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ORIGINAL PAPER

Inference to the best explanation and mathematical realism

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Abstract Arguing for mathematical realism on the basis of Field's explanationist version of the Quine–Putnam Indispensability argument, Alan Baker has recently claimed to have found an instance of a genuine mathematical explanation of a physical phenomenon. While I agree that Baker presents a very interesting example in which mathematics plays an essential explanatory role, I show that this example, and the argument built upon it, begs the question against the mathematical nominalist.

Keywords Mathematics · Explanation · Realism · Nominalism

1 Introduction

In a recent paper, Alan Baker (Baker, 2005) argues that there are genuine mathematical explanations of physical phenomena. Baker's insightful paper is a new intervention in the ongoing debate between mathematical realists and mathematical nominalists with regard to a new version of the indispensability argument for mathematical realism[1] advanced by Field (1989). Field noted that even if, contrary to what he argued

[1] The indispensability argument I refer throughout this paper is well known, but in the benefit of clarity let me present it very briefly. In Maddy's version (see Maddy, 1997, p. 133) it features three premises, roughly stated as follows:

(1) Our best physical theories make an indispensable use of mathematics—Indispensability
(2) The confirmation of a theory is wholesale, and since theories require much mathematics, confirmation includes the mathematical parts of the theory—Quine's Confirmational Holism and
(3) Our theories are committed to those things taken to be values of the variables our theories quantify (existentially) over—Quine's Criterion of Ontological Commitment.

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in his (1980), mathematical posits turn out to be indispensable to scientific theorizing, they still can't be granted ontological rights until they are shown to be indispensable in a stronger, more specific sense; in particular, the realists should be able to show that the mathematical posits are indispensable for scientific explanations (Field, 1989, pp. 14–20). More recently, Melia (2002, p. 75) argues along the same lines, claiming that even if mathematical entities may be useful, even indispensable to scientific theorizing, this is still not enough to justify belief in their existence. Thus, as he puts it, the challenge for realists is to show that there are convincing scientific examples in which positing mathematical *abstracta* "results in an increase in the same kind of utility as that provided by the postulation of theoretical entities" (Melia, 2002, p. 75). If the realists can find such examples, then they may be entitled to claim full ontological rights for the mathematical posits, via an "inference to the best explanation", or IBE (Field, 1989, pp. 17).

Baker's paper is an attempt to meet the nominalist challenge and thus to tip the balance in realists' favor. Before presenting his own example, Baker addresses the work of the mathematical realist Colyvan (2001, Ch. 4 and 2002), who also advanced a response to the Field–Melia challenge by producing a series of examples of mathematical explanations of physical phenomena.[2] Yet, despite his realist commitments, Baker is not swayed by Colyvan's examples. Two important objections, argues Baker, plague these examples. The first objection is that some of these examples are not instances of genuine explanations, but of predictions (Baker, 2005, p. 226).[3] The second is that other of Colyvan's examples of explanations are in fact geometrical explanations[4] and, given the notorious ambiguity in the subject matter of geometry (i.e., is this subject matter mathematical or physical?), it is unclear whether these geometrical explanations are genuinely mathematical (Baker, 2005, p. 228).[5] After pointing out these difficulties, Baker comes up with his own, different example of a genuine mathematical explanation of a physical phenomenon. Unlike Colyvan, whose examples are basically from physics, Baker examines an example taken from evolutionary biology.

My main objective in this short paper is to show that Baker's example, and the argument he builds upon it, beg the question against the nominalist. The plan of the paper is as follows. In Sect. 2 I review briefly Field's IBE realist strategy. In Sect. 3, after I sketch out Baker's example and argument, I explain how his example attempts to illustrate this strategy. In Sect. 4 I argue that it fails to do this. I close in Sect. 5 with some more general remarks about this strategy.

Footnote 1 continued
It follows that once we adopt a physical theory, we are committed to the existence of mathematical objects. The original argument is scattered in Quine's and Putnam's writings; see in particlular Quine (1981, p. 149–150) and Putnam (1971, p. 347; 1979, p. 74). M. Resnik (1995) proposes a 'pragmatic' version of the argument, but this version has no bearing on my position here.

2 To the best of my knowledge, Steiner (1978a) is the first paper to present such examples.

3 Baker refers to Colyvan's example from meteorology: given a certain moment of time, why are there two antipodal points P_0 and P_1 on the earth's surface with the same temperature and barometric pressure? Note, however, that nobody has ever found such points; yet, in case they will be found, the Borsuk-Ulam theorem in algebraic topology is an essential part of an explanation as to why such points exist. See Colyvan (2001, p. 49) for details.

4 Baker discusses Colyvan's example involving Minkowski's mathematical–geometrical explanation of certain relativistic effects (such as Lorentz contraction). See Colyvan (2001, p. 50).

5 For this objection, see also Melia (2002, p. 76).

2 The IBE strategy for mathematical realism

Field (1989, pp. 14–20) envisages the realist as attempting to argue for the existence of mathematical posits by highlighting their indispensable role in explanations of physical phenomena.[6] This argument for mathematical realism cleverly combines the well known scientific realist technique of "inference to the best explanation" with the fundamental insight of the Quine–Putnam Indispensability argument, that mathematics is indispensable to science. In a nutshell, Field grants that if the realist can show that mathematical posits are indispensable to *explanations* of physical phenomena, we should believe in their existence via an inference to the best explanation. The strategy Field considers is as follows. Suppose we hold some observational belief (i.e., about a physical phenomenon; henceforth: the explanandum) and we advance (what we take to be) the best explanation of this phenomenon. Furthermore, suppose that part of this explanation is claim S and that we have strong reasons to assume that no explanation of the phenomenon in question is possible without S. Now, remarks Field, "if a belief [S] plays an ineliminable role in explanations of our observations, then other things being equal we should believe it, regardless of whether that belief is itself observational, and regardless of whether the entities it is about are observable" (1989, p. 15). Importantly, this line of thinking leads to beliefs about observable and unobservable entities alike. Consequently, the relevance of this idea for mathematical realism is immediate.[7] If a certain physical phenomenon can be best explained by making a series of assumptions (henceforth: the explanans), and among these explanans we find a mathematical claim S which turns out to be ineliminable, then IBE counsels us to believe that the mathematical statement S is true and that the mathematical posits featuring in it exist.[8] In the next sections, after I sketch out Baker's example, I show that his argument fails to illustrate this realist strategy.

3 Baker's argument

Baker's case study is taken from evolutionary biology and involves periodical North American cicadas, a species of large fly-like insects having 13- or 17-year-periods. Biologists note that one of the aspects of cicadas' life in need of explanation is their prime-numbered-year life-cycle length. An explanation of this period length has been proposed[9] and, claims Baker, it is this explanation that can provide a better example of a genuine mathematical explanation of a physical phenomenon. The explanation is simple but ingenious: obviously, it is evolutionary advantageous for cicadas to intersect as rarely as possible with predators and to avoid hybridization with similar subspecies. The frequency of intersection and hybridization is minimized when cicadas' period is prime (Baker, 2005, p. 231). For instance, as was pointed out in biological literature, if cicadas had a 12-year cycle, they would clash with properly synchronized predators

[6] Field holds this argument in high esteem, noting that "arguments for the indispensability of mathematical entities *in explanations of the physical world* seem in some way more compelling to a scientific realist than other indispensability arguments." (1989, p. 17)

[7] So, if we believe in electrons on the basis of something like IBE, it looks like we should believe in numbers on the basis of the same methodology. For criticisms of IBE, see van Fraassen (1980) and Cartwright (1983).

[8] This is what I call 'Field's explanationist version of the Quine–Putnam Indispensability argument'.

[9] Baker cites recent literature in evolutionary biology. For these references, see Baker (2005).

every 1, 2, 3, 4, 6 and 12 years. By comparison, a species of cicadas with a period of 13 years will meet fewer predators. Baker begins by spelling out biologists' argument and then underscores the explanatory role of primeness (and, more generally, of mathematics) within this argument. The argument runs as follows (p. 233):

(1)	Having a life-cycle period which minimizes intersection with other periods is evolutionarily advantageous.	[Premise 1. Biological 'law'.]
(2)	Prime periods minimize intersections.	[Premise 2. Number theoretic theorem.]
(3)	Therefore, organisms with periodic life-cycles will have prime periods.	[Conclusion. This is a 'mixed' biological/mathematical law.]

This three-step argument is, claims Baker, an example of explanation of a physical (biological) phenomenon. Premise 2, the number theoretic theorem, is "essential to the overall explanation" (p. 233) and plays the role of claim S in Field's IBE strategy. Moreover, the phenomenon cited in the conclusion is "external" to mathematics (p. 225). Baker points out that this is "a key strategic point": had the fact to be explained been a mathematical fact, the case for realism would be vulnerable to "charges of circularity" (p. 225).

That the explanandum must be a phenomenon external to mathematics will be of central importance in what follows, so I now consider the force of the circularity charge. That is, I wish to clarify what is the force of this charge by explaining why the phenomenon being "outside the realm of pure mathematics" (p. 233) is so important for the realist. I proceed to this clarification because in Sect. 4 I will argue that Baker fails to escape this charge and thus his example fails to illustrate the effectiveness of this strategy, by begging the question against the nominalist.

In a recent paper, Leng (2005) touches on the circularity issue after she discusses and accepts the existence of mathematical explanations of mathematical phenomena (as proposed by Steiner, 1978b). She wonders why can't realists apply the IBE strategy to these mathematical explanations and establish mathematical realism: if the explanans are mathematical statements (what else?), and in so far as these explanations are genuine explanations (as they seem to be), their explanans must be true—hence it would immediately follow that the mathematical posits featuring in these statements must exist. So, why bother and require that the explanandum be a *physical* phenomenon? Leng writes:

> Given the form of Baker and Colyvan's argument, one might wonder why it is mathematical explanations of physical phenomena that get priority. For if there are (...) some genuine mathematical explanations of mathematical phenomena, then these explanations must also have true explanans. The reason that this argument can't be used is that, in the context of an argument for realism about mathematics, it is question-begging. *For we also assume here that genuine explanations must have a true explanandum, and when the explanandum is mathematical, its truth will also be in question.* (Leng, 2005, p. 174, fn. 2. Italics added.)

The key point here is that the IBE strategy assumes that both the explanans *and* the explanandum are true statements: if one doubts that the explanandum is true, then one sees no point in explaining it. The requirement about the truth of the explanandum is trivial indeed, and in his original presentation (see Field, 1989, pp. 14–20) Field never mentions it explicitly, as he discusses cases in which the explanandum is an observable, unproblematically true physical phenomenon (such as the appearance of stains on the wall-paper, etc). Now given that the IBE strategy works only if the explanandum is a true statement, we can understand why the explanandum can't be a mathematical statement. Suppose it were; because we also had to assume the explanandum were true (in order to make sense to advance an explanation of it), the entities it features exist. But this is just to assume that realism is correct, i.e., to beg the question against the nominalist. As Leng points out, if the explanandum is mathematical, "its truth will also be in question", and this would block the attempt to use the IBE strategy.

In the next section I'll argue that the central problem for Baker's argument arises along this line. Briefly put, my main concern is that Baker faces a major difficulty when claiming that his explanandum is true. As I will detail in the next section, the problem is that the truth of his explanandum presupposes, or depends on the truth of a mathematical statement—hence, his case for realism is question-begging. But before I present this objection, let me sort out an ambiguity in Baker's account of the cicada example. He begins by identifying the question that scientists set out to answer as "Why are the life-cycle periods prime?" (Baker, 2005, p. 230). Later on, however, after he spells out the argument and underscores the role of the number theoretic theorem in the explanatory story, he re-identifies the phenomenon to be explained as "the period length of cicadas" (Baker, 2005, p. 233). That is, the phenomenon to explain now seems to be why cicadas' life-cycle period is specifically 13 years. So, it becomes unclear whether the main question to answer is "Why are the life-cycle periods prime?" or "Why is the period 13 years?" Fortunately, there is a way to read Baker's argument such that this ambiguity is rendered harmless. Given certain ecological constraints Baker mentions (that the periods can range from 12 to 15 years; see p. 233), once we know that the period has to be prime—i.e., once we answered the first question—the number 13 comes out as the only acceptable answer. So, the first question "Why are the life-cycle periods prime?" is more basic and the central question to answer. Consequently, the thing to explain (the explanadum) is "The primeness of cicadas' life-cycle period." This reading of Baker's point also squares well with the evidence he brings in from biologists' practice; as he notes, they are puzzled about primeness, not about any specific numerical value, be it 13 or 17 (Baker, 2005, p. 230, item (v)).

4 The objection

The conclusion of the previous section was that the explanandum in the cicada example is the primeness of the life-cycle period. Moreover, if the IBE strategy is to be effective, this explanandum must be true; more explicitly: the statement 'the life-cycle of cicadas (in years) is prime' must be true. The stage is now set to argue that this requirement undermines Baker's case for realism.

To begin with, it is crucial to note that the explanandum is not a pure physical (biological) phenomenon. It seems natural to regard the explanandum as a kind of mixture,[10] involving several elements:

[10] I take it that Baker endorses the idea that his explanandum is a mixture, since he presents the partial conclusion (3) on p. 233 as "a mixed biological-mathematical fact". He reendorsed this reading in personal correspondence.

(i) a physical (biological) phenomenon, or physical 'object'—the time interval between two successive occurrences of cicadas,
(ii) a description of this phenomenon, or, in other words, a concept under which the physical object falls—the concept in question here being, obviously, 'life-cycle period (in years)',
(iii) a mathematical object (a certain number, 13 in this case) associated with the description, i.e., a number ascribed to the concept and, finally
(iv) a mathematical property (primeness) of the number involved.

To put it in a more general (even if more baroque) form, our wonder is about the relevance of *a mathematical property of a mathematical object attributed to the concept under which the physical object in question falls*. So, if the explanandum is the relevance of the primeness of a certain number, since primeness is a mathematical property, it is not surprising that we have to advance a mathematical explanation of its relevance, in terms of specific theorems about prime numbers.

Now, as this quasi-Fregean analysis suggests, the explanandum 'The life-cycle period of cicadas is prime' is to be cashed out as "The number attributed to the concept [life-cycle period of cicadas measured in years] *is prime*". Obviously, this is not a purely mathematical claim—it is *about* something physical, a time span. Yet, upon analysis, it undeniably involves a mathematical claim, that a certain number is prime. Now, as we saw, in order to apply the IBE strategy, we have to assume that this explanandum is true. By doing this, I argue, Baker begs the question against the nominalist. For (part of) the explanandum consists in a property-attribution claim, where the property in question is primeness, a paradigmatic example of a mathematical property. And if mathematical properties apply to anything, they apply prima facie to mathematical objects; hence, if "The number attributed to the concept [life-cycle period of cicadas measured in years] *is prime*" is taken to be true, this can't hold unless there is a mathematical object (specifically: a number) to which the property 'is prime' applies. Therefore, by taking the explanandum as being true (to comply with the requirements of the IBE strategy), Baker assumes realism before he argues for it.

A possible problem for this approach could be that biologists themselves take the explanandum to be true. Don't they say, one might wonder, "Look, the life cycle period of cicadas is prime—why is this so?" How can it be a problem that Baker has to assume that the explanandum is true, if he was supposed to follow scientific practice as closely as possible? This point is connected with an important methodological advantage of Baker's example over other examples (for instance, Colyvan's examples mentioned at the outset), namely that it is extracted directly from scientists practice. That is, as Baker's reference to recent biological literature shows, the primeness of cicadas' life-cycle periods is a genuine puzzle among evolutionary biologists, a regularity they think cries out for an explanation; moreover, the advanced explanation (correctly reconstructed in Baker's paper) is very convincing and it does involve mathematics essentially. Now I grant that from biologists' perspective it is unproblematic to take the explanandum to be true. Yet, as it often happens in conceptual analysis, what can be taken for granted by the scientist can't be so taken by the philosopher. Unlike their biologist colleagues, philosophers attempting to establish mathematical realism via the IBE strategy cannot unreflectively take the explanandum to be true—if they do, given the structure of this explanandum, they beg the question against the nominalists. In this case, blindly following in biologists' footsteps and simply assuming the truth of the explanandum is fatally damaging for the mathematical realist. Of course there is nothing wrong with the biologists' taking the explanandum to be true and then attempting to explain it. My point is that the realist philosopher should perceive this as a problem in this particular context (the IBE strategy), when the truth of the explanandum is part of an argument for mathematical realism. So, I submit that this type of example and this explanandum in particular are ill suited to illustrate the IBE strategy, which was supposed to endorse a philosophical (metaphysical) thesis, mathematical realism.

5 Conclusion

Summing up, I think that the realist project does not succeed because of Baker's failure to provide an explanandum that is unproblematically physical, as he correctly set out to provide, well aware of the requirements of the IBE strategy. Although the explanandum was difficult to characterize, it minimally involved a mixture of physical elements and mathematical assumptions, so it was "outside of the realm of pure mathematics" indeed—as Baker wanted, in order to ensure the application of the IBE strategy. However, the requirement to assume the truth of this mixed explanandum bound Baker to assume the truth of the mathematical part of the mixture, and hence to beg the question against the nominalist.

Let me close by noting that the analysis of the failure of Baker's insightful example-argument brings to light a more general dilemma, suggesting that something could be wrong with the IBE strategy itself. First, note that in order to have mathematical claims among explanans, we need to find an explanandum that is not purely mathematical, but involves some mathematical assumptions or at least some mathematical terms—otherwise, one needs a further argument to see how the mathematical explanans can *in principle* have any explanantory relevance for an explanandum that is purely physical, free of any traces of mathematical vocabulary.[11] Now, to apply the IBE strategy, this (necessarily mixed) explanandum has to be true, and the question arises as how can the realist deal with the mathematical part of this explanandum, in so far as this part has to be true as well. So, in broad strokes, the dilemma is as follows. Either the realist takes the mathematical assumption(s) appearing in her explanandum to be true and thus begs the question against the nominalist (as I argued that Baker did); or, if she doesn't take them to be true and suspends her judgment over them, this will presumably be further reflected in her overall judgment about the truth-value of the explanandum—and, if she can't take it to be true, she can't apply the IBE strategy. It follows that either the IBE strategy is question-begging or is not applicable in the first place.

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[11] In connection to this, see Balaguer (1998, pp. 130–142) who discusses in detail whether or not all apparently mixed mathematical/physical facts supervene on purely mathematical and purely physical facts.

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