# DESCRIPTION AND THE PROBLEM OF PRIORS

JEFFREY A. BARRETT

Abstract. Belief-revision models of knowledge describe how to update one's degrees of belief associated with hypotheses as one considers new evidence, but they typically do not say how probabilities become associated with meaningful hypotheses in the first place. Here we consider a variety of Skyrms-Lewis signaling game [Lewis (1969)] [Skyrms (2010)] where simple descriptive language and predictive practice and associated basic expectations coevolve. Rather than assigning prior probabilities to hypotheses in a fixed language then conditioning on new evidence, the agents begin with no meaningful language or expectations then evolve to have expectations conditional on their descriptions as they evolve to have meaningful descriptions for the purpose of successful prediction. The model, then, provides a simple but concrete example of how the process of evolving a descriptive language suitable for inquiry might also provide agents with effective priors.

## Description, Prediction, and Expectation

Belief-revision models of knowledge describe how to update one's degrees of belief as one considers new evidence. On a Bayesian model, for example, one fixes a descriptive language, sets coherent prior probabilities over a set of hypotheses expressed in the language, then updates one's degrees of belief as one conditions on new evidence. While such an account of reflective inquiry has many virtues, it has nothing to say concerning how to assign prior probabilities to meaningful hypotheses. This is the problem of priors.

On reflection, one might, however, find the problem of priors itself puzzling. While it is indeed unclear what procedure one should adopt in assigning prior expectations to hypotheses expressed in a fixed descriptive language, it is similarly unclear how one might ever come to use such a language without already having a rich set of expectations. The symmetry of these reflections suggests a strategy.

We will consider how it might be possible for basic expectations to coevolve with a simple descriptive language. More specifically, we will consider how simple

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descriptive and predictive practice might coevolve with basic expectations in the context of a sender-predictor game, a variety of Skyrms-Lewis signaling game.[1]

Rather than assigning prior probabilities to hypotheses then conditioning on new evidence, the modeled agents begin with no meaningful language or expectations, then come to form basic expectations conditional on their simple descriptions as they coevolve descriptions that may be used as the basis for successful prediction. These basic conditional expectations reflect the degree of success in fact exhibited in their descriptive and predictive practice and, as such, may evolve to inform other decisions.

Reflective Bayesian inquiry is only possible for agents who have the capacity to represent alternative possible states of the world and associate expectations with these representations. The story here concerns how agents might evolve such capacities in the first place. With regard to the problem of priors, it provides a simple but concrete example of how the evolution of a descriptive language suitable for coordinated prediction might also also provide a core set of well-tuned posterior expectations that might then be available to play the role of priors should the agent turn to reflective inquiry.

We will consider the story in three parts.

## Part I: The Coevolution of Description and Prediction

A sender-predictor game is a variety of Skyrms-Lewis signaling game.[2] In a sender-predictor game, however, the agents coevolve both descriptive and predictive dispositions. We will refer to these are the agents' first-order dispositions. Their second-order dispositions determine how they update their first-order dispositions as they learn from experience.[3]

In a two-state/two-signal/two-act sender-predictor game, the sender observes a prior state of nature, then sends a signal. The probability of particular signal being sent is determined by the state of nature and the sender's first-order dispositions.

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[1]See [Lewis (1969)] for Lewis' characterization of signaling games and [Skyrms (2006)], [Skyrms (2010)], [Barrett (2009)], and [Barrett (2007)] for examples of such games in an evolutionary context.

[2]David Lewis [Lewis (1969)] introduced signaling games in the context of classical game theory as a way to study the possible nature of linguistic convention. Brian Skyrms [Skyrms (2006)] [Skyrms (2010)] later considered Lewis' signaling games in the context of evolutionary game theory. Such evolutionary games have a number of virtues over Lewis' conventional games as one need not assume common knowledge nor any special rational faculties. Rather, the agents have only simple conditional dispositions. The sender-predictor games described in the present paper were coded and run as C++ simulations.

[3]The distinction between first- and second-order dispositions is less clear for agents who might learn how to learn. The agents here are not so subtle. Also note that the evolutionary story here involve the evolution of the dispositions of particular agents rather than the evolution of types of agent in a population. It is often possible to translated results between such models. See [Skyrms (2010)] for a discussion of this point.

On observing the signal, the receiver performs a predictive action that is either successful or unsuccessful depending on the posterior state of nature at a later time. The probability of a particular predictive action being performed is determined by the signal and the predictor's first-order dispositions. If the predictive action is successful, which will depend on both the regularities exhibited by nature and what counts as success for the agents as determined by their second-order dispositions, then the sender and predictor reinforce the first-order dispositions that led to the action they just took conditional on the state of nature and the signal type, respectively. If the predictive action is unsuccessful, they might weaken the first-order dispositions that led to the actions they just took. Precisely how they reinforce or weaken their first-order dispositions is determined by their second-order dispositions.[4]

Consider a sender who checks the water temperature off Newport Pier each morning. If the water is cooler than normal, she draws a ball from one urn; and if it is warmer than normal, she draws a ball from another urn. Suppose that the cool urn and the warm urn each initially contain one blue and one green ball. The sender waves either a blue or green flag depending on the type of ball drawn. Her predicting friend, standing on the beach on Catalina Island, sees the color of the flag through his spyglass and draws a ball from an urn that corresponds to the flag color. Suppose that each of the predictor's urns begins with one ball each of two predictive action types corresponding to going night fishing or to staying home to repair fishing nets. He then performs the action corresponding to the type of ball he drew. Night fishing is successful if there are clear skies that evening; staying home to repair nets is successful if there is fog.[5] If the predictive action is successful, then each agent returns his ball to the urn from which it was drawn and adds a new ball of the same type; otherwise, each agent simply returns his ball to the urn from which it was drawn.[6]

[4]See [Barrett (2012)] for more details regarding the set-up and interpretation of such a games.

[5] In terms of the endogenous norms of the model, all it means for a predictive action to be successful is that, given the regularities between the prior and posterior states of nature and the agent's second-order dispositions to update their first-order dispositions, it produces an event that in fact leads to a reinforcement of the agents' first-order dispositions that led to the act. Otherwise, the predictive action is unsuccessful.

[6]This sort of simple reinforcement learning was introduced by Herrnstein in his discussion of the law of effect [Herrnstein (1970)]. More sophisticated learning dynamics also allow for punishment and forgetting. They typically do much better than simple Herrnstein reinforcement in games like those discussed in this paper [Barrett and Zollman (2009)], and, saliently, they often much better model the actual behavior of learners [Roth and Erev (1995)] [Bereby-Meyer, Yoella and Erev (1998)]. The methodological thought is that Herrnstein reinforcement learning requires only relatively weak dispositional resources and if it allows for successful coordinated action in a particular context, then one can expect a broad class of more sophisticated reinforcement dynamics to allow for similar success.

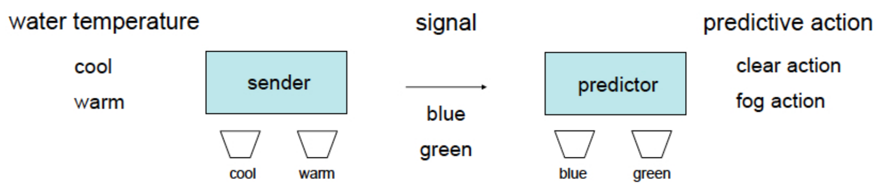


FIGURE 1. The water-fog sender-predictor game

While the sender's signals are initially meaningless and the receiver's predictive actions are correspondingly random, as the sender's dispositions to signal conditional on the state of nature and the receiver's dispositions to act conditional on the signal evolve, the sender's signals become meaningful precisely insofar as they may serve as the basis for successful coordinated predictive action. If there is in fact a natural correlation between morning water temperature and evening weather, the agents will be able to exploit this natural regularity for the purpose of successful action if they are able to coevolve appropriately interrelated descriptive and predictive dispositions.

Suppose that nature is such that the morning water temperature simply determines whether or not there will be fog in the evening. More specifically, suppose that cool water guarantees clear weather and warm water guarantees fog. In this case, if lower and higher than normal water temperatures are unbiased, independent, and randomly distributed in nature, then, in this particular game, the agents are guaranteed to coevolve successful descriptive and predictive dispositions in the limit. When they do, one type of signal will almost always be sent when the water is cool and will almost always lead to night fishing, and the other will almost always be sent when the water is warm and will almost always lead to staying home to repair nets.[7]

The game and the agents' evolved behavior is more subtle when the relationship between morning water temperature and evening fog is stochastic. This is also a natural context to describe how well-tuned posterior expectations might coevolve with the agents' successful descriptive and predictive practice.

## PART II: TRACKING PREDICTIVE SUCCESS

Consider the following implementation of the stochastic water-fog game. The sender has one urn for each possible prior state of nature labeled 0 and 1. Each of her urns begins with one ball of each type of signal she might send labeled *blue* and

[7]The deterministic case is formally equivalent to a two-state/two-signal/two action signaling game. See [Argiento, Pemantle, Skyrms, and Volkov (2009)] for a proof of convergence to a signaling system in this case. If there are more than two states, signals, and actions or if nature is biased, then the agents dispositions sometimes fail to converge to a signaling system. See [Barrett (2006)] and [Huttegger (2007)] for discussions of such suboptimal pooling equilibria.

*green*. The receiver has one urn corresponding to each possible signal type (color). Each of his urns begins with one ball of each type of predictive action he might take labeled $0'$ and $1'$.

Suppose that the prior states of nature are equally likely and randomly distributed. Suppose further that nature is such that when prior state 0 obtains, the most likely posterior state is $0'$, occurring with probability $T_{00'}$; and when prior state 1 obtains, the most likely posterior state is $1'$, occurring with probability $T_{11'}$. The probability of a transition from prior state 0 to posterior state $1'$ then is $T_{01'} = 1 - T_{00'}$, and the probability of a transition from prior state 1 to posterior state $0'$ is $T_{10'} = 1 - T_{11'}$. When the predictor gets a signal of type $\mathcal{S}$ and performs an action of type $\mathcal{A}'$, the action counts as a success for signal type $\mathcal{S}$ if and only if the posterior state of nature is of type $\mathcal{A}$.

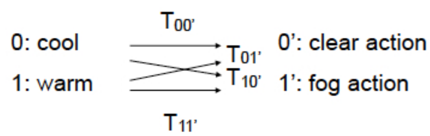


FIGURE 2. Crossover probabilities representing natural statistical regularities

In this version of the game, we will suppose as before that the agents learn by simple reinforcement. We will suppose further, however, that the predictor has expectation weights associated with each possible signal type and that evolve by bounded reinforcement with forgetting. More specifically, in addition to his action urns, the predictor has an expectation urn for blue signals and an expectation urn for green signals. Each expectation urn begins the game with a random assortment of a thousand white and black balls. If an act is successful, in addition to reinforcing the act he took, the predictor adds a white ball to the expectation urn corresponding to the signal type he just used; if the act was unsuccessful, he adds a black ball to that expectation urn. Then the predictor draws a ball at random out of the expectation urn he just put a ball in, and discards it. This gradual random forgetting maintains a constant number of balls in each expectation urn.

For now, we will simply suppose that the weight of white balls in each expectation urn represents the degree to which the predictor expects that his predictive action will be successful conditional on his receiving the corresponding signal. By stipulation, then, if the sender's signals evolve successful operational meanings for the purpose of prediction, then the expectation weights will evolve to indicate the predictor's conditional expectations on meaningful descriptions of prior states.

On simulation, the sender and receiver begin by randomly signaling and predicting, and any initial success is the result of blind luck. Over time, however, the

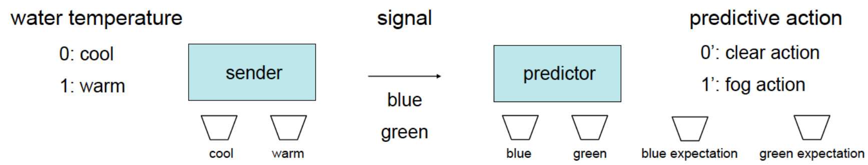


FIGURE 3. Water-fog game with expectations

agents very often learn to exploit the statistical regularities exhibited by nature. And when they do, the predictor's conditional expectations on the signal types typically coevolves to agree with the degree of predictive success the agents' evolved descriptive and predictive practice in fact affords.

In the symmetric case, where each prior state of nature is equally likely and the failure of the most likely posterior state occurring is independent of the prior state (i.e. $T_{01'} = T_{10'} = T$), the agents nearly always evolve on simulation to signal and to predict as well as theoretically possible.[8] More specifically, the sender evolves to associate a different signal type with each of the prior states and nearly always sends that type of signal when the corresponding prior state obtains; and, when the predictor receives the signal, he nearly always performs the predictive action that has been most likely to occur when the prior state corresponding to the signal has obtained.[9]

If clear weather has typically followed cool water and fog has typically followed warm water, then the sender evolves to nearly always send one type of signal whenever she observes cool water and the other whenever she observes warm water; and the predictor evolves to perform the predictive action that is most likely to be successful on a clear evening when he receives the signal associated with cool water and to perform the predictive action that is most likely to be successful on a foggy evening when he receives the signal associated with warm water. The agents then evolve a perfectly sharp descriptive language for prior states, their success rate in predicting posterior states approaches the theoretically optimal rate of $1 - T$, and the predictor comes to expect that the probability of an action being successful, conditional on each signal, is $1 - T$, which reflects the agents' actual success rate.[10] In this case, then, the agents evolve to make the best possible predictions

[8]Suppose $T_{01'} = T_{10'} = 0.25$. The theoretically best possible success rate is 0.75. On simulation, the agents evolve to do better than a success rate of 0.70 in 0.881 of the runs with $1 \times 10^6$ plays/run, 0.924 of the runs on $1 \times 10^7$ plays/run, and 0.959 of the runs on $1 \times 10^8$ plays/run with 1000 runs in each case. And the predictor's expectation conditional on each signal type evolves to agree well with their actual relative frequency of successes on each signal type.

[9]The conjecture here is that this is true for the symmetric case as an extension of the results of [Argiento, Pemantle, Skyrms, and Volkov (2009)].

[10]More precisely, the conditional expectations quickly evolve to approximate the agents' actual success rate, then fluctuate under the random mechanism for forgetting but spend most of their time close to the actual evolving success rate.

and their posterior conditional expectations in fact tell them just how reliable their predictions are. Finally, significant to the problem of priors, the evolved conditional expectations here are independent of the initial contents of the predictor's expectation urns. By the time the agents have evolved sharp meanings and clear predictive dispositions, the initial configuration of the expectation urns has been washed out under bounded reinforcement learning with random forgetting.[11]

In the asymmetric case, where the failure of the most likely posterior state occurring depends on the prior state (i.e. $T_{01'} \neq T_{10'}$), the simple-minded reinforcement learning implemented by the simulated agents here may lead them to a suboptimal equilibrium where they do not evolve to signal and predict as well as theoretically possible. But if the statistical asymmetry is modest, even agents who learn in this primitive way typically do evolve to signal and to predict as well as theoretically possible. And when they do, they also coevolve conditional expectations that match the asymmetric success rates for each of their evolved terms.[12] In particular, the receiver evolves to nearly always do predictive action $0'$ on the signal that is sent on prior state 0 and predictive action $1'$ on the signal that is sent on prior state 1; and he also evolves to expect that his action will be successful with probability $T_{00'} = 1 - T_{01'}$ in the first case and $T_{11'} = 1 - T_{10'}$ in the second.[13]

In both the symmetric and asymmetric cases, when successful, the agents coevolve intertwined linguistic and predictive dispositions. The signals are initially meaningless and the predictions made on the basis of these signals are consequently wildly unreliable. Successful prediction coevolves with the meanings of the signals, and the predictor's conditional expectation weights for the success of each type of predictive action conditional on each signal type reflect the type and degree of predictive success. Neither the evolved meaning of the agents' descriptive language nor their predictive dispositions nor the associated expectation weights are

[11]This is because bounded reinforcement learning with forgetting is much faster here at tracking the evolving success rate than simple reinforcement learning is at evolving successful descriptive and predictive dispositions. See [Barrett and Zollman (2009)] for a description of the relative virtues of learning dynamics that incorporate a relatively strong variety of forgetting.

[12]Even when they agents do not evolve the make predictions in a theoretically optimal way, they still do quite well. And, significantly for the issue of well-tuned expectations, their coevolved conditional expectations track their actual degree of predictive success on each signal type for both optimal and suboptimal evolutions. So while the agents may not know that their evolved descriptive and predictive practices are suboptimal, they do know here how reliable the predictions based on those practices are.

[13]Suppose $T_{01'} = 0.25$ and $T_{10'} = 0$. The theoretically best possible success rate is 0.875. On simulation, the agents evolve to do better than a success rate of 0.825 in 0.751 of the runs with $1 \times 10^6$ plays/run, 0.777 of the runs on $1 \times 10^7$ plays/run, and 0.773 of the runs on $1 \times 10^8$ plays/run with 1000 runs in each case. A little less than a quarter of the time, then, the agents fail to evolve those distinctions that would allow them to make optimal predictions. But when they do evolve to capture the relevant distinctions, they also learn that the term associated with prior state 0 leads to a successful predictive action about 0.75 of the time and the term associated with prior state 1 nearly always leads to a successful predictive action.

somehow prior; rather, the agents' descriptive and predictive practice and their expectations are interdependent and forged by precisely the same empirical evidence in the context of the same evolutionary process.

Since the predictor's expectation weights typically do in fact track the type and degree of predictive success afforded by the agents evolved language and predictive practice, whatever that success may be, if their past relative frequencies of successful coordinated prediction are representative of their future success, then their expectation weights can be expected to reliably indicate what their expectations should be.

## Part III: How Expectation Weights May Evolve to Represent Expectations

To this point we have simply supposed that the predictor's expectation weights represent the degree to which he expects that his predictive action will be successful conditional on his receiving the corresponding signal, but one would like to be able to tell a story concerning how expectation weights might come to play the role of expectations even as they evolve to better track the coevolving success of the agents' evolving descriptive and predictive practice.

One might suppose that this is easily explained by the fact that the predictor will do well when he uses his well-tuned expectation weights as expectations. The thought is right, but to say precisely how predictor might evolve to use expectation weights as expectations as opposed to doing something else, one must provide a specific potential use for his evolving expectation weights and at least one alternative to his not using the expectations weights in the standard way. While no concrete model can account for all of the behaviors an agent might exhibit or how they might evolve, a very simple concrete is perhaps instructive.

Suppose that the predictor is not only interested in whether he should go fishing or mend nets but also in making side wagers concerning the success of his predictive actions. Suppose further that his second-order dispositions are such that he begins by typically flipping a fair coin to determine whether he accepts a particular wager at odds but that he also begins with a small positive probability of using expectation weights as as if they were expectations to determine whether he accepts the wager. Suppose that the probability that he will flip a coin or use expectation weights to determine whether he accepts a wager in future plays increases or decreases slightly proportional to his winnings when each method is used to regiment his behavior. And, finally, suppose that the probability of regimenting his wagers in each way always remains positive.

Such dispositions might be represented as follows. The predictor has a decision urn that determines whether he flips a coin or uses expectation weights to regiment his betting behavior on a particular play. Specifically, the decision urn starts with $999,999$ balls that indicate that the predictor flip a coin and one ball that indicates that he use expectation weights as expectations. When presented with a wager, the predictor draws a ball from the decision urn. If the ball indicates that he flip a coin, he does so and accepts the wager on heads, whatever it is. If the ball indicates that he use expectation weights as expectations, he accepts a wager of $a$ of his dollars against $b$ of his friend's if and only if the ratio of the number of white balls to the total number of balls in the relevant expectation urn is greater than $a/(a+b)$. Suppose that the stakes are always \$10 and that each bet is whole valued and randomly determined with equal likelihood.

Suppose that the contents of the decision urn evolve by proportional reinforcement with forgetting. In particular, if the predictor flips a coin and wins the wager, he adds $b$ balls of the type that indicate that he flip a coin to the decision urn, then he discards $b$ balls from the urn at random. And if he flips a coin and loses, he adds $a$ balls of the type that indicate that he use expectation weights as expectations to the decision urn, then discards $a$ balls from the urn at random. Similarly, if the predictor uses expectation weights as expectations and wins, he adds $b$ balls of the type that indicate that he use expectation weights as expectations to the decision urn, then he discards $b$ balls from the urn at random. And if he uses expectation weights as expectations and loses, he adds $a$ balls of the type that indicate that he flip a coin to the decision urn, then discards $a$ balls from the urn at random. Finally, suppose that the agent never discards from the decision urn the last ball of any type.

While there are now three interacting parts to the composite model, its behavior is nevertheless relatively straightforward. As we have seen, if there are in fact regular statistical correlations between prior and posterior states, then the predictor typically evolves to make the most accurate statistical predictions possible and the expectation weights come to track how accurate his predictions are. Since expectation weights evolve to indicate the agents' actual type and degree of success, when they do, the predictor does better than even on average when he uses expectation weights as expectations to determine whether to accept a wager. And since flipping a fair coin leads to an unbiased acceptance of wagers and the wares offered are unbiased, the predictor is always even on average when he flips to determine whether to accept a wager. Hence the proportion of balls in the decision urn that indicate that the predictor use expectation weights as expectations tends to increase over time. The result is that the predictor comes to spend most of the time with a high probability of using expectation weights as expectations.[14]

It is perhaps worth making a final observation here for the sake of a modest variety of completeness. One might consider allowing the predictor's success on his wagers to affect both the evolution of his use of expectation weights as expectations *and* the evolution of the sender's and predictor's basic descriptive and predictive behavior. The short story is that such models can in general behave in complex and subtle ways. But if the effect of the outcome of each wager on the basic signaling and prediction urns is sufficiently small relative to the reinforcements for successful fishing or mending to begin (i.e. if the agents' descriptive and predictive actions are primarily in the service of making basic predictions rather than wagers to begin) or if the chance that the predictor uses his expectation weights to decide whether to accept a wager on a play of the game is sufficiently small and changes relatively slowly to begin (i.e. if the aim is to explain how it is possible for expectation weights gradually to evolve to be used as expectations), then it will behave much as the model just described.

The upshot is that the agents end up with relatively stable, well-tuned expectation weights that typically get used as expectations without ever having assigned anything like prior expectations over possible descriptions. Note, in particular, that the expectation weights are not initially associated with descriptions insofar as there are no meaningful descriptions to begin. Further, the expectation weights do not initially represent expectations at all inasmuch as they only gradually come to play such a role. Also salient to how one thinks of the expectation weights, since they track the evolving descriptive and predictive practice faster than it evolves here, any initial bias in the expectation weights washes out by the time the agents evolve a successful descriptive and predictive practice.

## Conclusion

The present model shows how it is possible for simple variety of descriptive and predictive dispositions to coevolve with the formation of a simple variety of expectations. The agents' descriptions are initially meaningless and their predictions are hence wildly unreliable. Reinforcement learning allows for the coevolution of successful descriptive and predictive practice and for the predictor's expectation weights to closely track the type and the degree of that success. And a reinforcement model allow one to explain how such expectation weights might come to play the role of expectations. Together they provide a account of how the successful

[14]Note, however, that since the predictor might use expectation weights as expectations to accept a wager or flip a coin to accept a wager and win, and since neither type of ball is ever allowed to go to extinction on the model, the predictor's behavior is never fully stable.

evolution of a simple descriptive language and associated predictive practice might at the same time provide agents with simple core expectations.

On this story, neither the agents' descriptive language nor their predictive dispositions nor the associated expectations are somehow prior. Rather, the agents' descriptive and predictive practice and the expectations emerge together in the context of the same evolutionary process.

The core posterior expectations that the agents acquire in the evolutionary process are associated with those descriptions that were in fact used in making the successful predictions as the descriptive and predictive practice evolved. Consequently, they can be expected to be well-tuned to whatever predictive success in fact allowed for the evolution of the descriptive language. Concerning the question at hand, such core posterior expectations would be available to play the role of prior probabilities should the agents turn to reflective inquiry.[15]

[15]I would like to thank Brian Skyrms, Jim Weatherall, and Seamus Bradley in particular for helpful discussions on this topic.

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