MEHMET BAC and PARIMAL KANTI BAG

COMMITTEE DECISIONS WITH PARTISANS AND SIDE-TRANSFERS

ABSTRACT. A dichotomous decision-making context in committees is considered where potential partisan members with predetermined votes can generate inefficient decisions and buy neutral votes. The optimal voting rule minimizing the expected costs of inefficient decisions for the case of a three-member committee is analyzed. It is shown that the optimal voting rule can be non-monotonic with respect to side-transfers: in the symmetric case, majority voting is optimal under either zero, mild or full side-transfer possibilities, whereas unanimity voting may be optimal under an intermediate side-transfer possibility. The side-transfer possibilities depend on the power of partisans (their ability or willingness to pay for neutral votes) relative to the corruptibility of neutral members (personal cost of deliberately casting a 'wrong' vote).

KEY WORDS: Committee, Partisan voting, Vote buying, Majority rule, Unanimity rule

JEL CLASSIFICATION NUMBERS: D71, D72.

1. INTRODUCTION

The literature on organizational decision making in committees[1] is broadly concerned with the selection of the 'correct' alternative among a limited number, often two, alternatives. This literature commonly makes two important assumptions. First, the committee members are all motivated by the same public motive of contributing to a correct decision when they vote (that is, there are no 'partisan' members). Second, the committee members do not sell their votes for cash, gifts, or other benefits. This paper investigates the impact of relaxing these two assumptions on the optimal voting rule.

We aim to capture a variety of decision-making contexts involving small size committees: a municipal committee evaluating various development projects, middle managers in a firm deciding on a contract to be awarded for the supply of key inputs, mid-level

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bureaucrats in developing countries implementing specific public policies, etc. Common to these examples is the propensity among the committee members to influence the outcomes of any selection process by casting biased votes and even 'buying' votes from other members.[2]

We consider a committee, consisting of three members, appointed by an organization (or principal) under imperfect information about the type of each member. Members can be partisans of the 'yes' or 'no' decision, or be neutral and motivated by the desire to cast a 'correct' vote to maximize the commonly known objective of the principal. The principal's objective depends on the 'state', which the committee members observe but the principal cannot. The principal determines the voting rule (2/3 majority or unanimity) to minimize the expected costs of wrongful committee decisions, where partisan members, if any, can influence the decision by buying out neutral members' votes. The side-transfer possibilities, that is, whether one or two partisans can buy one or two neutral votes, depend on the power of partisan members (ability or willingness to pay for neutral votes) and corruptibility of neutral members (reservation price or personal cost of modifying their votes).

We allow for asymmetric selection bias, as reflected in the common prior belief that 'yes' or 'no' is the correct decision, asymmetric costs for the two types of decision errors, and asymmetric probabilities of partisan voting in each direction. The *symmetric case*,[3] however, provides a clear ranking of two voting rules; we show that the optimal voting rule can be *non-monotonic* with respect to side-transfers: majority voting is optimal under either *zero*, *mild* or *full* side-transfer possibilities, whereas unanimity voting may be optimal under an *intermediate* side-transfer possibility. The intuitions for the optimal voting rule in the symmetric case are as follows.

Majority voting has a natural advantage over unanimity voting when side-transfers are *impossible* because unanimity voting puts too strong a bias in favor of rejection (under unanimity voting a single partisan of 'no' triggers the decision 'no'). When side-transfers are *mild* — it takes two partisans of the same type to buy a neutral vote — the case for majority voting is even stronger: compared to the case of no side-transfers, the principal's expected payoff under unanimity voting falls (whereas it remains unchanged under majority voting) because two partisans of the 'yes' type can now alter a neutral member's (correct) negative vote. However, introducing the *intermediate* possibility that one or two partisan members buy only one neutral vote, we find that unanimity voting performs better if the probability of nonpartisan voting exceeds the probability of partisan voting. The quality of decision making under majority voting falls sharply in this case because whenever the committee consists of two neutral members who intend to vote the opposite direction of the third, partisan, member, the partisan buys one neutral vote, secures the support of the majority and induces his preferred decision. Finally, if a partisan is powerful enough to buy two neutral votes, majority voting becomes, again, the optimal voting rule. In this case of *full* side-transfer possibilities, the relative advantage of majority voting stems from the correct decision it induces in committees comprised of two partisans with opposing interests and one neutral member (in such committees unanimity voting always induces the decision 'no'.) The partisan whose interest coincides with the correct decision can induce the neutral member to keep his 'correct' vote unchanged at a lower price; the opposite partisan would also have to compensate the neutral member for modifying his vote in the wrong direction.

Our result that majority voting is superior in most cases largely corroborates the familiar wisdom in the political science literature on jury decision making, that of *Condorcet Jury Theorem* (Condorcet, 1785).[4] Our setting differs in that it includes the additional features of partisan voting and side-transfer possibilities. The presence of partisan members with strong interests in the voting outcome will always *decrease* the quality of committee decisions and generate inefficiency. This is a particular manifestation of a general organizational problem, termed 'influence activities' by Milgrom and Roberts (1988). Given the potential presence of partisan members, we show that whether the possibility of side-transfers within the committee improves upon the efficiency of decisions depends on the voting rule and expected type profile of the committee. The same possibility of side-transfers generates the possibility that a pivotal neutral member is offered benefits for keeping his (correct) vote unchanged. An organization may therefore choose a voting rule that generates a wider scope for internal transfers of cash or other benefits in committees.

The paper is organized as follows. The next section outlines the model. In Section 3, the optimal voting rule is derived for the benchmark case of zero side-transfer. In Section 4, different (non-zero) side-transfer possibilities are considered and their implications for the optimal voting rule assessed relative to zero side-transfer scenario. Section 5 concludes. The Appendix contains the proofs of propositions.

2. THE MODEL

A principal appoints a committee of three members to advise him regarding a project's suitability for the organization. Each member separately and simultaneously votes $x = 1$ for acceptance, or $x = -1$ for rejection. The principal's main problem is to choose a voting rule, $V(\cdot)$, that maps the number of votes favoring acceptance into a final decision R, with $R = 1$ denoting acceptance of the project and $R = -1$ denoting rejection of the project. The principal's desirable decision depends on a random variable ω, the state, which the principal cannot observe. It is common knowledge that the value of ω is 1 with probability α, -1 with probability $1-\alpha$. The desirable decision in state ω is ω. We let $B(R : \omega)$ denote the principal's payoff from decision R in state ω. $B(1 : 1) > B(-1 : 1)$ and $B(-1 : -1) > B(1 : -1)$. Define $B(1) = B(1 : 1) - B(-1 : 1)$ as the benefit of avoiding a type I error and $B(-1) = B(-1 : -1) - B(1 : -1)$ as the benefit of avoiding a type II error.

There are three potential types of committee members. Two of these types are 'partisans'. We define a partisan as a member who has a predetermined vote regardless of his perception of the desirable decision from the principal's viewpoint, and never alters his vote in exchange for a payment;[5] a partisan will 'buy' votes (that is, offer cash, gift or any benefit), if worthwhile and possible, to induce his preferred decision. A type-(1) partisan always votes $x = 1$, and a type-(-1) partisan always votes $x = -1$. The third member type, called type-(0), is 'neutral' in that he would vote for what he perceives the desirable decision of the principal. Though type-(0) members never buy votes, we allow them to be corruptible, that is, to vote according to the preference of a partisan member who offers a sufficiently high side-transfer. A neutral member incurs a disutility of c by modifying his vote for a side-transfer. The variable c thus captures 'corruptibility' or 'price' of neutral members. The maximum that a partisan would pay to have the final decision *altered* according to his own preference by buying neutral vote(s) is v, which we assume to be common for type-(1) and type-(-1). The variable v captures the power of partisan members.[6] Members' types are independently drawn from a common distribution. Each member is of type $t \in \{-1, 0, 1\}$ with probability $\beta_t > 0$, and $\beta_{-1} + \beta_0 + \beta_1 = 1$.

Our model thus differs in style from standard jury decision making models where the jurors (along with the organization/society they represent) share the same costs/benefits of collective decisions. Here, the committee members are appointed to observe the true state and vote according to the principal's preferences and produce the desirable decision, but potential presence of partisan members and side-transfers for votes can generate inefficient outcomes. The sequence of events is as follows:

- (Stage 0) Nature draws the state (hence, the desirable decision) and the type configuration in the three-member committee.
- (Stage 1) The principal determines the voting rule, majority voting ($V = M$) or unanimity voting ($V = U$). Under majority voting, $R = 1$ only if the number of $x = 1$ votes is at least two. Under unanimity voting all three votes must be $x = 1$ to have the decision $R = 1$; otherwise $R = -1$.[7]
- (Stage 2) Committee members learn the entire type configuration and observe the true state ω (hence, the environment is one of complete information); there is no human fallibility. The partisan members, if any, can individually or as a group offer side-transfers to buy neutral members' votes.
- (Stage 3) The committee members individually vote and a final decision R is made according to the voting rule determined in Stage 1.

The assumption that the members know the true state serves to simplify the analysis, and should not be considered literally applicable. Furthermore, by abstracting from the problem of human fallibility, it allows us to highlight the pure role of potential presence of partisans and side-transfers in determining the principal's choice of the voting rule. The assumption that the committee members learn each others' types is not too restrictive given the purpose of the analysis. If types are private knowledge, partisan members would engage in lobbying and gathering information about the types of other members, and they would identify the committee members who would be willing to sell their votes. This should not be a difficult task in especially small-size committees.

Below we compare the effectiveness of the two voting rules, majority voting and unanimity voting, under different sets of assumptions regarding the extent of potential side-transfers.

3. NO SIDE-TRANSFERS: THE CASE $2V \leq C$

We compare the two voting rules in this section under the assumption that side-transfers are impossible, which corresponds to the case $2v \leq c$: even two partisans of the same type cannot buy a neutral vote. Now the strategy of a type-(0) member is to vote according to the observed state, $x = \omega$. We shall write the principal's expected payoff under voting rule $V = M, U$ as

$$\begin{aligned} W^V = &\alpha[\mathcal{A}^V B(1:1) + \mathcal{B}^V B(-1:1)] \\ &+ (1-\alpha)[\mathcal{C}^V B(1:-1) + \mathcal{D}^V B(-1:-1)], \end{aligned} \tag{1}$$

where $\mathcal{A}^V$ and $\mathcal{D}^V$ denote the probabilities that the committee makes the right decision when the state is respectively $\omega = 1$ and $\omega = -1$, and $\mathcal{B}^V$ and $\mathcal{C}^V$ denote the probabilities that the committee makes the wrong decision when the state is $\omega = 1$ and $\omega = -1$, respectively. $\mathcal{A}^V$, $\mathcal{B}^V$, $\mathcal{C}^V$ and $\mathcal{D}^V$ will be functions of β_{-1}, β_0 and β_1, and obviously of the voting rule. The two voting rules will generate, by inducing different probabilities of accurate/inaccurate decisions, different expected payoffs. Below we summarize the optimal voting rule due to partisan voting only.

PROPOSITION 1. *Assume $v \leq c/2$ or that side-transfers are impossible. Then majority voting should be preferred ($W^M \geq W^U$) if and only if*

$$\alpha(\beta_1 + \beta_0)^2 \beta_{-1} B(1) \geq (1-\alpha)\beta_1^2(\beta_{-1} + \beta_0)B(-1). \tag{2}$$

The left-hand side of (2) corresponds to the net benefit (or the cost that is avoided) under majority voting:[8] When one member is of type-(-1) while the other two members are either of type-(1) or -(0) and the true state is $\omega = 1$, the wrong decision $R = -1$ (which is made under unanimity voting) is avoided under majority voting. The right-hand side of (2) corresponds to the net benefit under unanimity voting: when the observed state is $\omega = -1$ and there are two members of type-(1), the presence of one type-(-1) or -(0) member triggers the correct decision under unanimity voting, while majority voting generates the wrong decision $R = 1$.

We obtain a clear-cut comparison in a special case of Proposition 1.

COROLLARY 1 (Symmetric case). If the two partisan types are equally likely ($\beta_1 = \beta_{-1} < 1/2$), there is no prior selection bias ($\alpha = 1/2$), and avoiding the two types of errors are equally beneficial ($B(1) = B(-1)$), then majority voting performs strictly better ($W^M > W^U$).

The intuition for this result is that in the symmetric case unanimity voting generates a bias too large in favor of rejection ($R = -1$). The optimality of the majority rule in the symmetric case has also been established by Ben-Yashar and Nitzan (1997), but in a different setting, where all voters are assumed to be neutral, incorruptible but fallible (as opposed to our partisan voters who observe the true state but may deliberately recommend the wrong alternative).[9] The design of the selection rule in our Proposition 1 mainly addresses the difficulties from partisan voting, whereas the objective of Ben-Yashar and Nitzan is to minimize the costs of unintentional human errors of judgement.

4. INTRODUCING SIDE-TRANSFERS

We now investigate how the possibility of side-transfers can change the relative performance of the two voting rules. With three members and three potential types for each member, the number of possible type configurations is 27. Thus, even in this simple case there is a large number of possible voting coalitions to ‘win’ a favorable decision. Depending on the committee’s type configuration, one or two partisan members can buy a neutral vote, or one partisan member alone can buy two neutral votes. There are nine type configurations where side-transfer cannot occur. These involve all-partisan committees and one in which all members are of type-(0). Below we identify the type configurations in which side-transfers are possible, to prepare for the analysis that follows.

Six type configurations involve members of three different types. In these cases there will be no side-transfer under unanimity rule because the presence of one type-(-1) member suffices to induce the decision $R = -1$. This is not so under majority rule. Two (opposite) partisans will now compete for the neutral vote to have the decision set according to their preference.

In other six type configurations two neutral members are combined with a partisan member. The following cases may arise. If the observed state is $\omega = 1$ and the partisan is of type-(1), there will be no side-transfer under either voting rule. If the partisan's type is changed to (-1), under majority voting the partisan has to buy at least one neutral vote to induce $R = -1$, while under unanimity voting $R = -1$ is guaranteed, hence, no need for buying votes. On the other hand, if the observed state is $\omega = -1$ and the partisan is of type-(-1), side-transfer will not occur. If the partisan is of type-(1) then he has to buy one vote under majority voting, two votes under unanimity voting, to induce $R = 1$.

Finally, in six type configurations a neutral member is matched with two partisans of the same type. Under majority voting the partisans get their preferred decision anyway. Under unanimity voting, the same is true if the partisans are of type-(-1). If the partisans are of type-(1), however, they have to offer a transfer of size at least c to win the neutral member's vote when the state is $\omega = -1$.

4.1. *Mild side-transfers:* $c/2 < v \leq c$

We begin with the *mild* side-transfer possibility, $c/2 < v \leq c$, where it takes two partisans of the *same* type to buy a neutral vote.

As discussed in the final paragraph preceding this subsection, side-transfer can occur only under unanimity voting, if and only if either one of the type configurations $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$ obtains and $\omega = -1$ is observed. We obtain the following result.

PROPOSITION 2. *Assume $c/2 < v \leq c$, so that it takes two partisans to buy a neutral vote. Introducing the possibility of side-transfer lowers the principal's expected payoff under unanimity voting, W^U, by* $3(1-\alpha)\beta_1^2\beta_0 B(-1)$, *while the expected payoff under majority voting remains unchanged. Thus, $W^M \geq W^U$ if and only if*

$$\alpha(\beta_1 + \beta_0)^2 B(1) \geq (1-\alpha)\beta_1^2 B(-1). \qquad (3)$$

If majority voting yields a higher expected payoff under no side-transfer, it also does so under the mild side-transfer possibility considered in Proposition 2. In particular,

COROLLARY 2 (Almost symmetric case). *Majority voting rule is optimal in the absence of prior selection bias and symmetric error costs but for mostly arbitrary member types, that is, if $\alpha = 1/2$, $B(1) = B(-1)$, $\beta_t > 0$.*

Note that the optimal voting rule may switch from unanimity voting to majority voting as we move from no side-transfer to the case of mild side-transfer. The intuition is as follows. In the no side-transfer case, the advantage of unanimity voting principally lies in preventing two type-(1) partisans from altering the decision when $\omega = -1$. But this also has a cost: with two neutral types and one type-(-1) partisan and $\omega = 1$, the wrong decision $R = -1$ will be induced; if instead majority voting is applied, the correct decision $R = 1$ will be induced because in the mild side-transfer case one type-(-1) partisan will fail to buy two neutral votes. Thus, under the mild side-transfer possibility, the principal's approach is mostly to discourage these transfers by opting for majority voting. However, as we will show in sections 4.2 and 4.3, there is *no simple, monotonic relationship* between the optimal voting rule and the extent of potential side-transfers (Corollaries 2.1, 3.1 and 4.1), *nor is side-transfer necessarily an impediment to efficient decision making* (Lemma 1).

4.2. *Intermediate level of side-transfers: $c < v \leq 2c$*

Increasing v above c brings in a much larger set of side-transfer possibilities. Now, one or two partisan members can buy a neutral vote though one partisan cannot buy two neutral votes. Below is the list of type configurations in which side-transfers may change the committee decision under majority rule. Side-transfers will not occur under unanimity rule for these type configurations.

- *Two type-(0)s and one type-(1).* Under majority voting the type-(1) member will buy one neutral vote only if the observed state is $\omega = -1$.
- *Two type-(0)s and one type-(-1).* Under majority voting the type-(-1) will buy one neutral vote only if the observed state is $\omega = 1$.
- *Three different types*. Under majority voting, there will be competition for type-(0)'s vote regardless of the observed state. As we show in Lemma 1, the partisan whose type coincides with the correct decision (observed state ω) wins the neutral member's vote, because the other partisan type has the disadvantage of inflicting the cost c on the neutral member by modifying his intended vote. That is, the partisan of type-(ω) can keep the neutral member's vote unchanged at a lower price.

LEMMA 1. *Suppose three voters are respectively one each of three different types, and $v > c$. Under majority voting, if $\omega = 1$ (resp. $\omega = -1$) then type-(1) (resp. type-(-1)) member wins the vote buying contest at an equilibrium bid price of $v - c$ and the correct decision $R = 1$ (resp. $R = -1$) is induced.*

Proof. The bidding contest between the two partisans to buy the neutral member's vote is a complete information game, so we analyze the corresponding Nash equilibrium.

The neutral member casts his vote in favor of the partisan whose bid yields him the higher net utility. The tie is broken by the neutral member casting his vote in favor of the partisan whose vote coincides with the true state ω. As we will see below (footnote 10), the tie-breaker is critical to ensure that a Nash equilibrium exists.

Consider the case $\omega = 1$. The neutral member's intention is to vote $x = 1$, which combined with the vote $x = 1$ by the type-(1) partisan, would result in $R = 1$. However, the partisan of type-(-1) would offer a (maximum) bid of $b_{-1} = v$ to buy the neutral member's vote and alter the final decision to $R = -1$. The net gain to the neutral member is thus $v - c > 0$. In response to $b_{-1} = v$, the partisan type-(1) would offer at least $b_1 = v - c$ to win back the neutral member's vote so that $R = 1$. It is easy to see that $b_1 > v - c$

cannot be an equilibrium, since the type-(1) partisan can win the contest with a lower bid of $b_1' = b_1 - \epsilon > v - c$. Nor is $b_1 < v - c$ an equilibrium, since the partisan type-(-1) would then offer $b_{-1} > b_1 + c + \epsilon$, for ϵ small, which would be accepted by the neutral member. Finally, any bid $b_{-1} < v$ will always lose to $b_1 = b_{-1} - c + \epsilon$ that drives the partisan of type-(-1)'s bid up to $b_{-1} = v$. Thus, $b_{-1} = v$ and $b_1 = v - c$ will constitute the unique equilibrium and the correct decision $R = 1$ will be implemented.[10]

Similarly, when $\omega = -1$, the type-(-1) partisan will win the contest by bidding $b_{-1} = v - c$ while the type-(1) partisan will bid $b_1 = v$, and the decision $R = -1$ will be implemented. □

Thus, when all three members are of different types, unanimity voting does not induce side-transfers but it is worse than (from the principal's point of view) majority voting which induces side-transfers. Majority voting, by creating an opportunity of side-transfer, generates the 'right' price to align private incentives with the social (i.e., principal's) objectives.

We now summarize the effect of introducing intermediate level of side-transfers on the principal's expected payoffs.

PROPOSITION 3. *Assume $c < v \leq 2c$ so that one partisan (or two) can buy one neutral vote. The change in the principal's expected payoffs due to the possibility of side-transfers is*

$$\Delta W^U = -3(1 - \alpha)\beta_1^2\beta_0 B(-1)$$

under unanimity voting (that is, same as in Proposition 2), and

$$\Delta W^M = -3\beta_0^2[\alpha\beta_{-1}B(1) + (1 - \alpha)\beta_1 B(-1)] \tag{4}$$

under majority voting. Thus, $W^M \geq W^U$ if and only if

$$\alpha\beta_{-1}(\beta_1 + 2\beta_0)B(1) \geq (1 - \alpha)(\beta_0^2 + \beta_{-1}\beta_1)B(-1). \tag{5}$$

The intuition behind Proposition 3 can be grasped by identifying the committee type configurations generating different decisions under the two voting rules: (i) a committee of two type-(0)s and one type-(1) induces $R = 1$ under majority voting, the correct decision $R = \omega$ under unanimity voting; (ii) a committee of two type-(1)s and one type-(-1) induces $R = 1$ under majority voting, $R = -1$ under unanimity voting; (iii) a committee of three different types induces $R = \omega$ under majority voting (by Lemma 1), while $R = -1$ is induced under unanimity voting. Majority voting has the potential benefit $B(1)$ of avoiding type I error in cases (ii) and (iii), with probability $3\alpha\beta_{-1}\beta_1(\beta_1 + 2\beta_0)$, while unanimity voting has the potential benefit $B(-1)$ of avoiding type II error in cases (i) and (ii), with probability $3(1 - \alpha)\beta_1(\beta_1\beta_{-1} + \beta_0^2)$. Combining the benefits of majority voting on the left-hand side and of unanimity voting on the right-hand side yields condition (5).

COROLLARY 3 (Symmetric case). *If* $\alpha = 1/2$, $B(1) = B(-1)$ *and* $\beta_1 = \beta_{-1}$ $(< 1/2)$*, the condition in* (5) *reduces to* $\beta_1 + \beta_{-1} \geq \beta_0$ *: Majority voting should be preferred if a committee member is more likely to be a partisan than neutral. However, if* $\beta_0 > \beta_1 + \beta_{-1}$ *then the unanimity voting is optimal.*

Notice that the above corollary contrasts with the more general optimality of majority voting under mild side-transfers (Corollary 2.1). Introducing the possibility that one partisan buys a neutral vote generates a sharp fall in the expected payoff under majority voting and this may alter the optimal voting rule to unanimity voting: compared with the case of mild side-transfers, now the committee will generate the wrong decision whenever two partisans of type-$(-\omega)$ are matched with a neutral member.

4.3. *Full side-transfer possibilities:* $v > 2c$

The last case presents the largest side-transfer possibilities: even one partisan member can buy two neutral votes. The new possibility of side-transfer arises when the committee consists of two type-(0) members and one type-(1) member. Under majority voting the type-(1) member does not need to buy votes when $\omega = 1$ is observed, while the case $\omega = -1$ is considered in the previous subsection (it then suffices to buy one neutral vote). Under unanimity voting, the type-(1) member has to buy both neutral votes to induce the decision $R = 1$ when the state is $\omega = -1$.[11]

Thus, the only new element to be considered here is the possibility that a type-(1) member buys two neutral votes when the state is $\omega = -1$, under unanimity voting. This leads to the following result:

PROPOSITION 4. *Assume $v > 2c$ so that one partisan can buy two neutral votes. The possibility of side-transfer induces the change*

$$\Delta W^U = -3(1-\alpha)\beta_1\beta_0(\beta_1+\beta_0)B(-1) \tag{6}$$

in the principal's expected payoff under unanimity voting, while the change in W^M is as given in Proposition 3. Thus, $W^M \geq W^U$ if and only if

$$\alpha(\beta_1+2\beta_0)B(1) \geq (1-\alpha)\beta_1 B(-1). \tag{7}$$

When a single partisan can buy the rest of the committee's neutral votes, the voting rules induce different decisions in the following cases: (i) a committee of two type-(1) members and one type-(-1) member will induce $R = -1$ under unanimity voting, and $R = 1$ under majority voting; (ii) a committee of three different types will induce $R = -1$ under unanimity voting, and $R = \omega$ under majority voting. Thus, majority voting has an advantage in case (ii), while in case (i) either voting rule can induce an inaccurate decision depending on the state. Thus, unless $B(-1)$ is too large relative to $B(1)$ and/or α is too small, majority voting should be preferred over unanimity voting. In particular:

COROLLARY 4 (Almost symmetric case). *If $\alpha = 1/2$ and $B(1) = B(-1)$, majority voting yields a higher expected payoff to the principal for all member types.*

Note that the optimal voting rule may switch from unanimity voting back to majority voting as we move from intermediate to full side-transfers.

5. CONCLUSION

With potential presence of partisan members who can vote, and buy vote(s), for inefficient decisions, the optimal voting rule in the case of dichotomous decision making committees becomes a function of the extent of side-transfers, that is, partisan members' power relative to corruptibility of neutral members. We considered small committees of size three. We have shown, in the symmetric case

TABLE I

Optimal voting rule

Side-transfers	Optimal rule	Condition
Zero	Majority rule	$\beta_1 = \beta_{-1} < 1/2$
Mild	Majority rule	$\beta_t > 0$
Intermediate	Either rule (*)	$\beta_1 = \beta_{-1} < 1/2$
Full	Majority rule	No restriction

(* See Corollary 3.1.)

(where approval and disapproval are equally likely to be the correct decision, the costs of two potential types of errors are the same and a partisan member is equally likely to be for or against the proposal), that majority voting has a relative advantage over unanimity voting except possibly under an intermediate side-transfer possibility. Table 1 summarizes the optimal voting rule in the symmetric case where $\alpha = 1/2$, $B(1) = B(-1)$, as a function of side-transfer possibilities.

Our restriction to three-member committees should not be considered too serious a restriction given many real-life decision contexts where important decisions are often delegated to small committees of size three. Deriving a compact formula for the optimal voting rule in the general case of n committee members as a function of the extent of potential side-transfers is much beyond the scope of this paper. Such an objective seems to be intractable because of the need to consider a large number of side-transfer possibilities and compare a large number of alternative voting rules.[12]

6. APPENDIX

Proof of Proposition 1. We derive below the expression of the principal's expected payoff under majority voting. The committee's decision will be $R = 1$ in the following cases:

- At least two members are type-(1) partisans. The probability of such type configurations is $\beta_1^3 + 3\beta_1^2\beta_{-1} + 3\beta_1^2\beta_0$;

- The members are all of different types and $\omega = 1$ is observed (thus the type-(0) member votes $x = 1$). The probability of this type configuration is $6\beta_1\beta_{-1}\beta_0$;
- One type-(1) member and two type-(0) members, and $\omega = 1$ is observed. The probability of this type configuration is $3\beta_1\beta_0^2$;
- One type-(-1) member and two type-(0) members, and $\omega = 1$ is observed. The probability of this type configuration is $3\beta_{-1}\beta_0^2$;
- Three type-(0) members who observe $\omega = 1$. The probability of this type configuration is β_0^3.

The list of the cases in which the decision $R = -1$ is induced can be derived similarly. The expected payoff under majority voting is given by (1) where

$$\begin{aligned}
\mathcal{A}^M &= \beta_1^3 + 3\beta_1^2\beta_{-1} + 3\beta_1^2\beta_0 + 6\beta_1\beta_{-1}\beta_0 + 3\beta_1\beta_0^2 \\
&\quad + 3\beta_{-1}\beta_0^2 + \beta_0^3, \\
\mathcal{B}^M &= \beta_{-1}^3 + 3\beta_{-1}^2\beta_1 + 3\beta_{-1}^2\beta_0, \\
\mathcal{C}^M &= \beta_1^3 + 3\beta_1^2\beta_{-1} + 3\beta_1^2\beta_0, \\
\mathcal{D}^M &= \beta_{-1}^3 + 3\beta_{-1}^2\beta_1 + 3\beta_{-1}^2\beta_0 + 6\beta_1\beta_{-1}\beta_0 + 3\beta_{-1}\beta_0^2 \\
&\quad + 3\beta_1\beta_0^2 + \beta_0^3.
\end{aligned}$$

Under unanimity voting the decision will be $R = -1$ whenever at least one type-(-1) member is present in the committee or there is at least one type-(0) member and the state is $\omega = -1$. Otherwise the decision will be $R = 1$. The expressions for $\mathcal{A}^U$, $\mathcal{B}^U$, $\mathcal{C}^U$ and $\mathcal{D}^U$ are as follows. When the state is $\omega = 1$, $R = 1$ with probability

$$\mathcal{A}^U = \beta_1^3 + 3\beta_1^2\beta_0 + 3\beta_1\beta_0^2 + \beta_0^3,$$

and $R = -1$ with probability

$$\begin{aligned}
\mathcal{B}^U &= \beta_{-1}^3 + 3\beta_{-1}^2\beta_1 + 3\beta_{-1}^2\beta_0 + 6\beta_1\beta_{-1}\beta_0 + 3\beta_1^2\beta_{-1} \\
&\quad + 3\beta_{-1}\beta_0^2.
\end{aligned}$$

When the state is $\omega = -1$, $R = 1$ with probability

$$\mathcal{C}^U = \beta_1^3,$$

and $R = -1$ with probability

$$\begin{aligned}\mathcal{D}^U =& \beta_{-1}^3 + 3\beta_{-1}^2\beta_1 + 3\beta_1^2\beta_0 + 3\beta_{-1}\beta_0^2 + 6\beta_1\beta_{-1}\beta_0 \\ &+ 3\beta_1\beta_0^2 + 3\beta_{-1}^2\beta_0 + 3\beta_{-1}\beta_1^2 + \beta_0^3.\end{aligned}$$

So, we have

$$\begin{aligned} \mathcal{A}^M - \mathcal{A}^U &= 3\beta_{-1}(\beta_1 + \beta_0)^2 \\ \mathcal{B}^M - \mathcal{B}^U &= -3\beta_{-1}(\beta_1 + \beta_0)^2 \\ \mathcal{C}^M - \mathcal{C}^U &= 3\beta_1^2(\beta_{-1} + \beta_0), \\ \text{and} \quad \mathcal{D}^M - \mathcal{D}^U &= -3\beta_1^2(\beta_{-1} + \beta_0). \end{aligned}$$

Using these in the principal's expected payoff W^V in Eq. (1), and comparing W^M with W^U, establishes the result. □

Proof of Proposition 2. The only differences in the mild case of side-transfers, compared to the benchmark scenario of no side-transfer in Proposition 1, are in the probabilities $\mathcal{C}^U$ and $\mathcal{D}^U$ while the probabilities $\mathcal{A}^U$ and $\mathcal{B}^U$ remain unchanged. The modified probabilities are as follows:

$$\begin{aligned} \mathcal{C}^U &= \beta_1^3 + 3\beta_1^2\beta_0, \\ \mathcal{D}^U &= \beta_{-1}^3 + 3\beta_{-1}^2\beta_1 + 3\beta_{-1}\beta_0^2 + 6\beta_1\beta_{-1}\beta_0 + 3\beta_1\beta_0^2 \\ &\quad + 3\beta_{-1}^2\beta_0 + 3\beta_{-1}\beta_1^2 + \beta_0^3. \end{aligned}$$

(Thus, $3\beta_1^2\beta_0$ is shifted from $\mathcal{D}^U$ to $\mathcal{C}^U$.) Using the modified probabilities in the principal's expected payoff W^V in Eq. (1), and comparing W^M with W^U, establishes the result. □

Proof of Proposition 3. The cases in which the committee's decision may change (relative to the no side-transfer scenario in Proposition 1) as a result of side-transfers are listed in the text. No change occurs in W^U while W^M will change due to the change in R in two cases. In a committee with two type-(0) members and one type-(1) member, when $\omega = -1$ the decision $R = -1$ will change into $R = 1$. Thus, $3(1 - \alpha)\beta_0^2\beta_1$ should be subtracted from $\mathcal{D}^M$ and added to $\mathcal{C}^M$. In a committee with two type-(0) members and one type-(-1) member, when $\omega = 1$, $R = 1$ will change into $R = -1$. We must therefore subtract $3\alpha\beta_0^2\beta_{-1}$ from $\mathcal{A}^M$ and add it to $\mathcal{B}^M$. Performing this exercise and rearranging terms yields the change in the expected payoff under majority voting: $\Delta W^M = -3\beta_0^2[\alpha\beta_{-1}B(1) + (1-\alpha)\beta_1 B(-1)]$.

Thus, $W^M \geq W^U$ if and only if

$$\begin{aligned}
&- 3\beta_0^2[\alpha\beta_{-1}B(1) + (1-\alpha)\beta_1 B(-1)] \\
&\quad + 3\alpha(\beta_1 + \beta_0)^2\beta_{-1}B(1) \\
&\quad \geq -3(1-\alpha)\beta_1^2\beta_0 B(-1) + 3(1-\alpha)\beta_1^2(\beta_{-1} + \beta_0)B(-1),
\end{aligned}$$

which, on simplification, yields (5).[13] $\square$

Proof of Proposition 4. Moving from the case $c < v \leq 2c$ to $2c < v$ modifies the principal's expected payoff only under majority voting, when a type-(1) member is matched with two type-(0) members and the state is $\omega = -1$. The correct decision $R = -1$ will become $R = 1$ and the principal's additional loss will be $3(1-\alpha)\beta_1\beta_0^2 B(-1)$, which yields the total loss in (6).

The principal's expected payoff under the two voting rules can be compared by using the same procedure as in the proofs of Propositions 2 and 3. We skip the details. It can be shown that

$$\mathcal{A}^M - \mathcal{A}^U = 3\beta_{-1}\beta_1^2 + 6\beta_1\beta_{-1}\beta_0 = -(\mathcal{B}^M - \mathcal{B}^U),$$

$$\mathcal{C}^M - \mathcal{C}^U = 3\beta_{-1}\beta_1^2 = -(\mathcal{D}^M - \mathcal{D}^U).$$

Using these expressions in (1) yields the result in (7). $\square$

NOTES

1. See Nitzan and Paroush (1982, 1984), Shapley and Grofman (1984), Nitzan and Procaccia (1986), Sah and Stiglitz (1988), Gradstein and Nitzan (1988), Gradstein, Nitzan and Paroush (1990), and more recently Ben-Yashar and Nitzan (1997).
2. These contexts differ from traditional vote buying models of legislatures (see, for example, Buchanan and Lee (1986), Weiss (1988), Kochin and Kochin (1998), Tullock (1998)) — while bribery among legislative members may not be uncommon in less developed countries, such practices are likely to be rare in the developed world.

3. The symmetric case may be considered more representative of a decision-making context where the principal does not know, ex-ante, what proposal(s) the committee will vote and therefore a priori a selection bias and/or asymmetric costs of errors may not seem justifiable. Given its importance, a similar symmetric case received an exclusive attention by Nitzan and Paroush (1982). However, they did not consider partisan voting or side-transfers but instead allowed the skills of the committee members in choosing the right alternative to differ.
4. There are more than one interpretation given to Condorcet's views but the general theme centers around the efficiency of the majority decision rule (see, for example, Young (1988) or McLean and Hewitt (1994)).
5. More precisely, a partisan has a very high personal cost of changing his vote, thus cannot be 'bribed' to this effect.
6. If vote transfer is prohibited by law or internal regulations, then we consider v as net of the expected penalty of vote buying, and c as inclusive of the expected penalty of vote selling.
7. The unanimity voting stipulating all members vote $x = -1$ for a decision $R = -1$ constitutes the symmetric opposite of the unanimity voting we study below. The results for this opposite unanimity voting can be obtained by interchanging the partisan type subscripts (1) and (-1) in the analysis.
8. The multiplicative factor, 3, has been cancelled out from both sides.
9. The efficiency of a (qualified) majority rule was previously established also by Nitzan and Paroush (1982), but in a less general setting than the one in Ben-Yashar and Nitzan (1997).
10. If the opposite tie-breaker is used so that, when indifferent, the neutral member casts his vote in favor of the partisan whose vote differs from the true state, then to win against $b_{-1} = v$ the type-(1) partisan will have to bid $b_1 > v - c$. But then no such b_1 can be sustained in equilibrium as it can always be lowered while still exceeding $v - c$.
11. There is another possibility of side-transfer, in a committee of two type-(0) members and one type-(-1) member. Under majority voting the type-(-1) has to buy one neutral vote, only if the state is $\omega = 1$. But this case is considered in the previous subsection. The type-(-1) has no need to buy votes under unanimity rule.
12. With $n = 3$, we have four potential side-transfer possibilities, including the case in which side-transfer is impossible, and two voting rules (the 2/3 majority rule and the unanimity rule). With $n = 5$ the number of potential cases of side-transfers goes up to eight, and the number of potential voting rules goes up to three (the 3/5 and 4/5 majority rules and the unanimity rule).
13. The expression on the left-hand side of the inequality, W^M, is derived by adding ΔW^M to the W^M of Proposition 1; W^U, the expression on the right-hand side, is same as in Proposition 1.

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Addresses for correspondence: Mehmet Bac, Sabanci University, Faculty of Arts and Sciences, Orhanli, Tuzla 81474, Istanbul, Turkey

Tel: 90-212-483-9000; Fax: 90-212-483-9005
E-mail: bac@sabanciuniv.edu
and
Parimal Kanti Bag, Department of Economics and Finance, Birkbeck College, University of London, 7-15 Gresse St., London, W1T 1LL, U.K.
Tel: 44-20-7631 6424; Fax: 44-20-7631-6416
E-mail: pbag@econ.bbk.ac.uk