Comparativism with Mixed Relations

David John Baker
djbaker@umich.edu

April 9, 2013

Abstract

Comparativism–the view that comparative relations like mass ratios are fundamental and intrinsic values of quantities are not–faces a challenge from physics. In its standard form, comparativism predicts indeterminism in physical theories that are ordinarily understood as deterministic. I explore an option for saving comparativism from this objection: the introduction of "mixed" relations that compare values of unlike quantities. Although tenable, this revised version of comparativism lacks some of the theoretical virtues of the standard version.

IMPORTANT NOTE: This essay is not up to date on the literature, and in particular Dasgupta's views are not portrayed with sufficient accuracy. But I think the positive arguments are correct and important, and I have too many other things on my plate to revise it in a timely manner, so I'm making it publicly available for now. Permission is granted to cite *only the discussion of my own views,* not the discussion of other authors.

1 Introduction

The metaphysics of quantity is a place of direct and obvious contact between physics and fundamental metaphysics. On the one hand, absolutist theories of quantity–according to which absolute values of quantities (like the property of having a mass of five grams) are fundamental–correspond closely to the language in which physics is normally written. On the other hand, comparativist theories–in which relationships like the ratio of the masses of two objects are fundamental–appear to exceed absolutist theories on an important metric of scientific success: ontological parsimony. Comparativist theories draw fewer unobservable distinctions between possibilities, which would seem to earn them Occam's blessing.

But comparativist theories also face a significant challenge from physics. Some significant physical theories, described in ordinary absolutist terms, appear able to "tell the difference" between initial states of the world which any comparativist would count as identical. These identical (for the comparativist) initial states will evolve into different (even for the comparativist) final states. So the comparativist seems committed to the claim that these theories are indeterministic, although they do not assign objective chances to the different physical possibilities. And as a further–perhaps even worse–consequence of this indeterminism, the comparativist must understand these examples as involving a sort of temporal action-at-a-distance.

These problems can be traced to a commitment of extant comparativist theories: their scale-independence. These theories identify possibilities related by the doubling (or tripling, or multiplication by any constant) of the quantities' values. This commitment can be scaled back, however, while retaining at least some of the core of comparativism, if fundamental *mixed relations* are introduced.

By a mixed relation, I mean a relation that compares objects' values for distinct quantities. A relation like "more massive than" encodes a comparison between two objects' values of the same quantity. When I note that my brother is more massive than I am, I'm comparing his mass with mine – comparing our respective values of the same quantity. A mixed relation instead compares values of two different quantities. For example, a possible mixed relation between me and my brother might compare my mass with his height. Although mixed relations may sound unnatural, there is no problem making formal sense of them. But comparativists have generally avoided positing fundamental mixed relations.

I'd like to explore the reasons for departing from this standard comparativist picture by introducing mixed relations, and the implications of doing so. Mixed relations are certainly less intuitively familiar than un-mixed relations. I imagine this will strike some as extremely problematic, and others as not at all problematic. It does make the comparativist's interpretation of physics holistic in a way that the absolutist's is not. While the absolutist may consistently hold that the only metaphysically distinguished relationships between masses and distances are given by the laws of nature, the comparativist must include such relationships in his fundamental ontology of particular, occurrent facts. Metaphysical novelty of this sort is preferable to physical pathology, which is the price of standard comparativism. So it seems to me that comparativists should believe in mixed relations. Unfortunately, comparativism with mixed relations fares worse than standard comparativism on Occamist grounds, undermining the best arguments for comparativism at least to some extent.

We should begin by establishing why standard comparativism (with no mixed relations) is a popular, natural and appealing form for a comparativist theory of quantity.

2 Standard comparativism

Besides the notion that comparative relations and not absolute values are fundamental, comparativists of all stripes share a commitment to what I'll call *scale independence.* A comparativist theory is scale-independent if multiplying the values of a quantity (when it's represented numerically) can't possibly change any of the fundamental relations. Of course it is trivial that we can change the numerical representation of a quantity without changing anything fundamental, even under absolutism, if we change which units we're using to describe the quantity. Switching from units of kilograms to units of grams will multiply the numerical value of every object's mass by 1,000. But comparativist scale independence is something more than this. On extant comparativist theories, a possibility in which the mass of every object, measured in kilograms, is twice its actual value is not fundamentally distinct from the actual world.

Two broad varieties of comparativism have been proposed, which we may call ratio-based and congruence-based comparativism. The former variety has been advanced by Dasgupta (2013) and Bigelow and Pargetter (1988). It is a Platonist approach according to which the fundamental relations involve numbers, for example:

Rxyn**.** For objects x and y and real number n, $Mxyn$ iff x is n times as massive as y.

In congruence-based comparativism, on the other hand, the fundamental relations only relate concrete physical objects. One example is:

C$wxyz$**.** The difference in mass between w and x is the same as the difference in mass between y and z.

This form of comparativism has been proposed by Field (1980) as a means of formulating his nominalist version of Newtonian gravitation.

The thing to notice about these proposed fundamental relations is that they're all scale-independent. Take a comparativist possible world and assign numerical values to the mass quantity in a way consistent with all the fundamental relations. (In measurement theory this assignment is called a *homomorphism* for mass.) Then double or triple the numerical mass assigned to every object. This operation will change nothing about the fundamental relations, on either ratio-based or congruence-based comparativism. Since all extant comparativist theories share this feature, let's use the label *standard comparativism* for comparativism with scale-independent fundamental relations.

Scale independence is the basis for the most promising direct argument in comparativism's favor. This case is made most clearly by Dasgupta (2013), who argues that comparativism enjoys a marked advantage by Occamist lights, due to its more parsimonious space of possibilities. In the form Dasgupta appeals to, Occam's razor prefers theories with less unobservable structure. We are to prefer theories whose fundamental ontology can "tell the difference" between two or more putative possibilities with no scientifically detectable difference. The fewer undetectable differences between distinct possible worlds, the better the theory on Occamist grounds.

An analogy with the theoretical advantages of special relativity is helpful here. In Lorentz's ether theory, the rest frame of the ether is undetectable–for all we can know, it could be any inertial frame whatever. By denying the existence of a preferred frame, special relativity dispenses with this unobservable "surplus structure." On Dasgupta's view, the absolutist's ontology of intrinsic values for quantities is a similar sort of surplus structure, and (standard) comparativism is to be preferred over absolutism because it dispenses with the extra structure.[1]

Dasgupta's argument would fail to convince if comparativist theories also exhibit unique disadvantages which threaten counterbalance their Occamist advantages. I will now show that under standard comparativism, these extenuating circumstances obtain. In certain examples, standard comparativist laws for Newtonian physics exhibit troubling non-probabilistic

[1]This argument makes it a little surprising that Dasgupta is apparently in favor of positing fundamental mass relations between objects at different times (Dasgupta, 2013, 8), since these are presumably just as undetectable as absolute values.

indeterminism and temporal action-at-a-distance.

3 Indeterminism and temporal non-locality

Let's take a more detailed look at Dasgupta's argument for the Occamist virtues of comparativism, including the undetectability of absolute mass values. I will follow Dasgupta in restricting attention to the most basic example: Newton's laws of motion. But it is worth mentioning that the problems for standard comparativism I raise here have direct analogues in more realistic classical theories like Newtonian gravity and relativistic electromagnetism (Baker, 2020).

Dasgupta proceeds by offering an alternative to Newton's Second Law, $F = ma$, which could be true of comparativist worlds as well as absolutist ones. His suggestion, formulated in terms of ratio-based comparativism, is:

(L2) For any material things x and y,

(a) For any reals r_1 and r_2, if x is r_1 times as massive as y and is accelerating r_2 times the rate of y, then x has $r_1 r_2$ times as much force acting on it than y.

(b) For any real r_3, if x has r_3 times as much force acting on it than y, then there are reals r_4 and r_5 such that $r_4 r_5 = r_3$, and such that x is r_4 times as massive as y and is accelerating r_5 times the rate of y. (Dasgupta, 2013, 18-19)[2]

Dasgupta then argues that, since (L2) is logically weaker than $F = ma$, any evidence that confirms $F = ma$ also confirms (L2) (Dasgupta, 2013, 21). This provides the basis for his argument that absolute values of mass are undetectable. Suppose for the moment that (L2), and not Newton's Second Law, exhausts the laws of motion for our world. Then there is a mass-doubled duplicate of our world with the same laws. Any putative "mass detector" in our world has a counterpart in the mass-doubled world which displays the same values for every object it examines. So for example, if the mass detector in my world says "1 kg" when I put a one-kilogram object in it, the mass detector in the duplicate world says "1 kg"

[2]Note that additional direction relations between forces and accelerations will be needed in more than one dimension; Dasgupta ignores these for brevity.

when a two-kilogram object is inserted. Since the mass-doubled world is physically possible according to (L2), the reliability of the mass detector in our world is simply a coincidence. And since we have no way of confirming $F = ma$ over (L2), the object's mass is not detectable by us even using the detector.[3]

It is then a minor step to conclude that (standard) comparativism is favored over absolutism by Occam's razor. The absolutist posits undetectable differences between physically possible worlds, while the comparativist does not. If one is serious about accepting simpler theories, this argument is quite compelling. But it does assume that there are no good theoretical reasons to reject (L2) in favor of $F = ma$. If there were, it wouldn't matter that the two laws are equally well confirmed by all possible experiments.

In fact, there are a couple of important theoretical virtues of $F = ma$ which (L2) does not share. We don't like our physical laws to exhibit indeterminism without at least assigning some probabilities to the different possible outcomes of an indeterministic event. That sort of behavior seems pathological–for example, it's been seen as a reason to reject the view that the electromagnetic potential is a fundamental quantity (Belot, 1998). We also like our laws to exhibit temporal locality (Lange, 2002, 7-17). If events in the past are allowed to cause or necessitate future events without said causation or necessitation being mediated by the present state of the world, this also appears pathological. A simple example will suffice to show that (L2) is pathological in both of these ways.

3.1 Friction World

The example I have in mind is Friction World (Fig. 1). A projectile of mass m is initially moving toward the left of the diagram with velocity v. Once it enters the shaded region (of width L), it will be subject to a force toward the right of the diagram until it stops moving or leaves the region to the left. We may imagine a projectile initially moving in a vacuum, then entering a liquid-filled region and being slowed by friction with the liquid.

There are two broad classes of possibilities for Friction World's future evolution. If it's

[3] One might object that the mass detector in the doubled world is not really the same device as the one in the actual world, since its mass is after all doubled along with everything else's. Dasgupta might reply that we still cannot use the detector to reliably detect mass, since the objection simply shows that we have no way of telling whether our detectors are reliable or not. I'm sure sure Dasgupta is right that what matters for scientific detectability is detectability by us using our devices, as opposed to detectability by our devices themselves (Dasgupta, 2013, 26), but I won't pursue this point further.

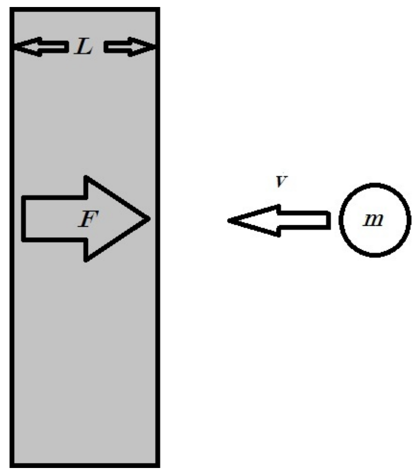

Figure 1: Friction World

massive enough and moving fast enough, the projectile will eventually make it through the shaded area and continue moving to the left forever at a reduced velocity. Otherwise it will come to a stop somewhere within the shaded region. If $F = ma$ is our law of motion, it is straightforward to determine which of these results will occur. Assume for the moment that the projectile will stop. This will take an amount of time $t = v/a$. Since its acceleration inside the shaded region is a constant $a = F/m$, its average velocity inside the region will be half its initial velocity, $v_{avg} = v/2$. The total distance it covers will be $v_{avg} * t = v^2/2a$. Thus it *would* have made it past the region if L were less than this distance. So (dropping our initial assumption) the projectile will make it past the region iff

$$\frac{v^2}{2a} = \frac{v^2}{2(F/m)} > L. \tag{1}$$

Thus Friction World's initial conditions–the values of v, F, m, L–determine whether the projectile will make it.

What about if (L2) is the law of motion, not $F = ma$? For the moment we'll assume absolutism is true (remember, this is compatible with (L2)). Since there is only one object in Friction World that undergoes a force, (L2) is trivially satisfied no matter what a is.[4]

[4]Ideally, perhaps we should consider the shaded region as a massive object that feels an equal and opposite force $-F$. This will not restore determinism, however. The initial conditions will now tell us the ratio of the shaded region's acceleration to the projectile's, but they will not determine the magnitude of these

As before, the projectile will make it past iff $v^2/2a > L$, but this time $a = F/m$ is not guaranteed by the law, which tells us nothing about the value of a. So the initial conditions do not determine whether the projectile will make it past. Unlike $F = ma$, (L2) makes Friction World indeterministic!

The same goes for a standard comparativist world governed by (L2). In this case, the initial conditions will consist of some scale-independent relations which hold at the initial time–between L and the projectile's initial distance from the region, perhaps between the projectile's mass and that of the region if we consider it to be massive, etc.[5] The important thing to note is that all these relations will be scale-independent. This means they will be unchanged by doubling the value of m (along with all the other masses in the universe, if we suppose there are any). But as Inequality (1) clearly shows, doubling m can change whether the projectile will make it past the region. For suppose v is just barely too small for the projectile to make it past. Then doubling m will reduce a by half, doubling the total distance the projectile can cover and allowing it to escape the region. So the scale-independent facts about the initial conditions do not determine whether the projectile will make it.

Moreover, the comparativist cannot protest that the difference between these possible outcomes is only intelligible on absolutist terms. There are scale-independent differences between a world in which the projectile stops within the region and a world in which it continues on to the left. For one thing, it is a scale-independent fact whether the projectile is inside the region or not. In the case where it stops, it will remain within the region forever. In the case where it escapes, it will not. On either absolutism or comparativism, (L2) makes these facts indeterministic.

Interestingly, this indeterminism "goes away" the moment the projectile enters the shaded region. Consider Inequality (1), $v^2/2a > L$. If this holds, the projectile will make it through; if not, it won't. Whether this inequality holds is a scale-independent fact. For when we transform distance by doubling it, for example, we must also double velocity and acceleration, since they are just derivatives of position. So when we double all distances, we must multiply both sides of the inequality by 2. Since multiplying the relevant quantities by a constant doesn't change whether the inequality holds, it is scale-independent and will be de-

accelerations. And we need to know the magnitude of a to determine whether the projectile will stop.

[5]Depending on our theory of velocity, we may want to consider the initial conditions to be the relations between objects in an infinitesimal temporal neighborhood of the initial time (Baker, 2020).

terminable from a sufficiently rich system of comparativist relations. So while the projectile is accelerating, its motion is deterministic even under comparativism.

This might seem like a good thing, but in fact it leads to the other pathology I mentioned: temporal action-at-a-distance. Suppose we modify the example as follows: split the shaded region of force into two regions, each of length $L/2$. Now put some empty space between the two of them. The projectile will either stop in the first region, pass through the distance between them and stop in the second region, or go on forever after passing through both regions. If we ask whether the projectile will stop in one of the regions, again the answer is that it will iff Inequality (1) is not satisfied. As we've seen, this is not determined by the scale-independent facts before the projectile enters the first region, but it is determined by the scale-independent facts while the projectile is in the first region. But what about after the projectile gets past the first region (assuming it does) and enters the empty space between the two regions? Once again, its acceleration will be zero, so the scale-independent facts about its present state will not determine whether it will make it past the second region. The indeterminism has returned.

But while the instantaneous state during the projectile's passage between the regions will not determine its future, the state of the entire past will. For as we saw before, the moment the projectile enters the first region, the scale-independent facts determine whether Inequality (1) is satisfied, and hence whether it will make it past both regions. So its past motion through the first region will necessitate its future motion in the second region, even though its present instantaneous state cannot. In this sense, the past state of this modified Friction World is able to determine its future state without this determination being mediated by its present state. So we have a surprising example of temporal action-at-a-distance.

This all makes (L2) look quite deviant by comparison with $F = ma$. First, we have a case of indeterminism in which the theory doesn't even assign objective probabilities to the different possible outcomes. The law simply tells us that multiple future states are possible, given the present state of affairs. Second, this indeterminism can be made to disappear and reappear if we doctor the example. Finally, as a consequence of this second pathology, the chain of cause and effect is no longer required to pass from the past through the present to the future. Instead (L2) allows the past to necessitate the future without the present state mediating that necessitation. Although (L2) and $F = ma$ are empirically indistinguishable, $F = ma$ would appear to be a much more acceptable law on theoretical grounds.

These problems are not unique to (L2). They arise for any standard comparativist interpretation of Newtonian gravity or classical electromagnetism, as a consequence of scale independence (Baker, 2020). But if we abandon scale independence, it is possible to avoid problems with determinism while retaining much of the core notion of comparativism. The natural way to accomplish this is with fundamental mixed relations.

4 Introducing mixed relations

To save determinism and temporal locality for the comparativist, we will have to relax the requirement of scale independence. There is a danger of relaxing it too far, and thereby recognizing as much undetectable structure as the absolutist. For example, one way to fix the problem would be to posit fundamental relations like

Knab**.** a is n kilograms more massive than b.

But then the comparativist will recognize as many distinct, undetectably different possibilities as the absolutist (assuming, as is natural, that zero is a physically privileged value of mass). Mixed relations, on the other hand, promise to restore determinism and temporal locality while still permitting the comparativist to count certain undetectable absolute differences as fundamentally unreal.

How would this work in an example like Friction World? As we saw earlier, via Inequality (1), the ratio $\frac{v^2m}{2FL}$ will determine the behavior that is not determined by the scale-independent initial conditions of Friction World. So, while there may be simpler relations that will do the trick, introducing the following fundamental mixed relation will permit a deterministic interpretation of the example:

Rxyn**.** The ratio $\frac{v^2m}{2FL} = n$, where v is x's velocity, m is x's mass, F is the force exerted by y and L is the length of y along the direction parallel to x's velocity (with all dimensionful quantities measured in SI units).

For purposes of brevity I've gone with a ratio-based, Platonist relation rather than a congruence-based one, but both are clearly possible.

Crucially, this relation will not be preserved by the doubling of force or mass. It will be preserved by the doubling of all the quantities, but so will escape velocity. So adding this sort of mixed relation allows the comparativist to reproduce the predictions of ordinary Newtonian mechanics within a comparativist interpretation, while allowing fewer possibilities than the absolutist (and hence staying true to the spirit of comparativism in at least one respect).

On the other hand, mixed comparative relations are less familiar than the standard comparativist's relations. It may be harder to grasp how they could constitute the basic furniture of the world. This may seem to threaten the ontological intelligibility of comparativism. The mixed relations may not seem like the sort of property that could fundamentally constitute a world like ours. The standard comparativist ontology consists of relations that seem fairly natural, compared with mixed relations. "This dumbbell is ten times more massive than that one" seems a much better candidate for a fundamental fact than the ratio involving velocity, mass, force and distance.

To some extent, this is an objection that even the standard comparativist must confront. After all, the common-sense understanding of quantity has it that the mass ratios (or other comparative relations) between massive objects hold because of those objects' intrinsic masses. Any comparativist metaphysics faces this seeming drawback. This makes comparativism most natural within an overall framework that gives little or no evidential weight to pre-theoretic intuitions. For example, this is the position of Dasgupta (2013, see esp. 5-6).

Whether intuitions have evidential weight or not, some candidate pictures of fundamental reality just don't make sense. For example, a modern-day Heraclitus who insisted that the universe is fundamentally made up of flames would not be advancing a coherent ontology for a world like ours. Could a similar criticism be levied against the comparativist who posits fundamental mixed relations? It may seem so, especially if the mixed relations are thought of as analogous to the relations that hold between masses and masses. For example, it sounds like nonsense to say that a planet is "twice as wide as it is massive."

But it should be clear that our definition of $\mathbf{R}xyn$ does not express this sort of bizarre comparison. Instead it expresses exactly the sort of ratio that we use in ordinary applications of physics: a ratio of the numbers assigned to quantities when those quantities are given in a particular system of units. To extend our analogy with the case of just length and mass, while it may sound unintelligible to claim that an object is "twice as wide as it is massive," it is certainly intelligible to suppose that its width in meters is twice its mass in kilograms. And the ratio that exists when $\mathbf{R}xyn$ holds is a ratio of this latter sort. It may be counterintuitive to say that the world is fundamentally made up of complicated constructions like $\mathbf{R}xyn$, rather than good old familiar values of mass and distance. But there's nothing incoherent about that position.

I don't see much of an argument against comparativism from the unfamiliar nature of fundamental mixed relations like $\mathbf{R}xyn$, unless one is very concerned with preserving our intuitions about what sort of facts can ground fundamental quantities. Since this sort of intuition counts against comparativism in general, this is hardly a new or surprising theoretical cost of the view. We have abandoned one of standard comparativism's tenets, namely the commitment to scale independence. But a system of mixed relations clearly achieves at least one of the goals of ordinary standard comparativism: it makes do with worlds that have less structure than absolutist possible worlds. The comparativist who accepts fundamental mixed relations still recognizes fewer distinctions between possibilities than the absolutist does.

That said, there are a couple of other ways for the comparativist to accommodate Friction World-style examples. As I'll argue in the next section, though, neither one is as promising as the mixed-relation approach.

5 Alternatives to mixed relations

I see two potential options for the standard comparativist who wants to restore determinism to Friction World and similar cases without resort to mixed relations. The first is to posit a concrete object which takes on a privileged role in the laws of nature. This option is suggested by an analogy with the treatment of handedness in spacetime theories. The second option is to deny that the relevant laws of physics govern fundamental properties. This one fits naturally with Dasgupta's picture of comparativist modality.

The idea of positing a concrete object to solve the comparativist's determinism problem may sound surprising, but it can seem natural in light of a deep similarity between the indeterminism of Friction World and another sort of indeterminism explored by Pooley (2003). Pooley considers how to accommodate parity-violating laws–laws in which fundamental particles of differing handedness are disposed to act differently–into a framework of relationism about handedness. This is the view that there is no fundamental property of right- or left-handedness, but merely a relation of opposite handedness. Essentially, it is the analogue of comparativism about the “quantity” of handedness.

Parity-violating laws may seem incompatible with this sort of relationism, but in fact the two are consistent. One example of a parity-violating relationist law is the following:

> all red hands which decay into green ones are *handed in the same way* (Pooley, 2003, PAGE)

which can account for experimental data that might ordinarily be interpreted as *red hands only decaying into green ones if they're right hands.* This law exhibits the same sort of indeterminism and non-locality as Friction World does under standard comparativism. If we consider a red hand in this toy theory at some initial time, the law does not determine whether it will decay into a green hand. But if it does, the law does determine that no oppositely-handed hand will ever decay into a green hand. So the initial, indeterministic decay event determines future facts in a non-local way (since any instantaneous future stage of a red hand could decay into a green one without violating the laws, if the past decay had been different).

Preferring a local explanation, Pooley suggests that we posit a fundamental mirror-asymmetrical orientation field which has the same handedness at every point in spacetime. The relationist can then replace the parity-violating law above with something like: all red hands which decay into green ones are handed in the same way as the orientation field. This law is deterministic and local. It requires additional ontology in the form of the orientation field, but this seems a small price to pay for these virtues. It is most natural to understand the orientation field as describing properties of spacetime, which makes it more natural that it should take on such a privileged role in the laws.

Inspired by this, the comparativist about mass, distance and other quantities may hope to posit a similar privileged object to make the laws of motion temporally local and deterministic in cases like Friction World. To do the job, the privileged object would have to exist with physical necessity. And just as the orientation field is itself handed (which is to say, according to relationism, that it bears relations of same or opposite handedness to other handed objects), the privileged object would have to be related in mass, size and so on to all the ordinary, physically contingent objects.

This feature is problematic, however, for it means the privileged object will behave rather differently than Pooley's orientation field. The orientation field doesn't interact with any material objects except via the parity-violating law. In particular, it doesn't exert forces or respond to them. But if the comparativist's privileged object is related in mass to ordinary objects, this is all it takes for it to have a mass itself, by comparativist lights. So in theories with a gravitational force law, the privileged object will be attracted to other massive objects and attract them in turn. Its existence will in principle be detectable.

This is not to say that the existence of such a privileged object has been empirically ruled out. It might after all be extremely small, far away and hard to detect. Or it might share its physical characteristics with the ordinary contingent objects we measure all the time (for example, it might have all the same properties as an ordinary quark, aside from its necessary existence). But the physically necessary existence of such an object is an extremely implausible posit for a scientific theory to make. (One is reminded of Russell's orbiting teapot.) In the case of Pooley's orientation field, the physically necessary object is probably best understood as a part of spacetime, and it does not interact directly with other concrete objects–both of which make its existence far more plausible than the privileged object required for the comparativist about mass, distance and other quantities. As a solution to the comparativist's determinism problem, I think it's fair to say that the privileged object has very little going for it in comparison with mixed relations.

Let's move on to the second option: physical laws governing non-fundamental properties. To see how this option might work, we must look at Dasgupta's views on modality for the comparativist. Dasgupta develops these views as an answer to arguments against standard comparativism that begin from our modal intuitions:

> It seems, for example, that there could be a pair of worlds w_1 and w_2, such that the same pattern of mass-betweenness and mass-congruence relations obtains between the objects in w_1 and their counterparts in w_2, yet the mass of each particle in w_1 is double that of its counterpart in w_2. From a relationalist [standard comparativist] point of view, it seems difficult to make sense of such possibilities. (Hawthorne, 2006, 230)

Dasgupta maintains that the comparativist can indeed make sense of this sort of possibility, without recognizing any difference between the possible worlds w_1 and w_2. In counterpart theory, it is possible to make sense of certain *de re* modal propositions only if we consider a possible world as "representing" more than one possibility. For example, if you are one of two qualitatively indistinguishable beings, you might intelligibly consider the possibility that you might have been your indistinguishable duplicate instead. In such a case, you are considering a possibility described by the actual world, but in which your duplicate is your counterpart and you are your duplicate's counterpart. There is one possible world, but two *de re* possibilities.

Dasgupta suggests that in addition to the counterpart relation for objects, the comparativist can make sense of a relation between "mass-counterparts." If we imagine the actual world to contain two equal-mass objects A and B, and consider a different possible world in which A's counterpart has twice the mass of B's counterpart, there are a couple of different possibilities this world could represent. In one possibility, A's mass is double its actual value (or in Dasgupta's terms, A is a mass-doubled counterpart of the actual A). In another possibility, B has half its actual mass (i.e., is a half-mass counterpart of the actual B). As with the counterpart relation for objects, which of these mass counterpart relations holds will depend on the context in which the world is considered.

Using this machinery, Dasgupta suggests that the possibility envisaged by Hawthorne, in which everything's mass is doubled, can be accounted for given comparativism

> by using the actual world along with a suitable mass-counterpart relation. To see how, note that the possibility of uniform doubling is a possibility for all the material bodies taken together. If we let S be an ordered set of all those bodies, then according to Lewisian counterpart theory a world represents a possibility for those bodies by containing a counterpart of S. Well, surely S can be its own counterpart in normal contexts. Moreover, S can also be its own [mass-doubled] counterpart in normal contexts. This is because Sï¿½s mass role–the pattern of mass-relations entered into by the members of S–resembles its mass role perfectly modulo a factor of 2: put intuitively, the pattern of mass-relations are exactly as they would be were everything doubled in mass! And relative to these counterpart and mass-counterpart relations, our mass-counterpart theoryï¿½suitably generalized to apply to ordered setsï¿½implies that the actual world itself represents the possibility of uniform doubling. (Dasgupta, 2013, 13)

So for the comparativist, the mass-doubled world imagined by Hawthorne is a metaphysical possibility. It is a possibility in which all the fundamental facts are the same as in the actual world–another possibility represented by the actual world, just like the *de re* possibility in which you are your duplicate and vice versa.

If Dasgupta is right, this picture of mass counterpart theory may allow the comparativist to understand $F = ma$ itself, and not just (L2), as a law of nature. This makes sense only if we understand physical possibility as applying, not to worlds, but to the finer-grained possibilities Dasgupta introduces. This is perhaps not terribly puzzling. More puzzling is the fact that on this picture, what we would ordinarily call the fundamental laws of physics must govern something other than the fundamental objects and properties. This is obvious given that the mass-doubled duplicate of the actual state of affairs is physically impossible according to $F = ma$, despite agreeing with the actual state of affairs on all the fundamental facts (since both these possibilities are represented by the same world).

On what seems to me the best way of understanding this Dasgupta-inspired picture, the most basic laws of physics must govern non-fundamental properties. These laws will be given by equations like $F = ma$, which relate the absolute values of quantities like force, mass and acceleration, and these absolute values are not fundamental according to the comparativist. This strikes me as an objectionable feature of this option for saving standard comparativism. Some will be tempted by the thought that it is incoherent to posit basic laws which govern non-fundamental facts. Others may suspect that the overall picture being suggested here is simply absolutism stated in different terms–redefining which possible states of affairs we call "worlds," and what we mean by "fundamental" properties.

Even assuming these worries are groundless, I see a more serious problem in the violation of a broad epistemic principle. The principle I have in mind is simply that we have good reason to believe a property is fundamental or perfectly natural if it is indispensable to the best theory of our world's physical laws.[6] Moreover, if we are scientific realists and accept a physical theory that must treat two possibilities as distinct in order to do some important predictive and explanatory work, we have excellent reason to suppose that those two possibilities are fundamentally different. But on this Dasgupta-inspired picture, comparativists must deny this. They must accept that $F = ma$ is the fundamental equation describing the physics of Friction World, but deny in some cases that a possibility permitted by this law is fundamentally different from a possibility disallowed by the law. This seems inconsistent

[6] Here I mean to be considering a case, rather unlike the actual situation of physics at present, in which a theory is known to be accurate for all phenomena in all domains of application.

with any reasonable form of scientific realism, especially one based around the principle of inference to the best explanation.

This proposal is clearly quite radical compared with the suggestion that we use mixed relations to formulate a non-standard version of comparativism. I conclude that comparativism works best if we allow fundamental mixed relations. At any rate, let's assume this and see where things stand.

6 Re-evaluating arguments for comparativism

In the literature, comparativism has thus far been motivated in two ways: as a way of implementing nominalism Field (1980) and via Occamist simplicity considerations (Dasgupta, 2013). The nominalist rationale is, of course, unchanged by the abandonment of scale independence and the introduction of mixed relations. But let's look more closely at the second sort of Occamist justification.

It may be that mixed relations exhibit a sort of complexity that undermines the best reasons to be a comparativist in the first place. Indeed, it seems to me that Dasgupta's Occamist argument does face new difficulties when standard comparativism is abandoned. His argument begins from a principle of simplicity that is best understood as a special case of Occam's dictum to prefer simpler theories. By Dasgupta's lights, "positing undetectable structure is a vice, in the sense that if one theory of the material world posits undetectable structure that another does not then all else being equal we should prefer the latter." (Dasgupta, 2013, 2)

For those who embrace this principle, Dasgupta's argument provides considerable reason to prefer a standard comparativist metaphysics if one is available. Unfortunately, as we've seen, standard comparativism leads to physical pathologies like temporal action-at-a-distance and indeterminism without chance. But the argument can perhaps be extended to provide at least some evidence for a non-standard form of comparativism with fundamental mixed relations. In most (and perhaps all) cases, the absolutist interpretation will still posit some undetectable structure that a viable, non-standard comparativist interpretation does not. In Newtonian mechanics there will remain some transformations of mass, distance and time that generate undetectable differences between possibilities for the absolutist, and not for the comparativist who posits mixed relations.

But this amounts to much less undetectable structure than the absolutist posits in Dasgupta's simplified example. This lessens the force of the argument, and opens the way for a more general concern about the Occamist stratagem. Dasgupta prefers comparativism on the grounds that undetectable structure is a disadvantage, "all else being equal." But is all else really equal?

The real imperative of Occam's razor, as it's usually understood, is not simply to eliminate undetectable structure. Rather, it is to make do with the simplest overall picture of the world that can be made to work. It may serve this goal to introduce undetectable structure. For example, if we can formulate our physics using fewer fundamental quantities and less complicated laws, but only by introducing undetectable differences between possible worlds, it may be worth our while to do so even by Occam's lights.

Now, in a sense the comparativist (whether standard or not) makes do with the same number of fundamental quantities as the absolutist. But the comparativist's quantities are grounded in a system of relational structures that seems rather baroque and complicated by comparison with the absolutist's roster of intrinsic properties. This is especially true when mixed relations of an intuitively unfamiliar sort are introduced, as they must be once we realize that standard comparativism cannot do the job of interpreting realistic physics. The satisfaction of scale independence by the standard comparativist was another simple, elegant-looking feature of the theory, but this advantage too seems to have gone by the wayside thanks to mixed relations. Mundy (1987), on the other hand, has put forward an elegant theory of quantity that proceeds from absolutist assumptions. Comparativist alternatives to Mundy's theory must, of course, be evaluated on an individual basis. But I suspect that, when the need for mixed relations is recognized, the resulting comparativist theories will be markedly inferior to extant absolutist alternatives–on Occamist grounds.

I am generally suspicious of arguments like Dasgupta's that endorse some metaphysical interpretation of physics on the basis of a single theoretical virtue like simplicity or detactability-in-principle. Scientific theories are generally accepted only on the basis of a large body of interrelated virtues, many of which are only considered salient by scientists at moments of conceptual or experimental crisis (Sklar, 2002). I would expect successful metaphysical theories to demand an equally nuanced body of evidence. But even restricting ourselves to simplicity alone, as Dasgupta does, comparativism's need for mixed relations weakens his argument for the view.

7 Conclusions

To be a comparativist is to believe in mixed relations, on pain of introducing severe pathologies into some of our most basic physical theories. I've suggested that this weakens our reasons for believing comparativism, but I would hardly say that no reasons remain. Rather than refuting comparativism, the main achievement of my arguments has been to transfigure the view. Comparativism has become more complicated than the view its proponents put forward. The comparativist's world now consists fundamentally of relations comparing unlike quantities. Things like the ratio of my height to my mass are now among the basic building blocks. I'd like to close by examining one surprising consequence of this new direction for comparativism.

In many ways, standard comparativism was much less of a departure from absolutism than the forms of comparativism that fit with what we know about physics. Like the absolutist, the standard comparativist could coherently allow that the fundamental facts about unlike quantities are metaphysically distinct. For example, the spatial facts can be understood as metaphysically independent of the mass facts on both the standard comparativist and absolutist pictures. A standard comparativist can tell you all of the spatial facts about objects – the ratios of their distances, velocities and accelerations – while leaving completely open the relations that determine their masses.

Once mixed relations are allowed in, this independence of the mass facts from the spatial facts becomes impossible. Suppose that I set the values of all the fundamental relations which involve distance. If the fundamental relations include a mixed relation comparing mass with distance, for example, I will leave out some spatial facts unless I tell you the ratio of each spatial distance to the mass of every object in the world. But if I do that, I've given you enough information to figure out the ratio of any two masses. If the ratio of my car's mass to my height is 700 kilograms per meter, while the ratio of my desk's mass to my height is 7 kilograms per meter, then my car must be 100 times as massive as my desk.

What to make of this holism? I'm sure some readers will count it among the theoretical disadvantages of comparativism. For the committed epistemological conservative, a theory is unattractive insofar as it departs from our prior conception of its subject matter. And plausibly, on our prior conception of mass and distance, the facts about these two quantities are independent in a way that now appears incompatible with comparativism.

But the staunch methodological conservative would not have much interest in comparativism–a revisionist theory to the core–in the first place. More likely, those who prize scientific detectability in their epistemology will be attracted to comparativism. My examples aside, Dasgupta's main point remains: the absolutist does posit at least some undetectable structure. A realist stance toward only detectable entities has been advocated as one way of combatting scientific anti-realist arguments (Giere, 1985). Earman and Roberts (2005a,b) have argued for a Humean metaphysics in which everything supervenes on the detectable facts. On this sort of approach, comparativism will thrive, and my arguments here will simply imply that our best theory of the world is holistic in an unexpected way.

References

Baker, David John (2020), "Some Consequences of Physics for the Comparative Metaphysics of Quantity," *Oxford Studies in Metaphysics* .

Belot, Gordon (1998), "Understanding Electromagnetism," *British Journal for the Philosophy of Science* 49:531–555.

Bigelow, John and Robert Pargetter (1988), "Quantities," *Philosophical Studies* 54:287–304.

Dasgupta, Shamik (2013), "Absolutism Vs Comparativism About Quantity," *Oxford Studies in Metaphysics* 8:105–150.

Earman, John and John Roberts (2005a), "Contact with the Nomic: A Challenge for Deniers of Humean Supervenience about Laws of Nature (Part I)," *Philosophy and Phenomenological Research* 71:1–22.

Earman, John and John Roberts (2005b), "Contact with the Nomic: A Challenge for Deniers of Humean Supervenience about Laws of Nature (Part II)," *Philosophy and Phenomenological Research* 71:253–286.

Field, Hartry (1980), *Science Without Numbers: A Defense of Nominalism*, Princeton: Princeton UP.

Giere, Ronald N. (1985), "Constructive Realism," in Churchland, Paul M. and Clifford A. Hooker (eds.), *Images of Science*, Chicago: University of Chicago Press, 75–98.

Hawthorne, John (2006), "Quantity in Lewisian Metaphysics," in *Metaphysical Essays*, Oxford: Oxford UP, 229–238.

Lange, Marc (2002), *An Introduction to the Philosophy of Physics*, Oxford: Blackwell.

Mundy, Brent (1987), "The metaphysics of quantity," *Philosophical Studies* 51:29–54.

Pooley, Oliver (2003), "Handedness, parity violation, and the reality of space," in Brading, Katherine and Elena Castellani (eds.), *Symmetries in Physics: Philosophical Reflections*, Cambridge: Cambridge UP, 250–280.

Sklar, Lawrence (2002), *Theory and Truth*, Oxford: Oxford UP.