Nature and our descriptions of it: Predicate, function, symmetry

Sunny Y. Auyang

Objectivity – to account for nature as it is, free from subjective biases – is a standard of science and commonsense. However, postmodernists distort it into God's perspective. Because God's position is beyond human reach, they dismiss objectivity as a sham and relegate all science and knowledge to be nothing but social constructions relative to specific cultures. Their forced choice between two polar options – an illusory absolute stance and arbitrary cultural fashions – is unwarranted. Objectivity has clear meaning within the bounds of human understanding.

Physical things do not contain concepts, which are our intellectual contributions to knowledge of them. Perhaps God can comprehend without concepts, but humans cannot. Without concepts, the world would be unintelligible to us and objectivity itself loses meaning. Therefore, the demand of objectivity cannot be the absolute purge of concepts; it can only be a critical analysis of concepts to eliminate extraneous and arbitrary elements. Its standards are reasonably achieved in science, although a healthy dose of skepticism is always useful to prevent complacency.

We will examine objectivity and conventionality in three forms of description: the subject-predicate form in common language, the functional form in most mathematical theories, and symmetry form in certain modern physical theories. Our languages, mathematics, and ways of thinking have their own structures, which may not fit perfectly the structures of reality that they are used to represent. Arbitrary elements do burden our representations. The question is whether they can be teased out, criticized, and, if not purged, have their detrimental effects neutralized. Symmetry, developed to handle delicate phenomena addressed by modern physics, offers an answer. It explicitly ensures that the objective features described are independent of the conventional representations used in the theories.

Science, commonsense, philosophy

When I was a graduate student, once I went to my thesis professor after working for weeks on a calculation. I proudly presented him with a pile of papers covered with Feynman diagrams. He glanced at it and asked: "Where is the physics in all these?" Sure, he told me, a physicist must be proficient in mathematics, but that is not enough. He would do poorly if he were engrossed by calculation to the neglect of its meaning. Its meaning is the physics. *What is the physics?* That question, which I soon found to be a favorite among physicists, sticks to my mind. So, when I turned from physics to philosophy, I constantly ask myself: *What is the philosophy?* That question prevents me from stir-frying technical physics, just as I once ground out Feynman diagrams, and garnishing the dish with a few pieces of naive philosophy.

The question I am interested in is central to traditional Western philosophy. What is the relation between the physical world and our thoughts, concepts, and theories about it?

This problem, explicitly formulated at least since Descartes, was tackled by a long list of philosophers. Quine's worry about words and objects and Putnam's question of how words hook on to word are only two of the latest additions. In a word, this is the problem of *reality*, because the real world is intelligible to us only through our concepts and theories. According to Einstein, the question of reality is also the central to the interpretation of modern physics.

I try to extract philosophical answers to the problem of reality mostly, but not only, from physical theories, especially gauge field theory. To avoid confusion, allow me to make two points at the beginning:

1. I examine only the *logical forms* of thinking and representation, not the *substantive contents* of physical theories. You will not hear about the physical structures of electrons and quarks. What you will hear are the logical form of theories that describe the physical properties. Logical forms convey *general concepts* such as object, entity, property. We will examine how the symmetry form asserts the objectivity of theoretical descriptions.

2. The subject matter of physical theories may be different and remote from the things familiar in our daily life, but the general structures of the theories are close to that of commonsense. I will compare the two in seeking answers to the philosophical problem of reality.

Albert Einstein once wrote: “The whole of science is nothing more than a refinement of everyday thinking. Therefore the critical thinking of the physicist cannot be restricted to concepts in his special field, but must include an analysis of the nature of everyday thinking.” Why must we analyze commonsense? If science is the development of everyday thinking, what advantages can philosophers gain in analyzing physical theories?

Scientists are first human beings and then scientists. Modern physics investigates exotic things remote from our everyday experience, but these strange things are not fantasies but part of the real world. When physicists distinguish between *physics* and *mathematics*, they have already understood that some parts of their theories have objective meaning while other parts are merely theoretical or instrumental. The thinking of physicists has already *presupposed* many concepts in our everyday thinking, general concepts such as *reality* and *objectivity*. These general concepts are indispensable in any interpretation of physical theories, but they attract little attention, because they are what we tacitly understand and automatically use in our everyday life. Everyone knows them, but our knowledge is not distinct but confused. Augustine remarked about time: "if no one asks me, I know what it is. If I wish to explain it to someone who asks, I do not know." The same can be said of most general concepts.

Our vague ideas of objects and reality may work in ordinary situations, but the vagueness hurts when modern physics confronts the unfamiliar and counterintuitive subatomic world. In the difficult situation, one cannot evade the task of analyzing the concepts we have already presupposed in our everyday thinking and carried over to physical theories. Without analyzing presuppositions, interpretations of physical theories tend to be philosophically naive.

Two classical forms of description

"The sun revolves around the earth." "The earth revolves around the sun." These two statements anchor the debate that launched the Scientific Revolution. They also make a paradigmatic case for science superseding commonsense. Actually, what science refutes is the *substantive content* of a common belief, a folk wisdom. Science has not touched the *logical form* of common thinking. Both statements assert a relation between two entities, the sun and the earth, and both use the same logical form *Rab* (read relation between *a* and *b*).

Forms of description

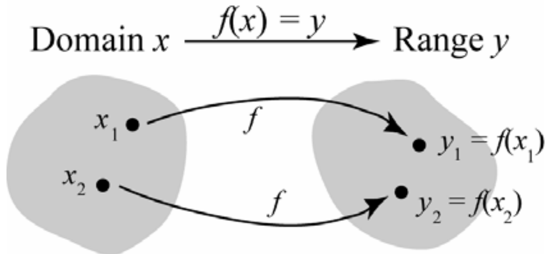
English	Mathematical theory	Predicate logic
Subject-predicate propositions:		
It has some color.	$f(x) = y$	
It is red.	$f(x) = y_0$	$\exists xFx$
The apple is red.	$f(x_0) = y_0$	Fa
Statistical description:		
Whatever that is red.	$f^{-1}(y_0) = x$	$\exists xFx$

Consider the simple statement "The apple is red." Its content is about the apple's color. Its form is the *subject-predicate proposition*, *Fa*. We refer to a subject, the apple or generally *a*, and describe it by a predicate, red or generally *F*.

Besides proper nouns or definite articles for picking out subjects of discourse, common languages also provide pronouns. "It" is a variable that designates different things in different contexts. Variables are widely used in mathematical theories. Those that pick out subjects of discourse are usually called *independent variables*. The *x* in the table below is an independent variable with meaning akin to "it." A variable can take on any one of a range of values, e. g., *x* can take on the values x_0, x_1, x_2, . . ., which can be number, vectors, things, anything at issue.

Science is superior in describing properties systematically. In mathematical theories, both systematic predicates and systematic assignments find powerful representation in the *functional form* of description $f(x) = y$.

A mathematical *function f* is rule that systematically maps from a *domain* to a *range*. The domain is represented by the independent variable *x*, the range by the dependent variable *y*. The function *f* systematically assigns to each value x_0 of *x* in its domain an unique value of $y_0 = f(x_0)$ in its range. A value of *x* maps into only one *y*, but more than one value of *x* can map into one value of *y*.



Functions perform a million jobs in the mathematical sciences. Among the most common is to represent broad *types* of property, e.g., position or energy. In this capacity $f(x) = y$ is akin to a subject-predicate proposition; it systematically assigns specific properties within the type to various entities. Suppose f represents the property type of color, then the values of its range represent specific colors in the color spectrum, y_0 = red, y_1 = green, etc., and $f(x) = y$ reads "it (x) has some color (y)," where both x and y are variables. If we fix a value for x by specifying an entity, e.g., x_0 for *this* apple, and if the color function f assigns the color red represented by y_0 to x_0, then $f(x_0) = y_0$ reads "this apple is colored red."

The functional form is more complicated than the canonical form of predication in predicate logic, $\exists xFx$ (read, "there is an apple and it is red.") The predicate-logic form $\exists xFx$ has only two elements, the subject x and the predicate F, which can be anything arbitrary. The functional form $f(x) = y$ has three elements. Besides the subject x and predicate y, it also indicates f, the *type* to which y belongs. Thus, it eliminates ambiguities in assertions such as "he is red" by indicating whether "red" describes a color or an ethnicity or a political inclination.

Philosophers may be interested to note that the functional form is sophisticated enough to resolve a dispute about two interpretation of $\exists xFx$. The traditional interpretation corresponds to the functional form. Quine's interpretation of $\exists xFx$ as "whatever that are red" corresponds to the inverse functional form, which not a subject-predicate statement but a *statistical* statement, which is coarser grained.

Saying too much

The functional form is successful in many scientific theories. It represents almost all quantities in classical physics and other mathematical sciences. Why is it not sufficient in modern physical theories? There are two major reasons. Both deal with the fact that the functional form is too definite. It says too much.

First, many functions are capable of describing the same thing. For example, a classical particle is completely characterized by six parameters, i.e., six values of y. The six parameters may be the three components of position and three components of velocity; or they can be the positions, energy, and two components of momentum; or they can be the positions, energy, angular momentum, and a component of momentum. Note that not only the predicates y, the function f itself differ in the three descriptions. We need the velocity function in one case, the energy function and the momentum function in the other, and so on. Which functions we use is our choice. The particular choice, however, is usually decided by theoretical convenience and has nothing to do with the objective reality. It introduces a conventional factor into our theories, which may produce spurious results. Furthermore, in view of multiple representations, we need to be sure that they are equivalent, which is usually not as obvious as in the classical mechanics case.

Another reason for the inadequacy of the functional form is that the range of the function, which represents the properties of the entities, usually has mathematical structures of its own that are not exactly fit for properties. If the representative system has too little structure, the solution is relatively easy, we can construct upon it. If it has too much structure already, it is much harder to erase the unwanted feature; it is difficult to eat one's words.

Most physical quantities are represented quantitatively, in numbers. The real numbers provide a systemic way of creating infinitely many predicates. However, the number system itself has rich structures: it has a zero, it has an ordering, it a direction in ascending number, it has a quantitative measure. These structures may *not* hold for all objective states of affairs. They are too much spacetime, and their excess posed a problem that the theories of relativity eventually solved – by introducing a new form of predication.

In short, functional form is often too heavy handed and surreptitious projects the structures of our logical thinking into the objects describe. (Ordinary languages are even worse). We have to find some ways that separate clearly the *objective* and the *conventional* elements in description. Symmetry provides an answer. It does not discard functions but embeds them in a larger conceptual framework that is capable of unsaying some of what the functions say. Symmetry introduces a new form of description that incorporates the functional form, brings out its weakness, compensates for it, and makes sure that it does not contaminate the objective statement about physical motion.

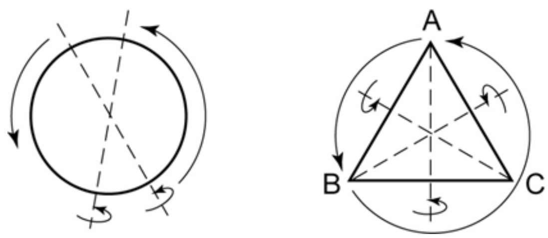
Symmetry and symmetry breaking

Technically, the *symmetry* of an object is defined in terms of the transformations that bring the object back into itself, or that leave the object unchanged or invariant. For example, a figure has bilateral symmetry if it returns to itself when it flips about the axis. If no transformation besides the identity can bring a figure back into itself, then the figure is asymmetric.

The mathematics for suitable for presenting symmetry is group theory. A symmetry is defined by the group of symmetry transformations such that the initial and final configurations of the transformation are indistinguishable from each other. The features that are *invariant* under the transformation are locus of the symmetry.

For example, an equilateral triangle returns to itself by any of six transformations: rotations through 120, 240, and 360 degrees about the center, and rotations through 180 degrees about the three bisectors. The group of these six transformations constitutes the symmetry that characterizes the property of being an equilateral triangle. Conversely, instead of defining an equilateral triangle as something with three equal sides or three equal angles, we can define it by symmetry as something that is invariant under the group of six transformations. The two definitions are not equivalent, however. The symmetry definition is more abstract and general. As far as symmetry is concerned, the triangle is not different to a circle with three equidistance points on it.

The more transformations that leave it unchanged, the more symmetric the object is. A circle has higher symmetry than a triangle. Only six transformations can bring an equilateral triangle back into itself. Infinitely many ways exist to transform a circle into a position indistinguishable from the original: rotating about the center through any angle, or flipping about any axis.



An entity with higher symmetry has fewer features. A circle lacks the three corners featured by the triangle. Additional features *breaks* symmetry. Add three equidistance dots on the circle, and its symmetry reduces to that of a circle. The group of its symmetry transformation also reduces correspondingly.

Symmetry was first introduced in the special theory of relativity, where it characterizes spatio-temporal properties. Since then it has proliferated. Physicists retrofit it into Newtonian physics. Here are some of the symmetries in modern physics. The Galilean group of classical mechanics contains fewer transformations than the Poincaré group of special relativity, which in turn is much smaller than the group of diffeomorphism for general relativity. The objective features they define are corresponding simpler and more general. The points in the manifold defined by diffeomorphism are applicable to all spatio-temporal theories, whereas the distance defined by the Galilean group is not applicable to relativity. Similarly, the unitary group $SU(2) \times U(1)$ for the electroweak interaction is larger than the $U(1)$ and $SU(2)$ groups for the electromagnetic and the weak interactions separately, for the unification captures what is common to the two interactions.

Symmetry in modern physics

Theory	Symmetry group	Objective features
Newtonian mechanics	Galilean group	distance, time interval
Special relativity	Poincaré group	proper time interval
General relativity	$Diff(\mathrm{M}^4)$	points in manifold
Electromagnetism	$U(1)$	
Weak interaction	$SU(2)$	
Electroweak	$SU(2) \times U(1)$	
Strong interaction	$SU(3)$	

Symmetry descriptions are the contrast to ordinary descriptions in one way. Ordinarily, the more objective features there are, the more complex the description. That is intuitive. In symmetry, more objective features entails less descriptions in the sense of requiring smaller groups of transformations. The most featureless entity, a manifold of points with no interval of any kind, requires the most complex symmetry transformations. Why is symmetry so perverse?

Symmetry transformations erase features. If the characteristics of an entity is defined by a group of transformations, then any feature that are changed by any of the transformations is excluded or erased. A circle with random points on it has low symmetry. If you want to erase the points, simply add more transformations; rotate the circle a bit, and the points are at a different position; they are no longer invariant, so they are expelled. We have seen earlier that the functional form and other descriptions are too heavy handed and inadvertently introduce conventional elements into descriptions of nature. The ability to erase extraneous features makes symmetry most powerful in objective descriptions. The more extraneous features to erase, the more work is required of symmetry descriptions.

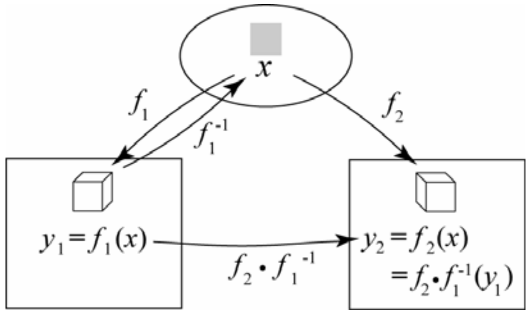
The symmetry form

Wolfgang Pauli remarked that Einstein introduced not merely a specific group of transformations but a new way of thinking, which changed "the general way of thinking of the physicists of today." This way of thinking is a new *form* of predication, the *symmetry* form in lieu of the functional form. It is the symmetry *form* of description that physicists retrofit into classical mechanics. The retrofit does not change the objective contents.

Remember that a *symmetry transformation* brings a system into a *final* state that is identical to its *initial* state. This definition of symmetry invokes difference and identity in a single breath; it distinguishes the initial and final configurations and immediately identifies the two. Thus it involves three concepts: We distinguish the initial from the final state by certain representations, for instance some labels or coordinate systems. Symmetry transformations relate the different states. The identity of a transformation's initial and final state defines the invariant features of the system. The concepts of representations, transformations among representations, the features invariant under transformations form the triad of a symmetry.

This is a schematic for the form of symmetry. It is a mess not easily rendered in words, because it has woven several important ideas into an integral conceptual structure. The important thing is that the whole structure works as a unit in physical theories.

Symmetry includes the subject-predicate or functional form of description. We have a system with various states x. According to the functional form of description, we use a function f_1 that assign a predicate $y_1 = f_1(x)$. This is usually called a *representation* of the object. This is the functional form of description in classical mechanics.



There is nothing unique or necessary about the particular representation. There are many ways of saying the same thing. We can use another function f_2, which assigns another predicate y_2 to describe the same x. It is like you can say "snow is white" in English or Chinese or German, and the different words for white does not affect the color of the snow at all. Simple examples such as snow is white give the wrong impression that it is a trivial matter to translation among the languages; but philosophers are familiar with the difficulty of translation. Translations among different representations are not always easy. This is apparent in the familiar example of the desk; what is a plain metal desk to most people is a sea of electrons in an ionic lattice for physicists.

In short, there is certain arbitrariness in the way we represent things. The arbitrariness is not absent in physical theories. A coordinate system in classical mechanics presents a particular representation, and which system we pick is arbitrary. Physicists have long acknowledged the arbitrariness, as they routinely transformed from Cartesian to spherical coordinates or back. However, the knowledge of the arbitrariness and the coordinate transformation were not stated explicitly *within* classical physical theories; they belonged to the intuitive practice of physicists. The intuition suffices for the rather simple subject matter of classical physics, but when modern physics becomes so complicated and strange, intuition is readily confused. When there are many options, the choice of a particular option involves some information, and this information may be extraneous to the objective state of affairs. In complicated and unfamiliar situations we cannot rely on intuition to ensure that the extraneous information dose not contaminate our description of the objective states of affair. The arbitrariness of representation systems needs to be explicitly acknowledged within physical theories, so that means can be instituted to safeguard against their unwanted effects.

We make reality intelligible by our words and concepts. Concepts are general and rule like, and to be so, they must have some structures of their own. The structures are sometimes explicit, sometimes tacit, and they may not all be suitable for the structures of the objective state of affairs that they are used to describe. Whenever we say something definite and give a definite description of something, we are saying too much. Our description hides something peculiar to the specific condition of assertion; our viewpoint, perspective, intellectual tool; our choice of a particular representation. This peculiarity is not objective and unsuited for the objective description. A big job of symmetry is to *erase* whatever extraneous and nonobjective elements in our definite descriptions. Symmetry transformations erase features, which is why a larger group of transformation produce a system with less features.

The form of symmetry collects *all possible* representations of a system, f_1, f_2, etc., and all possible transformations among them. It abstracts what are common to all representations, what are *invariant* under the transformations, and attributes only the invariant features to the system as its objective properties. Because the invariant features abstract from all representations, they are explicitly independent of any representation and answer to the intuitive notion of objectivity.

If only the invariant features are objective, why do we not try to represent them abstractly? Why do we need the various representations and the symmetry transformations? We need them because even in the most general physical theories, we are not merely stating what is universal. We also need to say definite things about *particular* systems because we need to do experiments. Without experiment, physical theories would cease to be empirical but be reduced to abstract mathematics. Experiments, however, are always performed in particular conditions. To specify experimental conditions so that their data can be compared to theories, we must use representations. That is why we need the entire complicated symmetry structure in physical theories.

If the particular representations $y_1 = f_1(x)$ and $y_2 = f_2(x)$ are deemed "subjective" because they are peculiar to the persons or the experimentalists who choose to use it, then the symmetry transformation $y_2 = f_2 \bullet f^{-1}{}_1(y_1)$ ensures intersubjective agreement by erasing the differences. Various representations translate into each other. The translation is not merely conventional. When you analyze it you find that it breaks into two mappings, an inverse function $f^{-1}{}_1(y_1) = x$ followed by $f_2(x) = y_2$. The inverse of the first representation, $f^{-1}{}_1$, points back to the object x, which the second representation represents. The object x, being common and same to all representations, is independent of any representation and whatever "subjective" elements it may carry. Its features that remain invariant through transforming among all representation are *objective* because the transformations have erased all extraneous elements.

Intelligibility and objectivity

Modern physical theories have adopted symmetry as the major form of characterization. Symmetry is a complicated conceptual structure that incorporates several major strands of ideas. Separately, the ideas themselves are common sense. We take for granted that we rely on concepts to bring things within our knowledge, that we have many ways of describing things, and that things are what they are independent of our concepts, our descriptions, our mind. But so far, the notion that things are independent of our description remains intuitive. Hence there is the idealist doctrine that it is senseless to say things are independent of mind, because even in asserting the independence, we are already thinking about it.

Symmetry answers the philosophical question. The objective world that is intelligible to us is not *absolutely* mind independent, because being intelligible is imposes a certain condition. However, this condition is very general and abstract. Objective features of the world are independent of any *substantive* concepts by which we describe them, but to be intelligible at all, they are not independent of *general* concepts such as objectivity. The general concept of objectivity is theoretically represented by the symmetry form. Substantive descriptive concepts are represented by functional form within the symmetry form. They are retained so that we can think about and observe the properties and hence to make them intelligible to us. On the other hand, symmetry transformations ensure that objective features abstract from them. Thus, symmetry articulates clearly the intuitive idea that objective properties are independent of our substantive concepts and representations. The symmetry form elucidates in what sense things are independent of us and yet intelligible to us. It elucidates the *general concept of objects*, which, as Kant argued, is the most general concept of our understanding and theoretical reason.

Part of a talk presented at the Philosophy Department
Concordia University, Montreal.
April 3, 1998.