}}$.

Before we prove the claim, let us state three useful properties of $\Delta^{\mathcal{I}}$:

1) $\Delta^{\mathcal{J}} \subseteq \Gamma^{\mathcal{I}}_C$, as it is easily verified by considering the definitions of both $A^{\mathcal{I}}_{j,\ell}$ and $\star g^{\mathcal{I}}$.

2) For all $d \in \Delta^{\mathcal{J}}$, paths p not containing the features $f_0, \dots, f_n$, and $i \in \{0, \dots, n\}$, the following holds: if there is a d' with $p^{\mathcal{J}}_{t_i}(d) = d'$, then $(d, t_i) \in \Gamma^i_C(p)^{\mathcal{I}}$.

The proof is by induction on the length of p, using Property 1. Details are left to the reader.

3) Let $d, d' \in \Delta^{\mathcal{J}}$, p be a path not containing the features $f_0, \dots, f_n$, and $i \in \{0, \dots, n\}$. Then $p^{\mathcal{J}}_{t_i}(d) = d'$ iff $(\Phi_i(p))^{\mathcal{I}}((d, t_i)) = d'$.

The proof is again by induction on the length of p. Details are left to the reader.

We now proof the claim by structural induction:

– D is a concept name. Then, $\Psi_i(D) = A$, and $(d, t_i) \in D^{\mathcal{I}}$ is an immediate consequence of the definition of $\mathcal{J}$.

– $D = \neg A$ (A is a concept name since C is in NNF). Then, $\Psi_i(D) = \neg A$. It is an immediate consequence of the definition of $A^{\mathcal{I}}$ that $(t_i, d) \notin A^{\mathcal{J}}$, which implies $(d, t_i) \notin A^{\mathcal{I}}$.

– $D = D_1 \sqcap D_2$. Easy using IH and the semantics.

– $D = D_1 \sqcup D_2$. Easy using IH and the semantics.

– $D = \exists R.E$. Then, $\Psi_i(D) = \exists R.(\Gamma_C \sqcap f_i : \Psi_i(E))$. Since $d \in D^{\mathcal{J}}_{t_i}$, there is a $d' \in \Delta^{\mathcal{J}}$ such that $(t_i, d, d') \in R^{\mathcal{J}}$ and $d' \in E^{\mathcal{J}}_{t_i}$. By definition of $R^{\mathcal{I}}$, we obtain $((d, t_i), d') \in R^{\mathcal{I}}$. By IH, we get $(d', t_i) \in E^{\mathcal{I}}$. By the interpretation of the f_i features, this yields $d' \in (f_i : \Psi_i(E))^{\mathcal{I}}$. By Property 1, we get $(d, t_i) \in \Psi_i(D)^{\mathcal{I}}$.

– $D = \forall R.E$. Then, $\Psi_i(D) = \forall R.(f_i : \Psi_i(E))$. Let $((d, t_i), d') \in R^{\mathcal{I}}$. By definition of $R^{\mathcal{I}}$, we have $d' \in \Delta^{\mathcal{J}}$ and $(t_i, d, d') \in R^{\mathcal{J}}$. Since $d \in D^{\mathcal{J}}_{t_i}$, we thus have $d' \in E^{\mathcal{J}}_{t_i}$. Thus, IH yields $(d', t_i) \in \Psi_i(E)^{\mathcal{I}}$. By the interpretation of the f_i features, this yields $d' \in (f_i : \Psi_i(E))^{\mathcal{I}}$ as required.

– $D = p : E$. Then, $\Psi_i(D) = \Phi_i(p) : f_i : \Psi_i(E) \sqcap \Gamma^i_C(p)$. Since $d \in D^{\mathcal{J}}_{t_i}$, there are is a $d' \in \Delta^{\mathcal{J}}$ such that $p^{\mathcal{J}}_{t_i}(d) = d'$ and $d' \in E^{\mathcal{J}}_{t_i}$. By Property 3, we have $\Phi_i(p)^{\mathcal{I}}((d, t_i)) = d'$ and IH yields $(d', t_i) \in \Psi_i(E)^{\mathcal{I}}$. Thus, by the interpretation of the f_i features we have $(d, t_i) \in (\Phi_i(p) : f_i : \Psi_i(E))^{\mathcal{I}}$. To verify that $(d, t_i) \in \Psi_i(D)^{\mathcal{I}}$, it thus remains to show that $(d, t_i) \in \Gamma^i_C(p)^{\mathcal{I}}$, which is true because of

Property 2.

– $D = p \downarrow q$. Then, $\Psi_i(D) = \Phi_i(p) \downarrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$. Since $d \in D^{\mathcal{J}}_{t_i}$, there are is a $d' \in \Delta^{\mathcal{J}}$ such that $p^{\mathcal{J}}_{t_i}(d) = q^{\mathcal{J}}_{t_i}(d) = d'$. By Property 3, we have $\Phi_i(p)^{\mathcal{I}}((d, t_i)) = \Phi_i(q)^{\mathcal{I}}((d, t_i)) = d'$. Thus, $(d, t_i) \in (\Phi_i(p) \downarrow \Phi_i(q))^{\mathcal{I}}$. To verify that $(d, t_i) \in \Psi_i(D)^{\mathcal{I}}$, it thus remains to show that $(d, t_i) \in (\Gamma^i_C(p) \sqcap \Gamma^i_C(q))^{\mathcal{I}}$, which is an easy consequence of Property 2.

– $D = p \uparrow q$. Then, $\Psi_i(D) = \Phi_i(p) \uparrow \Phi_i(q) \sqcap \Gamma^i_C(p) \sqcap \Gamma^i_C(q)$. Analogous to the previous case.

– $D = p\uparrow$. Then, $\Psi_i(D) = \Phi_i(p)\uparrow$. Let $d \in (p\uparrow)^{\mathcal{J}}$ and assume that $(d, t_i) \notin \Psi_i(D)^{\mathcal{I}}$. Then there is a $d' \in \Delta^{\mathcal{I}}$ such that $\Phi_i(p)^{\mathcal{I}}((d, t_i)) = d'$. By definition of $\Phi_i(p)$ and of $\mathcal{I}$, we have $d' \in \Delta^{\mathcal{J}}$. By Property 3, we obtain $p^{\mathcal{J}}_{t_i}(d) = d'$, which is a contradiction.

■

Since satisfiability of $\mathcal{ALCF}(A)$ concepts is PSPACE-complete [LUT 02c], satisfiability of $\mathcal{ALC}$-concepts is PSPACE-hard, and $\mathcal{ALC}$ is a fragment of $\mathcal{TL}$-$\mathcal{ALCF}$, we obtain the following theorem.

THEOREM 8. — *Satisfi ability of $\mathcal{TL}$-$\mathcal{ALCF}$ concepts is* PSPACE-*complete.*

5. Conclusions

We have discussed the relationship between the two interval-based temporal DLs $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$, and found that the gap between the two different knowledge representation paradigms suggested by these logics can be bridged by a suitable translation. Based on this translation, we have presented a reduction from $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability to $\mathcal{ALCF}(A)$ concept satisfiability that allowed us to determine the complexity of $\mathcal{TL}$-$\mathcal{ALCF}$ concept satisfiability as a PSPACE-complete problem. Moreover, the reduction allows to use the $\mathcal{ALCF}(A)$ tableau algorithm described in [LUT 02c] to be used for reasoning on $\mathcal{TL}$-$\mathcal{ALCF}$ concept expressions.

Concerning future work, the described reduction can be extended in at least two interesting directions:

(1) In this paper, we concentrated on the satisfiability of concepts. In description logics, an equally important reasoning task is the subsumption of concepts: a concept C is subsumed by a concept D if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all interpretations $\mathcal{I}$. In description logics with all Boolean operators, subsumption can be reduced to (un)satisfiability: C is subsumed by D iff $C \sqcap \neg D$ is unsatisfiable. Clearly, we cannot do this in $\mathcal{TL}$-$\mathcal{ALCF}$ since full negation is not available in the temporal part.[3] Moreover, our reduction cannot be used to decide $\mathcal{TL}$-$\mathcal{ALCF}$ subsumption. Consider, for example, the concepts

3. Indeed, adding full negation to $\mathcal{TL}$-$\mathcal{ALCF}$ would result in undecidability. Still, it was shown in [ART 98] that subsumption of $\mathcal{TL}$-$\mathcal{ALCF}$-concepts is decidable.

$$C = \Diamond(x)(\sharp \text{ before } x).A@x$$
$$D = \Diamond(x, y)(\sharp \text{ before } x)(y \text{ equals } x).A@y$$

Then C is subsumed by D (actually, they are equivalent concepts), but $\Psi(C)$ is not subsumed by $\Psi(D)$ since $\Psi(C)$ has only two "reserved features" f_0 and f_1, while $\Psi(D)$ has three: f_0, f_1, and f_2. It would thus be interesting to extend the correspondence between $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$ developed in this paper to concept subsumption.

(2) For the reduction, we consider the satisfiability of concepts without reference to so-called TBoxes. As modern DLs are usually equipped with TBoxes [BAA 03b], it would be worthwhile to add them to both $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$, and to extend our reduction accordingly. However, we cannot expect to obtain PSPACE-results: in [LUT 02b], it is proved that $\mathcal{ALCF}$ concept satisfiability w.r.t. general TBoxes (also known as GCIs) is undecidable. Thus, the same holds for both $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$. Undecidability may be overcome by resorting to so-called acyclic TBoxes [BAA 03b]. However, as proved in [LUT 99], $\mathcal{ALCF}$ concept satisfiability w.r.t. acyclic TBoxes is NEXPTIME-hard. Clearly, this lower bound is inherited by $\mathcal{TL}$-$\mathcal{ALCF}$ and $\mathcal{ALCF}(A)$. A matching upper bound for $\mathcal{ALCF}(A)$ has been proved in [LUT 02c]. A similar bound for $\mathcal{TL}$-$\mathcal{ALCF}$ is yet to be established.

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