Einstein, Bohm, and Leggett–Garg

Guido Bacciagaluppi*

18 May 2015

Abstract

In a recent paper (Bacciagaluppi 2015), I have analysed and criticised Leggett and Garg's argument to the effect that macroscopic realism contradicts quantum mechanics, by contrasting their assumptions to the example of Bell's stochastic pilot-wave theories, and have applied Dzhafarov and Kujala's analysis of contextuality in the presence of signalling to the case of the Leggett–Garg inequalities. In this chapter, I discuss more in general the motivations for macroscopic realism, taking a cue from Einstein's criticism of the Bohm theory, then go on to summarise my previous results, with a few additional comments on other recent work on Leggett and Garg. [To appear in: E. Dzhafarov (ed.), *Contextuality from Quantum Physics to Psychology* (Singapore: World Scientific).]

1 Introduction

Consider the following set of assumptions, which we shall collectively call 'macroscopic realism' (or 'macrorealism' for short):

*Department of Philosophy, University of Aberdeen; and UMR 8590 IHPST-Institut d'Histoire et de Philosophie des Sciences et des Techniques, Université Paris 1 Panthéon-Sorbonne, CNRS, ENS. Address for correspondence: Department of Philosophy, University of Aberdeen, Old Brewery, High Street, Aberdeen AB24 3UB, Scotland (e-mail: g.bacciagaluppi@abdn.ac.uk).

(a) macroscopic quantities have definite values at all times;

(b) they obey a well-defined (possibly stochastic) dynamics;

(c) at least under appropriate circumstances, measuring such a quantity reveals its value immediately prior to measurement (call this 'faithful measurability');

(d) at least under appropriate circumstances, measuring such a quantity does not change its value or its subsequent dynamics (call this 'non-disturbing measurability').

Leggett and Garg (1985, henceforth LG) take it that these conditions capture a common intuition,[1] and aim to provide an experimental test that may decide for or against it, specifically (as with Bell providing an experimental test for or against 'local realism'), by deriving inequalities from (a)–(d) that are violated by quantum mechanics. The test case they have in mind is the macroscopic flux in a SQUID tunnelling between two symmetric potential wells. Of course, the term 'macroscopic' is vague, but one could take conditions (a)–(d) as defining a specific theoretical sense in which a quantity would be 'macroscopic', and then test against it our pre-theoretical intuitions about what is or is not macroscopic ('Is the Moon there if we are not looking?').

Recent conceptual work on LG inequalities includes the papers by Kofler and Brukner (2013), Maroney and Timpson (2014), and Bacciagaluppi (2015).[2] All of these authors are critical of the significance of violations of the LG inequalities, and (independently) point out both that there are simpler necessary criteria for macrorealism than the satisfaction of the LG inequalities, and that pilot-wave theories seem to provide natural counterexamples to LG's assumptions (despite being 'realist' and even 'macrorealist' in suitable senses).[3] In particular, Kofler and Brukner point out that 'signalling in time', i.e. a (non-selective) measurement of some observable disturbing the statistics of subsequent measurements, already by itself rules out macrorealism. A

[1]Note that their formulation of the conditions is slightly different. In particular, their original condition of 'non-invasive measurability' appears to be an amalgam of conditions (c) and (d). Later on, Leggett (2002) distinguishes explicitly between these two components. In my earlier paper (Bacciagaluppi 2015), I call (d) 'non-invasive measurability', following Leggett and Garg, but a distinct terminology is preferable.

[2]For earlier literature, see e.g. the references in Maroney and Timpson (2014).

[3]Cf. footnote 10 below.

further point made by Maroney and Timpson is that 'non-invasive measurability' (to use LG's terminology) is not operationally well-defined (unlike the corresponding locality condition in Bell's case), and that if it is made properly operational, one sees that violations of the LG inequalities in fact rule out much less than expected. Finally, Bacciagaluppi follows Dzhafarov and Kujala's (henceforth DK) analysis of contextuality in the case of non-existence of pairwise joint distributions (e.g. Dzhafarov and Kujala 2014a,b), and shows that violation of the LG inequality is not always sufficient for contextuality in DK's sense (only when the initial state is sufficiently mixed).

In the following we shall discuss the motivation for macrorealism, taking the cue from Einstein's criticism of the Bohm theory (Section 2). We shall then briefly summarise LG's original argument (following the analysis in our earlier paper) and the simpler necessary criteria for macrorealism pointed out by the above authors (Section 3). We then discuss contextuality in the LG inequalities, summarising our analysis based on DK's work, and concluding with some additional remarks about analysing violations of the LG inequalities (Section 4).

2 Is macrorealism plausible?

In 1953 Max Born retired from the Tait Chair in Natural Philosophy at the University of Edinburgh, and among many others who contributed to the *Festschrift* that marked the event, Einstein wrote a short and beautiful paper on the interpretation of quantum mechanics (Einstein 1953). In it, Einstein argued for a statistical interpretation of quantum mechanics using a macroscopic example of a ball 1mm in diameter bouncing between two perfectly smooth walls.

The classical ('macrorealist') description of the ball is an *individual* description: the ball is somewhere between the walls, and it moves either to the left or to the right. The corresponding statistical description is that at any one time the ball has uniform probability for being anywhere in the box and equal probabilities for moving to the left or to the right.

The quantum mechanical description of the ball inside the box, according to Einstein, is instead given by the superposition of two plane waves with opposite momenta, $\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. In this way quantum mechanics gets the *statistical* predictions right, but it lacks the individual description of the ball. Therefore — so Einstein — the correct interpretation of quantum mechanics is one that takes the quantum state as a statistical, not a complete description of a system.

At this point, Einstein's contribution also includes a criticism of Bohm's theory. Recall that just the year before, Bohm (1952) had rediscovered and extended de Broglie's (1928) pilot-wave theory, in which point particles move in configuration space according to the guidance equation

$$m_i \dot{\mathbf{x}}_i = \nabla_i S(\mathbf{x_1}, \ldots, \mathbf{x}_n) \;, \tag{1}$$

where S is the phase of the quantum mechanical wave function.[4] De Broglie's theory (under the assumption that the position distribution of particles is given by $|\psi|^2$) gives a straightforward explanation for the results of diffraction and interference experiments. Indeed, it is while developing these ideas that de Broglie made the prediction of electron diffraction, soon quantitatively confirmed by Davisson and Germer.[5] Bohm extended the theory to include also measurements of observables that do not commute with position. Indeed, imagine that during the measurement some narrow wave packets describing pointer positions are coupled with the eigenstates of the measured observable: when the different components separate in the total configuration space, the system and pointer are 'trapped' inside one of the components, which (barring reinterference) is henceforth solely responsible for guiding the evolution of the total system. Assuming a Born distribution for the initial configurations, the theory predicts the appearance of collapse with the usual quantum probabilities.

At first one might think Bohm's theory is providing precisely the kind of individual description Einstein is arguing for. Indeed, in Bohm's theory the ball does have a well-defined position, which in an ensemble defined by the above wave function is distributed uniformly between the walls. But if one considers the momentum of the ball, then while Bohm's theory gets

[4] To be precise, in Bohm's papers the guidance equation is written in second-order form, while de Broglie generally writes it as the above first-order equation.

[5] For the development of de Broglie's ideas, see e.g. Bacciagaluppi and Valentini (2009, Chapter 2).

the same statistical predictions as quantum mechanics also for momentum measurements, *the value of the ball's momentum before the measurement is zero*, because the wave function is a standing wave, and so $\nabla S = 0$.

Einstein's objection to the Bohm theory is thus that the theory does not adequately describe the *macroscopic world*, specifically in the sense that, even for a macroscopic object, measuring its momentum in general does not yield its pre-existing value. In terms of the definition of macrorealism above, in the Bohm theory the ball always has well-defined position and momentum (condition (a)), which follow a well-defined dynamics (condition (b)), but faithful measurability for momentum is violated (condition (c)). One may imagine that Einstein would have endorsed also the condition of non-disturbing measurability (condition (d)), which the Bohm theory also violates: measuring the position of the stationary ball reveals its pre-existing position, but the ball starts moving, so its position and subsequent dynamics are disturbed.

At the time, the theory of decoherence had not been explicitly developed yet, but nowadays it provides a straightforward reply a Bohm theorist can make to Einstein. They will concede that the ball is 'macroscopic' by certain pre-theoretic criteria, but reply that it ought to behave in accordance with conditions (a)–(d) *only* in the appropriate 'macroscopic' (or classical) regime. And this regime is crucially characterised by the presence of decoherence. If one models Einstein's ball in a box to include decoherence interactions with the environment, then already prior to measurement (and in analogy to Bohm's own discussion of measurement) the ball and the particles in the environment will be trapped inside a component of the total wave function that projects down to a narrow wave packet bouncing between the walls of the box. Appropriately gentle measurements will both reveal and leave this motion unaffected, thus satisfying also conditions (c) and (d).[6]

The bone of contention is not whether or not macrorealism as set out in (a)–(d) is a reasonable constraint on a theory: it is whether it should be applied at a *fundamental* or an *emergent* level. Einstein (or at any rate an Einstein oblivious to decoherence) takes it as a fundamental constraint, while Bohm (or at any rate a Bohm appreciative of decoherence) takes it as a constraint

[6]For the role of decoherence in quantum mechanics, including the case of the Bohm theory, see in particular Bacciagaluppi (2012) and references therein.

on an appropriately emergent description of the macroscopic world.

This now reveals an essential difference between the case of local realism and that of macroscopic realism: while locality is the kind of notion that can indeed enter into a fundamental principle of nature, macroscopicity seems to be a notion that is intrinsically vague and limited in scope. Cell biology breaks down at 400K. Macrorealism may also break down under the wrong conditions. A cheetah in a zoo cut off from its natural environment will not run 70 mph.[7] A ball in a box cut off from *its* environment may well not be moving at all until measured. When Aspect performed his celebrated Bell experiments, we discovered that nature is fundamentally nonlocal. Should we perform a Leggett–Garg experiment on a SQUID and find a violation of the LG inequalities, we would not show that we were somehow mistaken in thinking that the Moon is there also when we are not looking. We would obtain a significant confirmation of quantum mechanics in a regime that we had not tested before, but as regards macroscopic realism we would simply confirm that decoherence furnishes a more reliable criterion for classical or indeed 'macrorealist' behaviour than mere particle number.[8]

3 The Leggett–Garg inequalities

Here is a simple derivation of an LG inequality. Take some two-valued quantity Q deemed to be macroscopic (e.g. the flux in a SQUID being in the left or right potential well, the Moon waxing or waning). From (a), we conclude that $Q = \pm 1$ at all times, e.g. t_1, t_2, t_3, which gives us eight possible sequences of values $\pm 1, \pm 1, \pm 1$. By inspection, this yields the following inequality (notation: Q_i is the random variable representing the value of Q at time t_i when no measurements are performed):

$$-1 \leq Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 \leq 3 \ . \qquad (2)$$

[7] The example is taken from Tolan (1996).

[8] I claim no novelty for this conclusion, which has been pointed out in various other ways before, ranging from e.g. Benatti, Ghirardi and Grassi (1994) through to Maroney and Timpson (2014).

From (b), there is a well-defined joint probability distribution for the various Q_i, thus we have also

$$-1 \leq \langle Q_1 Q_2 \rangle + \langle Q_2 Q_3 \rangle + \langle Q_1 Q_3 \rangle \leq 3 \ . \tag{3}$$

Now consider making measurements at times $t_i, t_j, t_k, \ldots$ Write $Q_n^{ijk\ldots}$ for the random variable representing the value of Q at time t_n if measurements of Q take place at $t_i, t_j, t_k, \ldots$ From (c) and (d) together we get

$$p(Q_i^{ij}, Q_j^{ij}) = p(Q_i, Q_j) \ , \tag{4}$$

thus

$$\langle Q_i^{ij} Q_j^{ij} \rangle = \langle Q_i Q_j \rangle \qquad \text{for all } i, j, \tag{5}$$

and

$$-1 \leq \langle Q_1^{12} Q_2^{12} \rangle + \langle Q_2^{23} Q_3^{23} \rangle + \langle Q_1^{13} Q_3^{13} \rangle \leq 3 \ . \tag{6}$$

This inequality now refers only to measured expectation values, and as such can be tested.[9]

Furthermore, it can be violated in quantum mechanics. Indeed, LG consider the example of the macroscopic flux in a SQUID, which in an appropriate regime simply tunnels between two symmetric potential wells, as discussed by Chakravarty and Leggett (1984):

$$|\psi(t)\rangle = \cos\left(\tfrac{\Delta E}{2\hbar}(t - t_0)\right)|\psi_R\rangle - i \sin\left(\tfrac{\Delta E}{2\hbar}(t - t_0)\right)|\psi_L\rangle \ . \tag{7}$$

Taking $Q := |\psi_R\rangle\langle\psi_R| - |\psi_L\rangle\langle\psi_L|$, one has

$$\langle Q_i^{ij} Q_j^{ij} \rangle = \cos\left(\tfrac{\Delta E}{\hbar}(t_j - t_i)\right) \tag{8}$$

(ΔE being the split between the symmetric and antisymmetric states). If one chooses

$$\tfrac{\Delta E}{\hbar}(t_3 - t_2) = \tfrac{\Delta E}{\hbar}(t_2 - t_1) = \tfrac{2\pi}{3} \ , \tfrac{\Delta E}{\hbar}(t_3 - t_1) = \tfrac{4\pi}{3} \ , \tag{9}$$

[9]We shall focus on the lower bound, but note that the upper bound can be made tighter. As in Suppes and Zanotti (1981), one has in fact

$$-1 \leq \langle Q_1^{12} Q_2^{12} \rangle + \langle Q_2^{23} Q_3^{23} \rangle + \langle Q_1^{13} Q_3^{13} \rangle \leq 1 + 2\min\{\langle Q_1^{12} Q_2^{12} \rangle, \langle Q_2^{23} Q_3^{23} \rangle, \langle Q_1^{13} Q_3^{13} \rangle\} \ .$$

all three cosine terms equal $-\frac{1}{2}$, and one obtains a maximal violation of the lower bound of (6).

There are, however, simpler ways of experimentally contradicting (a)–(d). One example is discussed by Maroney and Timpson (2014), who point out that (a)–(d) imply not only

$$p(Q_i^{ij}, Q_j^{ij}) = p(Q_i, Q_j) \ , \tag{10}$$

but also

$$p(Q_1^{123}, Q_2^{123}, Q_3^{123}) = p(Q_1, Q_2, Q_3) \ . \tag{11}$$

Since the $p(Q_i, Q_j)$ are marginals of $p(Q_1, Q_2, Q_3)$, if a $p(Q_i^{ij}, Q_j^{ij})$ fails to be a marginal of $p(Q_1^{123}, Q_2^{123}, Q_3^{123})$, that already refutes (a)–(d). This is the case e.g. if one observes interference such that

$$p(Q_1^{13}, Q_3^{13}) \neq p(Q_1^{123}, Q_2^{123} = +1, Q_3^{123}) \ + \ p(Q_1^{123}, Q_2^{123} = -1, Q_3^{123}) \ . \tag{12}$$

Maroney and Timpson note the analogy between (12) and the double-slit experiment, where interference rules out a (naive!) 'microrealistic' picture of the particle always going through the upper or lower slit.[10]

Even more simply, Kofler and Brukner (2013) point out that (a)–(d) imply

$$p(Q_j^j) = p(Q_j^{ij}) \ , \tag{13}$$

which they call 'no-signalling in time'. Since the dynamics is non-trivial, the LG inequality in fact contains distributions for results of successive measurements of pairs of incompatible observables. Thus we know that (13) is generally going to be violated, and this already refutes (a)–(d).[11]

[10]A non-naive trajectory-based picture is given by the Bohm theory, which violates (d): if the particle is measured as having gone through one of the slits, the wave that guides its further motion has effectively collapsed. Incidentally, this refutes LG's plausibility argument to the effect that (a)–(b) already imply 'non-invasive measurability'. Furthermore, as pointed out in my previous paper, the violation of (d) is generic for theories satisfying (a)–(c). Indeed, take any theory satisfying these three conditions with respect to some observable Q. It follows that $p(Q_i^i) = p(Q_i)$. If we add the requirement that the measured probabilities $p(Q_i^i)$ equal the quantum mechanical probabilities, then Bell (1986) has shown how to construct the most general Markovian dynamics for a (discrete) quantity Q in such a theory (a 'beable' in his terminology). The resulting theory is a stochastic version of pilot-wave theory, which will indeed generically violate (d), except in the appropriate decoherence regime.

[11]See also footnote 15 in Bacciagaluppi (2015). Extending these results, Clemente and

4 LG inequalities and contextuality

Take a Bell inequality, e.g. a CHSH inequality for bipartite correlations. Its violation can be interpreted as showing that quantum mechanical correlations are non-local (in the sense of violating Bell's factorisation condition), or equally as showing that the quantum mechanical probabilities are contextual, i.e. that there is no joint probability distribution for the mutually compatible pairs of observables.[12]

Since the LG inequalities are formally Bell inequalities (in the context of a single system), one might be tempted to interpret also their violation in terms of contextuality. But the non-existence of a joint probability distribution follows trivially from the non-existence of pairwise joint distributions, so again it seems that the violation of an LG inequality tells us nothing over and above the existence of simple signalling in time.

The following example, however, shows that there are special cases in which violation of an LG inequality tells us more. Take a standard CHSH inequality, and say we measure the spin-$\frac{1}{2}$ variables A^1 or A^2 on the left, B^2 or B^3 on the right (superscripts shall be explained):

$$-2 \leq \langle A^1B^2\rangle + \langle A^1B^3\rangle - \langle A^2B^2\rangle + \langle A^2B^3\rangle \leq 2 \; . \tag{14}$$

Let the state of the particle pair be the singlet, and take the spin directions A^2 and B^2 to be antiparallel (i.e. perfectly correlated). We then have

$$-2 \leq \langle A^1B^2\rangle + \langle A^1B^3\rangle - 1 + \langle A^2B^3\rangle \leq 2 \; , \tag{15}$$

i.e.

$$-1 \leq \langle A^1B^2\rangle + \langle A^1B^3\rangle + \langle A^2B^3\rangle \leq 3 \; . \tag{16}$$

The CHSH inequality is not sensitive to the timing of the measurements, so we may assume that the superscripts refer to the order of the measurements ($t_1 < t_2 < t_3$).

Kofler (2015) have recently given combinations of no-interference and no-signalling conditions that are equivalent to the LG inequalities, and combinations that are equivalent to the existence of $p(Q_1^{123}, Q_2^{123}, Q_3^{123})$.

[12] The necessity and sufficiency of the CHSH inequalities for the existence of a joint probability distribution was first shown by Fine (1982).

Now note that a measurement of A^1 on the left is an indirect measurement of the antiparallel B^1 on the right, and a measurement of A^2 on the left is an indirect measurement of B^2 on the right. Then (16) turns into

$$-1 \leq \langle B^1 B^2 \rangle + \langle B^1 B^3 \rangle + \langle B^2 B^3 \rangle \leq 3 \ . \tag{17}$$

And if we wish, we can arrange things such that B^1, B^2, B^3 can be measured by measuring the same observable Q at t_1, t_2, t_3 with suitable local unitaries in-between.

We see that in this case we have equivalence of an LG inequality with a CHSH inequality. Indeed, the maximal violation of the LG inequality corresponds to the well-known case of measuring spins such that the angle between A^1 and A^2 is 120 degrees, the angle between B^2 and B^3 is 120 degrees, and A^2 and B^2 are antiparallel.

This is not a typical case of LG violation, however, because the initial state is the maximally mixed state, and *in that state* the three observables are compatible (measuring any one will not disturb the statistics of any other).

Let us now return to the general case. The generic presence of signalling in time (or 'violation of marginal selectivity') has motivated the search for a more general definition of contextuality, applying also to the case where measuring one observable does affect the statistics of another. Such a generalisation has been provided by Dzhafarov and Kujala (e.g. 2014a,b).

According to DK, in the presence of temporal signalling one should not expect the LG inequality to be satisfied, at most a different inequality, to wit

$$-1 - 2\Delta_0 \leq \langle Q_1^{12} Q_2^{12} \rangle + \langle Q_2^{23} Q_3^{23} \rangle + \langle Q_1^{13} Q_3^{13} \rangle \leq 3 + 2\Delta_0 \ , \tag{18}$$

where

$$\Delta_0 := \tfrac{1}{2}\Big(|\langle Q_2^{12} \rangle - \langle Q_2^{23} \rangle| + |\langle Q_3^{13} \rangle - \langle Q_3^{23} \rangle| \Big) \tag{19}$$

is the measure of temporal signalling (given by a certain minimisation condition). To be precise, DK give the tighter Suppes–Zanotti form:

$$\begin{aligned} -1 - 2\Delta_0 \leq \langle Q_1^{12} Q_2^{12} \rangle + \langle Q_2^{23} Q_3^{23} \rangle + \langle Q_1^{13} Q_3^{13} \rangle \leq \\ \leq 1 + 2\Delta_0 + 2\min\{ \langle Q_1^{12} Q_2^{12} \rangle, \langle Q_2^{23} Q_3^{23} \rangle, \langle Q_1^{13} Q_3^{13} \rangle \} \ . \end{aligned} \tag{20}$$

We shall call such inequalities *DK inequalities*.

As pointed out in my previous paper, whatever the choice of t_0 in (7), and choosing $t_3 - t_2$ and $t_2 - t_1$ to maximally violate the LG inequality, we get

$$\Delta_0 = \frac{1}{2}\Big(| -\tfrac{1}{2}\cos(\eta) - \cos(\eta + \tfrac{2\pi}{3})| + | -\tfrac{1}{2}\cos(\eta) + \tfrac{1}{2}\cos(\eta + \tfrac{2\pi}{3})| \Big) \ , \quad (21)$$

where we have set $\eta := \frac{\Delta E}{\hbar}(t_1 - t_0)$. The range of (21) is

$$\tfrac{3}{8} \leq \Delta_0 \leq \tfrac{3}{4} \ , \quad (22)$$

so that the tightest lower bound of the DK inequality (18) or (20) becomes -1.75, and the inequality is satisfied for *all* choices of η, e.g. for all choices of the initial time t_0

That is, for all choices of the initial pure state in (7), there is no contextuality in the sense of DK, despite the violation of the LG inequality. This contrasts starkly with the case of the maximally mixed state, where $\Delta_0 = 0$.[13] In this case the lower bound is again -1, the LG inequality coincides with the DK inequality, and its violation is a sign of contextuality also in the sense of DK. More generally, the DK inequality will be violated whenever the initial state is sufficiently close to being maximally mixed.

This disappearance of temporal signalling for appropriately mixed states can be intuitively understood as follows. What effect a measurement has on the distribution of results of a later measurement depends on whether the initial state is, say, $|\psi_R\rangle$ or $|\psi_L\rangle$. If the state is always either one or the other, but we only know each has probability $\frac{1}{2}$, our choice of performing the measurement will still have an effect, but we do not know which. Accordingly, our ability to signal in time is only given by the average effect (which in this case we know to be zero). This is analogous to lack of signalling on average in Bohm's theory: if Alice knew the values of her hidden variables, she could signal across EPR pairs, but if she only knows they are distributed according to the Born rule, the effect washes out.[14]

[13]We can see this directly, or by noting the case is equivalent to the CHSH case, where we know there is no-signalling.

[14]See e.g. Valentini (2002), who extends this result to arbitrary deterministic hidden variables theories.

One can pursue the analogy with hidden variables theories further. In their paper, Maroney and Timpson use Spekkens' (2005) framework of ontic models, and succeed in providing a deep and detailed analysis of the violations of the LG inequality. In the rest of this section, I shall elaborate on the remark by Kofler and Brukner that macrorealism suggests an analogue of Bell's factorisation condition, namely

$$p_{\rho,\lambda}(Q_i^{ij}, Q_j^{ij}) = p_{\rho,\lambda}(Q_i^{i}) p_{\rho,\lambda}(Q_j^{j}) \ , \tag{23}$$

where ρ is the quantum state and λ is some suitable specification of the 'hidden' values of the macroscopic quantities of the system.

Consider the analogues of Shimony's (1986) conditions of outcome independence,

$$p_{\rho,\lambda}(Q_i^{ij}, Q_j^{ij}) = p_{\rho,\lambda}(Q_i^{ij}) p_{\rho,\lambda}(Q_j^{ij}) \ , \tag{24}$$

and parameter independence,

$$p_{\rho,\lambda}(Q_i^{ij}) = p_{\rho,\lambda}(Q_i^{i}) \quad \text{and} \quad p_{\rho,\lambda}(Q_j^{ij}) = p_{\rho,\lambda}(Q_j^{j}) \ , \tag{25}$$

which jointly imply (23).[15] Are these conditions motivated by macrorealism?

The intuitions we can invoke are that the values of the macroscopic quantities at each time t_i fully determine the probabilities for the results of measurements at that time (indeed, these probabilities are 0 or 1 in the case of faithful measurements), and that measurements do not affect the evolution of the macroscopic quantities.

Now suppose we take λ to be simply the initial value Q_0 of Q. If measurements do not affect the evolution of Q (condition (d)), then we may assume (25). However, unless the evolution of Q is deterministic, there is no reason for the value of Q_0 to screen off correlations between the values of Q_i and Q_j and therefore correlations between the measured values at those times. Thus, even assuming (c), (24) is unmotivated.

Alternatively, we can interpret λ as specifying the values of Q at all times. In this case, (24) will be justified (indeed it will follow trivially from (c)),

[15]Shimony's conditions are expressed in terms of macroscopic apparatus settings, averaging over any 'apparatus hidden variables'. The analogous conditions taking into account also the microscopic apparatus states are discussed by Jarrett (1984).

and so will (25). But the latter is now no longer related to (d). The role of (d) will be to ensure that the distribution of λ is independent of the chosen measurement settings, which in this case no longer follows by assuming the independence of the settings from the initial state.

Finally, we can make the simplifying assumption that the evolution of the complete state (ρ, λ) is deterministic, other than when measurements are performed (this is for instance the case when the complete state is just the quantum state ρ). In this case, the probabilities for measurement outcomes may be expected to depend only on the initial state and the settings and outcomes of previous measurements. If the state is indeed maximally mixed and arising from entanglement ('Schrödinger's SQUID'), parameter independence corresponds to no temporal signalling, and the violation of the DK inequality is due entirely to the violation of outcome independence.

Acknowledgements

I am especially grateful to Ehtibar Dzhafarov for the opportunity to present this material at the Weiner Memorial Lectures at Purdue University, 1–3 November 2014, and to publish it in the present volume, and to Johannes Kofler for that of presenting it at the Max Planck Institute for Quantum Optics in Munich in April 2015. I also wish to thank Ehti and Johannes, as well as Acacio de Barros, Lucas Clemente, Owen Maroney and Chris Timpson for extended discussions and correspondence, as well as audiences at Aberdeen, Purdue and especially Munich. Many thanks also to Vera Matarese and Davide Romano for finding an existing English translation of Einstein (1953).

References

Bacciagaluppi, G. (2012), 'The Role of Decoherence in Quantum Mechanics', in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy (Summer 2012 Edition)*, `http://plato.stanford.edu/archives/sum2012/entries/qm-decoherence/`.

Bacciagaluppi, G. (2015), 'Leggett–Garg Inequalities, Pilot Waves and Contextuality', *International Journal of Quantum Foundations* **1**, 1–17, `http://arxiv.org/abs/1409.4104`.

Bacciagaluppi, G., and Valentini, A. (2009), *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference* (Cambridge: Cambridge University Press).

Bell, J. S. (1986), 'Quantum Field Theory without Observers', *Physics Reports* **137**, 49–54. Reprinted as 'Beables for Quantum Field Theory' in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press, 1987), pp. 173–180.

Benatti, F., Ghirardi, GC., and Grassi, R. (1994), 'On some Recent Proposals for Testing Macrorealism versus Quantum Mechanics', *Foundations of Physics Letters* **7**(2), 105–126.

Bohm, D. (1952), 'A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables, I and II', *Physical Review* **85**, 166–179 and 180–193.

de Broglie, L. (1928), 'La nouvelle dynamique des quanta', in H. A. Lorentz (ed.), *Électrons et Photons: Rapports et Discussions du Cinquième Conseil de Physique Solvay* (Paris: Gauthiers-Villars), pp. 105–132. Translated as 'The New Dynamics of Quanta', in Bacciagaluppi and Valentini (2009), pp. 341–363.

Chakravarty, S., and Leggett, A. J. (1984), 'Dynamics of the Two-State System with Ohmic Dissipation', *Physical Review Letters* **52**, 5–8.

Clemente, L., and Kofler, J. (2015), 'Necessary and Sufficient Conditions for Macroscopic Realism from Quantum Mechanics', `http://arxiv.org/abs/1501.07517`.

Dzhafarov, E., and Kujala, J. V. (2014a), 'Probabilistic Contextuality in EPR/Bohm-type Systems with Signaling Allowed', `http://arxiv.org/abs/1406.0243`.

Dzhafarov, E., and Kujala, J. V. (2014b), 'Generalizing Bell-type and Leggett–Garg-type Inequalities to Systems with Signaling', `http://arxiv.org/abs/1407.2886`.

Einstein, A. (1953), 'Elementare Überlegungen zur Interpretation der Grundlagen der Quantenmechanik', in *Scientific Papers Presented to Max Born on his Retirement from the Tait Chair of Natural Philosophy in the University of Edinburgh* (Edinburgh: Oliver and Boyd), pp. 33–40. Transl. by D. Karanth as 'Elementary Considerations on the Interpretation of the Foundations of Quantum Mechanics', `http://arxiv.org/abs/1107.3701`.

Fine, A. (1982), 'Hidden Variables, Joint Probability, and the Bell Inequalities', *Physical Review Letters* **48**, 291–295.

Jarrett, J. P. (1984), 'On the Physical Significance of the Locality Conditions in the Bell Arguments', *Noûs* **18**, 569–589.

Kofler, J., and Brukner, Č. (2013), 'Condition for Macroscopic Realism beyond the Leggett–Garg Inequalities', *Physical Review* **A 87**(5), 052115, `http://arxiv.org/abs/1207.3666`.

Leggett, A. J. (2002), 'Testing the Limits of Quantum Mechanics: Motivation, State of Play, Prospect', *Journal of Physics: Condensed Matter* **14**, R415–R451.

Leggett, A. J., and Garg, A. (1985), 'Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?', *Physical Review Letters* **54**, 857–860.

Maroney, O., and Timpson, C. (2014), 'Quantum- vs Macro-Realism: What does the Leggett–Garg Inequality Actually Test?', `http://arxiv.org/abs/1412.6139`.

Shimony, A. (1986), 'Events and Processes in the Quantum World', in R. Penrose and C. Isham (eds.), *Quantum Concepts in Space and Time* (Oxford: Clarendon Press), pp. 182–203. Repr. in A. Shimony, *Search for a Naturalistic World View*, Vol. II (Cambridge: Cambridge University Press, 1993), pp. 140–162.

Spekkens, R. (2005), 'Contextuality for Preparations, Transformations and Unsharp Measurements', *Physical Review* **A 71**, 052108, `http://arxiv.org/abs/quant-ph/0406166`.

Suppes, P., and Zanotti, M. (1981), 'When are Probabilistic Explanations Possible?', *Synthese* **48**, 191–199.

Tolan, S. S. (1996), 'Is it a Cheetah?', `http://www.stephanietolan.com/is_it_a_cheetah.htm`.

Valentini, A. (2002), 'Signal-Locality in Hidden-Variables Theories', *Physics Letters* **A 297**, 273–278, `http://arxiv.org/abs/quant-ph/0106098`.