Interpersonal Comparisons of What?

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Abstract

I examine the once popular claim according to which interpersonal comparisons of welfare are necessary for social choice. I side with current social choice theorists in emphasizing that, on a narrow construal, this necessity claim is refuted beyond appeal. However, I depart from the opinion presently prevailing in social choice theory in highlighting that on a broader construal, this claim proves not only compatible with, but even comforted by, the current state of the field. I submit that all in all, the most accurate philosophical assessment consists not in flatly rejecting this necessity claim, but in accepting it in suitably revised form.

1 Introduction

Across politics, philosophy, and economics, various evaluation mechanisms permit ranking social options, such as public policies, based on how they would affect the welfare of the members of society, assuming this can be measured. The utilitarian and the maximin approaches are simple—albeit diametrically opposed in their distributional implications—examples. Casual inspection suggests that to go beyond the unproblematic cases where unanimity prevails, i.e., to propose principled decisions in the face of disagreement within society, any such mechanism must rely on some interpersonal comparisons of welfare. After all, in terms of the examples above, without such comparisons, the maximin could not identify the worst-off to alleviate her plight, nor could utilitarianism give meaning to its precept that the gains of the ones should outweigh the losses of the others.

The claim that interpersonal comparisons of welfare are necessary for satisfactory forms of social evaluation—herein: *the necessity claim*—is not only plausible on intuitive grounds. It has also long been taken to be the ultimate lesson to draw from Arrow's impossibility theorem (Arrow, 1951), with which social choice theory was born. To be sure, this landmark result relies on several assumptions, all of which contribute to the impossibility. Such is the case, for instance, of Arrow's so-called "Independence from Irrelevant Alternatives" assumption, to the effect that the social ranking of any pair of options depends only on how these options would affect the welfare of each member of society. Nevertheless, it was Arrow's additional exclusion of interpersonal comparisons of welfare that many pioneers of social choice theory—most prominently, though each in his own way, Arrow himself when introducing his result (e.g., Arrow, 1951, p. 59), and later Sen when rejuvenating the field to which it had given birth (e.g., Sen, 1970, p. 125)—singled out as the main culprit for the impossibility.[1]

However, most current social choice theorists would consider this a significantly outdated interpretation of their fundamental result (see, for example, Fleurbaey and Hammond, 2004, p. 1204). Informed by a rich variety of social evaluation mechanisms, they take it that the possibility of social choice

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[1]Famously, social choice theorists and philosophers of economics have also debated *the methodology* of interpersonal comparisons of welfare—who might conduct them, how, and why (for a representative sample of these debates, see Elster and Roemer, 1991). More often than not, they have reached rather aporetic conclusions on this issue. For this reason, it is worth highlighting that the question of the necessity of such comparisons is independent from, and in many respects preliminary to, these further debates. In particular, two authors might very well agree that interpersonal comparisons of welfare are necessary for social choice, but disagree on whether this amounts to another impossibility (for instance, because these comparisons themselves would be, according to some standard or another, impossible) or not (would they be, on the contrary, perfectly possible and commonplace). For a sophisticated philosophical take on the possibility of interpersonal comparisons of welfare, see List, 2003.

hinges in fact *either* on making interpersonal comparisons of welfare, *or* on relaxing the binary approach imposed by Arrow's independence assumption. By default, there seems to be no reason to highlight either branch of the alternative at the expense of the other. Evidently, this is a much more subtle and open diagnosis than the sweeping necessity claim with which we started.

My main contribution will be to show that the preceding alternative does not most clearly convey what makes social choice possible and that our stance towards the necessity claim should be further updated, specifically, in a more positive direction. While there is no disagreeing with current social choice theorists that on a narrow construal, the necessity claim is refuted beyond appeal, one should also emphasize that on a broader construal, this claim proves not just compatible with, but even comforted by, the current state of the field. Indeed, based on a thorough study of representative counterexamples to the necessity claim, I will detail two main reasons, succinctly announced next, for such a reappraisal. The first reason is that the set of interpersonal comparisons to be taken into account is larger than many current social choice theorists assume. In particular we shall see that, on the one hand, interpersonal comparisons of welfare are possible even when welfare is not assumed interpersonally comparable (an apparent paradox which we will dissipate), on the other hand, interpersonal comparisons of welfare are not the only interpersonal comparisons that matter for a thorough evaluation of the necessity claim. The second reason is that bringing in so-called "irrelevant alternatives" proves tantamount to exploiting that larger domain of interpersonal comparisons. Indeed, as we shall also see, relaxing Arrow's independence condition amounts to allowing for some interpersonal comparisons—of welfare or otherwise—that are precluded when the full force of the condition is imposed. This is not to argue against the independence condition on the grounds that one should be able to make such interpersonal comparisons; it is merely to say that however the independence condition is to be normatively appreciated, relaxing it should not be rigidly opposed to introducing interpersonal comparisons. For those two main reasons taken together, I will submit that the current state of social choice theory calls not for flatly rejecting the traditional necessity claim once endorsed by Arrow, Sen, and other pioneers of the field, but for accepting it in suitably revised form. As I will explain, the claim is not fundamentally contradicted by the current state of the field; it is merely made more precise, in a way that the currently established alternative between relaxing either non-comparability or independence does not sufficiently clearly reflect. To the best of my knowledge, this nuanced position has not yet been articulated in the literature, let alone systematically defended. In fact (as more fully explained and referenced therein), I consider it merely a change in philosophical emphasis with respect to the opinion currently prevailing in the literature. But this change certainly matters to our understanding of social choice across politics, philosophy, and economics.

The rest of the paper is organized as follows. Section 2 gathers the necessary preliminaries. This includes introducing the social-choice-theoretic framework that—unlike Arrow's own framework—is appropriate for studying comparability or the lack thereof, and stating Arrow's impossibility theorem within that framework. Section 3 exposes the traditional claim that interpersonal comparisons of welfare are necessary for social choice, and the now standard rebuttal of that claim. Section 4 details the previously announced two main reasons to qualify this rebuttal and spells out the revised necessity claim that, contrary to the traditional one, survives scrutiny. Section 5 sketches an analytical framework in which interpersonal comparisons of welfare and other information can be explicitly articulated. Section 6 briefly concludes.

2 Preliminaries: Arrow's Impossibility

I first introduce a simplified version of the framework popularized by Sen (Sen, 1970) to refine Arrow's earlier analysis of social choice (Arrow, 1951).[2] N is a finite set of *agents*, with $\#N = n \geq 2$, and generic element i. X is a finite set of *options*, with $\#X = m \geq 3$, and generic element x. U is the set of all possible *welfare* functions on X, with generic element $u : X \to \mathbb{R}$.[3] For simplicity, I will assume throughout this paper that all welfare functions induce a strict order over the set of options, i.e., for all $u \in U$, distinct $x, y \in X$, $u(x) \neq u(y)$. This will considerably simplify parts of the presentation and, for my purposes, entail little loss of generality. When for some $x, y \in X$, functions u and $v \in U$ are such that $u(x) = v(x)$ and $u(y) = v(y)$, I write $u|_{\{x,y\}} = v|_{\{x,y\}}$. More generally, when for some $x, y \in X$, $u, v \in U$ are such that $u(x) > u(y)$ if and only if $v(x) > v(y)$ and $u(x) = u(y)$ if and only if $v(x) = v(y)$, I write $u|_{\{x,y\}} \equiv v|_{\{x,y\}}$. With u_i the welfare function attached to agent i, the set $\mathcal{U}$, with generic element $\mathbf{u} = (u_1, \ldots, u_n)$, is the *universal domain* of all possible welfare *profiles* $\mathbf{u} : X \to \mathbb{R}^n$. Finally, R is the set of all weak orders, or *rankings*, over X. Its generic element is denoted $\succcurlyeq$, with asymmetric and symmetric parts $\succ$ and $\sim$, respectively. When for some $x, y \in X$, weak orders $\succcurlyeq, \succcurlyeq' \in R$ are such that $x \succ y$ if and only $x \succ' y$ and $x \sim y$ if and only $x \sim' y$, I write $\succcurlyeq |_{\{x,y\}} = \succcurlyeq' |_{\{x,y\}}$. The statement $\succcurlyeq = \succcurlyeq'$ means that the previous condition holds for all $x, y \in X$. Our central objects of interest will be so-called *social welfare functionals* (henceforth: SWFLs), i.e., functions f defined from $\mathcal{U}$ (or, if necessary, some

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[2]It is a refinement of Arrow's original—so-called *social welfare function*—framework under the assumption that all the preference binary relations postulated therein are numerically representable by a welfare function.

[3]Interchangeably in most of the background literature: *utility* functions. Whenever necessary when referring to the literature, I will replace "utility" by "welfare". This is for terminological homogeneity and not to be distracted from the main topic of this paper, which is interpersonal comparisons, not the conceptual links between "utility" and "welfare".

subset thereof) to R. Given a profile $u \in \mathcal{U}$, $f(u)$ is denoted $\succcurlyeq_u^f$. The following are familiar examples of SWFLs, with markedly different distributional implications.

THE CLASSICAL UTILITARIANISM (CU) SWFL. For any $u \in \mathcal{U}$, $x, y \in X$, $x \succcurlyeq_u^{\mathrm{CU}} y \Leftrightarrow \sum_{i \in N} u_i(x) \geq \sum_{i \in N} u_i(y)$.

THE CLASSICAL MAXIMIN (CM) SWFL. For any $u \in \mathcal{U}$, $x, y \in X$, $x \succcurlyeq_u^{\mathrm{CM}} y \Leftrightarrow \min_{i \in N} u_i(x) \geq \min_{i \in N} u_i(y)$.

This paper will focus on SWFLs that, like Classical Utilitarianism and the Classical Maximin, can be defined over abstract environments like the one just introduced. This is certainly a significant restriction. In particular, by construction, a wide range of non-welfare information—say, data about rights—cannot be articulated in that approach, let alone taken into account. Whatever information is given by the individual welfare functions, then, is the only information available. That being acknowledged, as will be seen shortly, the key objections to the necessity claim can be and have been raised even in that context. It is sufficiently rich for our philosophical purposes.

The following conditions are standard tools for the axiomatic analysis of SWFLs, starting with but not limited to the Classical Utilitarianism and Maximin SWFLs. The first of these conditions is satisfied by the latter but not the former, while the second is satisfied by the former but not the latter.

ORDINAL FULL COMPARABILITY (OFC). For any $u, v \in \mathcal{U}$, $\succcurlyeq_u^f = \succcurlyeq_v^f$ whenever for all $i \in N$, $v_i = \phi \circ u_i$ for some strictly increasing transformation ϕ.

CARDINAL UNIT COMPARABILITY (CUC). For any $u, v \in \mathcal{U}$, $\succcurlyeq_u^f = \succcurlyeq_v^f$ whenever for all $i \in N$, $v_i = \phi_i \circ u_i$ for some strictly increasing transformation of the form $\phi_i(z) = az + b_i$.

In the literature, assumptions like OFC or CUC are called *informational bases* for SWFLs. Each informational basis can be effectively decomposed in two assumptions. The first pertains to the *intra-personal measurability* of welfare. It answers the question: When do two welfare functions $u, v \in U$ contain the same intra-personal information? For instance, under OFC, u and v contain the same information if and only if they are related by a strictly increasing transformation—the so-called *ordinal* case—while under CUC, this holds if and only if the strictly increasing transformation is affine—the so-called *cardinal* case. The second assumption pertains to the *interpersonal comparability* of welfare. It provides an answer to the question: When do two welfare profiles $u, v \in \mathcal{U}$ contain the same interpersonal information? Under OFC, u and v contain the same information if they are related by a

common strictly increasing transformation (witness that ϕ does not depend on i), while under CUC, such is the case if they are related by a strictly positive affine transformation with a common unit (witness that a does not depend on i). The former requirement is equivalent to preserving the interpersonal order which a given profile induces over *welfare levels* (as the very definition of the Classical Maximin requires), while the latter suffices to preserve the order interpersonally induced over *welfare differences* (as Classical Utilitarianism requires).[4]

The previous two informational bases, which incorporate comparability assumptions, are best appreciated when contrasted from the following two informational bases, in which no such assumption is imposed.

> ORDINAL NON-COMPARABILITY (ONC). For any $u, v \in \mathcal{U}$, $\succcurlyeq_u^f = \succcurlyeq_v^f$ whenever for all $i \in N$, $v_i = \phi_i \circ u_i$ for some strictly increasing transformation ϕ_i.

> CARDINAL NON-COMPARABILITY (CNC). For any $u, v \in \mathcal{U}$, $\succcurlyeq_u^f = \succcurlyeq_v^f$ whenever for all $i \in N$, $v_i = \phi_i \circ u_i$ for some strictly increasing transformation of the form $\phi_i(z) = a_i z + b_i$.

Thus, like OFC, ONC supposes individual welfare ordinally measurable, but unlike OFC, it does not suppose welfare levels interpersonally comparable (witness that ϕ_i depends on i). Similarly, like CUC, CNC supposes individual welfare cardinally measurable, but unlike OFC, it does not suppose some interpersonal comparability of welfare differences (witness that a_i depends on i). From the definitions, it follows that ONC implies CNC which in turn implies CUC, while none of the converse implications holds. Similarly, ONC implies OFC but the converse does not hold. Finally, neither OFC nor CUC implies the other.

To state Arrow's impossibility theorem, three more conditions are needed. They are given next. The first of them (which I will call Welfare Independence of Irrelevant Alternatives) requires that the social ranking of any two options depends only on the individual welfare information regarding these

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[4]To preserve the interpersonal order which a profile induces over welfare differences, strictly positive affine transformations with a common unit are sufficient but, absent certain special assumptions, not necessary. Classical Utilitarianism satisfies CUC, but in general not the more demanding axiom requiring that a SWFL be invariant under *any* transformation preserving the order interpersonally induced over welfare differences (see especially Bossert, 1991). This qualification should be kept in mind whenever—as will happen in the present paper, like elsewhere in the literature—some connection is alluded to between Classical Utilitarianism and interpersonal comparisons of welfare differences.

two options, not on any other information.[5] As I will comment on this independence condition rather extensively in what follows, I refrain from elaborating more at this stage. I simply note that this condition is satisfied by the Classical Maximin and Classical Utilitarianism, for instance. The subsequent condition (Weak Pareto) states that the social ranking of any pair of options is positively related to the individual rankings of these options. The last condition (Non-Dictatorship) states that the social ranking is not always trivially determined by the ranking of only one single agent. These two conditions are also respected by the Classical Maximin and Classical Utilitarianism, among many other SWFLs.

> WELFARE INDEPENDENCE OF IRRELEVANT ALTERNATIVES (WIIA). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever for all $i \in N$, $u_i|_{\{x,y\}} = v_i|_{\{x,y\}}$.[6]

> WEAK PARETO (WP). For any $u \in \mathcal{U}$, $x, y \in X$, $x \succ_u^f y$ whenever $u_i(x) > u_i(y)$ for all $i \in N$.

> NON-DICTATORSHIP (ND). There is no $j \in N$ such that for all $u \in \mathcal{U}$, $x, y \in X$, $x \succcurlyeq_u^f y \Leftrightarrow u_j(x) \geq u_j(y)$.

Arrow's original theorem can now be formulated as follows.

> **Theorem 1** (Arrow, 1951). No SWFL $f : \mathcal{U} \to R$ can jointly satisfy ND, WP, WIIA, and ONC.

There are several other variants of Arrow's impossibility result, than the one just presented. For our purposes, the most relevant fact is that in the preceding statement, ONC can be weakened to CNC, i.e., the measurability

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[5]This is, more specifically, the standard interpretation for *welfarism* in social choice theory. For the classical definition, that involves WIIA, see, e.g., Bossert and Weymark, 2004, Sec. 2. Over a universal domain, the other pillar of welfarism is *Pareto Indifference*, not explicitly presented in the present paper but satisfied by all the SWFLs discussed therein. (A more thorough discussion of interpersonal comparisons of welfare would not only involve non-abstract frameworks, but also explicitly consider the role of Pareto Indifference.) For more refined typologies of various kinds of welfarism and departures therefrom, see especially Fleurbaey, 2003, Blackorby et al., 2005, and Morreau and Weymark, 2016.

[6]The condition has been given various names. The one given here coheres with the terminological choice previously explained (see fn. 3). As we will eventually return to, transposed to our framework, Arrow's stronger *Independence of Irrelevant Alternatives* (IIA) condition reads with $u_i|_{\{x,y\}} \equiv v_i|_{\{x,y\}}$ instead of $u_i|_{\{x,y\}} = v_i|_{\{x,y\}}$. IIA is not the most enlightening condition to start with here since it implies not only WIIA, but also the conceptually distinct ONC. Indeed, over a universal domain, IIA is equivalent to the conjunction of WIIA and ONC. WIIA permits a finer analysis of SWFLs inasmuch as it is compatible with not only ONC, but also generalizations of ONC such as OFC or CUC. In this sense, contrary to IIA, WIIA allows for non-comparability and comparability alike. That being said, more on IIA and related conditions in the second half of Section 5.

requirement strengthened from ordinality to cardinality. This is because assuming a universal (more generally: a sufficiently rich) domain, under WIIA, CNC implies ONC, thus, ONC and CNC are equivalent. Therefore, as Sen first proved:

> **Theorem 2** (Sen, 1970). No SWFL $f : \mathcal{U} \to R$ can jointly satisfy ND, WP, WIIA, and CNC.

3 Comparability—Sufficient, but not Necessary?

According to a longstanding interpretation of Arrow's discovery, the real key to the impossibility is the non-comparable approach common to Thm. 1 (under ONC, i.e., Ordinal *Non-Comparability*) and Thm. 2 (under CNC, i.e., Cardinal *Non-Comparability*).[7] For instance, after stating Thm. 2 and comparing it to Thm. 1, Sen comments: "[this] confirms the suspicion that mere cardinality without any comparability may not be helpful"; more colourfully: "cardinality alone seems to kill no dragons, and our little St. George must be sought elsewhere" (Sen, 1970, p. 130, 125). There is little doubt that on Sen's analysis, only comparability could play such a heroic role for social choice theory. Indeed, Sen's revival of this field in the 1970s consisted partly in showing that in contrast to Arrow's non-comparability approach, that led to a resounding impossibility, comparability assumptions such as OFC or CUC allowed for rich possibilities. Most distinctively for our purposes, this gave rise to the research program of seeking SWFL characterizations for the like of Classical Utilitarianism or the Classical Maximin, and eventually to providing, with these characterizations, as many possibility results. For instance, the Classical Maximin satisfies all the conditions of the ordinal variant of Arrow's theorem at the exception of ONC, to be weakened to OFC; similarly, Classical Utilitarianism satisfies all the conditions of the cardinal variant of the theorem at the exception of CNC, to be weakened to CUC (e.g., d'Aspremont and Gevers, 2002). In a nutshell, merely by weakening non-comparability to comparability, i.e., by introducing some interpersonal comparability while keeping all other assumptions—including the intra-personal measurability assumption—fixed, one can oppose possibilities to each variant of Arrow's impossibility theorem.

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[7]Singling out one of Arrow's conditions as "the key" to the impossibility he discovered is, of course, a matter of philosophical appreciation. Mathematically speaking, all conditions (whether explicit in Thms. 1 and 2, or implicit in the underlying framework) play a role. As is well known, possibilities arise when relaxing each of Arrow's conditions, holding the others fixed. It is understood, then, that not all possibilities are equally significant from a conceptual point of view. For example, to overcome Arrow's impossibility, mere domain restrictions suffice (e.g., Sen, 1966). But the light that domain restrictions shed on impossibilities is only indirect, and rather limited upon reflection (e.g., Sen, 1976, p. 233).

As a matter of fact, Sen's take on the impossibility discovered by Arrow echoed Arrow's own initial interpretation of his result.[8] For Arrow, too, had emphasized interpersonal comparisons of welfare in presenting the following as a mere "restat[ement]" of his discovery (Arrow, 1951, p. 59): "if we exclude the possibility of interpersonal comparisons of [welfare], then the only methods of passing from individual tastes to social preferences which will be satisfactory and will be defined for a wide range of sets of individual orderings are either imposed or dictatorial." On the joint influence of the diagnosis initially presented by Arrow and the cure later recommended by Sen, several social choice theorists would repeat in the 1970s the motto that either social choice relies on interpersonal comparisons of welfare, or else it is dictatorial (e.g., Parks, 1976, p. 450). This alleged dilemma was meant as a mathematically selective, but philosophically perceptive presentation of the impossibility result fundamental to their field.

However, such a take on Arrow's impossibility theorem would strike any current social choice theorist as significantly outdated. Specifically, the claim that interpersonal comparisons of welfare are necessary for social choice seems directly refuted by the existence of various and widely applicable SWFLs that, among Arrow's conditions, depart from and only from WIIA. The key fact is that such SWFLs reach interesting forms of social choice despite maintaining Arrow's non-comparable approach—a possibility which the necessity claim seems committed to denying. To explain and investigate this in more detail, I will focus on two canonical counterexamples, viz. the Borda Count and Relative Utilitarianism, that pertain to each of the ordinal and the cardinal variant of Arrow's impossibility theorem. Admittedly, many more examples could be examined. The ones introduced next are conveniently elementary. Furthermore I contend that, as far as comparability is concerned, they permit raising any important discussion which more sophisticated SWFLs would call for.[9]

The first of these counterexamples, then, is the *Borda Count* SWFL (Borda, 1781; see Nitzan and Rubinstein, 1981 for an axiomatization, Chebotarev and Shamis, 1998 for a comprehensive literature review, as well as Saari, 1994 for a book-length spirited defense). For any $u \in \mathcal{U}$, $i \in N$, $x \in X$,

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[8]Rather, the interpretation Arrow initially emphasized and his immediate successors (Sen included) typically picked on. For more nuances on the matter, see fn. 15.

[9]Admittedly, in the case of the Borda Count, one might also worry that the SWFL is merely a voting method, not a credible mechanism of social evaluation (e.g., d'Aspremont and Gevers, 2002, p. 473). This, too, proves immaterial for our purposes. The core insights which we will reach examining the Borda Count transpose to more sophisticated SWFLs that share the same informational basis while seeming ethically more attractive—e.g., the so-called *Egalitarian Equivalence* SWFL (Pazner and Schmeidler, 1978; Fleurbaey, 2005).

define $r_{u_i}(x) = \#\{z \in X | u_i(x) > u_i(z)\}$.[10] Thus, $r_{u_i}(x)$ stands for the cardinality of the lower contour set induced by u_i for x, so that the function r_{u_i} associates i's top option with the number $m - 1$, i's next-to-top option with the number $m - 2$, and so on. Consequently, r_{u_i} is the unique element of U that is ordinally equivalent to u_i, ranges over $[0, m - 1]$, and satisfies the condition that for any immediately consecutive distinct $x, y \in X$, $r_{u_i}(x) - r_{u_i}(y) = 1$. The Borda Count proceeds by taking the sum (or any strictly increasing function thereof), over all agents, of the welfare levels thus calibrated.

THE BORDA COUNT (BC) SWFL. For any $u \in \mathcal{U}$, $x, y \in X$, $x \succcurlyeq_u^{\mathrm{BC}} y \Leftrightarrow \sum_{i \in N} r_{u_i}(x) \geq \sum_{i \in N} r_{u_i}(y)$.

At the exception of WIIA, the Borda Count SWFL satisfies all the conditions of the ordinal variant of Arrow's theorem. WIIA must be violated since on the Borda Count, the social ranking of any pair of options x and y depends not only on the individual welfare information regarding x and y, but also on the (number of distinct) options between x and y. To see that the satisfied conditions include ONC, notice that as defined above, the function r_{u_i} is unchanged by any strictly increasing transformation of the underlying welfare function u_i. The result of counting options does not depend on the unilateral scale potentially adopted by each agent. Consequently, the sum of the r_{u_i} is unchanged by unilateral ordinal transformations of the u_i, so that the SWFL satisfies ONC. Thus, the Borda Count stands as a direct counterexample to the necessity claim highlighted at the beginning of the present section. To obtain another example with essentially the same features, one could consider a different but related SWFL by replacing, in the definition above, the sum by the min operator, as in the so-called *Simpson-Kramer Method* (see Simpson, 1969; Kramer, 1977). This means that the specific, broadly speaking utilitarian-inspired distributional convictions with which one could be tempted to associate the Borda Count are entirely unessential to the argument.

The second counterexample is the *Relative Utilitarianism* SWFL (Dhillon and Mertens, 1999; Börgers and Choo, 2017). For any $u \in \mathcal{U}$, $i \in N$, $x \in X$,

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[10]Would ties be allowed, i.e., would there be some $u \in U$ such that $u(x) = u(y)$ for some distinct $x, y \in X$, straightforward generalizations of the above definition would apply. Would X be infinite, however, it would become necessary to add the domain restriction that there is, for each agent, finitely many indifference classes in X. Judging from the interpersonal diversity this would preserve (and the fact that such diversity is the key aspect in any domain assumption), this is a rather innocuous restriction.

define $s_{u_i}(x) = \left(u_i(x) - \min_{z \in X} u_i(z)\right) / \left(\max_{z \in X} u_i(z) - \min_{z \in X} u_i(z)\right)$.[11] Thus, s_{u_i} is the unique element of U that is cardinally equivalent to u_i and satisfies the condition that i's top and bottom options are associated with the welfare levels 1 and 0, respectively. The Relative Utilitarianism SWFL proceeds by taking the sum (or any strictly increasing function thereof), over all agents, of the welfare levels thus calibrated.

THE RELATIVE UTILITARIANISM (RU) SWFL. For any $u \in \mathcal{U}$, $x, y \in X$, $x \succcurlyeq_u^{\text{RU}} y \Leftrightarrow \sum_{i \in N} s_{u_i}(x) \geq \sum_{i \in N} s_{u_i}(y)$.

At the exception of WIIA, the Relative Utilitarianism SFWL satisfies all the conditions of the cardinal variant of Arrow's theorem.[12] WIIA is violated because the social ranking of x and y depends on (the difference between) $\max_{z \in X} u_i(z)$ and $\min_{z \in X} u_i(z)$. To see that the satisfied conditions include CNC, notice that as defined above, the function s_{u_i} is unchanged by any strictly positive affine transformation of the underlying welfare function u_i. The relative place of each option on each welfare scale is not influenced by the unit or the constant which each agent could unilaterally adopt. Consequently, the sum of the s_{u_i} is unchanged by unilateral cardinal transformations of the u_i, so that the SWFL satisfies CNC. Thus, Relative Utilitarianism seems to be another direct counterexample to the necessity claim under scrutiny.[13] Noteworthily, an equally compelling counterexample would obtain if one replaced, in the definition above, the sum by the min operator, thus yielding, instead of Relative Utilitarianism, a form of *Relative Egalitarianism* (Sprumont, 2013). Here again, the point is that the specific distributional convictions expressed by the Relative Utilitarianism SWFL are, for our purposes, entirely immaterial.

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[11]Under our assumption that U admits of no tie, given that $\#X \geq 3$, these ratios are always well-defined. Modulo the additional definitional clause $s_{u_i}(x) = c$ for all x in case $\max_{z \in X} u_i(z) = \min_{z \in X} u_i(z)$, ties would raise no particular difficulty. In the general case of an infinite X, however, a domain restriction is necessary to ensure that for all $i \in N$, $\max_{z \in X} u_i(z)$ and $\min_{z \in X} u_i(z)$ exist. This is, arguably, another innocuous domain restriction.

[12]Surprisingly perhaps, Relative Utilitarianism seems yet to axiomatize under CNC. The references previously given contain axiomatizations under ONC—but assuming the additional structure of probability distributions over the set of options, and expected utility under risk. (See further Dhillon, 1998; Karni, 1998; Segal, 2000; Pivato, 2009; Sprumont, 2019; Brandl, forthcoming.) For our purposes, the CNC take on Relative Utilitarianism is the right one. In particular, it permits abstracting away from the debates on the links between welfare under risk and welfare under certainty (Weymark, 1991; Fleurbaey and Mongin, 2016). These debates are certainly important, but orthogonal to our main topic here. That being said, for more on Relative Utilitarianism under ONC, see fn. 26 below.

[13]Dhillon and Mertens, who provided the first axiomatization of Relative Utilitarianism, were keenly aware of this implication. Introducing their result, they write: "if until Arrow [the] ordinalist position was almost the consensus, apparently his theorem itself, together with the very influential work of Harsanyi, turned the tide partially, and led to the conclusion that interpersonal comparability was a must to obtain SWF's. The present theorem proves this conclusion false" (Dhillon and Mertens, 1999, p. 473).

The existence of SWFLs like the Borda Count or Relative Utilitarianism is the background for the current consensus view among social choice theorists. As expressed in an authoritative survey (Fleurbaey and Hammond, 2004, p. 1204), "the most promising escape from Arrow's 'impossibility' theorem [. . .] [may be] to abandon [WIIA] while retaining the other axioms. If ONC can be abandoned as well because individual good is measurable in an interpersonally comparable way, so much the better. But this informational demand should not be regarded as a sine qua non." Succinctly put, then, interpersonal comparisons of welfare are sufficient but not necessary for the possibility of social choice. Accordingly, the dilemma previously mentioned should be rejected, and a more descriptive dilemma should be proposed in its stead. In light of SWFLs like the Borda Count or Relative Utilitarianism, the analysis currently prevailing is that "the blame of the dictatorship conclusion in Arrow's theorem can be put either on the lack of interpersonal [welfare] comparisons captured by [ONC], or on the binariness property captured by [WIIA]" (Fleurbaey and Mongin, 2005, p. 399). In other words, would any dilemma be fundamental to social choice theory, it would not be between upholding either Arrow's non-dictatorship assumption, or his non-comparability assumption; it would be between upholding either non-comparability, or independence.[14] As previously noted, this is a much more subtle and open diagnosis than the sweeping necessity claim with which we started.

4   Interpersonal Comparisons under Non-Comparability

I now explain why the account of social choice just reported must be revisited and the role of interpersonal comparisons should be reevaluated accordingly. I will do so by showing that the previous analysis of the Borda Count and the Relative Utilitarianism SWFLs can be deepened.

Let me highlight, by way of anticipation, the main insights to be reached from the more thorough analysis detailed next. First (to be illustrated by Relative Utilitarianism), interpersonal comparisons of welfare are possible even when no comparability assumption is imposed. While this may at first sound like a paradox, I will explain that no paradox is at stake here—only the under-appreciated role of intrapersonal measurability assumptions. Second (to be illustrated by the Borda Count), among all interpersonal comparisons, not just interpersonal comparisons of welfare matter, but also interpersonal comparisons of other personal data. One way or another, then, the bottom line is that there are more interpersonal comparisons than the ones explicitly considered hitherto. Third (to be illustrated by both Relative Utilitarianism and the Borda Count), the main function of relaxing independence, and

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[14]Here, I take Fleurbaey and Mongin to target not just the ONC axiom specifically, but the non-comparable approach more generally—thus also encompassing, for instance, CNC.

not just that of relaxing non-comparability, is to allow for some interpersonal comparisons—of welfare or otherwise—that are precluded when the full force of the condition is imposed. In a nutshell, so-called "irrelevant alternatives" are, in fact, relevant tools for exploiting the larger domain of interpersonal comparisons alluded to above. The overall lesson of the three points taken together will be that the Borda Count and Relative Utilitarianism are weak counterexamples to the necessity claim and that, despite first appearances, a sufficiently general construal of that claim stands up to scrutiny. To show this in sufficient detail, Subsection 4.1 will focus on the interplay between measurability and comparability and Subsection 4.2, on the interplay between independence and comparability. The next section, while recapitulating more synthetically the main conclusions to be reached in the present section, will also sharpen them in sketching a general analytical framework in which interpersonal comparisons of welfare and other information can be explicitly articulated.

Brief acknowledgements on the originality of what follows are in order. The role of measurability assumptions has been highlighted before in the literature (see in particular Fleurbaey and Hammond, 2004, p. 1215-1218). My claim here will be to a greater systematicity and some novel conceptual implications, to be highlighted shortly. The idea that relaxing independence is tantamount to allowing for more interpersonal comparisons can, remarkably, be traced back to (most thoroughly: the second edition of) Arrow's foundational monograph.[15] Yet what one finds there is only this general insight, together with examples that are, on close inspection and with the benefit of hindsight, partially irrelevant; the conceptual and formal elaboration provided here will be mine. The idea that interpersonal comparisons extend beyond interpersonal comparisons of welfare is already present in the literature (see, again, Fleurbaey and Hammond, 2004). But as will be seen, its philosophical appreciation is substantially more negative than need be—or so I will argue. Besides, it has not yet been connected, as will be the case here, to an explicit analysis of how "irrelevant alternatives" can in effect act as relevant tools for the exploitation of a larger range of interpersonal comparisons. Finally, however exploratory, the general analytical framework sketched in the next section based on the insights gathered in the present one currently has, to my knowledge, no equivalent in the literature.

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[15]By Arrow's own mature admission: "the potential usefulness of irrelevant alternatives is that they may permit empirically meaningful interpersonal comparisons" (Arrow, 1967, p. 19; as in Fleurbaey and Mongin, 2005, fn. 29, p. 415). This was actually already noted (though not elaborated upon) in the first edition of his monograph, where Arrow observed that interpersonal comparisons could proceed "by some form of direct measurement *or by comparison with other alternative social states*" (1951, p. 59; my emphasis). To my knowledge, Section VIII.4 of the second edition of Arrow's monograph (1963), entitled "The Independence of Irrelevant Alternatives and Interpersonal Comparisons of Intensity", contains his most extensive remarks on the interplay between independence and comparability.

4.1 More on Measurability

It is helpful to start by discussing Relative Utilitarianism, more generally, the CNC informational basis to which it belongs. To be sure, given a profile, under CNC, no interpersonal order is induced over either welfare levels or welfare differences. Witness, in the definition of CNC, the presence of the agent-dependent constant b_i and unit a_i, respectively. But despite what the very name of the CNC informational basis suggests (to repeat: Cardinal *Non-Comparability*), this does not mean that *no* interpersonally comparable welfare information is available. For recall it is a property of cardinal forms of measurement that given any cardinally equivalent functions u, v and elements w, x, y, z in their domain, whenever this ratio is defined, $(u(w) - u(x))/(u(y) - u(z)) = (v(w) - v(x))/(v(y) - v(z))$. Thus, under cardinality, the ratios of welfare differences are constant across all admissible transformations of the individual welfare functions. Accordingly, under CNC, although the admissible transformations of a given profile do not induce any interpersonal order over either welfare levels or welfare differences, they induce—it bears emphasis: without any comparability assumption whatsoever—an interpersonal order over both the levels and the differences of *ratios of welfare differences*.[16] This includes, in particular, the relative welfare value of each option, thereby referring to the value given to it by the function s_{u_i} featured in the definition of Relative Utilitarianism.

Noteworthily, Relative Utilitarianism builds only on the latter part of this rich information (viz. what pertains to the differences between the ratios of welfare differences, not their levels), while Relative Egalitarianism would build only on the former (the levels, not the differences). But under CNC, both pieces of information are available in all cases. They can be incorporated by any SWFL that is sufficiently emancipated from the unrestricted constraint of the full WIIA condition. Specifically, as regards Relative Utilitarianism, it would suffice to respect, for instance, a weakened WIIA constraint that would also hold fixed, for all $i \in N$, the difference between (though not necessarily the levels of) $\min_{z \in X} u_i(z)$ and $\max_{z \in X} u_i(z)$.[17] Formally, this weakening, which I will call *Range* WIIA, reads as follows.

> RANGE WIIA (RWIIA). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever, for all $i \in N$: i) $u_i|_{\{x,y\}} = v_i|_{\{x,y\}}$; and ii) $\max_{z \in X} u_i(z) - \min_{z \in X} u_i(z) = \max_{z \in X} v_i(z) - \min_{z \in X} v_i(z)$.

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[16] For instance, it is readily checked that for any $v_i = a_i u_i + b_i$, $v_j = a_j u_j + b_j$, and all q, r, s, t, w, x, y, z, $(u_i(q) - u_i(r))/(u_i(s) - u_i(t)) \geq (u_j(w) - u_j(x))/(u_j(y) - u_j(z))$ if and only if $(v_i(q) - v_i(r))/(v_i(s) - v_i(t)) \geq (v_j(w) - v_j(x))/(v_j(y) - v_j(z))$, since the u_i and the v_i ratios are equal to one another, and so are the u_j and the v_j ratios.

[17] As regards Relative Egalitarianism, it would suffice to further weaken WIIA by holding constant for each i not just $\min_{z \in X} u_i(z) - \max_{z \in X} u_i(z)$, but the $\min_{z \in X} u_i(z)$ and $\max_{z \in X} u_i(z)$ levels.

A more radical but—for our present purposes—less informative weakening of WIIA would state more simply that $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever for all $i \in N$, $s_{u_i}|_{\{x,y\}} = s_{v_i}|_{\{x,y\}}$; more on this later in the paper.

Meanwhile, two general lessons may be drawn from the foregoing.[18] First, *measurability and comparability assumptions are not independent from one another.* Crucially, some measurability requirements on individual welfare functions can be sufficiently strong to enforce, by themselves, the interpersonal comparability of some welfare information. With the benefit of hindsight, consider first the limit case of absolute measurability. Willy-nilly, it then trivially holds that all welfare information is comparable. Less trivially and starting from the other end of the measurability spectrum, as already highlighted, going from ordinal to cardinal scales, ratio of welfare differences—and, more generally, any function thereof—necessarily become interpersonally comparable. But similarly, going from cardinal to ratio scales, in addition to ratio of welfare differences, ratio of welfare levels (and any function thereof) become necessarily interpersonally comparable.[19] Still other, less immediate examples could be added to that list (Bossert and Weymark, 2004, p. 1120-1121; more completely, Fleurbaey and Hammond, 2004, p. 1215-1218). In a nutshell, to secure the presence of interpersonally comparable welfare information, comparability assumptions are sufficient but not necessary; pure measurability assumptions can suffice, too. Quite simply, increasing the measurability requirements decreases the room there is for non-comparability.

Now, few SWFLs may be as directly or transparently tied as Relative Utilitarianism or Relative Egalitarianism are to the welfare information their measurability requirements render necessarily comparable. Nevertheless, these two SWFLs illustrate a critical point about measurability and comparability that applies broadly.[20] This fact may be further stressed and clarified by considering the informational basis defined next, wherein individual welfare is supposed ratio-scale measurable but no comparability assumption is imposed.

> RATIO NON-COMPARABILITY (RNC). For any $u, v \in \mathcal{U}$, $\succcurlyeq_u^f = \succcurlyeq_v^f$ whenever for all $i \in N$, $v_i = \phi_i \circ u_i$ for some strictly increasing transformation of the form $\phi_i(z) = a_i z$.

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[18]Drawing the second lesson is, in fact, a prerequisite to articulating the first in a fully consistent way. Nevertheless, it is more intuitive to expose the two lessons in reverse order.

[19]For instance, it is readily checked that for any $v_i = a_i u_i$, $v_j = a_j u_j$, and all w, x, y, z, $u_i(w)/u_i(x) \geq u_j(y)/u_j(z)$ if and only if $v_i(w)/v_i(x) \geq v_j(y)/v_j(z)$, since the u_i and v_i ratios are equal to one another, and so are the u_j and v_j ratios.

[20]An analysis similar to the one presented here for Relative Utilitarianism applies, albeit within limits, to the *Nash Bargaining* SWFL (see Nash, 1950; under CNC, Roberts, 1980, Thm. 7; under ONC, Kaneko and Nakamura, 1979; and especially Roth, 1979, Sec. I.D for what may be, as regards comparability, the key mathematical insight into the Nash SWFL).

Next, let $U_{>0}$ denote the set of all possible welfare functions on X such that for all $u \in U_{>0}$, $x \in X$, $u(x) > 0$, and let $\mathcal{U}_{>0}$ denote the resulting domain of welfare profiles. Judging from the interpersonal diversity thus preserved, this domain restriction is rather innocuous. Now, as already mentioned, ratio-scale measurability entails that ratio of welfare levels are constant across all admissible individual transformations, hence necessarily comparable across individuals. This is the main explanation for the following, under-appreciated possibility result[21] (on which see—yet without the conceptual implications highlighted here—Boadway and Bruce, 1984, Sec. 6.2; Fishburn, 1987, Sec. 9.1; Tsui and Weymark, 1997).

Theorem 3. Some SWFLs $f : \mathcal{U}_{>0} \to R$ jointly satisfy ND, WP, WIIA, and RNC.[22]

Under further technical restrictions, all such SWFLs are Cobb-Douglas with positive individual exponents. Accordingly, when I will need to refer to the SWFLs covered by Thm. 3, for brevity and concreteness, I will simply mention *Cobb-Douglas Utilitarianism*.

The main implication of Thm. 3 is that one should reject the updated dilemma with which we closed the preceding section.[23] To wit, starting from Arrow's impossibility theorem, possibilities can be reached while maintaining *both* Arrow's non-comparability approach and his independence assumption. The key reason for this is that there is a third, under-appreciated major player, viz. measurability. Crucially, measurability requirements can be sufficiently restrictive to *both* enforce the interpersonal comparability of some welfare information (like in Relative Utilitarianism) and make this information available without any departure from WIIA (unlike in Relative Utilitarianism). Although the paper will later provide a better replacement, as an

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[21]Revealingly, this result is not covered in Fleurbaey and Hammond, 2004.

[22]Incidentally, Thm. 3 fully clarifies that it would be mistaken to interpret Thm. 2 as establishing that under WIIA and non-comparability, any measurability requirement stronger than ordinality inexorably leads back to ONC. This can also be seen by inspecting how one proves that, under WIIA, CNC implies ONC. One crucially needs the two degrees of freedom that are available in a cardinal scale, but not in a ratio scale, for instance.

[23]Admittedly, in Thm. 3, a domain restriction is imposed. But so is there in the prime examples based on which the defenders of the second dilemma make their case—e.g., the already mentioned Egalitarian Equivalence SWFL. For our purposes, none of these domain restrictions should be considered conceptually significant anyhow. Relatedly, Thm. 3 may be compared with Thm. 3.1 in List, 2001. Unlike Thm. 3, List's result does not even require a domain restriction. But it does invoke a non-standard informational basis (ONC enriched with the measurability of a so-called "zero line") and, as List himself stresses, it covers rather limited possibilities. Yet, for our purposes, the conceptual implications of List's result would be the same as the one highlighted here based on Thm. 3. (The interpersonally comparable information exploited in List's result is the sign of certain welfare differences.) Because his focus is different, List does not highlight that his possibility result flies in the face of the alleged dilemma between upholding either non-comparability or independence.

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intermediary improvement, what one could say now is that Arrow's impossibility reveals not a dilemma between independence and non-comparability, but a trilemma between independence, non-comparability, and weak measurability assumptions.

The second lesson may seem less central to the present discussion, but (as the last section will confirm) it is in some respects even more fundamental than the first one as far as the structure of the SWFL approach is concerned. *Even when only interpersonal comparisons of welfare are considered, the understanding of comparability structural to standard SWFL theory is too narrow.* The construal of non-comparability officially underlying, e.g., the CNC or the RNC appellations consists in effect in the requirement that "any transformation (permitted by the measurability assumption) of any individual's welfare function leaves the social ordering unchanged" (Sen, 1970, p. 124). Accordingly, on this construal, comparability obtains only if the transformations permitted by the measurability assumptions are somehow coordinated from one agent to another—as in, say, the definitions of OFC and CUC. Now, there is no questioning that this is a consistent analytical framework. But notice that (pace Bossert and Weymark, and even more so the more complete Fleurbaey and Hammond, in the passages previously referred to) this analytical framework would *not* permit articulating the fact—however salient and undeniably relevant to our discussion—that some measurability requirements make some welfare information necessarily interpersonally comparable. Indeed, on the official construal, modelling comparability does not generally coincide with tracking whether some interpersonal welfare information is invariant across all admissible transformations of a given profile. Nevertheless, for the purpose of building SWFLs, only the latter point truly matters. Imposing that the individual transformations be coordinated from one agent to another is not an end in itself; it is only one of several possible means to secure, when desired, the availability of such invariant information. More on this in the final section of the paper.

4.2   More on Independence

Let us now temporarily focus away from Relative Utilitarianism and turn to the Borda Count. To be sure, the Borda Count has one aspect in common with Relative Utilitarianism. Under the applicable measurability assumption (however weak it may seem: ordinality), some information becomes necessarily interpersonally comparable. To wit, the cardinality of lower contour sets necessarily compares across agents. This can be exploited by a SWFL—provided the constraint of the unrestricted WIIA condition is suitably relaxed. Indeed, on the Borda Count, any pair of options x and y can be socially ranked based on some interpersonally comparable information; but this is only because other options than x and y have been allowed to bear on this social ranking, namely, for each agent, all the options in between x

and y.

More precisely, to obtain a revised WIIA condition which the Borda Count would respect, it would suffice, for instance, to also hold constant for each agent the number (though not necessarily the identity) of these options. Formally, for any profile u, agent i, and pair of distinct options x, y, let $B_{u_i}(x, y)$ denote the set of options situated in between x and y according to u_i, i.e. under our strict order assumption, $B_{u_i}(x, y) = \{z \in X | u_i(x) > u_i(z) > u_i(y) \text{ or } u_i(y) > u_i(z) > u_i(x)\}$. Then, the relevant weakening of WIIA, which I will call *Betweenness* WIIA, reads as follows.[24]

> BETWEENNESS WIIA (BWIIA). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever, for all $i \in N$: i) $u_i|_{\{x,y\}} = v_i|_{\{x,y\}}$; and ii) if $u_i(x) \neq u_i(y)$, $\#B_{u_i}(x, y) = \#B_{v_i}(x, y)$.

A more radical but—once again: for our purposes at this stage—less informative weakening of WIIA would state more simply that $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever for all $i \in N$, $r_{u_i}|_{\{x,y\}} = r_{v_i}|_{\{x,y\}}$; more on this later as well.

Now, where the Borda Count substantially differs from Relative Utilitarianism is in the nature of the interpersonally comparable information. Remarkably enough, in the case of Relative Utilitarianism as we defined it (viz. under CNC), it is difficult to dispute that the interpersonally comparable data is, more specifically, *welfare* data. Information about ratios of welfare differences may be somewhat less familiar, in a social choice context, than information about welfare levels or differences; but welfare information it still seems to be.[25] By contrast, in the Borda Count case, how to classify the underlying interpersonally comparable information is a considerably more delicate matter. On the one hand, information about the cardinality of lower contour sets is—and must be, given the primitives of the SWFL approach—encoded within the welfare function apparatus. In this respect it differs from actual *non*-welfare information—such as, for instance, information about rights—that other contexts could make available. On the other hand, it is clear that preferential rank levels or differences should not be simply assimilated to welfare levels or differences (the latter being even entirely meaningless under ONC). In fact, as I will explicitly articulate in the next section, the Borda Count and related SWFLs proceed by selecting one special representant for each welfare function. However meaningful (with respect to

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[24]As regards notation, under our strict order assumption, for any welfare function u and option x, $r_u(x) = 0$ if $x = \arg\min_{z \in X} u(z)$ and otherwise $r_u(x) = \#B_u(x, \arg\min_{z \in X} u(z)) + 1$. As for the BWIIA condition stated next, it is Saari's idea of preserving "intensity levels" (see especially Saari, 1994, Sec. 4.4.9). In the case of the Simpson-Kramer method, it would suffice to further weaken WIIA by holding constant for each i the number of the options not just between x and y, but also between each of x and y and $\arg\min_{z \in X} u_i(z)$.

[25]For more nuances on the matter, see fn. 51.

the admissible transformations of the individual welfare functions) this selection, there remains a gap in generality between this special representant and any other admissible one. This suffices to refuse that the resulting interpersonal comparisons be assimilated to interpersonal comparisons of welfare.[26] All in all, for lack of a better terminology, one may say that such SWFLS rely on interpersonal comparisons of *other personal data* than welfare data. Thus, although the Borda Count SWFL relies on interpersonal comparisons *of something*, it does not rely on interpersonal comparisons of welfare.[27]

However, even if one simply granted all of the foregoing, it would not automatically follow that ipso facto, SWFLs such as the Borda Count are nothing but flat counterexamples to the necessity claim. Yet this is, in effect, the prevailing opinion among current social choice theorists. For concreteness, I will take Fleurbaey's authoritative assessment as a guide for the rest of our discussion in this section. In joint work with Hammond, Fleurbaey notes that a detailed analysis of the Borda Count and the like eventually "refutes the broad claim that social choice is impossible without interpersonal comparisons of [*welfare*], but [. . .] [not] the weaker claim that social choice is impossible without interpersonal comparisons of *something*" (Fleurbaey and Hammond, 2004, p. 1197; emphasis in original). But this is immediately to suggest that the latter claim is an uninformative triviality, adding that "after all, even the choice of a dictator must rely on the comparison of something", and commenting that "the weaker claim that social choice is impossible without interpersonal comparisons of something may not be very profound." In joint work with Mongin (Fleurbaey and Mongin, 2005, p. 394), Fleurbaey further stresses that proponents of the Borda Count or more sophisticated ONC-based SWFLs "have in mind comparisons of wealth, economic positions, or indifference curves, not [. . .] [welfare] figures", thus highlighting the

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[26]The ONC analysis of Relative Utilitarianism referred to in fn. 12 would actually call for a similar analysis. The interpersonally comparable information would then consist in indifferent probability shifts (see especially Börgers and Choo, 2017, Sec. 3 and, for instance, Hammond, 1998, Sec. 2.3 for the relevant background in decision theory under risk). Similarly with the Egalitarian Equivalence SWFL, that is defined over a so-called "economic domain" (for more details on such domains, see Le Breton and Weymark, 2011). The relevant interpersonally comparable information would then consist in indifferent fractions of the total available amount of each commodity.

[27]Incidentally, this framing offers a clarification for the (at first intriguingly) non-committal statement of one of the most forceful advocates of the Borda Count: "[s]ome readers of earlier drafts of this paper claim that the Borda Count has nothing to do with interpersonal comparisons, others claim it does" (Saari, 1998, p. 258).

contrast with the traditional necessity claim.[28]

To repeat, the non-traditional interpersonal comparisons underlying the Borda Count or related ONC-based SWFLS should certainly be firmly distinguished from the traditional interpersonal comparisons of welfare differences or levels underlying Classical Utilitarianism or the Classical Maximin. On this, there is no disagreeing with Fleurbaey and co-authors. Emphatically, despite some occasional mistaken claims to the contrary (which they document), the Borda Count or comparable SWFLs do *not* vindicate the traditional necessity claim explicit in Arrow's early work, implicit in Sen's subsequent refoundation of social choice theory, and diffuse in the social choice community throughout the 1970s at least.

However, based on an inductive inspection of the Borda Count, Relative Utilitarianism, and other SWFLS departing from and only from WIIA, the following fact becomes salient. In the general quest to overcome Arrow's impossibility, *relaxing WIIA opens up possibilities only because and inasmuch as it allows, within a certain range of measurability assumptions, for some interpersonal comparisons that are otherwise not possible.* Naturally, this is not to argue against the independence condition on the grounds that one *should* be able to make such interpersonal comparisons. It is merely to say that, irrespective of whether one is inclined to defend or to criticize the condition, one should not oppose relaxing it to introducing interpersonal comparisons.[29] The interpersonal comparisons permitted by relaxing WIIA may or may not be, more specifically, interpersonal comparisons of welfare. As illustrated by Relative Utilitarianism and the Borda Count, respectively, this will depend on the cases. Either way, at the end of the day, strikingly much is in common between the two most popular ways to overcome Arrow's impossibility, viz. preserving WIIA while relaxing Arrow's non-comparability assumption (as in Classical Utilitarianism or the Classical Maximin) and relaxing WIIA while preserving Arrow's non-comparability assumption (as in

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[28]Actually, Fleurbaey's philosophy may have evolved since this joint work with Hammond or Mongin. He now indicates a more inclusive conception of what can be counted as interpersonally comparable welfare information. See, e.g., Fleurbaey and Maniquet, 2011, p. 13: "An important branch of social choice theory has sprung out of Arrow's impossibility by introducing *exogenous* interpersonally comparable utility functions (...). In comparison, (...) instead of taking welfare indices as exogenous data, [our approach] proposes to *construct* well-being indices out of ordinal and noncomparable preferences" (emphasis mine). The Borda Count would fit that description. While such is not its motivation and its conclusion will be somewhat different from these authors', the next section will, in effect, offer tools helping further articulate Fleurbaey and Maniquet's general idea stated above.

[29]Similarly, this is not to suggest that it is fruitless to discuss, on a case-by-case basis, whether such or such interpersonally comparable information (say, preferential ranks or $[0, 1]$-normalized values) forms an appropriate ethical basis for social choice—or, relatedly, whether such or such generalization of WIIA (say, BWIIA or RWIIA) is a normatively compelling constraint on social choice. My inquiry is evidently placed at a higher level of abstraction than such discussions, however interesting and important in their own right.

Relative Utilitarianism or the Borda Count).[30] In both cases, the initial diagnosis is in effect that the full set of the Arrow conditions forces SWFLs to disregard too much of the available information (on this general outlook on the problem of social choice, see especially the alternative formalism and the discussion in Fleurbaey, 2003). In both cases still, the proposed cure is to allow SWFLs to disregard less of the available information, and to do so precisely so as to be able to make some interpersonal comparisons. The only remaining difference pertains to the exact nature of the interpersonally comparable information—necessarily welfare, or possibly otherwise—, accordingly, how that information should be encoded and preserved. But from a philosophical point of view, what sets these two approaches apart strikes one as much less significant than what brings them together. Indeed, cutting across traditional divisions between measurability, comparability, or independence properties, axioms as heterogeneous as OFC, CUC, RWIIA, BWIIA, or RNC encapsulate, as it were, as many methods of interpersonal comparisons. More on this in the next, recapitulative section of the paper, where ways to make this insight more precise will be proposed.

Before proceeding to that recapitulation, one may already foresee what its main upshot should be. The current state of social choice theory calls not for rejecting (Fleurbaey and Hammond's position, among others'), but for duly revising the traditional necessity claim. Granted, its early proponents did not formulate—presumably: even conceive of—it at the required level of generality. But there was an important grain of truth in their position, which the current state of social choice theory enables one to grasp and express better than they could. Pace Fleurbaey and Hammond, downplaying it with reference to the interpersonal comparisons required to single out a dictator runs the risk of missing the philosophical big picture; i.e., this grain of truth is not a triviality. Indeed, if any alternative proves fundamental to social choice theory, it is neither the one between relaxing non-comparability or relaxing non-dictatorship, nor the one between relaxing non-comparability or relaxing independence—nor even the more descriptive one between relaxing non-comparability, relaxing independence, or strengthening measurability. Without essential reference either to dependence on irrelevant alternatives or to strong forms of measurability, that are in fact merely special cases, it is the alternative between relying on interpersonal comparisons of welfare

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[30]Recall a point established earlier in the present section, namely that these are the two most popular, but *not* the only escape routes that matter for our purposes (see Thm. 3).

data, or relying on interpersonal comparisons of other individual data.[31] Unbeknownst to at least some early defenders of the necessity claim, this was the grain of truth in their position. This duly updated necessity claim makes it clear that the current state of the field makes their basic tenet more precise, rather than it radically contradicts it. I now turn to more fully articulating and vindicating that updated necessity claim.

5   The Updated Necessity Claim

This last section synthetically recapitulates and sharpens the main conclusions reached in the previous one. One useful means of recapitulation is the tree displayed next in Figure 1. As examined in more detail in the sequel, the tree locates various Paretian, non-dictatorial[32] SWFLs by successively considering three axiomatic alternatives. The first is whether the SWFL imposes merely ordinal measurability assumptions on individual welfare—i.e., whether it assumes welfare only ordinally, or more uniquely (e.g., cardinally) measurable than that. The second is whether WIIA is respected—thus, whether only binary informational requirements are imposed on the SWFL, or instead its social ranking of x and y can require more information than individual welfare levels at x and y. The third alternative is whether some interpersonal co-measurability condition restricts the SWFL's intrapersonal measurability assumptions—in other words, whether the SWFL ostensibly refrains from or on the contrary explicitly commits to interpersonal comparisons of welfare.

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[31]Thus I agree with Mongin when he suggests that all solutions to the problem of social choice "involve two possibilities, i.e., to make interpersonal comparisons of either [welfare] values or other individual data" (Mongin, 2006, p. 41). Two significant disagreements remain, however. First, I contest that this new alternative *refines* the one currently prevailing in the literature, viz. the alleged dilemma between upholding non-comparability, or upholding independence. Indeed the former demonstrably differs from, and *supplants*, the latter (see especially Thm. 3 and its discussion). Second, a more conceptual disagreement, while Mongin presumes that only interpersonal comparisons of welfare can play a role in an assessment of the necessity claim, I contend that all interpersonal comparisons should. More on the latter point with the analytical tools introduced in the next section.

[32]Non-Dictatorship (or its strengthening to Anonymity) excludes that the SWFL rely on interpersonal comparisons of mere names or labels. Such interpersonal comparisons are, anyhow, only loosely related to interpersonal comparisons of welfare or generalizations. Unlike the latter, the former do not genuinely form a special topic in representational measurement theory, one that involves considerations of measurability or meaningfulness.
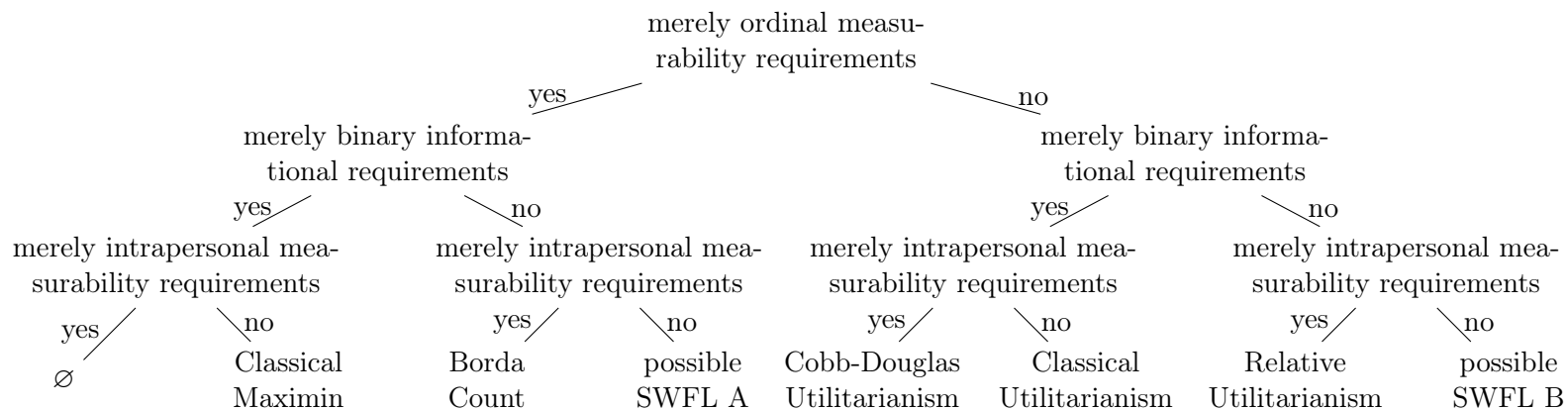
merely ordinal measu-
rability requirements

yes ╱   ╲ no

merely binary informa-
tional requirements

yes ╱   ╲ no

merely intrapersonal mea-
surability requirements

yes ╱   ╲ no

∅   Classical
           Maximin

Borda   possible
Count   SWFL A

merely binary informa-
tional requirements

yes ╱   ╲ no

merely intrapersonal mea-
surability requirements

yes ╱   ╲ no

Cobb-Douglas   Classical
Utilitarianism Utilitarianism

merely intrapersonal mea-
surability requirements

yes ╱   ╲ no

Relative   possible
Utilitarianism   SWFL B

Figure 1: A logical map of some Paretian non-dictatorial SWFLs[33]

While explaining the above tree in full detail, I will also explore abstract-
ing from all of the foregoing a more comprehensive theory of interpersonal
comparisons than currently available in the literature. Although what I will
provide here is admittedly only a sketch of that theory, I conjecture that
this sketch, together with the vindication of the revised necessity claim it
delivers, could be made both more precise and more complete.

Now, one may start by revisiting the very criterion for the presence of
interpersonal comparisons of welfare. Clearly, the most descriptive analy-
sis is as follows. A SWFL relies on interpersonal comparisons of welfare
whenever it counts as one of its existence conditions the meaningfulness,
under any of the transformations to which the individual welfare functions
may be subjected, of some interpersonal numerical inequalities over some
of the welfare quantities which the individual functions may meaningfully
order.[34] These include, most prominently, welfare levels, welfare differences,
and the ratios of the one or the other. This analysis generalizes the one
currently prevailing in SWFL theory, according to which a SWFL relies on
interpersonal comparisons of welfare just in case it requires that the admis-
sible transformations of the individual welfare functions be restricted, by
being inter-individually coordinated, beyond what their mere measurement

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[33]On the utmost left final position, by Arrow's theorem, no Paretian, non-dictatorial
SWFL is to be found. Recall that, in this paper, a universal domain assumption is built in
the definition of a SWFL. Assuming X finite as in the main text, the only mild departure
from that assumption in the above table is (as discussed in fn. 23) the Cobb-Douglas case.

[34]Recall that it is only under certain qualifications that Classical Utilitarianism can be
said to rely on interpersonal comparisons of welfare differences (see fn. 4). Yet observe that
quantified as it is, the above analysis of interpersonal comparisons of welfare can perfectly
accommodate these qualifications. Thus, to capture the fact that Classical Utilitarianism
does rely on interpersonal comparisons of welfare, this analysis can be rightfully invoked.

class (viz. ordinality, cardinality, and the like) would demand. Unlike the latter, the former analysis permits consistently stating, for instance, that however differently from Classical Utilitarianism or the Classical Maximin, Relative Utilitarianism (under CNC) and Cobb-Douglas Utilitarianism (under RNC), too, rely on interpersonal comparisons of welfare. Thus the new analysis rightfully puts on the side of interpersonal comparisons of welfare the whole right-hand side of the tree in Figure 1, and not just those of its final leaves that invoke co-measurability assumptions.

This fact and the difference between the two groups of SWFLs thus delineated may be further explained as follows. In any measurement class with sufficiently restrictive uniqueness properties, some welfare quantities are *necessarily* interpersonally comparable inasmuch as they are, in that measurement class, numerical constants. Indeed any standard departure from merely ordinal scales will imply introducing some such constants.[35] Exploiting them may demand departing from WIIA; as illustrated by Relative and Cobb-Douglas Utilitarianism, respectively, this will depend on the cases. Other welfare quantities are only *contingently* interpersonally comparable inasmuch as they are, in the given measurement class, numerical variables forbidding—absent any inter-individual coordination assumption—that any interpersonal order be meaningfully induced over them. For example, in the cardinal scale setting, ratios of welfare differences are necessarily comparable across agents, but welfare differences only contingently so. The bottom line of this analysis is that, either way, i.e. be it by strengthening either the intra-individual or the inter-individual measurability requirements, *any* standard departure from ONC—including CNC and RNC, and not just OFC and CUC—implies relying on some interpersonal comparisons of welfare, and vice versa. Therefore here is, fully motivated and articulated, a clear updated criterion: *A (Paretian non-dictatorial) SWFL relies on interpersonal comparisons of welfare exactly when it departs from ONC.*

This simple criterion has several immediate implications. First among them is that if a non-dictatorial, Paretian SWFL defined over a universal (or a sufficiently rich) domain respects WIIA, then it must rely on some interpersonal comparisons of welfare, since—by Arrow's theorem—it is specifically from ONC that it must depart.[36] Consider next the case of WIIA being violated. If, in addition, either the admissible transformations of the individual welfare functions are inter-individually restricted beyond what their mere measurement class requires, or this measurement class is not that of merely ordinal scales, then in light of the foregoing there is nothing left to argue for in order to establish the necessity claim.

The latter configuration is illustrated, in the tree, by the case of Relative

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[35]I here call "standard" any departure from ordinality invoking one of the standard uniqueness classes of representational measurement theory (e.g., cardinal or ratio scales).

[36]For more on this case at this level of generality, see Yamamura, 2017.

Utilitarianism, which we have already extensively discussed. To illustrate the ordinal and the more-than-ordinal variants of the former configuration, which we have not hitherto discussed, I have left in the tree possible SWFLs A and B as mere placeholders. This is because I am unaware of any simple, established SWFL in abstract social choice theory or welfare economics that would illustrate the axiomatic combinations in question. Indeed, co-measurability assumptions have been traditionally imposed on SWFLs that respect WIIA (specifically, so-called *social welfare orderings*) and, conversely, departures from WIIA have been traditionally explored assuming uncoordinated welfare scales (specifically, ONC). Nevertheless, as ad hoc SWFLs can illustrate, there is no logical inconsistency to the remaining combinations. For instance, SWFL A (respectively, B) could be the Borda Count (respectively, Relative Utilitarianism) enriched with a Classical Maximin (respectively, Classical Utilitarianism) tie-breaker.[37] Admittedly these ad hoc SWFLs are unlikely to be of significant intrinsic interest. But in any event they are well defined, and—a by-product of the present systematic approach—they indicate the existence of an apparently uncharted territory that could be worth exploring.

Only remains to be considered in some detail, then, the case of WIIA being violated under ONC, i.e., the class of all SWFLs represented in Figure 1 by the Borda Count. This more delicate case is analyzed in the rest of the section. Despite the overlap with what has already been said, since the analysis to be proposed formally extends to Relative Utilitarianism (under CNC) and more generally to any SWFL departing from WIIA without imposing any co-measurability assumption on the individual welfare functions, such other SWFLs will also be mentioned along the way. However they will be less emphasized, since unlike the Borda Count and the like, they have already been philosophically accounted for. Let me, by way of anticipation, announce the main claim towards which the rest of the section will be working in analyzing such SWFLs: *A (Paretian non-dictatorial) SWFL relies on interpersonal comparisons of other personal data than welfare data exactly when it departs from WIIA*.

Now, explicating SWFLs such as the Borda Count and Relative Utilitarianism, taking them all together, presents one with two main challenges. The first challenge is to identify a generalization of WIIA that is sufficiently encompassing to subsume as particular cases properties as diverse as BWIIA, RWIIA, and others. To this end, recall the following fact, that was highlighted when the Borda Count and Relative Utilitarianism were introduced. The r_{u_i} (respectively, s_{u_i}) functions based on which the Borda Count (respectively, Relative Utilitarianism) is defined are unchanged by any individually admissible (viz. strictly increasing and positive affine, respectively) transfor-

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[37]Naturally such tie-breakers are not guaranteed to break all social ties.

mation of the underlying u_i. This has the following, notable implication.[38] Given that the r_{u_i} (respectively, s_{u_i}) themselves are individually admissible transformations of the u_i, when each u_i is directly taken transformed to r_{u_i} (respectively, s_{u_i}), the social ranking of the Borda Count (respectively, Relative Utilitarianism) can be computed without any further transformation— i.e., $r_{r_{u_i}} = r_{u_i}$ (respectively, $s_{s_{u_i}} = s_{u_i}$).

Equipped with this observation, recall that a function f from a set X to itself is called *idempotent* if for any $x \in X$, $f(f(x)) = f(x)$. The identity function, for instance, is idempotent; but it is not the only function displaying that property. This leads to the generalization of WIIA stated next, that is new to the best of my knowledge.[39] It is an axiom schema. For the sake of generality, I state it leaving open the specific nature of the individually admissible transformations, i.e., the exact individual measurability assumption bearing on the SWFL. Most importantly, the schema can be fully specified only once some idempotent function $\theta : U \to U$ is given, such that for all $u \in U$, $\theta(u) = \phi \circ u$ for some individually admissible transformation ϕ.[40]

> θ-WIIA (θ-WIIA). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$
> whenever, for all $i \in N$, $\theta(u_i)|_{\{x,y\}} = \theta(v_i)|_{\{x,y\}}$.

θ-WIIA is respected by all the SWFLs mentioned in this paper and, more generally, by a great variety of SWFLs in the literature. Notice that the function θ thus indexing the axiom depends neither on the profile from which a given welfare function may be taken, nor on the agent holding that function. While this is admittedly a limitation, it seems that nothing essential in what follows depends on it.[41] Respecting WIIA amounts to respecting θ-WIIA with θ the identity function. The Borda Count and the Simpson-Kramer

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[38]This implication, shortly to be recognized as a form of *idempotence*, is naturally much weaker than the property from which it is here derived, whereby a unique representant is selected for each class of representationally equivalent welfare functions. Importantly however, it is the weaker property that permits the most seamless generalization of WIIA.

[39]A closely related property is called "individual quasi-welfarism" in Fleurbaey, 2003, Sec. 10.2. However, Fleurbaey does not axiomatically distill that property; specifically, he does not articulate an encompassing generalization of WIIA that could underpin it.

[40]Two brief technical clarifications are in order. First, it proves logically more convenient to work here with an axiom schema, rather than an existential axiom. Second, it proves clearer to state that schema with respect to a function $\theta : U \to U$, rather than admissible transformations $\phi : \mathbb{R} \to \mathbb{R}$ that one would need to index by each transformed $u \in U$.

[41]θ-WIIA is respected by most but not all SWFLs. Consider for instance economic SWFLs invoking competitive equilibria (such as the one tied to so-called *Egalitarian Walrasian* allocations), that illustrate what Fleurbaey (2003, Sec. 10.2) calls "non-individual quasi-welfarism". They would violate θ-WIIA, that is tied, by contrast, to "individual quasi-welfarism". It seems that Fleurbaey's more general brand of non-welfarism could be analyzed based on a simple generalization of θ-WIIA featuring a *profile-dependent* idempotent function ranging over the set of welfare functions. The essential conclusions reached here extend to this more general case; see especially fn. 49 for the key observation in this regard.

method (respectively, Relative Utilitarianism and Relative Egalitarianism) violate θ-WIIA when θ is the identity function but, with the individually admissible transformations being then determined as strictly increasing (respectively, positive affine), they respect it when θ is given, instead, by $\theta(u) = r_u$ (respectively, $\theta(u) = s_u$) for all $u \in U$. In words, whenever two profiles induce the same Borda scores (respectively, $[0,1]$-normalized relative values) for options x and y, the same social ranking of x and y obtains under the Borda Count or the Simpson Kramer method (respectively, Relative Utilitarianism or Relative Egalitarianism).

Therefore, to facilitate comparison with the previous theorems, one can state the following result. It simply takes stock, at the appropriate level of generality, of the existence of the aforementioned SWFLs.[42]

> **Theorem 4.** For some idempotent functions $\theta : U \to U$ such that for all $u \in U$, $\theta(u) = \phi \circ \theta(u)$ for some strictly increasing transformation ϕ (respectively, strictly increasing transformation ϕ of the form $\phi(z) = az + b$), there exist SWFLs $f : \mathcal{U} \to R$ jointly satisfying ND, WP, θ-WIIA, and ONC (respectively, CNC).

By Thms. 1 and 2, θ in Thm. 4 cannot be the identity function. To this extent (and barring the other escape routes already considered), overcoming Arrow's impossibility requires working with distinguished transformations of the individual welfare functions. To understand how idempotent functions departing from identity can thus deliver possibilities, especially under the full rigor of ONC, the following fact is key to appreciate. The set of all the admissible transformations of a given welfare representation may feature other constants—hence, *necessarily interpersonally comparable information*—than any of the welfare quantities previously mentioned to illustrate the properties of strongly unique forms of measurement. Indeed, even in an ordinal environment, the properties of the underlying framework allowing, examples of such other constants may include: the number of indifference classes in a representation; the amount of money or some physical resource solving some equality in the representation, i.e., for which a relevant welfare equivalence obtains for the agent; marginal rates of substitution across the probability weights to be distributed over some set of options. Such representation-independent quantities can be enlisted to define idempotent θ functions such that θ-WIIA is respected.[43]

This key fact calls for two clarifications. First and most obvious, these constants can be extracted only based on more than binary welfare in-

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[42]In the ONC variant of the result, it needs not be assumed that the idempotent function θ associates to any u an admissible transformation of u. Assuming idempotency alone, this could be derived from WP and θ-WIIA. Such is not the case under CNC, however.

[43]It is worth noticing that in a truly general abstract environment—i.e., one in which individual welfare functions could be unbounded, the option set would have no special structure, and so on—no such quantity could be relied on.

formation. This means that the values $\theta(u_i)(x)$ and $\theta(u_i)(y)$ depend not merely on the $u_i(x)$ and $u_i(y)$ values (for were this the case, the latter would—under ONC—carry only sign information, which is a long way to constancy) but also on further data contained in the rest of the u_i functions. The mechanism of such information extraction is made more transparent by less abstract generalizations of WIIA than θ-WIIA, such as BWIIA. Second, while representation-independent quantities like the ones just listed may not be best interpreted as conveying welfare but some other information (such as mere rank, wealth, or risk information), it is still in terms of the individual welfare functions u_i—the primitive of the SWFL approach—that they must be articulated, and with respect to the transformations of these functions that they qualify as constants. In fact, θ-WIIA can be seen to proceed by singling out distinguished representations—the fixed points of θ (that must have some non-fixed points since it departs from identity)—for the individual welfare functions. As the rest of the section will argue, this is exactly in such cases that one should say that a SWFL relies on interpersonal comparisons of other personal data than welfare data.

Precisely, the second challenge in explicating SWFLs such as the Borda Count and Relative Utilitarianism, taking them all together, is to identify generalizations of OFC, CUC, and related conditions, that could accompany the preceding generalization of WIIA to θ-WIIA. Among other things, one would like these generalizations to capture what can be, within the confines of SWFL theory, of the following intuitive analysis (which is often alluded to in the literature—see, e.g., Fleurbaey and Hammond, 2004, p. 1212-1213— but never developed, to the best of my knowledge). While the Borda Count (respectively, the Simpson-Kramer method) satisfies ONC with respect to the u_i, it effectively requires CUC and not OFC (respectively, OFC and not CUC) with respect to the r_{u_i}. A similar line of analysis can be proposed, under CNC rather than ONC, regarding Relative Utilitarianism and Relative Egalitarianism. Therefore the overall intuition also is, incidentally, that the Borda Count and Relative Utilitarianism (respectively, the Simpson-Kramer method and Relative Egalitarianism) have more in common than the traditional informational bases of SWFL theory can bring out.

A moment's reflection reveals that this second challenge will not be met directly, viz. by introducing schematic θ-generalizations of OFC, CUC, and the like.[44] However, as explained next, one natural indirect way for-

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[44]Succinctly put, if (as in all the examples considered hitherto) θ selects a unique representant for each class of representationally equivalent welfare functions, then any equality of the form $\theta(u) = \phi \circ \theta(v)$ with ϕ an individually admissible transformation implies that $\theta(u) = \theta(v)$, thus, that ϕ is the identity function. Consequently, the schematic θ-generalizations of OFC, CUC, and the like would add no axiomatic content, for their antecedent could be satisfied only when $\theta(u_i) = \theta(v_i)$ for all i, which by θ-WIIA alone implies $\succcurlyeq_u^f = \succcurlyeq_v^f$. In particular, such schematic generalizations could not capture the sense in which the Borda Count and the Simpson-Kramer Method (respectively, and Relative Utilitarianism) rely on different (respectively, similar) kinds of interpersonal comparisons.

ward remains. Notice that over a universal domain, the conjunction of WIIA and OFC is equivalent to the compound requirement according to which for any $u, v \in \mathcal{U}$, $x, y \in X$, $\succeq_u^f |_{\{x,y\}} = \succeq_v^f |_{\{x,y\}}$ whenever for all $i \in N$, $v_i|_{\{x,y\}} = \phi \circ u_i|_{\{x,y\}}$ with ϕ some agent-independent strictly increasing transformation. Replacing ϕ in this statement by ϕ_i some strictly increasing transformation of the form $\phi_i(z) = az + b_i$, one obtains a compound condition that is, under a universal domain assumption still, equivalent to the conjunction of WIIA and CUC. Call these compound properties IIA_1 (incorporating OFC) and IIA_2 (incorporating CUC), respectively.[45] A natural move, then, consists in considering, instead of the schematic of OFC and CUC, those of IIA_1 and IIA_2. These are the following axiom schemas. They are strengthenings of θ-WIIA that, like θ-WIIA itself, are fully specified only once some idempotent function $\theta : U \to U$ is given, such that for all $u \in U$, $\theta(u) = \psi \circ u$ for some individually admissible transformation ψ.

θ-IIA_1 (θ-IIA_1). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succeq_u^f |_{\{x,y\}} = \succeq_v^f |_{\{x,y\}}$ whenever, for all $i \in N$, $\theta(v_i)|_{\{x,y\}} = \phi \circ \theta(u_i)|_{\{x,y\}}$ for some strictly increasing transformation ϕ.

θ-IIA_2 (θ-IIA_2). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succeq_u^f |_{\{x,y\}} = \succeq_v^f |_{\{x,y\}}$ whenever, for all $i \in N$, $\theta(v_i)|_{\{x,y\}} = \phi_i \circ \theta(u_i)|_{\{x,y\}}$ for some strictly increasing transformation ϕ_i of the form $\psi_i(z) = az + b_i$.

θ-IIA_1 and θ-IIA_2 can, over a universal domain, be non-trivially evaluated even when θ departs from identity. For instance, when θ is given by $\theta(u) = r_u$ for all $u \in U$, the Borda Count can easily be illustrated to respect θ-IIA_2 but not θ-IIA_1 and the Simpson-Kramer Method, θ-IIA_1 but not θ-IIA_2.[46] In more detail, in the particular case of the Borda Count, θ-IIA_2 simply means that whenever the differences between the Borda scores for x and y are scaled up or down in the same proportion for all agents, then the same social ranking of x and y obtains. By contrast, when that difference is unilaterally changed for one agent only, or when a joint transformation preserves only the order interpersonally induced over the Borda score levels, there is no guarantee that the social ranking will be unchanged. This invariance and lack thereof evidence, possibly as much as can be from within SWFL theory,

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[45]Such compound conditions (some of which have been explored in Hammond, 1991) are term-to-term generalizations of Arrow's initial IIA condition—equivalently over a universal domain, as previously noted (fn. 6), the conjunction of WIIA and ONC.

[46]Notice that given Borda scores $r_{u_i}(x), r_{u_i}(y)$ and transformation parameters $a > 0$, b_i, the numbers $ar_{u_i}(x) + b_i$, $ar_{u_i}(y) + b_i$ are in general not Borda scores, i.e., integers ranging between 0 and $m - 1$. What θ-IIA_2 demands is that when the numbers $ar_{u_i}(x) + b_i$ and $ar_{u_i}(y) + b_i$ *are* Borda scores for all agents, the Borda Count induce the same social ranking of x and y. (A similar observation applies to θ-IIA_1 and the Simpson-Kramer Method.) Further notice that in the case of the Borda Count, if $u_i(x) \neq u_i(y)$ for all $i \in N$, then using previously introduced notation the key antecedent in θ-IIA_2 could be compactly stated (provided the restriction $u_i|_{\{x,y\}} \equiv v_i|_{\{x,y\}}$ is added) as $\#B_{u_i}(x, y) = a \# B_{v_i}(x, y)$.

the interpersonal comparability of the $\theta(u_i)$ differences.[47] With due changes, a similar analysis applies to the Simpson-Kramer Method, $\theta\text{-IIA}_1$, and the interpersonal comparability of the $\theta(u_i)$ levels. With further due changes, the opposition between Relative Utilitarianism and Relative Egalitarianism can be similarly put in perspective. Therefore it also follows that for suitable θ functions, both the Borda Count and Relative Utilitarianism (respectively, the Simpson-Kramer Method and Relative Egalitarianism) satisfy $\theta\text{-IIA}_2$ and not $\theta\text{-IIA}_1$ (respectively, $\theta\text{-IIA}_1$ and not $\theta\text{-IIA}_2$), thus highlighting in each pair of SWFLs more common points than the traditional ONC – CNC toolkit can bring out. This shortcoming of the standard toolkit notwithstanding, what the foregoing suggests is that the traditional approach to interpersonal comparisons is limited not so much because it relies on invariance conditions, than because it employs such conditions *too narrowly.* The limitation, if any, resides in preconceived ideas regarding what invariances are relevant or what kind of meaningful information these invariances can deliver. As illustrated above, invariance conditions *can* capture the non-traditional interpersonal comparisons underlying the Borda Count and the like.

Admittedly, a significant difference should be acknowledged between evaluating $\theta\text{-IIA}_1$ (respectively, $\theta\text{-IIA}_2$) with θ the identity function and evaluating it with θ some other function.[48] In the latter case, unlike in the former, the strictly increasing transformation ψ (respectively, positive affine transformation ψ_i) considered in the axiom does not generally correspond to an individually admissible transformation of the underlying welfare functions. This transformation alters the individual welfare information—although some equivalence obtains with respect to the $\theta(u_i)$, none does with respect to the u_i. In light of our preliminary remarks about specific idempotent functions such as those given by $\theta(u) = r_u$ and $\theta(u) = s_u$ for all $u \in U$, this is unsurprising. To the set of all admissible representations of a given welfare function u, these specific idempotent functions associate exactly one representant $\theta(u)$. Hence, any change made to the values of the latter function must break its representational equivalence with the former. Notwithstanding that difference in the kind of invariance at stake, as the preceding paragraph has illustrated, the general interpretation of $\theta\text{-IIA}_1$ and $\theta\text{-IIA}_2$ in terms of interpersonal comparisons of the $\theta(u_i)$, when θ may depart from identity, is naturally aligned with their interpretation in terms of interpersonal comparisons of the u_i, when θ is the identity function. One may speak of interpersonal comparisons of,

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[47]This statement holds under the same qualifications as those explained in fn. 4.

[48]Another difference is immediate from what has been previously noted in fn. 44. When θ differs from identity, $\theta\text{-IIA}_1$ (respectively, $\theta\text{-IIA}_2$) is generally not equivalent to, but stronger than, the conjunction of $\theta\text{-WIIA}$ and the schematic θ-generalizations of OFC (respectively, of CUC). When θ is given by $\theta(u) = r_u$ for all $u \in U$, for example, the Borda Count (respectively, the Simpson-Kramer Method) violates $\theta\text{-IIA}_1$ (respectively, $\theta\text{-IIA}_2$) although it respects both $\theta\text{-WIIA}$ and the schematic θ-generalizations of OFC (respectively, of CUC).

respectively, welfare data and other personal data than welfare data.

Now, considering compound conditions such as θ-IIA$_1$ and θ-IIA$_2$ naturally leads to the following question, that seems crucial for our discussion. It is well known that over a universal domain, the conjunction of WIIA and ONC is equivalent to Arrow's initial IIA condition, viz. that for any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever for all $i \in N$, $v_i|_{\{x,y\}} \equiv u_i|_{\{x,y\}}$, i.e., $v_i|_{\{x,y\}} = \phi_i \circ u_i|_{\{x,y\}}$ for some agent-dependent strictly increasing transformation ϕ_i. Furthermore, as it has been recalled in the remarks preliminary to Sen's Thm. 2, it is also well known that replacing ϕ_i in this statement by ϕ_i some agent-dependent positive affine transformation of the form $\phi_i(z) = a_i z + b_i$—i.e., considering the conjunction of WIIA and CNC— proves equivalent to Arrow's initial IIA condition. What about, then, the schematic θ-generalizations of these two compounds conditions? At stake with these further conditions is, as it were, the possibility that a θ-WIIA-abiding SWFL incorporate *no* assumption about the interpersonal comparability of the $\theta(u_i)$.

Notably for our philosophical purposes, it is readily seen, first, that the two generalizations are, like the initial conditions, equivalent to one another, second and more important, that the generalizations are equivalent to the initial conditions. To see this, let us state the schematic generalization of Arrow's original IIA, indexing it by some idempotent function $\theta : U \to U$ such that for all $u \in U$, $\theta(u) = \psi \circ u$ for some individually admissible transformation ψ.[49]

θ-IIA (θ-IIA). For any $u, v \in \mathcal{U}$, $x, y \in X$, $\succcurlyeq_u^f |_{\{x,y\}} = \succcurlyeq_v^f |_{\{x,y\}}$ whenever, for all $i \in N$, $\theta(v_i)|_{\{x,y\}} = \phi_i \circ \theta(u_i)|_{\{x,y\}}$ for some strictly increasing transformation ϕ_i.

First, Sen's lemma can still be invoked to establish that if $\theta(v_i)|_{\{x,y\}} = \phi_i \circ \theta(u_i)|_{\{x,y\}}$ for some strictly increasing ϕ_i, then $\theta(v_i)|_{\{x,y\}} = \chi_i \circ \theta(u_i)|_{\{x,y\}}$ for some positive affine χ_i. Consequently, whatever the exact index θ, replacing ϕ_i in θ-IIA by ϕ_i some agent-dependent positive affine transformation of the form $\phi_i(z) = a_i z + b_i$ again leads back to θ-IIA.[50] Second, since for all $u \in U$, $\theta(u) = \psi \circ u$ for some transformation ψ that has at least the property of being strictly increasing, $\theta(v_i)|_{\{x,y\}} = \phi_i \circ \theta(u_i)|_{\{x,y\}}$ holds if and only if $v_i|_{\{x,y\}} = \xi_i \circ u_i|_{\{x,y\}}$ for some strictly increasing ξ_i. This establishes the equivalence of θ-IIA and IIA—put differently, the fact that θ-IIA holds with θ any suitable idempotent function if and only if it holds with θ the identity function. Given that IIA implies WIIA, this entails that no WIIA-

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[49] It is worth noticing that in the reasoning to be detailed next, nothing depends on the idempotent function θ being profile- or agent-independent.

[50] To see why this conclusion would not extend to uncoordinated ratio (rather than merely positive affine) transformations, recall the explanations in fn. 22. The analysis provided when discussing Cobb-Douglas Utilitarianism could then be cautiously adapted.

violating SWFL can respect θ-IIA. As a result, the space of all possible WIIA-violating, θ-WIIA-respecting SWFLs is restricted to those satisfying θ-IIA$_1$, θ-IIA$_2$, or similar weakenings of θ-IIA incorporating, one way or another, what is best interpreted as interpersonal comparability assumptions about the $\theta(u_i)$. Since only such SWFLs remained to be accounted for in our overall recapitulative argument, this completes the sketch of a vindication of the updated necessity claim that is qualified—mainly because however general, flexible, and widely respected an axiom, θ-IIA is assumed—but systematic.

Indeed, under the aforementioned qualifications, one may as previously announced conclude that a (Paretian non-dictatorial) SWFL relies on interpersonal comparisons of other personal data than welfare data exactly when it departs from WIIA. Bringing together the two halves of the present section therefore gives support to the updated necessity claim with which we tentatively concluded the previous one, viz. that the possibility of social choice hinges on either interpersonal comparisons of welfare data (departing from ONC) or interpersonal comparisons other individual data (departing from WIIA). These two kinds of interpersonal comparisons are not incompatible with one another (witness SWFLs A and B in Figure 1, together with the accompanying explanations on p. 24)[51] and neither implies the other. The only impossibility is that *none* be present—which is nothing else than the statement of Arrow's theorem.

6 Conclusion

The core domain of interpersonal comparisons may pertain to welfare levels, welfare differences, and the other standard targets for the traditional comparability assumptions of social choice theory. Nevertheless, this core domain naturally extends in two directions. First, interpersonal comparisons of welfare are possible even when no comparability assumption is imposed. While this may at first sound like a paradox, I have explained that no paradox is at stake here—only the under-appreciated role of intrapersonal measurability assumptions. Interpersonal comparisons of welfare simply extend beyond levels, differences, and other such basic targets.

Second and no less important, interpersonal comparisons extend beyond welfare altogether. Admittedly, it then becomes a matter of conceptual appreciation whether to emphasize the continuity or the discontinuity between the classical interpersonal comparisons, that concern welfare information,

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[51]The case of Relative Utilitarianism (as defined here, viz. under CNC) is especially interesting since on the above analysis, whether the SWFL relies on interpersonal comparisons of welfare or interpersonal comparisons of other personal data will be a matter of perspective. By its departing from ONC, as emphasized in the main text, Relative Utilitarianism relies on interpersonal comparisons of welfare. By its departing from WIIA and selecting distinguished representants for each u_i, it relies on interpersonal comparisons of other personal data.

and any of the non-classical ones, that may be best interpreted otherwise. Presumably in a reaction proportional to earlier excessively sweeping claims, the current literature strongly emphasizes discontinuity. The time of excesses being now arguably over, one may start appreciating continuity, which I have highlighted in various ways.

All the more so, that a third and last point should be stressed. Allowing for one kind or another of dependence on (so-called) irrelevant alternatives is just a means for allowing, within the appropriate range of measurability assumptions, for some interpersonal comparisons that are otherwise precluded. These interpersonal comparisons may or may not be, more specifically, interpersonal comparisons of welfare; this will depend on the cases. In any event, irrelevant alternatives are, in fact, relevant tools for exploiting the two aforementioned extensions of the core domain of interpersonal comparisons.

All in all, then, there is qualified but extensive support for the claim once endorsed by Arrow, Sen, and others, to the effect that interpersonal comparisons of welfare are necessary for social choice. Granted, as it was initially stated, the claim proves incorrect. But in light of all the foregoing, the current state of social choice theory calls less for outright rejecting it, than for accepting it in suitably revised form.

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