Two notions of "convergence to truth"

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"Closure occurs in science when a consensus emerges that the 'truth' has been winnowed from the various interpretations."[1] More than once in library books I saw "*sic*" scribbled in the margin pointing to the scare quotation marks in this and similar texts. If the readers read on, they would discover that scare quotes around scientific truth, fact, reality, nature, technological progress, and similar terms are fashionable in postmodern literature and are spreading beyond it. Scientific results are "true." Scientists arrive at the "fact." What do the scare quotes mean? What are their effects?
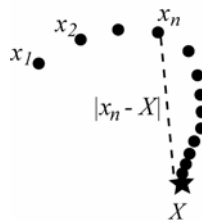
General concepts of truth, reality, and the like have puzzled philosophers since the ancient Greeks. They are extremely difficult to define precisely, because they involve the deepest metaphysical and epistemological problems, such as the nature of the human mind and its relation to things that are neither parts of it nor created by it. Efforts through the millennia have produced a plethora of philosophical theories, some clarification, but no perfectly satisfactory explanations.[2]

Skeptics complain that scientific results are often crude and susceptible to errors. They are correct, which is why scientists usually claim that their results are only approximately true. However, scientists add that as they continue to criticize their results, ferret out and correct errors, include minor factors previously neglected, the results become more accurate and reliable. Successively better approximations converging on truth are characteristic of scientific progress. Skeptics retort that the notion of "convergence on truth" is illusory, because it presupposes an Absolute Truth toward which various approximate versions converge. However, the Absolute Truth is accessible only to God's eye point of view and not to us mortals. Without ability to know Absolute Truth, convergence and consequently scientific progress are illusory. Some postmodernists dogmatically impute Absolute Truth to what they call "the scientist's account of science," an account that abhors scientists.[3] They are wrong. The notion of convergence does not necessarily presuppose the final point of convergence. Scientists do not need to know Absolute Truth to make sense of convergence on truth.
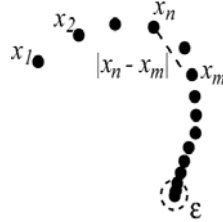
Clear definitions of convergences are available in mathematics. Mathematical convergence applies to sequences of many kinds of entities, for instance topological sets. For simplicity and familiarity, let us consider a sequence of real numbers. Here are two definitions of convergence:

> **Definition 1**. A sequence of real numbers $\{x_1, x_2, \ldots x_n, \ldots \}$ *converges* on a limit X if and only if for each error bound $\varepsilon > 0$, there exists a positive integer N such that for all $n > N$, the absolute difference between x_n and X is smaller than the error, $|x_n - X| < \varepsilon$.
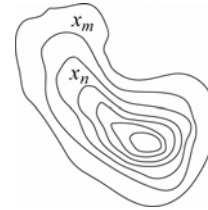
Definition 2. A sequence of real numbers $\{x_1, x_2, \ldots x_n, \ldots\}$ is *convergent* if and only if for each error bound $\varepsilon > 0$, there exists a positive integer N such that for all $n > N$ and $m > N$, the absolute difference between x_n and x_m is smaller than the error, $|x_n - x_m| < \varepsilon$.

| Definition 1. | Definition 2. | Definition 2. |

The first definition of convergence posits a limit of convergence in advance. Definition 2 does not. It defines convergence by comparing terms in the sequence and demanding the differences between them to diminish to vanishingly small. If the sequence converges, we can obtain the limit after we examine its terms. General definitions of convergence without presupposing a limit are available for mathematical entities other than numbers, for instance topological sets.

The limit X is akin to Absolute Truth in talks about convergence on truth. It is the base of much unjustified postmodern attack on science. However, the notion of convergence that scientists and people of common sense imply is closer to Definition 2. Scientists examine and compare various research results, which are akin to x_n and x_m. When theories with diverse representations and experiments performed with diverse methods yield closely similar results for a certain range of phenomena, we judge the results true, or very close to truth. There is no need for Absolute Truth at all.

1. Bijker, W. E., Hughes, T. P., and Pinch, T. J. eds. 1987. *The Social Construction of Technology Systems*. Cambridge: MIT Press. p.12.
2. Kirkham, R. L. 1992. *Theories of Truth: a Critical Introduction*. Cambridge: MIT Press.
3. See for example Pickering, A. 1984. *Constructing Quarks*. Chicago: University of Chicago Press.