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Comparing uncertainty aversion towards different sources

Aurélien Baillon[1] · Ning Liu[2] · Dennie van Dolder[3]

Abstract We propose simple behavioral definitions of comparative uncertainty aversion for a single agent towards different sources of uncertainty. Our definitions allow for the comparison of utility curvature for different sources if the agent's choices satisfy subjective expected utility towards each source. We discuss how our definitions can be applied to investigate ambiguity aversion in Klibanoff et al.'s (Econometrica 73(6):1849–1892, 2005) smooth ambiguity model, to study the effects of learning and situational factors on uncertainty preferences, and to compare uncertainty preferences between different agents.

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✉ Aurélien Baillon
baillon@ese.eur.nl
http://www.aurelienbaillon.com

Ning Liu
liu.ning@unibocconi.it
https://www.sites.google.com/site/ningliuhomepage

Dennie van Dolder
dennie.vandolder@nottingham.ac.uk
http://www.dennievandolder.com

1 Erasmus School of Economics, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands

2 Ettore Bocconi Department of Economics, Bocconi University, via Roentgen 1, 20136 Milan, Italy

3 Nottingham School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, UK

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1 Introduction

We often do not know the probabilities of uncertain events when making decisions. Research has shown that attitudes towards uncertainty do not just differ between individuals, but also depend on the source of the uncertainty (Abdellaoui et al. 2011). For example, people prefer to bet on sources for which they feel competent rather than on sources for which they feel incompetent (Heath and Tversky 1991; Keppe and Weber 1995; Tversky and Fox 1995; de Lara Resende and Wu 2010). It has been argued that such a preference can partially explain the *home bias* in financial markets, defined as the tendency of investors to hold nearly all their wealth in domestic assets despite the benefits of international diversification (French and Poterba 1991; Kilka and Weber 2000; Uppal and Wang 2003).

Preferences over different sources of uncertainty can be modeled through source-dependent utility functions, as axiomatized by Cappelli et al. (2016). Chew et al. (2008) investigated and found support for source-dependent utility in a neuroimaging experiment. Various models have been proposed in which the agent faces two consecutive stages of uncertainty and has a different utility function for each stage (Nau 2006; Ergin and Gul 2009; Strzalecki 2011).

In the present paper, we propose simple behavioral definitions of comparative uncertainty aversion of a single agent towards different sources of uncertainty.[1] We build on Yaari's (1969) seminal definition of comparative uncertainty aversion and purely rely on the agent's willingness to accept bets under each source.[2] Our definitions do not require that the agent's choices satisfy subjective expected utility (SEU), but allow for comparative statements regarding the agent's utility curvature if they do.

Our results follow directly from Yaari's mathematical result. Conceptually, however, they remove two fundamental limitations. Yaari's condition allows for the comparison of uncertainty aversion of a single agent across different situations, but only if the agent (1) faces the exact same events and (2) holds the same beliefs regarding the likelihood of these events across the two situations. These limitations make Yaari's definition inapplicable for most studies of source preference, which typically involve different events or different information levels about these events. By removing these

[1] We use the term *risk aversion* for preferences regarding acts with known probabilities, and *uncertainty aversion* for preferences regarding acts with unknown probabilities. The term uncertainty aversion is sometimes used to refer to the preference for acts with unknown probabilities over acts with known probabilities. We use the term *ambiguity aversion* for such preferences.

[2] A large body of work has used Yaari's definition or has expanded upon it by constructing comparative definitions of revealed risk aversion, ambiguity aversion, and loss aversion (Kihlstrom and Mirman 1974; Roth 1985; Epstein 1999; Ghirardato and Marinacci 2002; Nau 2003; Köbberling and Wakker 2005; Nau 2006; Olszewski 2007; Blavatskyy 2011; Jewitt and Mukerji 2011; Bommier et al. 2012; Chambers and Echenique 2012; Heufer 2014).

two limitations, our conditions not only allow for the comparison of uncertainty aversion of a single agent towards different sources, they also open up further possibilities to investigate the effects of learning and situational factors on uncertainty preferences, and to compare uncertainty aversion between agents.

The paper is organized as follows. Section 2 introduces the theoretical framework and presents our main definition for binary acts. Section 3 extends our definition to general acts. Section 4 shows how our definitions can be applied to investigate ambiguity aversion in Klibanoff et al.'s (2005) smooth ambiguity model, to study the effects of learning and the decision situation on uncertainty preferences, and to compare preferences between agents. Section 5 concludes.

2 Main result

We compare an agent's uncertainty aversion to two sources indexed by $j \in \{A, B\}$. Let a *source of uncertainty* S_j be a finite or infinite *state space* containing all *states of nature* s pertaining to j. The agent does not know which state of S_j is true. It is possible to consider a compound state space $S_A \times S_B$, but we do not need it for our main results. An event E is a subset of S_j. Let the set of events of S_j considered by the agent be a sigma-algebra denoted as Σ_j. The complementary event of E is denoted as E^c. The *outcome set* is X, an open interval of the reals. The agents can choose between *acts*, which are finite Σ_j-measurable mappings from S_j to X. Acts are typically denoted f or g and the set of all acts on S_j is $\mathcal{F}_j$. The *bet* $x_E y$ is a binary act yielding outcome x, if event E occurs, and y, otherwise. When $x > y$, we call $x_E y$ *a bet on* E and $y_E x$ *a bet against* E. Acts that yield the same outcome z for all $s \in S_j$ are referred to as z.

The agents have preferences $\succsim_j$ over $\mathcal{F}_j$, with $\sim_j$, $\succ_j$, $\prec_j$, and $\precsim_j$ defined as usual. We will say that $\succsim_j$ is represented by *subjective expected utility* (SEU) if there exists a countably additive subjective probability measure P_j and a utility function u_j uniquely defined up to an affine transformation such that $f \succsim_j g \Leftrightarrow \int_{S_j} P_j(s)u_j(f(s))\mathrm{d}s \geq \int_{S_j} P_j(s)u_j(g(s))\mathrm{d}s$. Throughout, we assume that the utility is continuous and strictly increasing. We say that u_A is *more concave than* u_B if there exists a concave function φ such that $u_A = \varphi \circ u_B$. Finally, P_j is *nonatomic* if for all $E \in \Sigma_j$ such that $P_j(E) > 0$, there exists $F \in \Sigma_j$ such that $F \subset E$ and $0 < P_j(F) < P_j(E)$. Nonatomicity is guaranteed by Savage's (1954) axiomatization of SEU.

Yaari (1969) defined comparative uncertainty aversion in terms of the agent's willingness to accept bets. His definition states that an agent is more uncertainty-averse in one situation than in another, if her acceptance set in the former situation is a subset of her acceptance set in the latter. Under SEU, this implies that the agent has more concave utility in the former situation than in the latter. For this definition to be meaningful, it is necessary that the agent (1) faces the same events and (2) holds the same beliefs regarding the likelihood of these events across the two situations. We propose an alternative way to compare uncertainty attitudes, which removes these constraints. The main point of our result is simple: to compare uncertainty aversion, one should not only consider bets on events, but also bets against them.

Consider two events: $E \in \Sigma_A$ and $F \in \Sigma_B$. Imagine that the agent is willing to accept a bet on E ($x_E y$), but not on F ($x_F y$). This can either be because the agent is more uncertainty-averse towards source S_B than she is towards source S_A, or because she believes that event E is more likely than event F. Beliefs, however, are unable to explain why the agent would simultaneously prefer bets on E to bets on F and prefer bets on E^c to bets on F^c. This logic leads to the following condition for the agent to be more uncertainty-averse towards source S_B than towards source S_A: whenever the agent would reject a bet $x_E y$ and its symmetric bet $y_E x$, she should always reject either $x_F y$ or $y_F x$, possibly both.

Theorem 1 *Assume that $\succsim_A$ defined over $\mathcal{F}_A$ and $\succsim_B$ defined over $\mathcal{F}_B$ are represented by subjective expected utility. The following statement* (i) *is necessary for statement* (ii). *It is also sufficient if there exist $E \in \Sigma_A$ and $F \in \Sigma_B$ such that $P_A(E) = P_B(F) = \frac{1}{2}$.*

(i) $\forall\, E \in \Sigma_A$, $F \in \Sigma_B$, *and* x, y, *and* z *in* X,
$(z \succsim_A x_E y$ *and* $z \succsim_A y_E x) \Rightarrow (z \succsim_B x_F y$ *or* $z \succsim_B y_F x)$.
(ii) u_B *is more concave than* u_A.

Statement (i) is necessary and sufficient to compare the agent's utility towards two sources of uncertainty if there is at least one event under each source to which the agent assigns probability $\frac{1}{2}$. Using Ramsey's (1931) concept of ethically neutral events, we only need one ethically neutral event under each source. This richness condition is automatically satisfied if P_A and P_B are nonatomic, as follows from Savage's (1954) axiomatization.

Corollary 1 *If, in Theorem* 1, *P_A and P_B are nonatomic, then* (i) *is equivalent to* (ii).

Ergin and Gul (2009) and Strzalecki (2011) call being more uncertainty-averse to one source than to another *second-order risk aversion* and propose preference conditions to characterize it. Unlike in Ergin and Gul (2009), our condition does not use concepts from the representation we assume. Unlike in Strzalecki (2011), our condition does not require the existence of acts whose outcomes depend on both sources of uncertainty.

Cappelli et al. (2016) proposed a condition for source preference based on utility midpoints. Such endogenous midpoints can be measured by techniques proposed by Abdellaoui et al. (2007), Fishburn and Edwards (1997), Ghirardato et al. (2003), Harvey (1986), Köbberling and Wakker (2003), and Vind (2003). Yet, they are more complex than the modification of Yaari's intuitive technique proposed here.

Kopylov (2016) proposed a condition with the same structure as condition (i) in Theorem 1 but in a (Anscombe–Aumann) setting where acts are mappings from the state space to lotteries. His condition drops the second limitation of Yaari's condition (same beliefs), but still suffers from the first limitation by requiring that the same events are considered before and after the implication symbol. Condition (i) generalizes it such that the events can differ and potentially stem from different sources.

Our definition allows for the ranking of sources of uncertainty on the basis of the agent's uncertainty aversion towards each source. It allows for comparing utility between different sources and, thereby, complements the definitions of Tversky and

Wakker (1995), which allowed for comparing decision weights between sources in non-expected utility models.[3]

3 Extensions to general acts

To extend the definition from bets to general acts, we can consider what happens to two events only for each agent and keep everything else (what happens on the other events) constant. Theorem 2 applies this approach. We denote $x_E y_F f$ the act yielding x on E, y on F, and $f(s)$ for all $s \notin E \cup F$.

Theorem 2 *Assume that* $\succsim_A$ *defined over* $\mathcal{F}_A$ *and* $\succsim_B$ *defined over* $\mathcal{F}_B$ *are represented by subjective expected utility. The following statement* (i) *is necessary for statement* (ii). *It is also sufficient if there exist* $E, F \in \Sigma_A$ *and* $G, H \in \Sigma_B$ *such that* $P_A(E) = P_A(F)$ *and* $P_B(G) = P_B(H)$.

(i) $\forall\ E, F \in \Sigma_A,\ G, H \in \Sigma_B,\ f \in \mathcal{F}_A,\ g \in \mathcal{F}_B,$ *and* x, y, *and* z *in* X,
$(z_{E\cup F} f \succsim_A x_E y_F f \quad and \quad z_{E\cup F} f \succsim_A y_E x_F f)$
$\Rightarrow (z_{G\cup H} g \succsim_B x_G y_H g \quad or \quad z_{G\cup H} g \succsim_B y_G x_H g).$
(ii) u_B *is more concave than* u_A.

Note that we have weakened the richness requirement on the state spaces: it is still necessary that there are two events that the agent finds equally likely under each source, but these events no longer have to be complementary.

In statement (i) of both Theorems 1 and 2, we permuted two outcomes. We can also permute all outcomes of the act. Consider $f \in \mathcal{F}_A$ and $g \in \mathcal{F}_B$, with $f = (E_1 : x_1, \ldots, E_n : x_n)$ and $g = (F_1 : x_1, \ldots, F_n : x_n)$. Hence, f and g yield the same n outcomes, but need not assign these outcomes to the same events. Let $\Pi(x_1, \ldots, x_n)$ denote a permutation of $(x_1, \ldots, x_n)$. For simplicity, we use the $\Pi(f)$ to denote the act assigning $\Pi(x_1, \ldots, x_n)$ to $(E_1, \ldots, E_n)$ and $\Pi(g)$ to denote the same permutation of the outcome of g.

Theorem 3 *Assume that* $\succsim_A$ *defined over* $\mathcal{F}_A$ *and* $\succsim_B$ *defined over* $\mathcal{F}_B$ *are represented by subjective expected utility. The following statement* (i) *is necessary for statement* (ii). *It is also sufficient if there exist, for some integer* n, *an* n*-fold partitions* $\{E_i\}_n$ *of* S_A *and an* n*-fold partition* $\{F_i\}_n$ *of* S_B, *such that* $P_A(E_i) = P_B(F_i) = \frac{1}{n}$ *for all* $i \in \{1, \ldots, n\}$.

(i) $\forall\ f \in \mathcal{F}_A$ *and* $g \in \mathcal{F}_B$ *yielding the same outcomes* $(x_1, \ldots, x_n)$ *and for all* $z \in X$,
$(z \succsim_A \Pi(f) \quad \forall \Pi) \Rightarrow (\exists \Pi$ *such that* $z \succsim_B \Pi(g))$.
(ii) u_B *is more concave than* u_A.

We can also decide not to consider all permutations, but only a subset of permutations that "maximally differ" from each other. We can do so by using a type of permutation that we will call cyclic. Consider an act $f = (E_1 : x_1, E_2 :$

[3] See Baillon et al. (2012) for a discussion of the descriptive appropriateness of utility to capture source preference.

$x_2, \ldots, E_{n-1} : x_{n-1}, E_n : x_n)$. Let π be a *cyclic permutation function*, defined as $\pi(f) = (E_1 : x_2, E_2 : x_3, \ldots, E_{n-1} : x_n, E_n : x_1)$. It moves each outcome one event to the left. We denote π^m the compound function that applies π m times. For instance, $\pi^3 = \pi \circ \pi \circ \pi$.

Theorem 4 *Assume that $\succsim_A$ defined over $\mathcal{F}_A$ and $\succsim_B$ defined over $\mathcal{F}_B$ are represented by subjective expected utility. The following statement* (i) *is necessary for statement* (ii). *It is also sufficient if there exist, for some integer n, an n-fold partitions $\{E_i\}_n$ of S_A and an n-fold partition $\{F_i\}_n$ of S_B, such that $P_A(E_i) = P_B(F_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$.*

(i) *$\forall f \in \mathcal{F}_A$ and $g \in \mathcal{F}_B$ yielding the same outcomes $(x_1, \ldots, x_n)$ and for all $z \in X$,*
$(z \succsim_A \pi^m(f) \;\; \forall m \in \{1, \ldots, n\}) \Rightarrow (\exists m \in \{1, \ldots, n\}$ such that $z \succsim_B \pi^m(g))$.
(ii) *u_B is more concave than u_A.*

In Theorems 3 and 4, the additional requirement for (i) to be sufficient for (ii) is stronger than that of Theorem 2. The approach of Theorem 2 is, therefore, the least demanding in terms of richness of the state spaces and beliefs of the agents. For all the three theorems, the requirements are trivially satisfied by nonatomic probability measures.

Corollary 2 *If, in Theorems 2, 3, and 4, P_A and P_B are nonatomic, then* (i) *is equivalent to* (ii).

4 Further applications

Until now, our theory has focused on a single agent facing different sources of uncertainty. Our results, however, also open up further possibilities to investigate ambiguity aversion in Klibanoff et al.'s (2005) smooth ambiguity model, to study the effect of learning on uncertainty preferences, to study how the decision situation affects preferences, and to compare preferences between agents. Here, we will briefly discuss these applications.

4.1 Application 1: Smooth ambiguity model

Ambiguity aversion can be seen as a special case of source preference. If one of the two sources of uncertainty that an agent faces is risky (known probabilities), then ambiguity aversion can be defined as being more averse towards the uncertain source than to the risky one. In his seminal work, Ellsberg (1961) provides convincing examples suggesting that individuals will prefer to bet on risk rather than uncertainty. In his simplest example, people prefer to bet on an urn with 50 red and 50 black balls over an urn containing a 100 balls that are either red or black in unknown proportions, irrespective of the winning color. While Ellsberg presented this as a thought experiment, subsequent work has convincingly shown his intuition to be correct (Camerer and Weber 1992).

We now show how our condition can be applied to study ambiguity aversion in the smooth ambiguity model introduced by Klibanoff et al. (2005). In their setup, the state space is the compound of a separable metric space S_B and a probability interval $S_A = (0, 1]$. Hence, $S = S_A \times S_B$. Let Σ_A and Σ_B be the Borel sigma-algebra of S_A and S_B, respectively, and define $\Sigma = \Sigma_A \otimes \Sigma_B$. The set of all countably additive probability measures over Σ that are congruent with the Lebesgue measure over S_A is denoted Δ. Let $\mathcal{F}$ be the set of all acts on S. An act is a *lottery* if it only depends on S_A and we denote $\mathcal{L}$ the set of lotteries. We denote $x_p y$ the lottery $x_{E \times S_B} y$ when the Lebesgue measure of E is p. The agent's preferences $\succsim$ are represented by the *smooth ambiguity model* if there exist a utility function u, a function φ defined over the image of u and a probability measure over Δ such that $f \succsim g$ is equivalent to $\int_\Delta \varphi(\int_S u(f)\mathrm{d}P)\mathrm{d}\mu \geq \int_\Delta \varphi(\int_S u(g)\mathrm{d}P)\mathrm{d}\mu$. We assume that the functions φ and u are continuous and strictly increasing.

Theorem 5 *Assume $\succsim$ is represented by the smooth ambiguity model. The following statement* (i) *is necessary for statement* (ii). *It is also sufficient if there exists $E \in \Sigma$ such that $\int_\Delta P(E)\mathrm{d}\mu = \frac{1}{2}$.*

(i) $\forall\, p \in [0, 1],\ F \in \Sigma$, *and* $x,\ y$, *and* z *in* X,
$(z \succsim_A x_p y$ *and* $z \succsim_A y_p x) \Rightarrow (z \succsim_B x_F y$ *or* $z \succsim_B y_F x)$.
(ii) φ *is concave.*

Our condition allows us to characterize the concavity of φ without using unobservable acts mapping Δ to X unlike Baillon et al. (2012), and without using u or μ in the preference condition unlike Klibanoff et al. (2005).

4.2 Application 2: Learning

Consider an agent who has the possibility to learn about a single source S (e.g., to receive signals). Will she become less uncertainty-averse after learning more about the uncertainty she faces? If she is Bayesian, she should update her beliefs based on the information she receives and we could not use Yaari's original definition to compare her utility before and after receiving information. By contrast, denote $\succsim_B$ her preferences *Before* receiving information and $\succsim_A$ her preferences *After* receiving information. Theorems 1–4 allow us to study the effect of learning on her uncertainty aversion and on her utility curvature.

Observation 1 *Theorems 1–4 can be used to compare an agent's utility curvature before and after receiving information about S.*

4.3 Application 3: Situational factors

In a similar vein, Yaari's definition does not allow for the comparison of an agent's utility curvature in different situations (decision context, material conditions, time pressure, etc.) if the agent's beliefs differ between them. It will often be difficult to rule out the possibility that beliefs are affected by situational factors, especially if these factors are influential enough to affect preferences. Theorems 1–4 allow for the study of situational influences on preferences, even if beliefs are also affected.

Observation 2 *Theorems 1–4 can be used to compare an agent's utility curvature between different situations, irrespective of whether these situations affect the agent's beliefs.*

An example of such a situational factor is the weather. It has been found that the weather affects traders' behavior, with market returns being lower on cloudy days (Saunders 1993; Hirshleifer and Shumway 2003). It is not clear whether this is because it affects traders' perceived likelihood of future events or because it affects their intrinsic willingness to bear uncertainty. Watson and Funck (2012) showed that short selling by (professional) traders increases on cloudy days, suggesting that they are more willing to bet *against* stocks going up. In the light of our definition, this suggests that the impact of cloudiness on stock returns is caused by a change in beliefs, rather than a higher degree of uncertainty aversion.

4.4 Application 4: Different agents

So far, we have focused on comparing uncertainty aversion of a single agent across different situations. The main focus of Yaari's (1969) original definition was to compare uncertainty aversion between different agents. To do this, however, agents needed to face the same events and their subjective probabilities should coincide. Especially, this latter requirement is overly restrictive as it denies uncertainty its fundamental property of potential disagreement on beliefs; requiring all agents to hold the same beliefs effectively reduces uncertainty to risk. Theorems 1–4 can also be used if $\succsim_A$ and $\succsim_B$ belong to different actors. As such, they can be used to compare preferences between agents who hold different beliefs or face different events entirely.

Observation 3 *Theorems 1–4 can be used to compare the utility curvature between different agents, possibly facing different events from different sources of uncertainty.*

Comparing utility curvature when beliefs differ can alternatively be obtained from requirements about endogenous midpoints as shown by Baillon et al. (2012). However, such midpoints are more complex to be observed than our modifications of Yaari's intuitive technique.

The ability to compare uncertainty preferences between agents facing different sources of uncertainty can be of great practical value in cross-cultural research. Consider comparing uncertainty preferences towards the stock market for investors from different countries. One option would be to compare their uncertainty preferences on a single market, such as the Dow Jones. However, such a market will potentially have a different meaning for investors from different countries. It will often be more informative to compare the investors' willingness to take risk on markets that carry similar meaning to them, such as their respective home markets.

5 Conclusion and discussion

We have introduced behavioral definitions of comparative uncertainty aversion for a single agent towards different sources of uncertainty. Our definitions generalize Yaari's (1969) intuitive definition of comparative uncertainty aversion and only depend on the agent's willingness to accept bets under each source. Although they do not require that the agent's choices satisfy subjective expected utility (SEU), our definitions allow for comparative statements regarding the agent's utility curvature towards each source if they do.

Our generalizations also open up new possibilities to investigate ambiguity aversion in Klibanoff et al.'s (2005) smooth ambiguity model, to study the effects of learning and situational factors on uncertainty preferences, and to compare uncertainty preferences between different agents.

Yaari (1969) already wrote that the major use of measures of risk aversion is to facilitate empirical applications. We hope that our definitions will facilitate the growing study of the rich domain of uncertainty.

Appendix A: Proofs

A.1 Proof of Theorem 1

A.1.1 (ii) $\Rightarrow$ (i)

Proof By the definition of "more concave", u_B more concave than u_A implies that there exists a concave function φ such that $u_B = \varphi \circ u_A$. Moreover, φ is strictly increasing because u_A and u_B are strictly increasing.

Consider any $z, x, y \in X$ and events $E \in \Sigma_A$ and $F \in \Sigma_B$. Without loss of generality, we assume $x \geq y$.

We first consider the case $P_A(E) \geq P_B(F)$.

$$
\begin{aligned}
& z \succsim_A x_E y \\
& \quad \Rightarrow u_A(z) \geq P_A(E)u_A(x) + (1 - P_A(E))u_A(y) \\
& \quad \Rightarrow u_A(z) \geq P_B(F)u_A(x) + (1 - P_B(F))u_A(y) \\
& \quad \Rightarrow \varphi(u_A(z)) \geq P_B(F)\varphi(u_A(x)) + (1 - P_B(F))\varphi(u_A(y)) \\
& \qquad \text{because } \varphi \text{ is strictly increasing and concave} \\
& \quad \Rightarrow u_B(z) \geq P_B(F)\, u_B(x) + (1 - P_B(F))\, u_B(y) \\
& \quad \Rightarrow z \succsim_B x_F y
\end{aligned}
$$

The case $P_A(E) < P_B(F)$ can be derived in the same way by starting from $z \succsim_A y_E x$. □

A.1.2 (i) $\Rightarrow$ (ii)

Below, we prove not (ii) $\Rightarrow$ not (i) if there exist $E \in \Sigma_A$ and $F \in \Sigma_B$ such that $P_A(E) = P_B(F) = \frac{1}{2}$.

Proof Remember that u_A and u_B are strictly increasing. We can, therefore, define φ over the image of u_A by $\varphi = u_B \circ u_A^{-1}$. Consequently, φ is also strictly increasing.

Not (ii) $\Rightarrow$ there exists b and c in the image of u_A such that $\varphi(\frac{1}{2}(b+c)) < \frac{1}{2}\varphi(b) + \frac{1}{2}\varphi(c)$.

Let $x, y, z \in X$ be uniquely defined by $u_A(x) = b$, $u_A(y) = c$, $u_A(z) = \frac{b+c}{2}$.

Consider an event $E \in \Sigma_A$ such that $P_A(E) = \frac{1}{2}$ (it must exist according to the richness condition). Consequently, we have $z \sim_A x_E y$ and $z \sim_A y_E x$.

Now, consider event $F \in \Sigma_B$ such that $P_B(F) = \frac{1}{2}$ (it must also exist according to the richness condition).

$$\begin{aligned}
u_B(z) &= \varphi(u_A(z)) = \varphi\left(\frac{1}{2}(b+c)\right) \\
&< \frac{1}{2}\varphi(b) + \frac{1}{2}\varphi(c) = \frac{1}{2}u_B(x) + \frac{1}{2}u_B(y) \\
&= P_B(F)\, u_B(x) + (1 - P_B(F))\, u_B(y) \\
&= (1 - P_B(F))\, u_B(x) + P_B(F)\, u_B(y) \\
&\Rightarrow z \prec_B x_F y \quad \text{and} \quad z \prec_B y_F x.
\end{aligned}$$

□

A.2 Proof of Theorem 2

A.2.1 (ii) $\Rightarrow$ (i)

Proof By the definition of "more concave", u_B more concave than u_A implies that there exists a concave function φ such that $u_B = \varphi \circ u_A$. Moreover, φ is strictly increasing because u_A and u_B are strictly increasing.

Consider any $z, x, y \in X$, $f \in \mathcal{F}_A$, $g \in \mathcal{F}_B$, and events $E, F \in \Sigma_A$ and $G, H \in \Sigma_B$. Without loss of generality, we assume $x \geq y$.

We first consider the case $\frac{P_A(E)}{(P_A(E)+P_A(F))} \geq \frac{P_B(G)}{(P_B(G)+P_B(H))}$.

$$\begin{aligned}
& z_{E\cup F} f \succsim_A x_E y_F f \\
&\Rightarrow u_A(z) \geq \frac{P_A(E)}{(P_A(E) + P_A(F))} u_A(x) + \frac{P_A(F)}{(P_A(E) + P_A(F))} u_A(y) \\
&\Rightarrow u_A(z) \geq \frac{P_B(G)}{(P_B(G) + P_B(H))} u_A(x) + \frac{P_B(H)}{(P_B(G) + P_B(H))} u_A(y) \\
&\Rightarrow \varphi(u_A(z)) \geq \frac{P_B(G)}{(P_B(G) + P_B(H))} \varphi(u_A(x)) + \frac{P_B(H)}{(P_B(G) + P_B(H))} \varphi(u_A(y))
\end{aligned}$$

because φ is strictly increasing and concave

$$\Rightarrow u_B(z) \geq \frac{P_B(G)}{(P_B(G)+P_B(H))} u_B(x) + \frac{P_B(H)}{(P_B(G)+P_B(H))} u_B(y)$$
$$\Rightarrow z_{G \cup H} g \succsim_B x_G y_H g$$

The case $\frac{P_A(E)}{(P_A(E)+P_A(F))} < \frac{P_B(G)}{(P_B(G)+P_B(H))}$, can be derived in the same way by starting from $z_{E \cup F} f \succsim_A y_E x_F f$. □

A.2.2 (i) $\Rightarrow$ (ii)

Below, we prove not (ii) $\Rightarrow$ not (i) if there exist $E, F \in \Sigma_A$ and $G, H \in \Sigma_B$, such that $P_A(E) = P_A(F)$ and $P_B(G) = P_B(H)$.

Proof Remember that u_A and u_B are strictly increasing. We can, therefore, define φ over the image of u_A by $\varphi = u_B \circ u_A^{-1}$. Consequently, φ is also strictly increasing.

Not (ii) $\Rightarrow$ there exists b, c in the image of u_A such that $\varphi\left(\frac{1}{2}(b+c)\right) < \frac{1}{2}\varphi(b) + \frac{1}{2}\varphi(c)$.

Let outcomes $x, y, z \in X$ be uniquely defined by $u_A(x) = b$, $u_A(y) = c$, $u_A(z) = \frac{b+c}{2}$.

Consider events $E, F \in \Sigma_A$ such that $P_A(E) = P_A(F)$ (they must exist according to the richness condition) and any act $f \in \mathcal{F}_A$. Consequently, we have $z_{E \cup F} f \sim_A x_E y_F f$ and $z_{E \cup F} f \sim_A y_E x_F f$.

Now consider events $G, H \in \Sigma_B$ such that $P_B(G) = P_B(H)$ (they must also exist according to the richness condition) and any act $g \in \mathcal{F}_B$.

$$\begin{aligned} u_B(z) &= \varphi(u_A(z)) = \varphi\left(\frac{1}{2}(b+c)\right) \\ &< \frac{1}{2}\varphi(b) + \frac{1}{2}\varphi(c) = \frac{1}{2}u_B(x) + \frac{1}{2}u_B(y) \\ &\Rightarrow (P_B(G)+P_B(H))\, u_B(z) < P_B(G)\, u_B(x) + P_B(H)\, u_B(y) \\ &\quad \text{and } (P_B(H)+P_B(G))\, u_B(z) < P_B(H)\, u_B(x) + P_B(G)\, u_B(y) \\ &\Rightarrow z_{G \cup H} g \prec_B x_G y_H g \quad \text{and} \quad z_{G \cup H} g \prec_B y_G x_H g. \end{aligned}$$

□

A.3 Proof of Theorem 3

A.3.1 (ii) $\Rightarrow$ (i)

Proof By the definition of "more concave", u_B more concave than u_A implies that there exists a concave function φ such that $u_B = \varphi \circ u_A$. Moreover, φ is strictly increasing because u_A and u_B are strictly increasing.

Consider any $f = (E_1 : x_1, \ldots, E_n : x_n) \in \mathcal{F}_A$ and $z \in X$.

$$
\begin{aligned}
& z \succsim_A \Pi(f) \quad \forall \Pi \\
& \Rightarrow \\
& u_A(z) \geq P_A(E_1)\, u_A(x_1) + P_A(E_2)\, u_A(x_2) + P_A(E_3)\, u_A(x_3) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_n) \\
& \text{and } u_A(z) \geq P_A(E_1)\, u_A(x_2) + P_A(E_2)\, u_A(x_1) + P_A(E_3)\, u_A(x_3) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_n) \\
& \vdots \\
& \text{and } u_A(z) \geq P_A(E_1)\, u_A(x_n) + P_A(E_2)\, u_A(x_{n-1}) + P_A(E_3)\, u_A(x_{n-2}) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_1)
\end{aligned}
$$

In total, there are $n!$ inequalities, and each outcome has been assigned to each event $(n-1)!$ times.

Summing up these $n!$ inequalities and dividing by $n!$, we have:

$$
\begin{aligned}
& u_A(z) \geq \frac{1}{n} u_A(x_1) + \frac{1}{n} u_A(x_2) + \cdots + \frac{1}{n} u_A(x_n) \\
& \Rightarrow \varphi(u_A(z)) \geq \frac{1}{n} \varphi(u_A(x_1)) + \cdots + \frac{1}{n} \varphi(u_A(x_n)) \text{ because } \varphi \text{ is strictly increasing and concave.} \\
& \Rightarrow u_B(z) \geq \frac{1}{n} u_B(x_1) + \cdots + \frac{1}{n} u_B(x_n).
\end{aligned}
$$

This inequality implies that there must exist a Π such that $z \succsim_B \Pi(g)$ for any $g = (F_1 : x_1, \ldots, F_n : x_n) \in \mathcal{F}_B$ whose outcomes are the same as those of f because otherwise:

$$
\begin{aligned}
& z \prec_B \Pi(g) \quad \forall \Pi \\
& \Rightarrow \\
& u_B(z) < P_B(F_1)\, u_B(x_1) + P_B(F_2)\, u_B(x_2) + P_B(F_3)\, u_B(x_3) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_n) \\
& \text{and } u_B(z) < P_B(F_1)\, u_B(x_2) + P_B(F_2)\, u_B(x_1) + P_B(F_3)\, u_B(x_3) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_n) \\
& \vdots \\
& \text{and } u_B(z) < P_B(F_1)\, u_B(x_n) + P_B(F_2)\, u_B(x_{n-1}) + P_B(F_3)\, u_B(x_{n-2}) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_1).
\end{aligned}
$$

Summing these $n!$ inequalities, and dividing them by $n!$ implies

$$u_B(z) < \frac{1}{n} u_B(x_1) + \cdots + \frac{1}{n} u_B(x_n).$$

□

A.3.2 (i) $\Rightarrow$ (ii)

Below, we prove not (ii) $\Rightarrow$ not (i) if there exist, for some integer n, an n-fold partitions $\{E_i\}_n$ of S_A and an n-fold partition $\{F_i\}_n$ of S_B, such that $P_A(E_i) = P_B(F_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$.

Proof Remember that u_A and u_B are strictly increasing. We can, therefore, define φ over the image of u_A by $\varphi = u_B \circ u_A^{-1}$. Consequently, φ is also strictly increasing.

Not (ii) $\Rightarrow$ there exists $u_1, \ldots, u_n$ belonging to the domain of u_A such that

$$\varphi\left(\frac{1}{n}(u_1 + \cdots + u_n)\right) < \frac{1}{n}\varphi(u_1) + \cdots + \frac{1}{n}\varphi(u_n)$$

The outcomes $x_1, \ldots, x_n$ are uniquely defined by $u_A(x_1) = u_1, \ldots, u_A(x_n) = u_n$ and z is defined by $u_A(z) = \frac{1}{n}(u_1 + \cdots + u_n)$.

Consider a partition $\{E_i\}_n$ of S_A such that $P_A(E_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$ (which is assumed to exist), the act $f \in \mathcal{F}_A$ assigning $(x_1, \ldots, x_n)$ to $(E_1, \ldots, E_n)$ and all its permutations $\Pi(f)$. The equality $u_A(z) = \frac{1}{n}u_1 + \cdots + \frac{1}{n}u_n$ implies $z \sim_A \Pi(f)$ for all Π.

Yet,

$$\begin{aligned} u_B(z) &= \varphi(u_A(z)) = \varphi\left(\frac{1}{n}(u_1 + \cdots + u_n)\right) \\ &< \frac{1}{n}\varphi(u_1) + \cdots + \frac{1}{n}\varphi((u_n)) = \frac{1}{n}u_B(x_1) + \cdots + \frac{1}{n}u_B(x_n). \end{aligned}$$

Consider a partition $\{F_i\}_n$ of S_B such that $P_B(F_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$ (which is also assumed to exist), the act $g \in \mathcal{F}_B$ assigning $(x_1, \ldots, x_n)$ to $(F_1, \ldots, F_n)$ and all its permutations $\Pi(g)$. The inequality $u_B(z) < \frac{1}{n}u_B(x_1) + \cdots + \frac{1}{n}u_B(x_n)$ implies $z \prec_B \Pi(g)$ for all Π, and, hence, not (i). □

A.4 Proof of Theorem 4

A.4.1 (ii) $\Rightarrow$ (i)

Proof By the definition of "more concave", u_B more concave than u_A implies that there exists a concave function φ such that $u_B = \varphi \circ u_A$. Moreover, φ is strictly increasing because u_A and u_B are strictly increasing.

Consider any $f = (E_1 : x_1, \ldots, E_n : x_n) \in \mathcal{F}_A$ and $z \in X$.

$$
\begin{aligned}
& z \succsim_A \pi^m(f) \quad \forall m \\
& \Rightarrow \\
& u_A(z) \geq P_A(E_1)\, u_A(x_1) + P_A(E_2)\, u_A(x_2) + P_A(E_3)\, u_A(x_3) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_n) \\
& \text{and } u_A(z) \geq P_A(E_1)\, u_A(x_2) + P_A(E_2)\, u_A(x_3) + P_A(E_3)\, u_A(x_4) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_1) \\
& \vdots \\
& \text{and } u_A(z) \geq P_A(E_1)\, u_A(x_n) + P_A(E_2)\, u_A(x_1) + P_A(E_3)\, u_A(x_2) + \cdots \\
& \quad + P_A(E_n)\, u_A(x_{n-1})
\end{aligned}
$$

In total, there are n inequalities, and each outcome has been assigned to each event a single time.

Summing up these n inequalities and dividing by n, we have:

$$
\begin{aligned}
& u_A(z) \geq \frac{1}{n} u_A(x_1) + \frac{1}{n} u_A(x_2) + \cdots + \frac{1}{n} u_A(x_n) \\
& \Rightarrow \varphi(u_A(z)) \geq \frac{1}{n} \varphi(u_A(x_1)) + \cdots + \frac{1}{n} \varphi(u_A(x_n)) \text{ because } \varphi \text{ is strictly increasing and concave.} \\
& \Rightarrow u_B(z) \geq \frac{1}{n} u_B(x_1) + \cdots + \frac{1}{n} u_B(x_n).
\end{aligned}
$$

This inequality implies that there must exist an $m \in \{1, \ldots, n\}$ such that $z \succsim_B \pi^m(g)$ for any $g = (F_1 : x_1, \ldots, F_n : x_n) \in \mathcal{F}_B$ whose outcomes are the same as those of f because otherwise:

$$
\begin{aligned}
& z \prec_B \pi^m(g) \quad \forall m \\
& \Rightarrow \\
& u_B(z) < P_B(F_1)\, u_B(x_1) + P_B(F_2)\, u_B(x_2) + P_B(F_3)\, u_B(x_3) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_n) \\
& \text{and } u_B(z) < P_B(F_1)\, u_B(x_2) + P_B(F_2)\, u_B(x_3) + P_B(F_3)\, u_B(x_4) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_1) \\
& \vdots \\
& \text{and } u_B(z) < P_B(F_1)\, u_B(x_n) + P_B(F_2)\, u_B(x_1) + P_B(F_3)\, u_B(x_2) + \cdots \\
& \quad + P_B(F_n)\, u_B(x_{n-1}).
\end{aligned}
$$

Summing these n inequalities, and dividing them by n implies

$$
u_B(z) < \frac{1}{n} u_B(x_1) + \cdots + \frac{1}{n} u_B(x_n).
$$

□

A.4.2 (i) $\Rightarrow$ (ii)

Below, we prove not (ii) $\Rightarrow$ not (i) if there exist, for some integer n, an n-fold partitions $\{E_i\}_n$ of S_A and an n-fold partition $\{F_i\}_n$ of S_B, such that $P_A(E_i) = P_B(F_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$.

Proof Remember that u_A and u_B are strictly increasing. We can, therefore, define φ over the image of u_A by $\varphi = u_B \circ u_A^{-1}$. Consequently, φ is also strictly increasing.

Not (ii) $\Rightarrow$ there exists $u_1, \ldots, u_n$ belonging to the domain of u_A such that

$$\varphi\left(\frac{1}{n}(u_1 + \cdots + u_n)\right) < \frac{1}{n}\varphi(u_1) + \cdots + \frac{1}{n}\varphi(u_n)$$

The outcomes $x_1, \ldots, x_n$ are uniquely defined by $u_A(x_1) = u_1, \ldots, u_A(x_n) = u_n$ and z is defined by $u_A(z) = \frac{1}{n}(u_1 + \cdots + u_n)$.

Consider a partition $\{E_i\}_n$ of S_A such that $P_A(E_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$ (which is assumed to exist), and the act $f \in \mathcal{F}_A$ assigning $(x_1, \ldots, x_n)$ to $(E_1, \ldots, E_n)$. The equality $u_A(z) = \frac{1}{n}u_1 + \cdots + \frac{1}{n}u_n$ implies $z \sim_A \pi^m(f)$ for all m.

Yet,

$$\begin{aligned} u_B(z) = \varphi(u_A(z)) &= \varphi\left(\frac{1}{n}(u_1 + \cdots + u_n)\right) \\ &< \frac{1}{n}\varphi(u_1) + \cdots + \frac{1}{n}\varphi(u_n) = \frac{1}{n}u_B(x_1) + \cdots + \frac{1}{n}u_B(x_n). \end{aligned}$$

Consider a partition $\{F_i\}_n$ of S_B such that $P_B(F_i) = \frac{1}{n}$ for all $i \in \{1, \ldots, n\}$ (which is also assumed to exist) and the act $g \in \mathcal{F}_B$ assigning $(x_1, \ldots, x_n)$ to $(F_1, \ldots, F_n)$. The inequality $u_B(z) < \frac{1}{n}u_B(x_1) + \cdots + \frac{1}{n}u_B(x_n)$ implies $z \prec_B \pi^m(g)$ for all m, and, hence, not (i). $\square$

A.5 Proof of Theorem 5

A.5.1 (ii) $\Rightarrow$ (i)

Proof Consider any $z, x, y \in X$, probability p, and event $F \in \Sigma$. Without loss of generality, we assume $x \geq y$.

We first consider the case $p \geq \int_\Delta P(F)\mathrm{d}\mu$.

$$\begin{aligned} & z \succsim x_p y \\ & \Rightarrow u(z) \geq pu(x) + (1-p)u(y) \\ & \Rightarrow u(z) \geq \left(\int_\Delta P(F)\mathrm{d}\mu\right)u(x) + \left(1 - \int_\Delta P(F)\mathrm{d}\mu\right)u(y) \\ & \Rightarrow u(z) \geq \int_\Delta (P(F)u(x) + (1-P(F))u(y))\,\mathrm{d}\mu \\ & \Rightarrow \varphi(u(z)) \geq \int_\Delta \varphi(P(F)u(x) + (1-P(F))u(y))\,\mathrm{d}\mu \end{aligned}$$

$$\begin{aligned} &\text{because } \varphi \text{ is strictly increasing and concave} \\ &\Rightarrow z \succsim x_F y \end{aligned}$$

The case $p \leq \int_\Delta P(F)\mathrm{d}\mu$ can be derived in the same way by starting from $z \succsim y_p x$. □

A.5.2 (i) $\Rightarrow$ (ii)

Below, we prove not (ii) $\Rightarrow$ not (i) if there exist $E \in \Sigma$ such that $\int_\Delta P(E)\mathrm{d}\mu = \frac{1}{2}$.

Proof Not (ii) $\Rightarrow$ there exists an interval $[b, c]$ in the image of u on which φ is convex. Let $x, y, z \in X$ be uniquely defined by $u(x) = b$, $u(y) = c$, $u(z) = \frac{b+c}{2}$.

Consequently, we have $z \sim x_{\frac{1}{2}} y$ and $z \sim y_{\frac{1}{2}} x$.

Now consider event E such that $\int_\Delta P(E)\mathrm{d}\mu = \frac{1}{2}$ (it must exist according to the richness condition).

$$\begin{aligned} \varphi(u(z)) &= \varphi\left(\frac{1}{2}(b+c)\right) \\ &< \frac{1}{2}\varphi(b) + \frac{1}{2}\varphi(c) \\ &= \left(\int_\Delta P(E)\mathrm{d}\mu\right)\varphi(b) + \left(1 - \int_\Delta P(E)\mathrm{d}\mu\right)\varphi(c) \\ &= \int_\Delta (P(E)\varphi(b) + (1 - P(E))\varphi(c))\,\mathrm{d}\mu \\ &< \int_\Delta \varphi(P(E)u(x) + (1 - P(E))\,u(z))\,\mathrm{d}\mu \\ &\Rightarrow z \prec x_E y. \end{aligned}$$

Similarly, using the same steps with $\int_\Delta P(E)\mathrm{d}\mu = \frac{1}{2}$,

$$\begin{aligned} \varphi(u(z)) &< \int_\Delta (P(E^c)\varphi(b) + (1 - P(E^c))\varphi(c))\mathrm{d}\mu \\ &< \int_\Delta \varphi(P(E^c)u(x)(1 - P(E^c))u(z))\mathrm{d}\mu \\ &\Rightarrow z \prec y_E x. \end{aligned}$$

□

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