The conventionality of simultaneity in Einstein's practical chrono-geometry

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ABSTRACT: While Einstein considered that *sub specie aeterni* the correct philosophical position regarding geometry was that of the conventionality of geometry, he felt that provisionally it was necessary to adopt a non-conventional stance that he called practical geometry. Here we will make the case that even when adopting Einstein's views we must conclude that practical geometry is conventional after all. Einstein missed the fact that the conventionality of simultaneity leads to a conventional element in the chrono-geometry, since it corresponds to the possibility of different space-time metrics (which, when changing, accordingly, the "physical part" of the theory, are observationally equivalent).

Keywords: practical geometry, conventionality of geometry, conventionality of simultaneity.

RESUMEN: Pese a que Einstein consideraba que *sub specie aeterni* el convencionalismo era la posición filosófica correcta respecto a la geometría, él pensaba que provisionalmente era necesario adoptar una posición no convencionalista a la que llamó geometría práctica. Aquí, defendemos la posición de que aunque se adopten las ideas de Einstein tenemos que concluir que la geometría práctica es convencional. Einstein no tuvo en cuenta el hecho de que la convencionalidad de la simultaneidad lleva a un elemento convencional en la chrono-geometría, pues corresponde a la posibilidad de distintas métricas del espacio-tiempo (que son equivalentes a nivel observacional, cuando se cambia adecuadamente la "parte física" de la teoría).

Palabras clave: geometría física, convencionalidad de la geometría, convencionalidad de la simultaneidad.

1. *Introduction*

According to Einstein, in a fully developed physics theorization it should be possible for one to adopt an axiomatic geometry ($G_{\text{non-standard}}$) different from whatever one standardly adopts (G_{standard}), being possible a change in the "physical part" of the theory (P) such that at an experimental level both formulations of the theory (G_{standard}) + (P_{standard}) and ($G_{\text{non-standard}}$) + ($P_{\text{non-standard}}$) give the same results. In a heuristic way we have (G_{standard}) + (P_{standard}) = ($G_{\text{non-standard}}$) + ($P_{\text{non-standard}}$). However, at the present stage of development of physics one needs to "coordinatize" the geometry (G) directly with measuring rods and clocks conceived as independent concepts that are not theoretically described within the theory (G) + (P). This prevents, according to Einstein, the adoption of a conventional position; i.e. the reference to material objects (the rods and clocks) fixes the geometry (G) in a non-conventional way.[1] Einstein did not address the conventionality of simultaneity

[1] We must notice that nowhere does Einstein makes explicit why this is so. There is no argument justifying the conclusion that the geometry is non-conventional when we coordinatize it with rods and

(which he knew about) in relation to his views on the conventionality of geometry or practical geometry. It seems that it went unnoticed to him the relevance of the conventionality of simultaneity in relation to his views on the non-conventionality of practical chrono-geometry. The metric of space-time is not settled even if we consider that with practical chrono-geometry we set the (mathematical) congruence of distant space intervals coordinated to identical rods, and the (mathematical) congruence of successive time intervals coordinated to the time reading of a clock. The coordinate time (and the metric) is conventionally defined depending on the adopted relation of distant simultaneity. In our view this implies that practical chrono-geometry is conventional.

This paper is structured as follows. In section 2 we will review Einstein's practical geometry and conventionality of geometry. In section 3 we will consider the thesis of the conventionality of simultaneity as developed by Reichenbach, Grünbaum, and others. In section 4 we will defend the view that due to the conventionality of simultaneity, practical chrono-geometry is not free of a conventional element.

2. *Einstein's practical geometry*

While Einstein wrote about geometry along the years in several of his papers,[2] it is in the well-known text "geometry and experience" from 1921 that Einstein presented clearly his views on the conventionality of geometry and his provisional position that he named "practical geometry".[3] Axiomatic geometry is to be distinguished from practical geometry on the

clocks taken to be independent concepts not described by the theory. As a reviewer remarked, this seems to be a sort of "theoretical trick", since "the trick Einstein does is to situate [the rod (or clock)] as an independent theoretical element, so it works as a supra-empirical element that allows the congruence of measurements". Here we will not address this issue. Our intention is not to discuss the "presuppositions" of practical geometry. Here we will make the case that even if accepting these presuppositions we can conclude that practical chrono-geometry is conventional.

[2] See, e.g., Einstein (1912, 28), Einstein (1914, 78), Einstein (1916, 148), Einstein (1917, 250-1), Einstein (1920, 144-6).

[3] It is beyond the scope of this work to present the background and context of Einstein's philosophy of geometry, in particular of his text "geometry and experience". Friedman wrote a paper on this issue regarding specifically "geometry and experience" (Friedman 2002), and several authors made important contributions which are helpful in this regard (see, e.g., Paty 1993, Ryckman 2005, Howard 2010, Howard 2014, Giovanelli 2013, and Giovanelli 2014). We will only mention briefly two points regarding this issue: a) While, e.g., Friedman (2002) and Paty (1993) consider that Einstein was influenced by Helmholtz's views, Giovanelli, considers that the "Helmholtzian" tradition "did not play any relevant role in the emergence of general relativity" (Giovanelli 2013, 3822), and that "Einstein's theory drew its conceptual resources exclusively from what we may call a "Riemannian tradition"" (p. 3822). It might be the case that Helmholtz's influence on Einstein's views might be in need of further scrutiny; b) In his paper, Friedman missed a crucial aspect of the background and context of "geometry and experience": Einstein's debate with Weyl. The importance of this debate was noticed, e.g., in Ryckman (2005), Giovanelli (2013), Giovanelli (2014), Howard (2010), and Howard (2014). This is in our view the most important element that differentiates Einstein's view on practical geometry as developed in "geometry and experience" from his previous remarks on geometry as a physical science. It is also in the context of this debate that arose Einstein's justification for the adoption, in practice, of practical geometry, to which we make reference in the main text.

account that the first has a "merely logical-formal character" (Einstein 1921, 211), while the second is a natural science (211). We can regard practical geometry as an extension of axiomatic geometry that is accomplished by what Einstein refers to as the "coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry" (p. 211). What this means is that we attribute to a practically-rigid body a particular mathematical length – it is represented by a straight segment, one of the elements of the conceptual schemata of axiomatic geometry. In this way, "the propositions of Euclid contain affirmations as to the behavior of practically-rigid bodies" (p. 211).

However, there are in principle reasons to "reject the relation between the body of axiomatic Euclidean geometry and the practically-rigid body of reality" (p. 211). This comes about for the following reason:

> Under closer inspection the real solid bodies in nature are not rigid, because their geometrical behavior, that is, their possibilities of relative disposition, depend upon temperature, external forces, etc. Thus the original, immediate relation between geometry and physical reality appears destroyed. (p. 212)

This led Einstein to advocate the following conventionalist view on geometry:

> [axiomatic] Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification. Thus (G) may be chosen arbitrarily, and also parts of (P); all these laws are conventions. All that is necessary to avoid contradictions is to chose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience. (p. 212)[4]

It seems that Einstein endorsed this view as late as 1949. In the text usually refered to as "reply to criticisms", there is an imaginary dialogue, between Poincaré (and at the end a non-positivist) and Reichenbach. In it Einstein's "Poincaré" mentioned:

> The empirically given bodies are not rigid, and consequently can not be used for the embodiment of geometric intervals. Therefore, the theorems of geometry are not verifiable … why should it consequently not be entirely up to me to choose geometry according to my own convenience (i.e. Euclidean) and to fit the remaining (in the usual sense "physical") laws to this choice in such a manner that there can arise no contradiction of the whole with experience? (Einstein 1949b, 677)

This dialogue seems to have the objective of defending a conventionalist view, even if in it, Einstein's "Reichenbach" endorsed a view compatible with Einstein's practical geometry. In fact, earlier in this text, as Howard (2010) noticed, Einstein endorsed a conventionalist/holistic stance:

[4] There are authors that defend the view that Einstein's conventionality of geometry is a form of Duhemian holism (see, e.g., Grünbaum 1962, Howard 1990). We will not go into this kind of detail here, since the only purpose of this brief section is to present the most relevant aspects of Einstein's conventionality of geometry and his practical geometry, and why he adopted it —provisionally— in spite of his preference for a conventionalist stance on geometry.

> The theoretical attitude here advocated is distinct from that of Kant only by the fact that we do not conceive of the "categories" as unalterable (conditioned by the nature of the understanding) but as (in the logical sense) free conventions. (Einstein 1949b, 674)

As we have a conventionality of geometry due to the, in principle, impossibility of considering a solid body as a true embodiment of a geometrical interval (straight segment), we face the same situation, mutatis mutandis, with the case of clocks and what we might call chronometry. According to Einstein, "the idea of the measuring rod [(i.e. solid body)] and the idea of clock … do not find their exact correspondence in the real world" (Einstein 1921, 212). This implies that the uniformity of time is conventional. In principle we should be able to choose a chronometry corresponding to a uniform time (C) to which corresponds the physical laws (P); or adopt an arbitrary chronometry (i.e. a non-uniform time), and chose (P) so that (C) and (P) are together in agreement with observations.

While the conventionality of geometry and chronometry was Einstein's in principle position; in the context of his theories —special and general relativity—, Einstein felt forced to admit provisionally practical geometry. This resulted, in a way, from an "incompleteness" of his theories.

Einstein had it clear what should be the role of the concepts of measuring rod (solid body) and clock in the theory:

> The solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which must not play any independent part in theoretical physics. (pp. 212-3)

The situation in practice was altogether different. According to Einstein:

> In the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. (p. 213)

This creates what we might call an inconsistency in the theory. As Einstein wrote in his Nobel lecture:

> It seems logically unjustified to base all physical considerations on the rigid, that is, the solid body and then, in the end, to reconstruct it atomistically again with the aid of the fundamental laws of physics that are, in turn, constructed with the aid of the concept of a rigid measuring-body. (Einstein 1923, 75)

The provisional adoption of practical geometry resulted from this limitation of his theories. As Einstein wrote in his autobiographical notes:

> Strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not as it were, as theoretically self-sufficient entities … it was better to permit such inconsistency – with the obligation, however, of eliminating it at a later stage of the theory. (Einstein 1949a, 59 and 61)

Having to treat measuring rods and clocks as theoretically self-sufficient entities and not as solutions of (G) + (P), makes it meaningless, at this point, to adopt the conventionality of geometry in the context of Einstein's theories, or so it seems. The measuring-rods and clocks are, in practice, coordinated directly with the geometry and chronometry, i.e. they are assumed to be practically-rigid rods and what we might call practically-uniform clocks. The non-conventional practical geometry is established as a necessity due to the fact that relativity (special and general) "does not yet exist at all as a finished product" (Einstein 1949b, 678).

While Einstein developed his views mainly mentioning the conventionality of Euclidean geometry or the practical geometry of Euclid, as we have seen his views apply also to the case of time – in what we might call the conventionality of chronometry and practical chronometry. In fact, Einstein's views do not apply only to these two restricted cases. According to him they apply also to the case of Riemannian geometry (Einstein 1921, 213).

While there is a sort of logical loophole in using directly measuring-rods and clocks as theoretically self-sufficient entities, nevertheless, their direct coordination with spatial intervals and temporal intervals is experimentally warranted. This is so because there are in nature physical systems – atoms, that enable the implementation of the notions of practically-rigid body and practically-uniform clock. In Einstein's words:

> All practical geometry is based upon a principle which is accessible to experience … suppose two marks have been put upon a practically-rigid body … we now assume that: if two marks are found to be equal once and anywhere, they are equal always and everywhere. (p. 213)

According to Einstein, "not only the practical geometry of Euclid, but also its nearest generalization, the practical geometry of Riemann, and therefore the general theory of relativity, rest upon this assumption" (p. 213). This assumption is experimentally warranted since in the theory we can relate a spatial interval to a time interval (p. 213), and atoms are natural clocks that emit "light" with frequencies that do not depend on their past history (pp. 213-4): two identical atoms with the same proper frequencies (i.e. two identical clocks with the same rate) will always have the same proper frequencies, "no matter where and when they are again compared with each other at one place" (p. 214).[5]

[5] We can define a practically-rigid body by what Darrigol called "chrono-optical control of rigidity" (Darrigol 2015, 167). According to Darrigol, "rigid rods can be constructed by clock-based optical control, exploiting the constancy of the [two-way] velocity of light" (p. 179). While Darrigol is not very explicit, one approach might be simply to check that a small measuring-rod is in fact practically-rigid by considering light emitted by an atomic clock located in one of the extremities of the rod, which is reflected back at the other extremity of the rod. As Synge mentioned, the length of a spatial interval (in this case corresponding to the measuring-rod) can be determined simply from the time measurements of one clock, taking into account events related to the emission and reception of a light signal that is reflected back (Synge 1960, 112-3). The basic point is that with a natural clock (atom) and light we can define spatial lengths previous to a notion of practically-rigid rod, and only after, from this, define the practically-rigid rod.

3. *The conventionality of simultaneity*

According to Einstein, in special relativity to establish a notion of coordinate time in an inertial reference frame we need to stipulate how two distant clocks can be considered to give the same time reading. This is made by applying a particular synchronization procedure and by considering by definition that two clocks synchronized in this way have the same time coordinate. In this way we have a definition of simultaneity of distant events; if an event (e.g. a flash of lightning) occurs at the location of one of the clocks when its time reading is t, then we consider this event to be simultaneous to all the other events occurring next to all the clocks of the inertial reference frame (i.e. at every location) when these give a time reading of t. According to Einstein:

> It is not possible to compare the time of an event at A with one at B without a further stipulation ... the latter can now be determined by establishing by *definition* that the 'time' needed for the light to travel from A to B is equal to the 'time' it needs to travel from B to A. For, suppose a ray of light leaves from A toward B at 'A-time' t_A, is reflected from B toward A at 'B-time' t_B, and arrives back at A at 'A-time' t_A'. The two clocks are synchronous by definition if $t_B - t_A = t'_A - t_B$. (Einstein 1905, 142)

Einstein's definition of coordinate time corresponds to adopting the rule $t_B = t_A + ½(t'_A - t_A)$. In his 1917 book, Einstein is clear regarding the conventional character of the definition of distant simultaneity (which goes hand in hand with the definition of coordinate time) made by assuming that the process of propagation of light in two opposite directions is symmetrical:

> That light requires the same time to traverse the path [AB] as for the path [BA] is in reality neither a *supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own freewill in order to arrive at a definition of simultaneity. (Einstein 1917, 272)[6]

In his detailed analysis of the definition of simultaneity, Reichenbach noticed that since it results from a stipulation we might as well choose a different definition of coordinate time. In particular Reichenbach considered the following definition:

> A definition of simultaneity ... according to the formula $t_2 = (t_1 + t_2)/2$ is not false because such a definition is arbitrary ... the inconvenience of such a notion of [coordinate] time resides in the fact that it would violate the principle of causality. (Reichenbach 1922, 115)

Reichenbach made an extension of Einstein's synchronization procedure in which the one-way speed of light was undefined and it was just imposed a causality condition corresponding to taking $t'_A > t_B > t_A$. This led to a definition of coordinate time in terms of the expression $t_B = t_A + \varepsilon(t'_A - t_A)$, where $0 < \varepsilon < 1$ (Reichenbach 1924, 31-40; see also Reichenbach 1927, 123-47; Grünbaum 1955, 451-6).

6 In 1924, in a letter, Einstein mentioned that there were physical hypothesis and conventions in the theory of relativity, one example of convention being the concept of simultaneity (see, e.g., Martínez 2005, 453).

Einstein's definition of simultaneity corresponds to setting $\varepsilon = ½$. Any other value of ε between 0 and 1 leads to a non-standard definition of simultaneity. In this case the one-way speed of light is not isotropic. To simplify, by considering a non-standard definition of simultaneity just in the x direction, this means that the one-way speed of light in the positive x direction is different from the one-way speed of light in the negative x direction. Grünbaum mentioned this point:

> The conventionality of simultaneity allows but does not entail our choosing the same value $\varepsilon = ½$ for all directions within every system. In each system this choice assures the *equality* of the one-way velocities of light in opposite directions by yielding *equal* one-way transit times ... for equal distances. The ratio of these one-way transit times is $\varepsilon/(1 - \varepsilon)$, and therefore, in the case of $\varepsilon \neq ½$, these one-way times are *unequal*. But ... no fact of nature ... would be contradicted if we choose values of $\varepsilon \neq ½$ for each inertial system, thereby making the velocity of light different from c in both senses along each direction in all inertial systems. (Grünbaum 1968, 308)

Reichenbach and Grünbaum's work on the conventionality of simultaneity has been extended by several authors (see, e.g., Jammer 2006, Janis 2014). In 1963, Edwards obtained generalized Lorentz transformations corresponding to the adoption of a non-standard definition of simultaneity. Edwards' approach was further extended by Winnie that developed kinematical relativity for any possible value of ε (Edwards 1963, Winnie 1970; see also Jammer 2006, 251-4). Edwards' work made it clear that when adopting an anisotropic one-way speed of light the corresponding space-time is anisotropic. Edwards made his case just in terms of the anisotropic one-way speed of light, not showing what kind of anisotropic space-time corresponded to each definition of simultaneity (Edwards 1963, 484-5). However, Edwards did mention that the light cone structure of space-time changed according to the adopted definition of simultaneity (pp. 488-9).

Applying Reichenbach's ε-definition of simultaneity (Reichenbach 1924, 31-40; see also Rynasiewicz 2003), the one-way speed of light in the positive direction of the x-axis is $c_+ = c/2\varepsilon$ and the one-way speed of light in the negative direction of the x-axis is $c_- = c/2(1 - \varepsilon)$, where c is the constant two-way speed of light (see, e.g., Winnie 1970, 83). The time coordinate at a distance x from the origin is given by $t_B = t_A + x/c_+$, where t_A is the time reading of the clock located at the origin (see, e.g., Zhang 1997, 8; Jammer 2006, 180).[7] The line element that corresponds to this definition of coordinate time is given by $ds^2 = (c\,\mathrm{d}t + q\,\mathrm{d}x)^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$, where $q = 2\varepsilon - 1$, to which is associated a non-orthogonal metric tensor (see, e.g., Zhang 1997, 82-5, Anderson *et al.,* 1998, 106 and 111).[8] Due to the non-standard definition of coordinate time we find that we have an anisotropic space-time. In contrast, when adopting Einstein's standard definition of coordinate time, the line element is given by $\mathrm{d}s^2 = (c\,\mathrm{d}t)^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$ and the metric tensor is orthogonal (see, e.g., Zhang 1997, 27).

That the conventionality of simultaneity is a consistent thesis was confirmed when in 1978 Giannoni developed a generalization of relativistic dynamics and electrodynam-

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7 This expression can be deduced from $t_B = t_A + \varepsilon(t'_A - t_A)$ by taking into account the expressions for c_- and c_+.

8 Making $\mathrm{d}s^2 = (c\,\mathrm{d}t + q\,\mathrm{d}x)^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = 0$ we obtain the expression describing the tilted light cone structure mentioned by Edwards (1963, 488-9).

ics for the case of a non-standard synchronization according to Reichenbach's prescription (Giannoni 1978). For us it is particularly interesting the development of an anisotropic electrodynamics compatible with the adopted definition of simultaneity. This is so because, as we have seen, a non-standard definition, corresponding to an $\varepsilon \neq ½$, implies "making the velocity of light different from c in both senses along each direction in all inertial systems" (Grünbaum 1968, 308); i.e., the one-way speed of light in the positive direction of the x-axis c_+ is different from the one-way speed of light in the negative direction of the x-axis c_-. However, the standard formulation of electrodynamics is made only in terms of the two-way speed of light, being implicit in its formulation that $c_+ = c_- = c$. If we had to adopt isotropic electrodynamics with an anisotropic space-time we would have an inconsistency: to the anisotropic space-time corresponds $c_+ \neq c_-$, and to the isotropic electrodynamics corresponds $c_+ = c_-$. Giannoni showed that a generalization of Maxwell-Lorentz equations is possible that is consistent with the anisotropic space-time. In particular, Giannoni showed that, while isotropic electrodynamics has solutions corresponding to a plane wave traveling in free space with a speed c in any direction, anisotropic electrodynamics predicts a wave traveling in the positive direction of the x-axis with a speed of c_+ and a wave traveling in the negative direction of the x-axis with a speed of c_- (Giannoni 1978, 33-8). According to Giannoni, "this solution is perhaps one of the most critical verifications of the thesis that Maxwell's equations are generalizable for arbitrary synchronization of clocks" (p. 38).

4. *The conventionalism of practical geometry*

While Einstein was the precursor of the thesis of the conventionality of simultaneity and he knew about Reichenbach's ideas,[9] it does not seem that Einstein ever took into account the conventionality of simultaneity in relation to his proposal of physical geometry. As we have seen, while Einstein was an advocate of the conventionality of geometry he felt it necessary, provisionally, to adopt practical geometry.

Practical geometry is supposed to supersede the conventionality of geometry, i.e. the geometry is supposed to be settled in a non-conventional way. Accordingly, there should be no conventional elements in practical geometry. Let us see an example of how practical geometry avoids conventional elements. Grünbaum gave a clear example of the conventionality of geometry at work (Grünbaum 1959, 205-6; Grünbaum 1962, 414-5):

> Differential geometry allows us to metrize a given physical surface, say an infinite blackboard or some portion of it, in various ways so as to acquire any metric compatible with its topology. Thus, if we have such a space and a net-work of Cartesian coordinates on it, we can just as legitimately metrize the portion *above* the x-axis by means of the metric $ds^2 = (dx^2 + dy^2)/y^2$, which confers a hyperbolic geometry on that space, as by the Euclidean metric $ds^2 = dx^2 + dy^2$. (Grünbaum 1959, 205)

[9] In 1927, Einstein wrote a review of Reichenbach's book of that year, emphasizing Reichenbach's treatment of this matter: "special care has been taken to ferret out clearly what in the relativistic definition of simultaneity is a logically arbitrary decree and what in it is a hypothesis, i.e. an assumption about the constitution of nature" (cited in Jammer 2006, 191).

With this non-standard metrization, the congruence relation between two distant segments is not the one we have with the Euclidean geometry:

> The lengths of horizontal segments whose termini have the same coordinate differences dx will be ds = dx/y and will thus depend on where they are along the y-axis ... this metrization commits [us] to regard a segment for which dx = 2 at y = 2 as *congruent* to a segment for which dx = 1 at y = 1. (p. 205)

This is possible because according to the conventionality of geometry thesis, "the congruence of two segments is a matter of convention, stipulation or definition and not a factual matter" (p. 205).

What about measurements made with a transportable measuring-rod? At this point we have not yet addressed how a geometrical segment is related to a solid body, under any of the possible conventions for the congruence of distant segments. According to Grünbaum:

> [We do] not say, of course, that a transportable solid rod will coincide, successively with the two hyperbolically-congruent segments but allows for this non-coincidence by making the length of the transported rod a suitable function of its position rather than a constant. (p. 205)

In terms of Einstein's heuristics of (G_{standard}) + (P_{standard}) = ($G_{\text{non-standard}}$) + ($P_{\text{non-standard}}$), what Grünbaum is saying is that if it turns out, as a factual matter, that with our chosen measuring-rod two hyperbolically-congruent segments are measured as different, we can accommodate this by considering a change in the physical laws, i.e. we can conventionally adopt hyperbolic geometry and consider that our measuring-rod changes its length under transport (when being moved).

With practical geometry we lose this conventional choice of geometry. The measuring-rod is taken to be a practically-rigid rod that maintains its length under transport; and since it is a theoretically self-sufficient entity, it is coordinated directly with the concept of straight segment, an element of the conceptual schemata of axiomatic geometry. That is, we do not treat the practically-rigid body as a physical system described by the theory and to which should correspond a solution of (G) + (P).

In a conventionalist context we are free to make an assumption regarding the congruence of distant spatial intervals independently of having or not having the same coordinate intervals. In the case of Grünbaum's example, a segment with a coordinate interval of dx = 2 (at y = 2) has the same ds as a segment with a coordinate interval dx = 1 (at y = 1). In the case of practical geometry, dx and dy correspond to the lengths of practically-rigid rods. Since the practically-rigid body is taken to maintain its length under transport it follows the equality (congruence) of spatial intervals dx and dy at different locations for the same or identical rods. If at a particular location with y = 1 we have, e.g., a dx = 1 (corresponding to a unit measuring-rod), then ds = 1; and if at any other location with a different y we also have a dx = 1 (considering an identical measuring-rod or the same rod transported to this location), then again ds = 1.

With practical geometry, by identifying dx (and dy) with a solid body (measuring-rod), geometry becomes, as Einstein mentioned, "the most ancient branch of physics" (Einstein 1921, 211). If we make experiments regarding the disposition of practically-rigid bodies we verify that they "are related, with respect to their possible dispositions, as are bodies in

Euclidean geometry" (p. 211). We are then enforced to adopt the Euclidean metric corresponding to the line element $ds^2 = dx^2 + dy^2$.

Regarding chronometry the situation is the same, mutatis mutandis: the dt is identified with what we might call the time reading of a practically-uniform clock, and the chronometrical variable t is taken to represent a uniform time. With the conventionality of chronometry we might consider that successive time intervals are not equal (congruent), i.e. that successive dt_1, dt_2, dt_3, ... do not all correspond to the same dt (see, e.g., Reichenbach 1927, 114-7). This would enable to adopt a non-uniform time scale (see, e.g., Grünbaum 1962, 418-9), which according to Einstein would lead to a change in the physical laws. This is not possible with practical chronometry.

We must notice that Grünbaum's and Reichenbach's views apply already at the "level" of what Einstein called an amended geometry (see, e.g., Einstein 1914, 78); i.e. the axiomatic geometry complemented by the coordination of a line segment with a "material straight line" (Einstein 1912, 28). As we have seen, in 1921 Einstein made the case that the amended, supplemented (Einstein 1917, 251), or completed geometry (Einstein 1921, 211) was a non-conventional practical geometry. In this way we cannot consider the examples taken from Grünbaum and Reichenbach as corresponding to what might be Einstein's "exact" views. However, we find them illustrative of how a conventionality of geometry and a conventionality of chronometry might operate along Einstein's heuristics of $(G_{\text{standard}}) + (P_{\text{standard}}) = (G_{\text{non-standard}}) + (P_{\text{non-standard}})$.

In what regards what we might consider as an example of Einstein's exact position, this is not to be found in his writings on the conventionality of geometry. According to Einstein the conventionality of geometry occurs at the level of axiomatic geometry; what we might call a non-interpreted abstract geometry in which, e.g., a line segment could not be related to an experimentally measured length. As we have seen, according to Einstein, "[axiomatic] geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so" (Einstein 1921, 212). Contrary to the cases of Grünbaum and Reichenbach (that locate the conventionality of geometry and chronometry at the level of an amended geometry), Einstein did not give any example of how the conventionality of geometry might unfold according to his "formula". It might be the case that since Einstein considered that only on a later stage of development of physics theories might comply with geometry conventionalism, he simply could not give any "toy theory" example of geometry conventionalism at play according to his heuristics.[10]

However, we should notice that it is implicit in Einstein's views that the conventionality of the congruence of spatial intervals and the conventionality of the congruence of time intervals are the two conventional elements that are superseded with practical geometry and chronometry, which, importantly, are according to Einstein applicable in the general case of a Riemannian geometry. We might then speak of practical chrono-geometry, since in the semi-Riemannian geometry of special and general relativity we have a four-dimen-

[10] This would also imply, even if Einstein was not explicit about it, that the space-time of classical mechanics was to him non-conventional; i.e. that a sort of practical geometry had implicitly been used in physics. In fact, Einstein recognized that the possibility of assuming the conventionality of geometry, "at the current state of science should be regarded as overly cautious" (Einstein 1924, 326).

sional space-time, not a three-dimensional space and a separated one-dimensional time. From these views it follows that Einstein did not consider that there was any other conventional element at play. The amended practical geometry (G) would be settled.

As we have seen, when adopting Einstein's synchronization procedure, the line element of the Minkowski space-time is given by $ds^2 = (c\,dt)^2 - dx^2 - dy^2 - dz^2$ and the metric tensor is orthogonal. The Minkowski space-time is isotropic. Notice that Einstein's views in terms of practical geometry have a bearing only on the identification of dx (dy, dz) with a practically-rigid body and dt with the reading of a practically-uniform clock. If we now adopt a non-standard definition of simultaneity (making $\varepsilon \neq ½$), since we do not have in this case an isotropic one-way speed of light, the non-standard definition of coordinate time leads to a line element given by $ds^2 = (c\,dt + q\,dx)^2 - dx^2 - dy^2 - dz^2$, where $q = 2\varepsilon - 1$. In this case the metric tensor is non-orthogonal, implying that the space-time is anisotropic. We have a case similar to that of Grünbaum's example of the conventionality of geometry. We can have different metrizations of space-time, all of them legitimate. We are allowed to metrize space-time in different ways so as to acquire orthogonal or non-orthogonal metrics, leading each case to an isotropic space-time or an anisotropic space-time. The non-standard metrization commits us to regard the one-way speed of light as anisotropic and to have a definition of coordinate time different from the one adopted in the standard case.

Here we adopt the position, compatible with Grünbaum's views, that the conventionality of the (amended) geometry is manifested in the metric when we have the possibility of different metrizations of space-time that due to a change in the "physical part" of the theory are observationally equivalent. Accordingly, practical geometry is still a conventional stance on geometry, since we have a the conventional element in the chrono-geometry, due to the conventionality of simultaneity, which occurs at the level of an amended geometry not as in Einstein's original conception at the level of axiomatic geometry. In this case, when adopting a chrono-geometry corresponding to the line element $ds^2 = (c\,dt + q\,dx)^2 - dx^2 - dy^2 - dz^2$, electrodynamics is generalized to the anisotropic case so that it predicts a plane wave traveling in the positive x-axis with a one-way speed of light identical to the one implied by the non-standard definition of simultaneity, with an equivalent result holding in the case of the negative x-axis.[11, 12] As mentioned, Einstein did not present any example of geometric conventionalism in a physical theory. Even if we say that strictly speaking we cannot consider this conventionality as an example of Einstein's conventionality of geometry as originally formulated by him, it is still

[11] Since in general relativity the space-time is locally Minkowskian these results also apply to the case of general relativity in which there is still a conventional element in its practical geometry. It is a fact that, depending on the particular space-time under consideration, we might not be able to establish a standard definition of simultaneity in a finite region (see, e.g., Jammer 2006, 271-285). However, here we are considering infinitesimal regions where the metric is locally Minkowskian (see, e.g., Norton 2015). It is in the local Minkowskian metric that we find the conventional element due to the conventionality of simultaneity.

[12] One way to avoid the presence of a conventional element in Einstein's practical geometry would be to deny the thesis of the conventionality of simultaneity (see, e.g., Janis 2014). Another possibility might be to consider that when adopting practical geometry one escapes standard skeptical arguments regarding the non-conventionality in the synchronization of distant clocks (Bacelar Valente 2017).

the case that we cannot consider practical chrono-geometry to be free of a conventional element.[13]

5. *Conclusions*

Practical geometry, the provisional view adopted by Einstein, was supposed to supersede, in practice, the conventionality of chrono-geometry – the view that Einstein endorsed in principle. It seems that it went unnoticed by Einstein that the thesis of the conventionality of simultaneity was relevant to his views regarding practical geometry versus the conventionality of geometry. We can see that, in a way, the surpassing of the conventionality of geometry was in an importance sense incomplete. The simple point is that practical geometry has a conventional element since depending on the stipulation of distant simultaneity we have a different definition of coordinate time and a different metric. The conventionality of simultaneity implies that the line element (or metric) of space-time is not uniquely determined; we can choose different chronogeometries (G_s) or (G_{ns}) corresponding to an isotropic space-time or anisotropic space-times. This goes hand in hand with a change in the physical laws; in particular with the adoption of an anisotropic electrodynamics (P_{ns}) instead of the standard isotropic electrodynamics (P_s). The presence of a conventional element in practical geometry implies that practical geometry is still a conventional stance on chrono-geometry.

Abbreviations

CPAE Albert Einstein (1987-). *The Collected Papers of Albert Einstein*. Ed. by John Stachel *et al.* 14 Vols. Princeton: Princeton University Press.

[13] A reviewer mentioned that "in the case of the conventionality of geometry it seems that we are talking about actually determining different geometries, while in the case of simultaneity what changes is the expression of the metric, not the actual geometry or chronogeometry". Because of this the conventionality still present in practical geometry would be of a "weaker type". We would have no quarrel with a somewhat nuanced version of this view. For example, when adopting a view on the conventionality of geometry along the lines of Grünbaum, we could say that practical geometry already fixes in a non-conventional way that the space-time is flat. The conventional element due to the conventionality of simultaneity does not change the geometry from a flat to a non-flat space-time. However, in our view, this is still a "legitimate" case of conventionality of geometry. This does not mean that there might not be views on the conventionality of geometry in which the reviewer's claim might be validated. It is beyond the scope of this work to address this issue. One last point we would like to make is that if we think in terms of Einstein's proposition it does not seem to be possible to defend the view that the conventionality still at play is of a "weaker type". Einstein's views are rather sketchy and he did not exemplified how actually the conventionality is supposed to unfold at the level of axiomatic geometry (besides his very general heuristic prescription). For a consistency reason (in relation to his views on practical geometry) we must consider that it should be related to (mathematical) space and time congruences but previous to their coordinatization with material standards of length and duration. However, we do not have enough elements to compare the eventual conventions at the level of axiomatic geometry with the conventional element still present in practical geometry.

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