***Mathematics of casino carnival games*, by Mark Bollman**

Every time a new book on gambling mathematics is published, the first question arising is whether its target audience is players or math students. Establishing one of these two options is not straightforward, and the criteria for putting a label on one or the other are dependent upon deep analysis since they have a cognitive, a pedagogical, and a gaming-behavioral dimension that are not independent of each other. In addition, generally a criterion that favors one audience disadvantages the other. This is why any possible claim of a publisher that a title in such a category has a double target audience (*both* players and math students) should be received with skepticism (such claims are actually present in the publishing industry). *Mathematics of casino carnival games* definitely falls within the narrow category of gambling-mathematics books from which both players and math students have something to learn in a systematic pedagogical way, and my arguments refer to content, organization, and approach.

It is a necessity for all gambling mathematics books to dedicate a first chapter to the pure mathematics behind the games and gambling. For those books whose target audience is declared to be that of players, the big challenge of their authors is to adapt this content to the non-mathematical audience. This assumes adaptation of the mathematical language beyond mathematical formalism and an extraction from the larger mathematical structures in which the presented concepts stand. Professor Mark Bollman in the *Mathematical background* section presents the basics of probability theory limited to the Laplacian discrete probability applicable to games of chance, in a language in which mathematical formalism is actually present only in the denotations in definitions. This elementary definition of probability, together with subsequent essential notions like conditional probability and the statistical notions of expectation and house advantage are all the basic concepts needed for applying mathematics in gambling from the perspective of a player. The properties of probability are presented in the form of 'counting rules' (addition rules, multiplication rule, complement rule, and the expression of conditional probability), while the law of large numbers (LLN) is expressed in its simplest form, in terms of frequencies, with no reference to the more general Central Limit Theorem. At this point, it is remarkable that LLN is further explained conceptually, through examples, not only with reference to the mathematical notion of limit, but also relative to the perception of how this law works in the real world of gambling. In fact, one of the strengths of this book is that this section, as well as those describing mathematically each game in part, is packed with plenty of examples of mathematical applications. The pure mathematics section comes with a section dedicated to combinatorics – so important for the probability calculations applied in games of chance – which is as well limited to the necessary notions and counting principles. Overall, such extraction is not something easy – for the content to be understandable and applicable, it has to be supplemented with relevant examples and explanations in a mixed language[1] making the conceptual connections with the mathematical notions and structures missing in the exposition; it is a pedagogical ability. The author successfully fulfills this task and it is clear that understanding of the non-mathematical audience is one of his main goals, along with the exposition of the mathematical facts as information. As such, the content of the *Mathematical background* section and all applications following in the sections dedicated

[1] language containing both mathematical and non-mathematical terms

to the games can be easily followed by a high school graduate. While this simplicity is effective and as such is a virtue, if I had to raise any objection to the mathematical content of this chapter, I would say that presenting probability also as a mathematical measure would have been useful, especially from the perspective of the player, who has to learn more about the *nature* of probability. Such a presentation is possible without going deeper into the complex notions of measure theory.

Another asset of this book is the section dedicated to the mathematics of casino game design, which completes the theoretical part on probability theory basics as a useful application for the didactical purposes. In this section, the author shows what math work is needed for developing a game of chance from the point of view of the house and how the mathematical results allow game designers to adjust parameters of a game to make it functional and profitable in the favor of the house. Here the reader is made aware that the mathematics of gambling is not limited to computing simple combinatorial probabilities and simple statistical means, but has some complexity that is not accessible to anyone without prior mathematical knowledge. Besides probabilities, expectation, and house advantage, there is one statistical indicator which is important for the casino game design, namely *variance*. The author not only defines variance in mathematical terms, but also explains how its role of measuring deviation of the data from the expected value is lucrative for the house. The practical examples supporting the math applications of this section were chosen from the so-called casino carnival games and not from classical games, and this choice may strengthen reader's belief in the principle that however simple or complex, popular or rare a game of chance may be, its mathematical description and all mathematical models behind it stand as the main premise of its existence.

Casino carnival games are presented as less (currently) popular variations of the classical casino games, having their own history, in which the variations come from changing the number of game elements (cards in the deck, pockets of a wheel, faces of a die, or number of dice), but also some rules of betting and payoff odds. These variations themselves represent a math lesson, as they show how the entire application of mathematics down to numerical results changes when changing the parameters in the description of a game. Changing the composition of the deck or the hand size, the number of the faces of a die or the number of dice to be rolled, all result in new formulas and calculations for the probabilities associated with the gaming events of those games and their statistical indicators. These are good exercises for those studying the probability theory applied in games, and they were not imagined by the instructor, but actually come from real games that were or are played. From this perspective, interested players may learn their own applied-mathematics lesson as well as a lesson related to gambling misconceptions: applied probability is a tricky field when it comes to subjective estimations based on experience, for rules of proportion do not usually apply in probability calculations, mainly due to the combinatorial nature of most games of chance.

The impressive collection of the so-called casino carnival games (numbering seventy individual different games) is grouped in three large categories (wheel and ball games, card games, and dice games). Each game's section holds game's description, with rules and parametric configuration, and a set of examples of calculation for probability, expected value, and house advantage for some gaming events and bets; each section ends up with proposed exercises in a mere schooling mode. While coverage seems to be

another virtue of this book, and the games collection itself a novelty (most of the games presented here did not have any book dedicated – partially or totally – at the time of the release of Mark Bollman's title), it worth noting that labeling the book with "mathematics of …" when referring to these games induces a broader coverage than that which the mathematical content actually does. Mathematics of a game (in fact, mathematics *applied* to that game) covers all possible gaming situations and events, all that can be described and modeled mathematically, including betting systems or strategies. Due to the vastness of all these possible situations and options (mainly originating in the combinatorial nature of most of the games of chance), it is no surprise for a "mathematics of …" book dedicated to a game of chance to fill pages in the order of hundreds and even more. As such, since 'mathematics of', say, Texas Hold'em Poker, would fill a book of a certain length, we should expect that any variation of poker (for instance, Mississippi Stud or Double Down Stud, present in the book), to have a similar length for their mathematics. Of course, it would not be possible for such a collection to submit to such ideal conditions and fit into a book of a decent length, and this note is in no way any kind of reproach for a title's not reflecting the content. On the contrary, it is remarkable that the author was able to find relevant examples of calculation for each game and touch on the essentials instead of laying down a systematic presentation of the mathematical facts of each game (again, not possible due to length limits). Moreover, the chosen examples not only stand as exercises for calculating probabilities, expected values, and house advantages as individual notions applied, but also show practically how gaming decisions may be made on the basis of balances between probability, expected value, and stake associated with a bet. Players are taught in this way that the probability of winning or expected value alone is not all that counts in weighing the chances for short- or long-run success, but also the way they are balanced, including relative to the stake. With these elementary notions, the author never misses the opportunity to talk about optimal play and strategy in concrete situations, without going into advanced concepts of game theory.

Overall, the mathematical simplicity, the informative-didactic style, and the unifying approach is what makes this book suitable for the double audience (players and math students), and any non-mathematical person interested in gambling mathematics may start with it as a primer. As for the players, this mathematical perspective not only aims at gaming strategy and information, but includes problem gamblers fairly in the target audience because by such perspective, the author offers them lessons too throughout the book, sometimes "between the lines" but mostly present in the applied-mathematics content.